MEDIUM EFFECTS ON THE TRANSPORT COEFFICIENTS OF AN INTERACTING PION GAS

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Plan of the talk

- **Introduction** explaining the origin of transport coefficients in an interacting system.
- **Evaluation of transport coefficients** by solving the relativistic transport equation.
- **Calculating the reaction cross-section** for a medium with non-zero temperature and chemical potential.
- **Results** showing the medium effect on the temperature dependence of transport coefficients and comparing with the existing ones.
- **Summary and outlook.**

Introduction

- When a system is slightly away from equilibrium collisions within the system restore it back. These collisions involve momentum transfer between different elements of the system which set up different dissipative processes.

- Since due to shear viscosity the fluid elements of the adjacent layers have a velocity gradient , there appears a distortion in the fluid distribution which modifies the distribution function of fluid elements by an amount δf. In case of bulk viscosity, since it results from compression or expansion of the fluid it also results in modification of distribution function.

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 E} p^\mu p^\nu [f_0 + \delta f]$$

- These viscous effects also modify the energy-momentum stress tensor. To first order in velocity gradient shear viscosity appears as the coefficient of the traceless part .

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \eta\left\langle \nabla^\mu u^\nu \right\rangle + \xi\Delta^{\mu\nu}(\nabla \cdot u)$$

- Comparing the above equations and obtaining δf by solving Boltzmann transport equation the value of shear and bulk viscosity has been estimated .

The relativistic Boltzmann transport equation for the phase space distribution function *f(x,p)* in a covariant frame

$$p^{\mu}\partial_{\mu} f^{(0)}(x, p) = C[f]$$

Collision term

For a system with no external forces and slightly away from equilibrium.

$$C[f] = \int d\Gamma_k d\Gamma_{p'} d\Gamma_{k'} \left[f(x, p') f(x, k') \{1 + f(x, p)\}\{1 + f(x, k)\} - f(x, p) f(x, k) \{1 + f^{(0)}(x, p')\}\{1 + f^{(0)}(x, k')\} \right] W$$

Reaction rate

The Uehling-Uhlenbeck collision term for a binary elastic collision p+k $\longrightarrow$p$^{/}$+k$^{/}$

$$W = \frac{s}{2} \frac{d\sigma}{d\Omega} (2\pi)^6 \delta^4 (p + k - p' - k')$$

Differential scattering cross-section

$$d\Gamma_p = \frac{d^3 p}{(2\pi)^3 p^0}$$

Phase-space factor

Solving transport equation by Chapman-Enskog approximation

This method expands the distribution function in a series in term of an ordering parameter.

$$f(x,p) = f^{0}(x,p) + f^{0}(x,p)\{1 + f^{0}(x,p)\}\varphi(x,p)$$

Equilibrium distribution function

A small deviation of distribution function from the equilibrium resulting from the correspondence between the non-equilibrium kinetic theory and viscous hydrodynamics.

Integral equation satisfied by $\phi(x,p)$

$$p^{\mu}\partial_{\mu} f^{(0)}(x,p) = -\Re[\varphi]$$

The deviation function ϕ satisfies the integro-differential equation.

$$\Re[\varphi] = f^{(0)}(x,p)\int d\Gamma_k d\Gamma_{p'} d\Gamma_{k'} f^{0}(x,k)\{1 + f^{(0)}(x,p')\}\{1 + f^{(0)}(x,k')\}[\varphi(x,p) + \varphi(x,k) - \varphi(x,p') + \varphi(x,k')]W$$

Solving the unknown function ϕ

The equilibrium distribution function for a Bosonic system ,

$$f_0 = \frac{1}{\exp\left\{\frac{p^\nu u_\nu(x) - \mu(x)}{K_B T(x)}\right\} - 1}$$

The thermodynamic quantities, T(x), μ(x), U^μ(x) upon which f_0 depends are again function of 4 time-space co-ordinates.

