Effect of flow on heavy quark damping rate in hot QCD plasma

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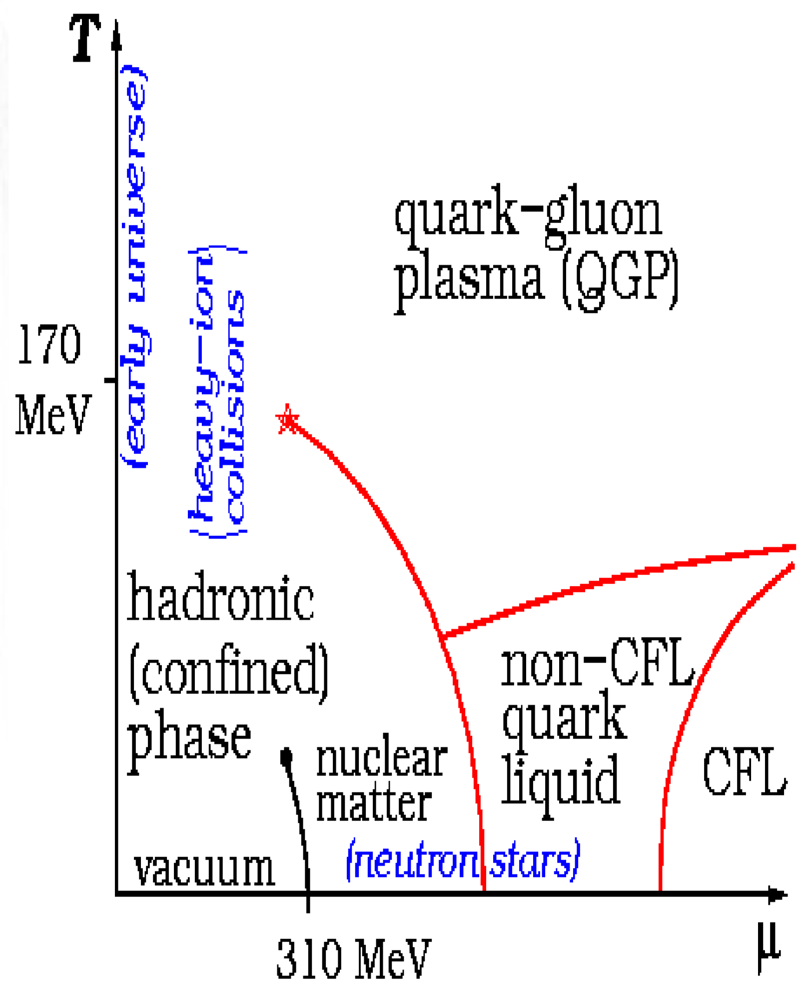
Outline



- Formalism → Ideal QCD plasma
- Non-ideal effects → shear viscosity
- Particle distribution function modified by shear flow
- Viscous QCD plasma and particle damping rate
- Summary and Conclusion

Ideal QCD plasma and particle damping rate

- Information particle life-time → the retarded propagator

$$S_R(t,p) \sim e^{-E_p t} e^{-\Gamma t}$$

- The life-time of the particle excitation → $\tau(p) \propto 1/\Gamma(p)$

Boltzmann kinetic equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v_p}.\nabla_\mathbf{r} + \mathbf{F}.\nabla_\mathbf{p}\right) f_p = -\mathcal{C}[f_p]$$

Collision integral

$$\begin{aligned} \mathcal{C}[f_p] &= \frac{1}{2E_p}\int \frac{d^3k}{(2\pi)^3 2k}\frac{d^3p'}{(2\pi)^3 2E_p'}\frac{d^3k'}{(2\pi)^3 2k'} \\ &\times f_p f_k (1 \pm f_{E_p'})(1 \pm f_{k'}) - f_{p'} f_{k'} (1 \pm f_p)(1 \pm f_k) \\ &\times (2\pi)^4 \delta^4 (P + K - P' - K')\frac{1}{2}\sum_{spin} |\mathcal{M}|^2 \end{aligned}$$

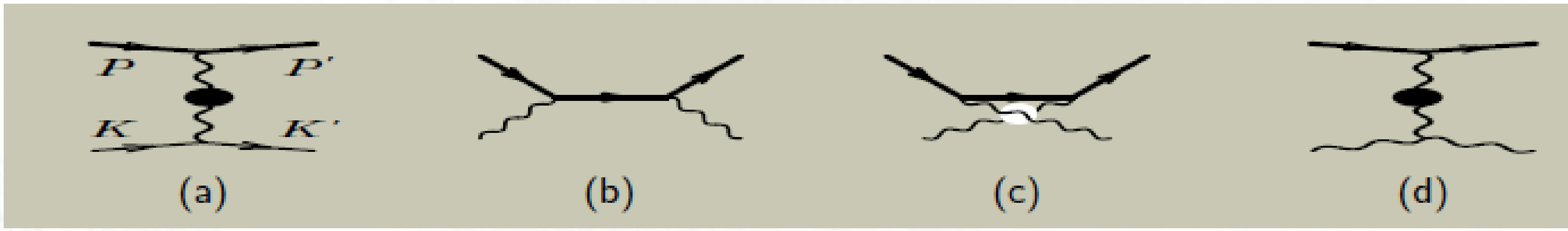
- In absence of any external force and inhomogenity

