Modification to kick velocity of neutron stars due to magnetic interactions in dense plasma

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Outline of the talk

- Introduction: Quark matter as the core of NS.
- Quark dispersion relations.
- Specific heat capacity of degenerate quark matter.
- Pulsar kick velocity.
- Results.
- Conclusion.

Quark matter as the core of the neutron star

- URCA : UnRecordable Cooling Agent process was first discussed by George Gamow and Mario Schoenberg while they were visiting a casino named Cassino da Urca in Rio de Janeiro.
- Direct URCA process:

$$n \to p + e^- + \bar{\nu}_e$$

- Modified URCA process:

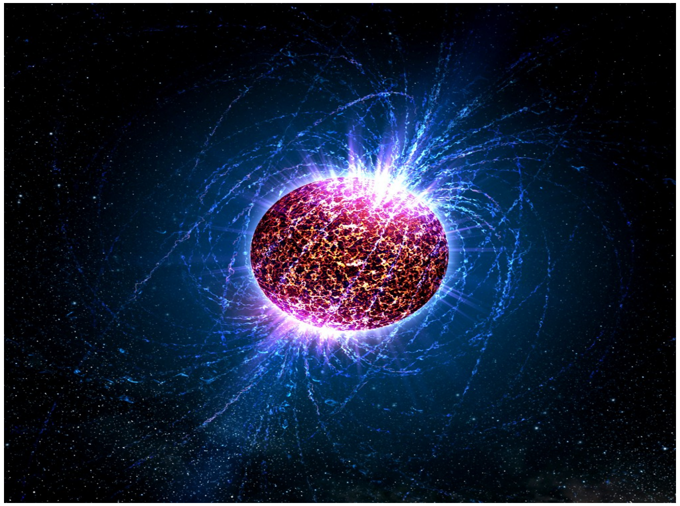
$$n + n \to n + p + e^- + \bar{\nu}_e$$

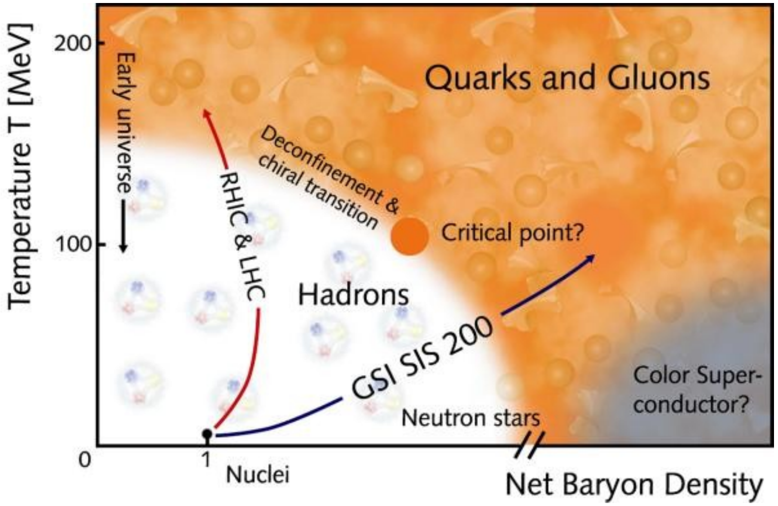
- Quark direct URCA process:

$$d \to u + e^- + \bar{\nu}_e$$

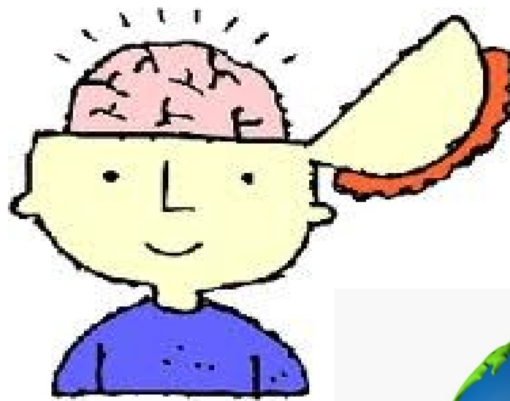
$$u + e^- \to d + \nu_e$$

[N.Iwamoto, Ann.Phys.(N.Y.)141, 1 (1982).]