The space-time gradient may be decomposed w.r.t. the 4-velocity U^μ(x) into a time-like and a space-like part.

$$\begin{tikzcd}
\partial^\mu = U^\mu & D \arrow[d] & + & \nabla^\mu \arrow[dd] & \\
 & U^\mu \partial_\mu & & & \\
 & & & \Delta^{\mu\nu}\partial_\nu \arrow[r, bend right] & g^{\mu\nu} - U^\mu U^\nu
\end{tikzcd}$$

In local rest frame,

$$U^\mu = \left(1, \vec{0}\right)$$

$D \rightarrow \frac{\partial}{\partial t}$ ➡ Pure time derivative

$\nabla_\mu \rightarrow \partial_i$ ➡ Pure spatial derivative

$$(p.u)\left[\frac{p.u}{T^2} DT + D\left(\frac{\mu}{T}\right) - \frac{p^\mu}{T} Du_\mu\right] + p^\mu\left[\frac{p.u}{T^2}\nabla_\mu T + \nabla_\mu\left(\frac{\mu}{T}\right) - \frac{p^\nu}{T}\nabla_\mu u_\nu\right] = -\frac{\Re[\varphi]}{f^{(0)}\left(1 + f^{(0)}\right)}$$

These derivatives should be expressed in terms of velocity and temperature gradients.

Needed to be eliminated using thermodynamic equilibrium laws.

Linear equation solved by the function ϕ

$$Q\partial_\mu u^\mu + \left(p^\sigma u_\sigma - h\right)\underbrace{p_\mu \Delta^{\mu\alpha}\left\{T\partial_\alpha T - Du_\alpha\right\}} - p_\mu p_\nu \left\langle \partial^\mu u^\nu \right\rangle = -k_B T\left(1 + Af_0\right)^{-1}\Re[\varphi]$$

Thermodynamic forces with different tensorial rank representing a scalar, a vector and a tensor respectively.

In order to be a solution of the previous equation ϕ must be a linear combination of thermodynamic forces,

$$\boxed{\varphi = A\partial_\mu u^\mu + B_\mu \Delta^{\mu\alpha}\left\{T\partial_\alpha T - c^{-2}Du_\alpha\right\} - C_{\mu\nu}\left\langle \partial^\mu u^\nu \right\rangle}$$

In order to keep the function ϕ a scalar quantity, the coefficients should be of appropriate tensorial rank.

Scalar coefficient

$$\boxed{A}$$

Term related to bulk viscosity

Vector coefficient

$$\boxed{B_\mu = B\Delta_{\mu\nu} p^\nu}$$

Term related to thermal conductivity

Tensor coefficient

$$\boxed{C_{\mu\nu} = Cp^\mu p^\nu}$$

Term related to shear viscosity

Linearized transport equations

Since the thermodynamic forces are independent,

Integral equation solved by A

$$\Re[A] = -\frac{1}{T} f^{(0)}(p)\{1 + f^{(0)}(p)\}Q$$

Integral equation solved by Bμ

$$\Re[B_\mu] = -\frac{1}{T} f^{(0)}(p)\{1 + f^{(0)}(p)\}\Delta_{\mu\sigma} p^\sigma \left(p_\nu u^\nu - h\right)$$

Integral equation solved by $C_{\mu\upsilon}$

$$\Re[C_{\mu\nu}] = -\frac{1}{T} f^{(0)}(p)\{1 + f^{(0)}(p)\}\langle p_\mu p_\nu \rangle$$

Solving the transport coefficients

Since ϕ is a small perturbation, close to equilibrium, the relation of energy four-flow and viscous pressure tensor with the thermodynamic forces can be approximated by linear laws.

$$I^{\mu} = \lambda\Delta^{\mu\alpha}\{\partial_{\alpha}T - TDu_{\alpha}\}$$

Thermal conductivity

$$\Delta T^{\mu\nu} = 2\eta\langle\partial^{\mu}u^{\nu}\rangle + \xi\Delta^{\mu\nu}\partial_{\alpha}u^{\alpha}$$

