Three Loop HTL Thermodynamics at finite T and μ

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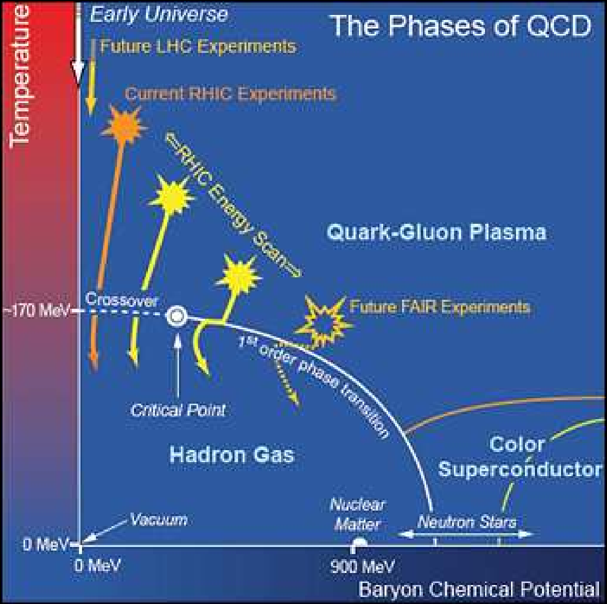


The Phases of QCD
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
~170 MeV
Crossover
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Vacuum
Nuclear Matter
Neutron Stars
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

Why EOS of QCD matter is Important?

- If either the temperature or the density of strongly interacting matter is increased enough, it undergoes a phase transition from the hadronic phase into deconfined quark-gluon plasma (QGP) phase .
- This high temperature/high baryonic situation can be achieve in Heavy-ion collisions experiment RHIC ,LHC and FAIR.
- Determination of Equation of State of such deconfined matter at finite temperature and finite chemical potential is extremely important.

Various Models

- There are various existing perturbative and non-perturabative (Lattice) thermodynamics calculations in Literature.
- The currently most reliable method for determining the EOS is lattice QCD.
- But lattice calculations can be performed at arbitrary temperature; however, they are restricted to relatively small chemical potentials.
- Perturbative calculation have problems like convergence and large dependence on the choice on renormalization scale.
- Hard Thermal Loop perturbation theory is resummed perturbation theory where problems of pure perturbation theory can be cured partially.

HTL perturbation Theory

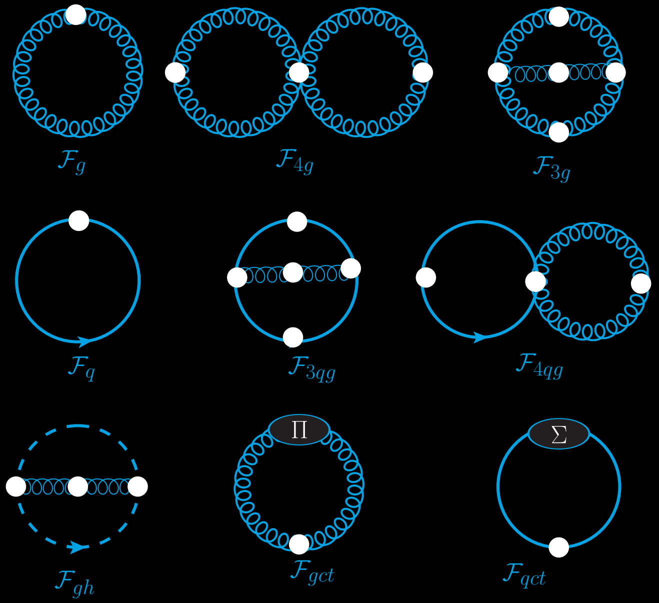
HTL perturbation theory is a reorganization of the perturbation series for thermal QCD. The Lagrangian density is written as

$$\mathcal{L} = (\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL})\Big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL}.$$

The HTL improvement term is

$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta)m_D^2 \,{\rm Tr}\left(G_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y G^\mu{}_\beta\right) + (1-\delta)\, i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{y^\mu}{y\cdot D}\right\rangle_y \psi\,,$$

HTLpt is defined by treating δ as a formal expansion parameter.