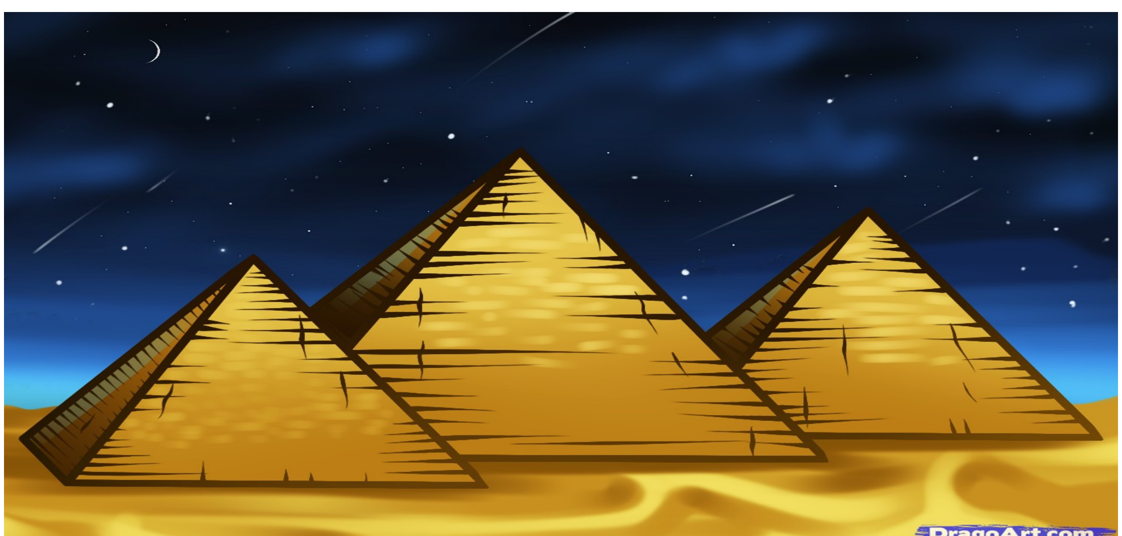
How much magnetic field are we talking about?



- Human brain: 1 - 10 nG
- Earth's magnetic field: 310 mG
- 50 G: refrigerator magnet
- 1.5 kG : sunspot

Some estimate of density in NS

- equivalent to the mass of a Boeing 747 compressed to the size of a small grain of sand.
- One teaspoon weighs about 900 times the mass of the Great Pyramid of Giza.



Advantages of quark matter

- The DURCA process cannot occur because it is not kinetically possible at such temperature of interest.
- The MURCA requires a bystander particle. The neutrino emission rate is found to be insignificant.
- Conditions are quite different for quark matter!!
  - quark – quark interactions
  - S quarks neglected
- Problem..
  - Not well known EOS of quark matter.
  - Dynamical properties (mass, inertia etc.)

quark stars ~ ordinary NS

{How to distinguish by observation?}

Quark dispersion relation

- Interactions within the medium severely modify the self-energy of the quarks. For quasiparticles with momenta close to the Fermi momentum pF, the one-loop self-energy is dominated by the soft gluon exchanges.
- The quasiparticle energy satisfies the relation,

$$\omega\pm = \pm(E_{p(\omega\pm)} + \mathrm{Re}\Sigma\pm(\omega\pm, p(\omega\pm)))$$

where ω is the quasiparticle/antiquasiparticle energy which is a solution of the dispersion relation and $E_{p(\omega)}$ is the kinetic energy.

- At $T \to 0$; $m = 0$ the distribution functions become step functions.

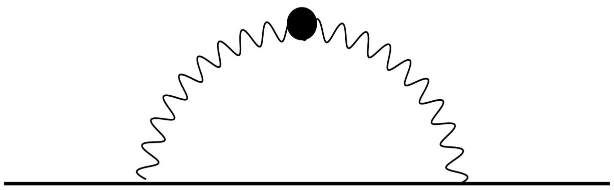
$$\bar{n}(k0) = \theta(\mu - E + q0) \text{ and } 1 + n(q0) = \theta(q0)$$

[C.Manuel, Phys.Rev.D 62, 076009 (2000).]

I am continuing...