Shear viscosity

Bulk viscosity

Again since,

$$I^{\mu} = \int\frac{d^3p}{(2\pi)^3p^0}f_0(1+f_0)B_{\mu}\Delta^{\mu\alpha}\{T^{-1}\partial_{\alpha}T - c^{-2}Du_{\alpha}\}p^{\mu}\left(p^{\sigma}u_{\sigma} - h\right)$$

$$\Delta T^{\mu\nu} = \int\frac{d^3p}{(2\pi)^3p^0}f_0(1+f_0)\langle p^{\mu}p^{\nu}\rangle C_{\mu\nu}\langle\partial^{\mu}u^{\nu}\rangle - \int\frac{d^3p}{(2\pi)^3p^0}f_0(1+f_0)QA\partial_{\mu}u^{\mu}$$

$$\xi = -\int d\Gamma_p f^0(p)(1+Af^0(p))QA$$

$$\lambda = -\frac{1}{3T}\int d\Gamma_p f^0(p)(1+f^0(p))(p^{\sigma}u_{\sigma} - h)B_{\mu}p_{\nu}\Delta^{\mu\nu}$$

$$\eta = -\frac{1}{10}\int d\Gamma_p f^0(p)(1+f^0(p))\langle p^{\mu}p^{\nu}\rangle C_{\mu\nu}$$

Expression for bulk viscosity

Expression for thermal conductivity

Expression for shear viscosity

Transport coefficients using Chapman-Enskog Approximation

- Bulk viscosity

$$\boxed{\xi = T\,\frac{\alpha_2^2}{a_{22}}}$$

$$a_{22} = 2z^2 I_3(z)$$

$$\alpha_2 = \frac{z^3}{2}\left[\frac{1}{3}\left(\frac{S_3^0}{S_2^1} - z^{-1}\right) + \left(\frac{S_2^0}{S_2^1} - \frac{3}{z}\frac{S_3^1}{S_2^1}\right)\left\{(1-\gamma'')\frac{S_3^1}{S_2^1} + \gamma''' z^{-1}\right\} - \left(\frac{4}{3} - \gamma'\right)\left\{\frac{S_3^0}{S_2^1} + 15z^{-2}\,\frac{S_3^2}{S_2^1} + 2z^{-1}\right\}\right]$$

- Thermal conductivity

$$\boxed{\lambda = \frac{T}{3m_\pi}\,\frac{\beta_1^2}{b_{11}}}$$

$$\beta_1 = -3z^2\left[1 + 5z^{-1}\frac{S_3^2}{S_2^1} - \left(\frac{S_3^1}{S_2^1}\right)^2\right]$$

$$b_{11} = 8z\left(I_2(z) + I_3(z)\right)$$

- Shear viscosity

$$\boxed{\eta = \frac{T}{10}\,\frac{\gamma_0^2}{c_{00}}}$$

$$\gamma_0 = -10\,\frac{S_3^{-2}(z)}{S_2^{-1}(z)}$$

$$c_{00} = 16\left[I_1(z) + I_2(z) + \frac{1}{3}I_3(z)\right]$$

Integral form of the bracket expression

$$I_\alpha(z)=\frac{z^4}{\left[S_2^{-1}(z)\right]^2}e^{(-2\mu_\pi/T)}\int_0^\infty d\psi\cosh^3\psi\sinh\psi^7\int_0^\pi d\Theta\sin\Theta\frac{1}{2}\frac{d\sigma}{d\Omega}(\psi,\Theta)$$

$$\int_0^\infty d\chi\sinh^{2\alpha}(\chi)\int_0^\infty d\varphi\int_0^\pi d\theta\sin\theta\underbrace{\frac{e^{2z\cosh\psi\cosh\chi}}{\left(e^E-1\right)\left(e^F-1\right)\left(e^G-1\right)\left(e^H-1\right)}}_{\Big\downarrow\; f^{(0)}(p)f^{(0)}(k)\{1+f^{(0)}(p')\}\{1+f^{(0)}(k')\}}M_\alpha(\theta,\Theta)$$