$$\frac{\partial f_p}{\partial t} = -\mathcal{C}[f_p]$$

- All the bath particles are at equilibrium except → $f_p = f_p^0 + \delta f_p$
- In relaxation time approximation → $\frac{\partial \delta f_p}{\partial t} = -\mathcal{C}[f_p] = -\delta f_p \Gamma_p^0$
- Damping is exponential in time → $\delta f_p = \delta f_p^0 e^{-\Gamma_p^0 t}$
- **Particle damping rate**

$$\Gamma_p^0 = \frac{1}{2E_p}\int \frac{d^3k}{(2\pi)^3 2k}\frac{d^3p'}{(2\pi)^3 2E_p'}\frac{d^3k'}{(2\pi)^3 2k'} f_k^0 (1 \pm f_{k'}^0)$$
$$\times (2\pi)^4 \delta^4(P + K - P' - K')\frac{1}{2}\sum_{spin}|\mathcal{M}|^2$$

Scattering matrices



Hard gluon exchange where the medium effects can be ignored

$$\Gamma_p^0 \simeq \frac{g^4 T^3}{4\pi} \int \frac{dq}{q^3}$$

In medium effect → **HTL resummed gluon propagator**

$$\Delta_{\mu\nu}(Q) = \delta_{\mu 0}\delta_{\nu 0} \frac{q^2 - q_0^2}{q^2} \Delta_L(Q) + A_{\mu\nu}\Delta_T(Q)$$

$$\Delta_{L,T} = \frac{-1}{q^2 - q_0^2 + \Pi_{L,T}}$$

- **Longitudinal and transverse polarization tensor**

$$\Pi_L(q,\omega) = m_D^2 \frac{q^2 - q_0^2}{q^2}\left[1 - \frac{q_0}{2q}\ln\left(\frac{q_0+q}{q_0-q}\right)\right]$$

$$\Pi_T(q,\omega) = m_D^2\left[\frac{q_0^2}{2q^2} + \frac{q_0(1-\frac{q_0^2}{q^2})}{4q}\ln\left(\frac{q_0+q}{q_0-q}\right)\right]$$

$$\frac{1}{2}\sum_{spin}|\mathcal{M}|^2_{qq} = 8A_{b,q}g^4\left[\frac{1}{(q^2+\Pi_L)} - \frac{v_{p,T}.v_{k,T}}{(q^2-\omega^2+\Pi_T)}\right]^2$$

- **In the static limit** $x \sim \frac{q_0}{q} \sim 0$, $\Pi_L = m_D^2 \Rightarrow$ Debye mass

- **In the weakly dynamical limit** $x \sim \frac{q_0}{q} \sim 0$, $\Pi_T = \frac{i m_D^2 \pi\omega}{4q} \Rightarrow$ Landau Damping

- **Debye screening in longitudinal interaction improves IR divergence**
- **Transverse interaction → weakly dynamical screening**

Ideal QCD plasma and particle damping rate

In high temperature plasma

$$\Gamma(p)^{0,L}_{qq} = \frac{g^4 A_b T^3}{96\pi}\left(\frac{1}{m_D^2} - \frac{1}{q^2_{max}}\right)$$

$$\Gamma(p)^{0,T}_{qq} = \frac{g^4 A_b T^3 v_p}{48\pi m_D^2}\int_0^{q^*}\frac{dq}{q}$$

$$\Gamma(p)^{0}_{qg} = \frac{g^4 T^2 A_f}{48\pi E_p}\left[\ln\left|\frac{4E_p T}{m_q^2}\right| + \mathcal{O}(1)\right]$$

- **In the magnetic sector screening occurs only for nonzero frequency. This dynamical screening is not sufficient to completely remove the infrared divergence of** $\Gamma(p)^{0,T}_{qq}$

- The leading divergences can be resummed in non-perturbative treatment based on generalization of the Bloch-Nordseick model giving non-exponential decay rate $S_R(t,p) \sim e^{-\alpha T t \ln \omega_p t}$

J. P. Blaizot and E. Iancu, Phys. Rev. Lett. 76, 3080 (1996)

Motivating Non-ideal effects

Heavy Ion Collision

Incorporation of viscosity required → $v_2(p_T)$ → $p_T > 2$ GeV

Shear flow modifies photon, dilepton spectra

System where non-ideal effects are present

↓

Relax the condition → **all the bath particles are at equilibrium**

Particle distribution function → $f = f_0 + \delta f \ (\delta f << f_0)$

Non-ideal fluid → Stress energy tensor → $T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$