- The one loop quark self-energy $\Sigma(\omega, p(\omega))$ is dominated by a diagram with a soft gluon in the loop.
- Collision of charged quasiparticle with the particles of the plasma are governed by gluon exchange, and that the gluon propagator is dressed(or r+e+s+u+m+m+e+d) by interactions

  ⟹Resummation case proposed by Braaten and Pisarski.



- Hard scale of momentum $\sim \mu$
- Soft scale of momentum $\sim g\mu$ ⟹Braaten and Yuan

  [R.D.Pisarski, Phys.Rev. Lett. 63, 1129 (1989); E.Braaten and R.D.Pisarski, Nucl.Phys.B 337,569 (1990).]

Non-Fermi liquid phenomenon

• Degenerate Fermi gas at low or zero temperature show different behavior under inclusion of magnetic interactions for the relativistic case.

• Specific heat of degenerate QED matter contains the anomalous $T \ln T^{-1}$ term.[1]

⟹ Damping rate and energy loss of quasiparticles.

⟹ Drag and diffusion coefficients.[2]

Q. How does NFL effect enter into the calculation?

Hint 1: Dis...

Hint 2: Modified (of course!)

Ans.: Through the modified Quark dispersion relation.

[1. T. Holstein, R.E. Norton and P. Pincus, Phys. Rev. B8, 2649 (1973).
2. S.Sarkar and A.K.Dutt-Mazumder, Phys.Rev.D 82, 056003 (2010).
3. K.Pal and A.K.Dutt-Mazumder,Phys.Rev.D 84, 034004 (2011).]

Quark self energy

$$\Sigma(P) = -g^2 C_F T \sum_s \int \frac{d^3q}{(2\pi)^3} \gamma_\mu S_f(i(\omega_n - \omega_s), \mathbf{p} - \mathbf{q}) \gamma_\nu \Delta_{\mu\nu}(i\omega_s, \mathbf{q})$$

- The analytical expression for the one-loop quark self energy (for $T \sim |E - \mu| \ll g\,\mu \ll \mu$) exhibits a logarithmic singularity close to the Fermi Surface.
- Low temperature expansion of the on-shell fermion self energy for the ultrarelativistic case is given as:

$$\Sigma_+(\omega) = -g^2 C_F m \left\{ \frac{\epsilon}{12\pi^2 m}\left[\log\left(\frac{4\sqrt{2}m}{\pi\epsilon}\right)+1\right] + \frac{i\epsilon}{24\pi m} + \frac{2^{1/3}\sqrt{3}}{45\pi^{7/3}}\left(\frac{\epsilon}{m}\right)^{5/3}(\mathrm{sgn}(\epsilon) - \sqrt{3}i) + \frac{i}{64\sqrt{2}}\left(\frac{\epsilon}{m}\right)^2 - 20\frac{2^{2/3}\sqrt{3}}{189\pi^{11/3}}\left(\frac{\epsilon}{m}\right)^{7/3}(\mathrm{sgn}(\epsilon)+\sqrt{3}i) - \frac{6144-256\pi^2+36\pi^4-9\pi^6}{864\pi^6}\left(\frac{\epsilon}{m}\right)^3\left[\log\left(\frac{0.928\,m}{\epsilon}\right) - \frac{i\pi\mathrm{sgn}(\epsilon)}{2}\right] + \mathcal{O}\left(\left(\frac{\epsilon}{m}\right)^{11/3}\right)\right\}$$

[A.Gerhold, A.Ipp and A.Rebhan, Phys.Rev.D 70, 105015 (2004); 69, R011901(2004).]

Not over yet..

•In the above expression, $\epsilon = (\omega - \mu) \sim T$ where NFL effects dominate.

•The Debye mass is given as $m^2 = mD^2/2$ where $m^2 = N_f\, g^2 \mu^2 / (4\pi^2)$.

•HDL (Hard Dense Loop) resummation for gluon propagator required because higher order diagrams can contribute to lower order in coupling constant which is missing in bare p-QCD; resummation done by means of Dyson-Schwinger eqn.

•It is interesting to note here that fractional powers in ϵ appear ⟹Dynamical screening of the transverse exchange of Gauge bosons

[C.Manuel, Phys.Rev.D 62, 076009 (2000); A.Gerhold, A.Ipp and A.Rebhan, Phys.Rev.D 70, 105015 (2004); 69, R011901(2004).].