$$E=z\left(\cosh\psi\cosh\chi\text{ - }sinh\psi\sinh\chi\cos\theta\right)$$

$$F=z\left(\cosh\psi\cosh\chi\text{ - }sinh\psi\sinh\chi\cos\theta'\right)$$

$$G=E+2z\sinh\psi\sinh\chi\cos\theta$$

$$H=F+2z\sinh\psi\sinh\chi\cos\theta'$$

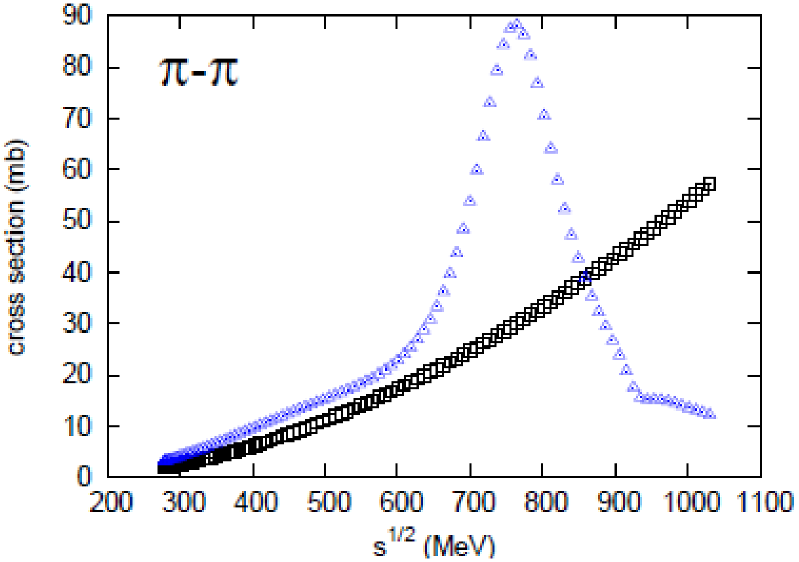
$$M_1(\theta,\Theta)=1-\cos^2\Theta$$

$$M_2(\theta,\Theta)=\cos^2\theta+\cos^2\theta'\text{ - }2\cos\theta\cos\theta'\cos\Theta$$

$$M_3(\theta,\Theta)=\left[\cos^2\theta\text{ - }\cos^2\theta'\right]^2$$

$$S_n^\alpha(z)=\sum_{k=1}^\infty k^\alpha K_n(kz)$$

Evaluating the ππ cross section



Ref: K. Itakura, O.Morimatsu, H.Otomo, Phys Rev. D 77,014014(2008)

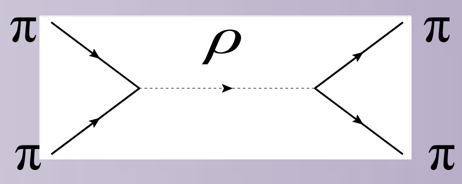
The ππ cross section calculated from effective field theory considering only the contact diagrams gives an amplitude,

$$\overline{|M_{\pi\pi}|^2} = \frac{1}{9f_\pi^4}\left\{21m_\pi^4 + 9s^2 - 24m_\pi^2 s + 3(t-u)^2\right\}$$

which does not match with the experimental data beyond 600 MeV.

- The invariant amplitude for the ππ scattering is evaluated using a ρ meson exchange between the pions using the following Lagrangian,

$$\mathcal{L}_{\rho\pi\pi} = g_\rho \bar{\rho}_\mu \cdot \left(\bar{\pi} \times \partial^\mu \bar{\pi}\right)$$



g_ρ =6.05 is fixed from ρ→ππ decay width.

- In order to describe ππ scattering at low energies the σ exchange diagrams are also included using the interaction

$$\mathcal{L}_{\sigma\pi\pi} = \frac{1}{2} g_\sigma m_\sigma \bar{\pi} \cdot \bar{\pi}\sigma$$