$\mathcal{F}_g$
$\mathcal{F}_{4g}$
$\mathcal{F}_{3g}$
$\mathcal{F}_q$
$\mathcal{F}_{3qg}$
$\mathcal{F}_{4qg}$
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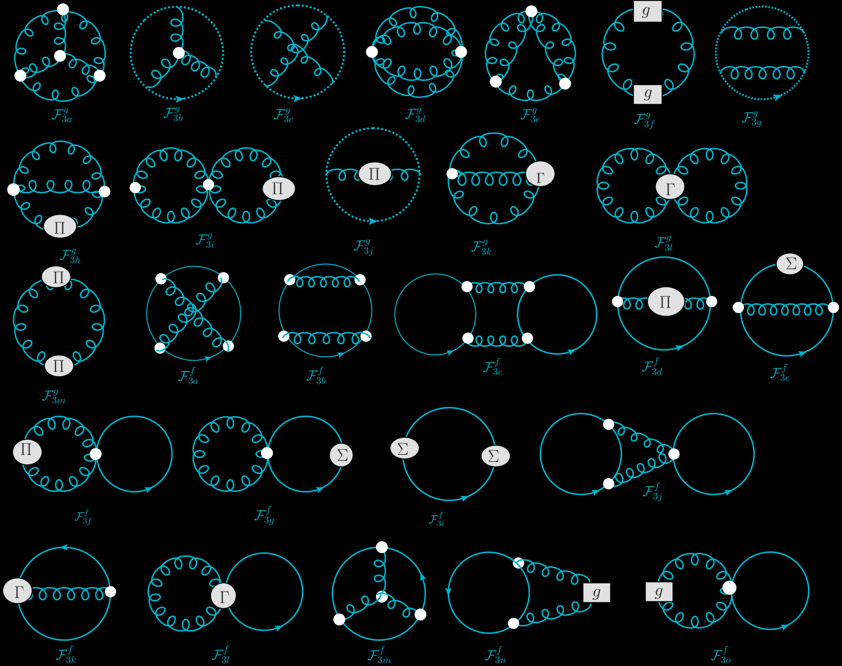
Π
$\mathcal{F}_{gct}$
Σ
$\mathcal{F}_{qct}$



$\mathcal{F}^g_{3a}$
$\mathcal{F}^g_{3b}$
$\mathcal{F}^g_{3c}$
$\mathcal{F}^g_{3d}$
$\mathcal{F}^g_{3e}$
g
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$\mathcal{F}^g_{3i}$
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Γ
$\mathcal{F}^g_{3k}$
Γ
$\mathcal{F}^g_{3l}$
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Π
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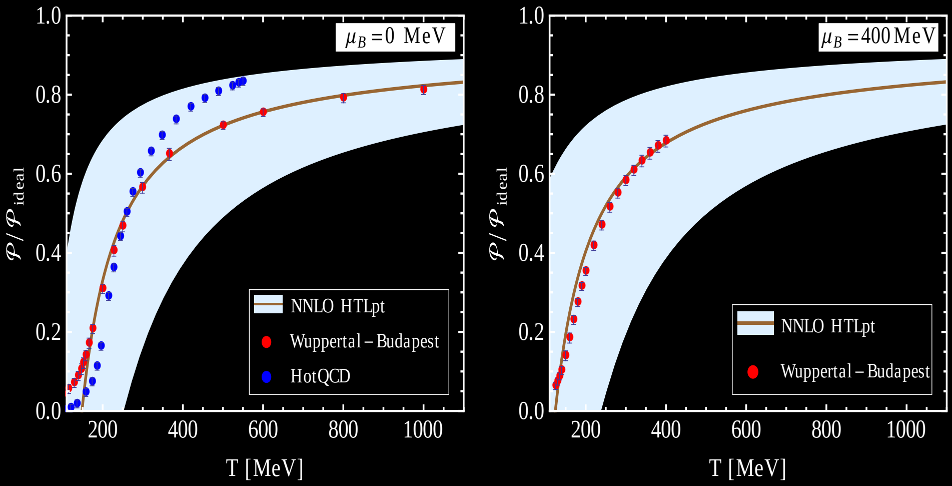
Γ
$\mathcal{F}^f_{3l}$
$\mathcal{F}^f_{3m}$
g
$\mathcal{F}^f_{3n}$
g
$\mathcal{F}^f_{3o}$