Dynamical screening

- The longitudinal and transverse HDL propagators are given as:

$$\Delta_L(q_0,q) = \frac{-1}{q^2+2m^2\left[1-\frac{q_0}{2q}\,log\left(\frac{q_0+q}{q_0-q}\right)\right]}$$

$$\Delta_T(q_0,q) = \frac{-1}{q_0^2-q^2-m^2\frac{q_0^2}{q^2}\left[1+\frac{q^2-q_0^2}{2qq_0}\,log\left(\frac{q_0+q}{q_0-q}\right)\right]}$$

$$\Delta_{L,T} = \frac{1}{q^2-\Pi_{L,T}}$$

$$\Pi_L(q,\omega) = m_D^2\left[1-\frac{q_0}{2q}\ln\left(\frac{q_0+q}{q_0-q}\right)\right]$$

$$\Pi_T(q,\omega) = m_D^2\left[\frac{q_0^2}{2q^2}+\frac{q_0\left(1-\frac{q_0^2}{q^2}\right)}{4q}\ln\left(\frac{q_0+q}{q_0-q}\right)\right]$$

- For $q_0 \to 0$ longitudinal photons acquire an effective mass $m_D^2=2m^2$ which screens IR singularities.
- For $q_0 \to 0$ transverse (or magnetic) interactions are NOT screened; only dynamical screening.
- Retaining the leading term for $(q_0/q \to 0)$ we obtain:

$$\Delta_T \simeq \frac{1}{q^2-\frac{i\pi m^2 q_0}{2q}}$$

$$q_C = \left(\frac{\pi m^2 q_0}{2q}\right)^{(1/2)}$$

- Frequency dependent screening with a frequency dependent cut-off.
- This cut-off is able to screen IR singularities so that finite results are obtained.

Specific heat capacity of degenerate quark matter

- The specific heat of normal(non-color superconducting) degenerate quark matter shows NFL behavior at low temperature.
- Thus, at low temperatures, the resulting deviation of the specific heat from its FL behavior is significant in case of normal quark matter and thus of potential relevance for the cooling rates of NS with a quark matter component.

..in absence of magnetic field..

- The Fermi liquid result:

$$C_v\Big|_{FL} = \frac{N_c N_f}{3}\mu_q^2 T$$

- The Non-Fermi liquid result (upto LO):

$$C_v\Big|_{LO} = N_g \frac{g_{eff}^2 \mu_q^2 T}{36\pi^2}\left(\ln\left(\frac{4 g_{eff}\mu_q}{\pi^2 T}\right) + \gamma_E - \frac{6}{\pi^2}\zeta'(2) - 3\right)$$

- ..and the NLO result:

$$C_v\Big|_{NLO} = N_g\Big[-40\frac{2^{2/3}\Gamma\left(\frac{8}{3}\right)\zeta\left(\frac{8}{3}\right)}{27\sqrt{3}\pi^{11/3}}T^{5/3}(g_{eff}\mu_q)^{4/3} + 560\frac{2^{1/3}\Gamma\left(\frac{10}{3}\right)\zeta\left(\frac{10}{3}\right)}{81\sqrt{3}\pi^{13/3}}T^{7/3}(g_{eff}\mu_q)^{2/3}$$
$$+\frac{2048 - 256\pi^2 - 36\pi^4 + 3\pi^6}{180\pi^2}T^3\Big[\ln\left(\frac{g_{eff}\mu_q}{T}\right) + \bar{c} - \frac{7}{12}\Big]\Big]$$

- A.Gerhold, A.Ipp and A.Rebhan, Phys.Rev.D 70, 105015 (2004); 69, R011901(2004).

..in presence of magnetic field..

Some playing around with phase space integrations..