Viscosity corrected distribution function

- Viscous effects modify the momentum dependence of the distribution function

$$\delta f_i^{\eta} = \chi(k) \frac{f_k^0(1 \pm f_k^0)}{T} \hat{k}_i \hat{k}_j \nabla_i u_j$$

Form of $\chi(k)$ depends on various microscopic theories

K. Dusling *et al* Phys. Rev. C **81**, 034907 (2010).

$$\begin{aligned} \delta f_i^{\eta}(k) &= f_i^0(1 \pm f_i^0)\Phi_i(k) \\ \Phi_i(k) &= \frac{1}{2T^3\tau}\frac{\eta}{s}\left(\frac{k^2}{3} - k_z^2\right) \end{aligned}$$

- **First order correction of shear part of the stress tensor.**

D. Teaney Phys. Rev. C **68**,034913 (2003)

Damping rate in presence of flow

- In a medium with non-zero flow gradient collision integral

$$C[f_p] = \frac{1}{2E_p}\int \frac{d^3k}{(2\pi)^3 2k}\frac{d^3p'}{(2\pi)^3 2E_p'}\frac{d^3k'}{(2\pi)^3 2k'}\sum_{i=1,2}\alpha_i$$
$$\times \; (2\pi)^4\delta^4(P+K-P'-K')\frac{1}{2}\sum_{spin}|\mathcal{M}|^2$$

Medium particles modified by viscous distribution function

$$\alpha_1 = \delta f_p f_k^0 (1 \pm f_{k'}^0)$$

$$\alpha_2 \simeq \delta f_p \left[\Phi_k f_k^0 (1 \pm f_{k'}^0) \pm \Phi_{k'} f_{k'}^0 f_k^0\right]$$

Ideal part

Non-ideal part

Continued….

- **Relaxation time approximation**

$$\mathcal{C}[f_p] \simeq \delta f_p \left(\Gamma^0(p) + \delta\Gamma^\eta(p)\right)$$

Hard gluon exchange in viscous plasma

$$\delta\Gamma^\eta_{qq} = \left(\frac{\eta}{s}\right)\left[\mathcal{C}_1 f_1(v_p)\int_0^\infty g_1(k)dk \int \frac{dq}{q} + \mathcal{C}_2 f_2(v_p)\int_0^\infty g_2(k)dk \int \frac{dq}{q^3}\right]$$

In viscous medium both logarithmic and algebraic divergences

- **Terms** $\mathcal{O}((\eta/s)^2)$ **neglected.**
- **Only small angle scatterings are considered.**

$$\begin{aligned} f(k') &= f(k-\omega) \simeq f_k - \omega f_k'^0 \\ \Phi_{k'} &= \Phi_{(k-\omega)} \simeq \Phi_k - \omega \Phi_k \end{aligned}$$

Continued....

HTL corrected viscous damping rate

$$\delta\Gamma_q^\eta = \delta\Gamma_{qq}^\eta + \delta\Gamma_{qg}^\eta = \left(\frac{\eta}{s}\right)\Bigg[C_1 f_1(v_p)\left(-\frac{1}{2} + \ln\left|\frac{q_{max}}{m_D}\right|\right)$$

$$+ \; C_2 f_2(v_p)\left(-\frac{16}{225} - \frac{4}{15}\ln\left|\frac{2q_{max}}{m_D\sqrt{\pi v_p}}\right|\right) + C_3 f_3(v_p)\int_0^{q^*}\frac{dq}{q}\Bigg]$$

$$+ \; \left(\frac{\eta}{s}\right)\frac{A_f g^4}{32T^3\tau\pi^3 E_p}\left[\frac{2\pi^4 T^4}{135} - \frac{T^3 m_q^2}{3E_p}\left(2\zeta(3)\ln\left|\frac{4E_p T}{m_q^2}\right| + 3\zeta(3) - 2\gamma\zeta(3) + 2\zeta'(3)\right)\right]$$

- Finite interaction rate → **one extra ω in the numerator** coming from the phase-space distribution function $\omega\partial f_k/\partial k$ cures the logarithmic divergence.

- The physical processes responsible for these divergences are the ***collisions involving the exchange of quasistatic, magnetic gluons, which are not screened by plasma effects*** and show logarithmic divergence.

S. Sarkar and A.K Dutt-Mazumder, Phys. Rev. D 88, 054006 (2013).

Summary & Conclusions

- Effect of viscosity enters into the damping rate calculation through the phase-space distribution functions.
- In viscous QCD medium hard gluon exchange gives both logarithmic and algebraic divergences.
- Infrared behaviour of the transverse damping rate has the same logarithmic divergence both for the viscous and the non-viscous part.
- Non-perturbative treatment can be used to obtain finite result in viscous medium.

THANK YOU