$$
\begin{aligned}
\mathcal{P}_{\rm NNLO} = \frac{d_A\pi^2T^4}{45}\Bigg[\!\!\Bigg[& 1+\frac{7}{4}\frac{d_F}{d_A}\Big(1+\frac{120}{7}\hat\mu^2+\frac{240}{7}\hat\mu^4\Big)-\frac{15}{4}\hat m_D^3-\frac{s_F\alpha_s}{\pi}\Bigg[\frac{5}{8}\Big(5+72\hat\mu^2+144\hat\mu^4\Big) \\
&+90\hat m_q^2\hat m_D-\frac{15}{2}\Big(1+12\hat\mu^2\Big)\hat m_D-\frac{15}{2}\Big(2\ln\frac{\hat\Lambda}{2}-1-\aleph(z)\Big)\hat m_D^3\Bigg]+s_{2F}\Big(\frac{\alpha_s}{\pi}\Big)^2\Bigg[-\frac{45}{2}\hat m_D\Big(1+12\hat\mu^2\Big) \\
&+\frac{15}{64}\Bigg\{35-32\Big(1-12\hat\mu^2\Big)\frac{\zeta'(-1)}{\zeta(-1)}+472\hat\mu^2+1328\hat\mu^4+64\Big(6(1+8\hat\mu^2)\aleph(1,z)+3i\hat\mu(1+4\hat\mu^2)\aleph(0,z) \\
&-36i\hat\mu\aleph(2,z)\Big)\Bigg\}\Bigg]+\Big(\frac{s_F\alpha_s}{\pi}\Big)^2\Bigg[\frac{5}{4\hat m_D}\Big(1+12\hat\mu^2\Big)^2+30\Big(1+12\hat\mu^2\Big)\frac{\hat m_q^2}{\hat m_D}+\frac{25}{12}\Bigg\{\frac{1}{20}\Big(1+168\hat\mu^2+2064\hat\mu^4\Big) \\
&+\Big(1+\frac{72}{5}\hat\mu^2+\frac{144}{5}\hat\mu^4\Big)\ln\frac{\hat\Lambda}{2}+\frac{3\gamma_E}{5}\Big(1+12\hat\mu^2\Big)^2-\frac{8}{5}(1+12\hat\mu^2)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{34}{25}\frac{\zeta'(-3)}{\zeta(-3)}-\frac{72}{5}\Big[3\aleph(3,2z) \\
&+8\aleph(3,z)-12\hat\mu^2\aleph(1,2z)-2(1+8\hat\mu^2)\aleph(1,z)+12i\hat\mu\left(\aleph(2,z)+\aleph(2,2z)\right)-i\hat\mu(1+12\hat\mu^2)\,\aleph(0,z)\Big]\Bigg\} \\
&-\frac{15}{2}\Big(1+12\hat\mu^2\Big)\left(2\ln\frac{\hat\Lambda}{2}-1-\aleph(z)\right)\hat m_D\Bigg]+\frac{c_A\alpha_s}{3\pi}\frac{s_F\alpha_s}{\pi}\Bigg[\frac{15}{2\hat m_D}\Big(1+12\hat\mu^2\Big)+90\frac{\hat m_q^2}{\hat m_D} \\
&-\frac{235}{16}\Bigg\{\Big(1+\frac{792}{47}\hat\mu^2+\frac{1584}{47}\hat\mu^4\Big)\ln\frac{\hat\Lambda}{2}-\frac{24\gamma_E}{47}\Big(1+12\mu^2\Big)+\frac{319}{940}\Big(1+\frac{2040}{319}\hat\mu^2+\frac{38640}{319}\hat\mu^4\Big)-\frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)} \\
&-\frac{144}{47}\Big(1+12\hat\mu^2\Big)\ln\hat m_D-\frac{44}{47}\Big(1+\frac{156}{11}\hat\mu^2\Big)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{72}{47}\Big[4i\hat\mu\aleph(0,z)+\Big(5-92\hat\mu^2\Big)\aleph(1,z)+144i\hat\mu\aleph(2,z) \\
&+52\aleph(3,z)\Big]\Bigg\}+\frac{315}{4}\Bigg\{\Big(1+\frac{132}{7}\hat\mu^2\Big)\ln\frac{\hat\Lambda}{2}+\frac{11}{7}\Big(1+12\hat\mu^2\Big)\gamma_E+\frac{9}{14}\Big(1+\frac{132}{9}\hat\mu^2\Big)+\frac{2}{7}\aleph(z)\Bigg\}\hat m_D\Bigg] \\
&+\frac{c_A\alpha_s}{3\pi}\Bigg[-\frac{15}{4}+\frac{45}{2}\hat m_D-\frac{135}{2}\hat m_D^2-\frac{495}{4}\Big(\ln\frac{\hat\Lambda_g}{2}+\frac{5}{22}+\gamma_E\Big)\Bigg]+\Big(\frac{c_A\alpha_s}{3\pi}\Big)^2\Bigg[\frac{45}{4\hat m_D}-\frac{165}{8}\Big(\ln\frac{\hat\Lambda_g}{2} \\
&-\frac{72}{11}\ln\hat m_D-\frac{84}{55}-\frac{6}{11}\gamma_E-\frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)}+\frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\Big)+\frac{1485}{4}\Big(\ln\frac{\hat\Lambda_g}{2}-\frac{79}{44}+\gamma_E-\ln 2-\frac{\pi^2}{11}\Big)\hat m_D\Bigg]\Bigg]\!\!\Bigg]
\end{aligned}
$$