- The FL case:

$$C_v\Big|_{FL}^{B} = \frac{N_c N_f T m_q^2}{6}\Big(\frac{B}{B_{cr}^q}\Big)$$

- The LO case:

$$C_v^{(i)}\Big|_{LO}^{B} = \Big(\frac{g^2 C_F}{24\pi^2}\frac{|q_i| g_{di} B}{2\pi^2}\Big)\sum_{\nu=0}^{\infty}\int_0^{\infty} d\epsilon \frac{\partial f(\epsilon)}{\partial T}(\epsilon-\mu) log\Big(\frac{m_B^2}{(\epsilon-\mu)^2}\Big)$$

$$C_v\Big|_{LO}^{B} \simeq \Big(\frac{N_c N_f C_f \alpha_s}{36\pi}\Big) m_q^2\Big(\frac{B}{B_{cr}^q}\Big) T\Big[(-1+2\gamma_E) + 2 log\Big(\frac{2m_B}{T}\Big)\Big]$$

- The NLO case:

$$C_v\Big|_{NLO}^{B} \simeq \Big(\frac{N_c N_f}{3}\Big)(C_f\alpha_s)\Big(m_q^2\frac{B}{B_{cr}^q}\Big) T\Big[c_1\Big(\frac{T}{m_B}\Big)^{2/3} + c_2\Big(\frac{T}{m_B}\Big)^{4/3} + c_3\Big(\frac{T}{m_B}\Big)^2\Big(c_4 - log\Big(\frac{T}{m_B}\Big)\Big)\Big]$$

Polarisation fraction of electrons

- We first choose the magnetic field strength to be much larger than the temperature, the chemical potential as well as the electron mass (μ_e, m_e , $T \ll 2eB$). The number density is given as:

$$\chi \sim 1 - \frac{4}{\ln(2)}\sqrt{\frac{\pi T}{2\sqrt{2eB}}}e^{-\sqrt{2eB}/T}.$$

- For the case of weak magnetic field, the number of Landau levels occupied is large. Hence the number density of electrons is given as:

$$\chi \simeq \frac{3}{2}\frac{m_e^2}{\mu_e^2 - m_e^2}\left(\frac{B}{B_{cr}^e}\right)$$

- I.Sagert and J.Schaffner-Bielich, arXiv:0708.2352.

Kick velocity including magnetic field effects

- The simple FL result:

$$v\Big|_{FL}^{B} \simeq \frac{4.15 N_C N_f}{3}\Big(\frac{\sqrt{m_q^2(B/B_{cr}^q)}}{400MeV}\frac{T}{1MeV}\Big)^2\Big(\frac{R}{10km}\Big)^3\frac{1.4M_\odot}{M_{NS}}\chi\frac{km}{s}$$

- The LO result (note the appearance of log term):

$$v\Big|_{LO}^{B} \simeq \frac{8.8 N_C N_f}{3}(C_f\alpha_s)\Big(\frac{\sqrt{m_q^2(B/B_{cr}^q)}}{400MeV}\frac{T}{1MeV}\Big)^2\Big(\frac{R}{10km}\Big)^3\frac{1.4M_\odot}{M_{NS}}\Big[0.0635+0.05log\Big(\frac{m_D^B}{T}\Big)\Big]\chi\frac{km}{s}$$

- The NLO result:

$$v\Big|_{NLO}^{B} \simeq \frac{8.3 N_C N_f}{3}\Big(\frac{B}{B_{cr}^q}\Big)\Big(\frac{m_q}{400MeV}\frac{T}{1MeV}\Big)^2\Big(\frac{R}{10km}\Big)^3\frac{1.4M_\odot}{M_{NS}}\chi(C_F\alpha_s)$$
$$\times\Big[a_1\Big(\frac{T}{m_B}\Big)^{2/3}+a_2\Big(\frac{T}{m_B}\Big)^{4/3}+\Big[a_3+a_4\ln\Big(\frac{m_B}{T}\Big)\Big]\Big(\frac{T}{m_B}\Big)^2\Big]\frac{km}{s}$$

Pulsar kick velocity

- The kick velocity is given as:

$$dv = \frac{\chi}{M_{NS}} \frac{4}{3} \pi R^3 \varepsilon dt$$

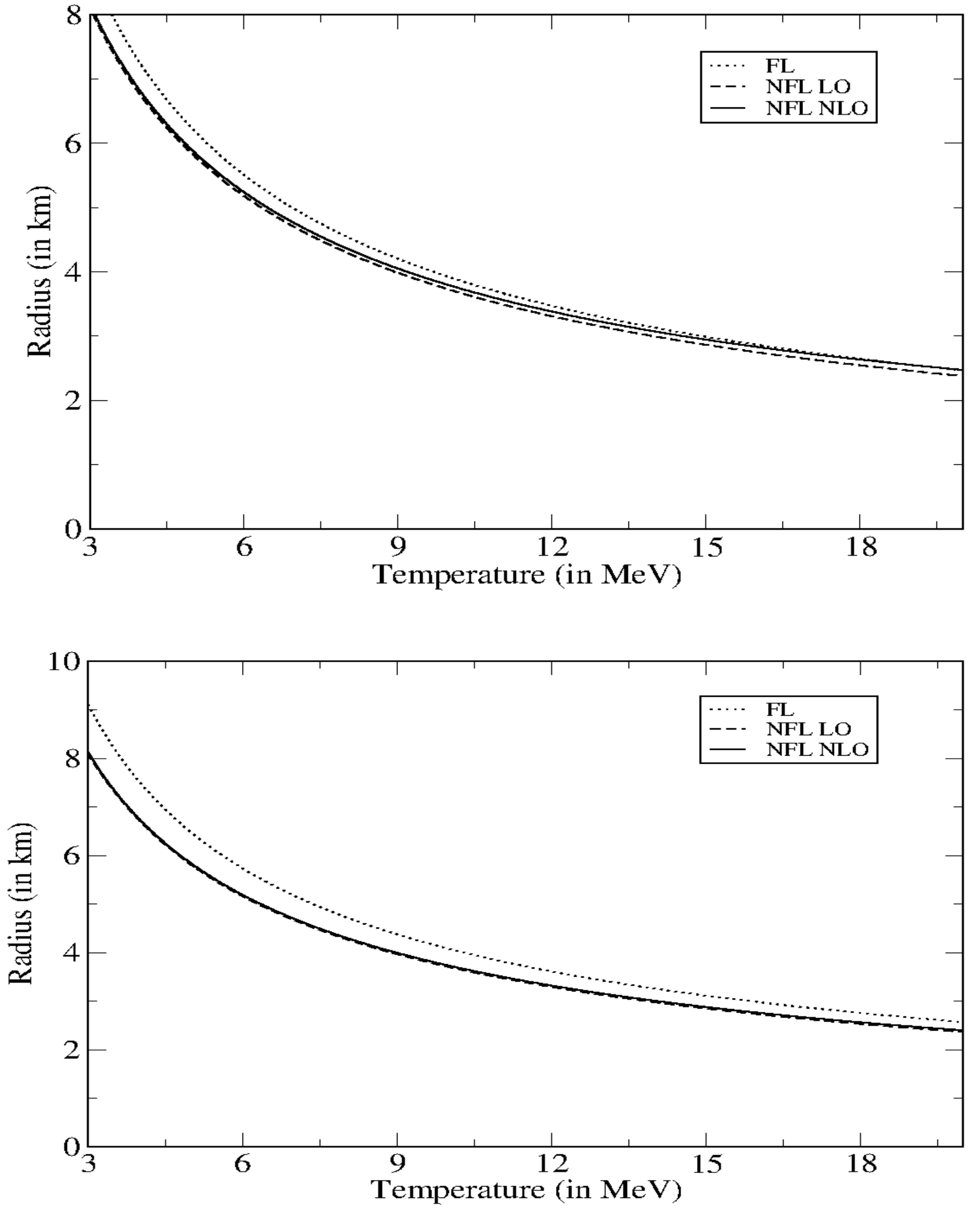
- The FL, NFL LO and NFL NLO results are given as:

$$v\Big|_{FL} \simeq \frac{8.3 N_C N_f}{3} \left(\frac{\mu_q}{400MeV} \frac{T}{1MeV} \right)^2 \left(\frac{R}{10km} \right)^3 \frac{1.4 M_\odot}{M_{NS}} \chi \frac{km}{s}$$

$$v\Big|_{LO} \simeq \frac{16.6 N_C N_f}{3} (C_F \alpha_s) \left(\frac{\mu_q}{400MeV} \frac{T}{1MeV} \right)^2 \left(\frac{R}{10km} \right)^3 \frac{1.4 M_\odot}{M_{NS}} \chi \left[c_1 + c_2 \ln \left(\frac{g \mu_q \sqrt{N_f}}{T} \right) \right] \frac{km}{s}$$

$$v\Big|_{NLO} \simeq \frac{16.6 N_C N_f}{3} \left(\frac{\mu_q}{400MeV} \frac{T}{1MeV} \right)^2 \left(\frac{R}{10km} \right)^3 \frac{1.4 M_\odot}{M_{NS}} \chi (C_F \alpha_s)$$
$$\times \left[a_1 \left(\frac{bT}{\mu_q} \right)^{2/3} + a_2 \left(\frac{bT}{\mu_q} \right)^{4/3} + \left[a_3 + a_4 \ln \left(\frac{\mu_q}{bT} \right) \right] \left(\frac{bT}{\mu_q} \right)^2 \right] \frac{km}{s}$$

Results



- FIG. 1. The figure shows the comparison between the FL, NFL LO and NFL NLO result for the radius and temperature dependence for the case of fully polarised electrons. Results have been plotted for the case of presence and absence of external magnetic field effect in the specific heat of degenerate quark matter. The top panel shows the relationship (FL,LO and NLO respectively) for the case where external magnetic field on the specific heat is ignored. The bottom panel shows the corresponding case when magnetic field effect in specific heat is included.

- S.P. Adhya, P. K. Roy and A.K.Dutt-Mazumder, arXiv:1303.6126 (2013).

Results

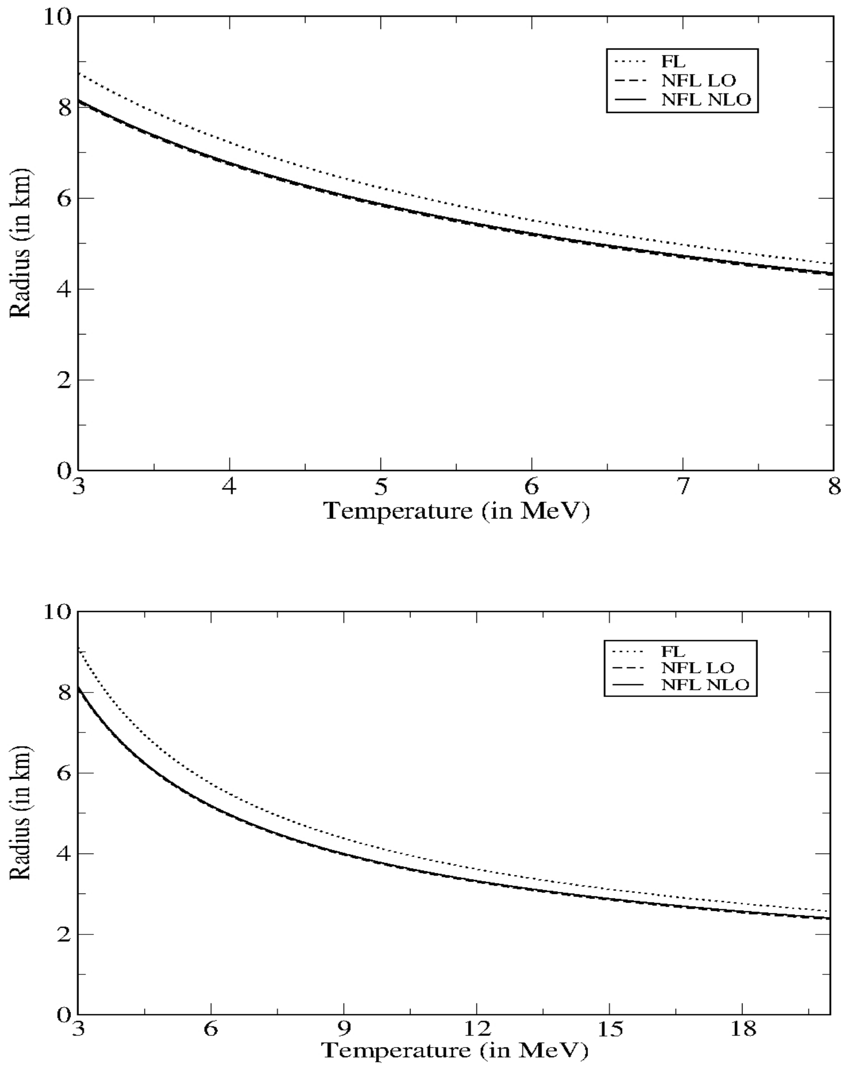


FIG. 2. The figure shows the numerical comparison where high magnetic field has been taken into account along with vanishing temperature for kick velocity of 100km/s. The top panel shows the relationship (FL,LO and NLO respectively) for the case where external magnetic field on the specific heat is ignored. The bottom panel shows the corresponding case when magnetic field effect in specific heat is included.