arXiv:1309:3968



Why BAND ?

Running coupling constant corresponding to one loop beta function

$$\alpha_s(\Lambda) = \frac{12\pi}{11C_A - 2N_f} \frac{1}{\ln\left(\frac{\Lambda^2}{\Lambda^2_{\overline{MS}}}\right)}$$

- $C_A =$ Color factor associated with gluon emmision from a gluon. For $SU(N_C)$ gauge theory, $C_A = N_c$.
- $N_f =$ Number of flavor,
- $\Lambda_{\overline{MS}} =$ QCD scale. For one loop beta function with $N_f = 3, \ \Lambda_{\overline{MS}} = 176$ MeV(from Lattice).
- $\Lambda =$ Renormalization scale which is $\sim 2\pi T$ at finite temperature. There are different choices for Λ as $2\pi T$ or $1.47 \times 2\pi T$. We choosen here the center value as $2\pi\sqrt{T^2 + \mu^2/\pi^2}$ and we varied the center value by a factor of 2.

$$\Delta P(T,\mu) = P(T,\mu) - P(T,0).$$

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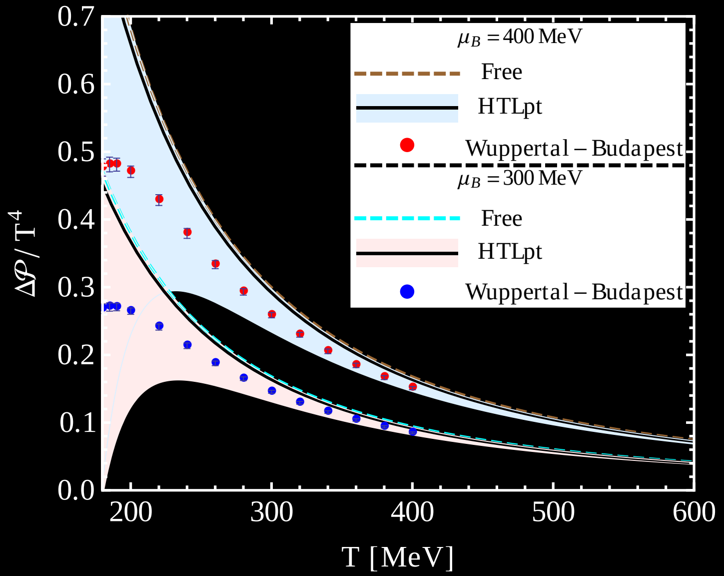