- S.P. Adhya, P. K. Roy and A.K.Dutt-Mazumder, arXiv:1303.6126 (2013).

Results

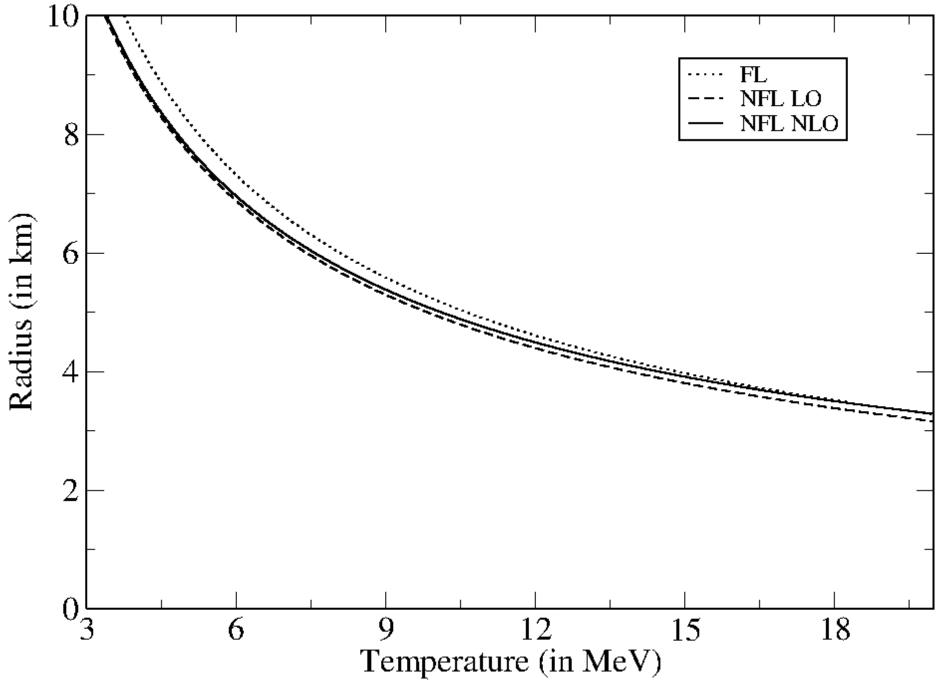


FIG. 3. The figure shows the comparison between the FL, NFL LO and NFL NLO result for the radius and temperature dependence for the case of partially polarized electrons in weak magnetic field. Results have been plotted for the case of absence of external magnetic field effect in the specific heat of degenerate quark matter.

- S.P. Adhya, P. K. Roy and A.K.Dutt-Mazumder, arXiv:1303.6126 (2013).

Conclusions

- In this work, we have derived the expressions of the pulsar kick velocity including the NFL corrections to the specific heat of the degenerate quark matter core.
- The contributions from the electron polarization (χ) for different cases has also been taken into account to calculate the velocities.
- We have included the effect of the external magnetic field into the specific heat of the degenerate quark matter for the calculation of the pulsar kick velocity. The calculation of the specific heat of the degenerate quark matter in magnetic field for the NFL LO and NLO are new.
- We have found that the NFL LO contributions are significant while calculating the radius-temperature relationship as seen from the graphs presented for the case of the neutron star with moderate and high magnetic field. The anomalous corrections introduced to the pulsar kick velocity due to the NFL (LO) behavior increases appreciably the kick velocity for a particular value of radius and temperature. However, for all the cases, no appreciable change in the R-T relationship has been observed for the NLO correction with respect to the LO case.

- S.P. Adhya, P. K. Roy and A.K.Dutt-Mazumder, arXiv:1303.6126 (2013).

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- S.P. Adhya, P. K. Roy and A.K.Dutt-Mazumder, AIP Conf. Proc. (2013).
- S.P. Adhya, P. K. Roy and A.K.Dutt-Mazumder, arXiv:1303.6126 (2013).

Recent work (2012-2013)

- S.P. Adhya, P. K. Roy and A.K.Dutt-Mazumder, arXiv:1303.6126 (2013).
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Thank you !!