Other Thermodynamic quantities

$$
\begin{aligned}
\text{Entropy density} \qquad \mathcal{S}(T,\mu) &= \frac{\partial \mathcal{P}}{\partial T}\,, \\
\text{Number density} \qquad n_i(T,\mu_i) &= \frac{\partial \mathcal{P}}{\partial \mu_i}\,, \\
\text{Energy density} \qquad \mathcal{E}(T,\mu) &= T\frac{\partial \mathcal{P}}{\partial T} + \mu\frac{\partial \mathcal{P}}{\partial \mu} - \mathcal{P} \\
\text{Speed of sound} \qquad c_s(T,\mu)^2 &= \frac{\partial \mathcal{P}}{\partial \mathcal{E}} \\
\text{Trace anomaly} \qquad I(T,\mu) &= \mathcal{E} - 3\mathcal{P}
\end{aligned}
$$

Fluctuations of conserved charges

- Fluctuations and correlations of conserved charges are sensitive probes of deconfinement
- Quark Number fluctuation for three flavor system is defined as

$$\chi^{uds}_{ijk}(T) = \frac{\partial^{i+j+k}\mathcal{P}}{\partial\mu_u^i\partial\mu_d^j\partial\mu_s^k}$$

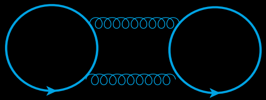
- We can also define Baryon Number fluctuations as

$$\chi^B_n(T) = \frac{\partial^n\mathcal{P}}{\partial\mu_B^n}$$

with $\mu_B = \mu_u + \mu_d + \mu_s$ for three-flavor system.

Fluctuations of conserved charges

In three loop HTLpt case, we have a diagram:



The flavor of two fermionic loop are not same always.

⇒ Off-diagonal susceptibilty is non-zero.

⇒ Quark number flutuations and baryon number fluctuations are not proportional to eache other.

Second order fluctuations

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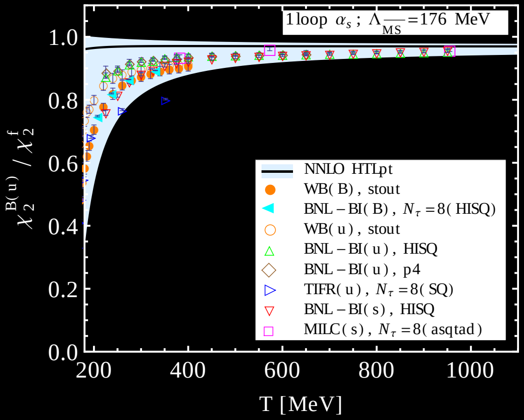
- As we are interested in fluctuation of the system near zero chemical potential, second order off-diagonal susceptibility is zero.
  $\Rightarrow$ Second order quark number and baryonic number fluctuations are proportional.



Fourth order baryon number fluctuations

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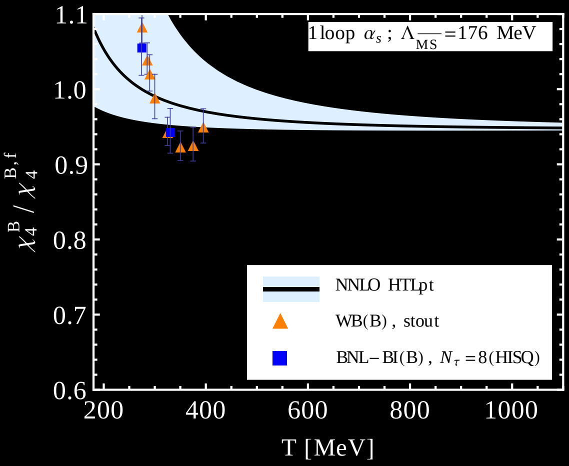

Fourth order diagonal quark number fluctuations

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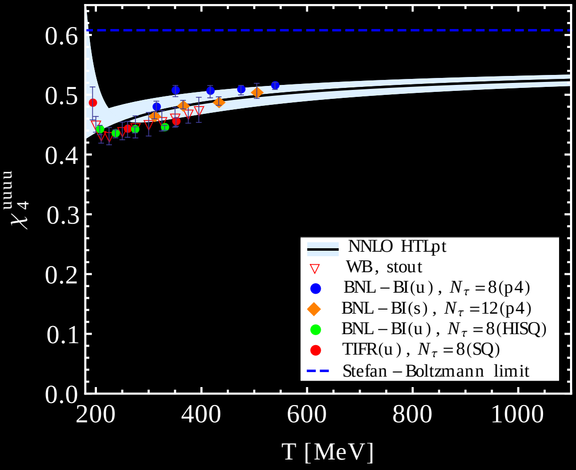

Speed of Sound

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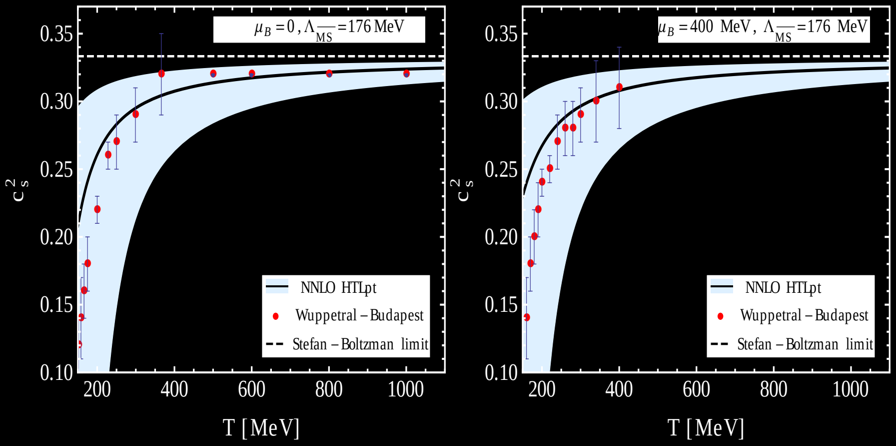
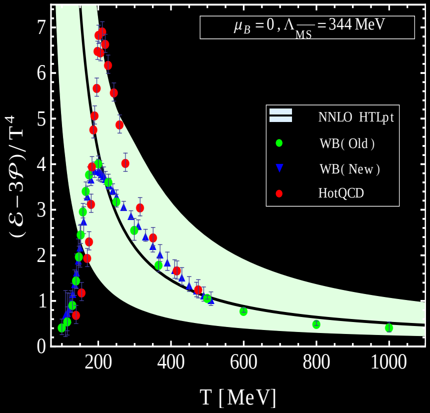
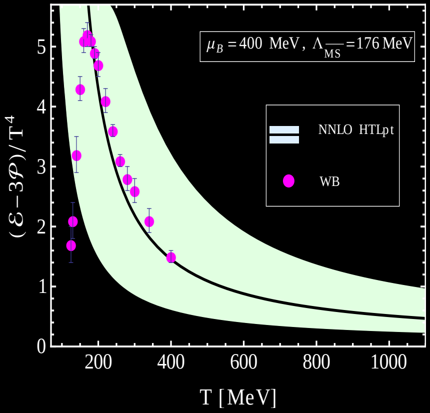


Trace Anomaly





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Thank You for your attention.