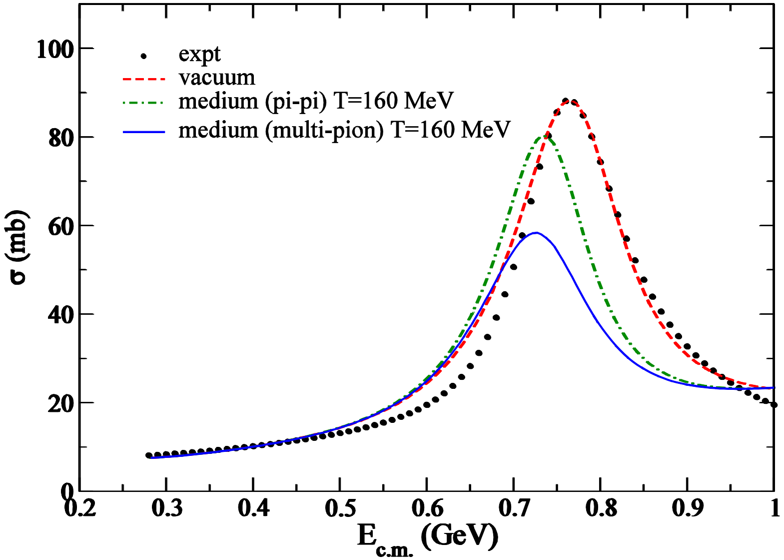
Iso-spin averaged amplitudes from effective field theory

$$M_{I=0}=2g_\rho^2\left[\frac{s-u}{t-m_\rho^2}+\frac{s-t}{u-m_\rho^2}\right]+g_\sigma^2 m_\sigma^2\left[\frac{3}{s-m_\sigma^2+im_\sigma\Gamma_\sigma}+\frac{1}{t-m_\sigma^2}+\frac{1}{u-m_\sigma^2}\right]$$

$$M_{I=1}=g_\rho^2\left[\frac{2(t-u)}{s-m_\rho^2+im_\rho\Gamma_\rho(s)}+\frac{t-s}{u-m_\rho^2}-\frac{u-s}{t-m_\rho^2}\right]+g_\sigma^2 m_\sigma^2\left[\frac{1}{t-m_\sigma^2}-\frac{1}{u-m_\sigma^2}\right]$$

$$M_{I=2}=g_\rho^2\left[\frac{u-s}{t-m_\rho^2}+\frac{t-s}{u-m_\rho^2}\right]+g_\sigma^2 m_\sigma^2\left[\frac{1}{t-m_\sigma^2}+\frac{1}{u-m_\sigma^2}\right]$$

$$\overline{|M|^2}=\frac{1}{9}\sum(2I+1)\overline{|M_I|^2}$$



- The medium modification of the ρ in hot and dense matter is expected to modify the cross-section and as well as the value of η.

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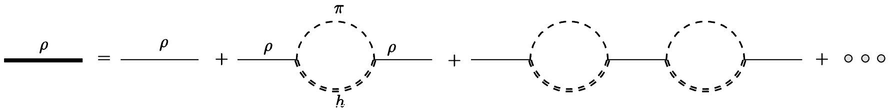
- In this calculation the ρ and σ propagator is modified by introducing the two pion decay width,

$$\Gamma_\rho(s)=\frac{g_\rho^2}{48\pi s}\left(s-4m_\pi^2\right)^{3/2}$$

- This modification in done only in s-channel ρ-exchange diagrams ,which contributes in I=1 and in s-channel σ exchange diagrams, which contributes in I=0 state.

Introducing medium effect in the ππ cross section



The exact ρ propagator with π-meson loop diagrams from Dyson equation

$$D_{\mu\nu} = D^{(0)}_{\mu\nu} + D^{(0)}_{\mu\rho}\Pi^{\rho\lambda}D_{\lambda\nu}$$

The full ρ meson propagator in the medium

Vacuum propagator

One loop self energy function

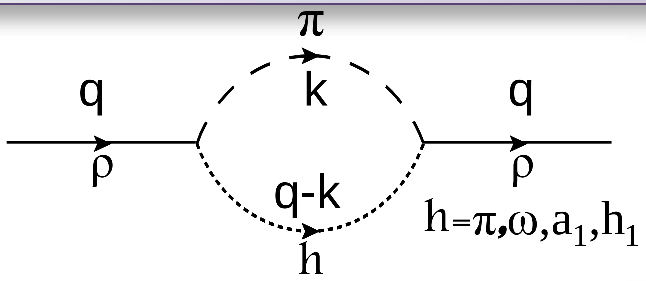
$$D_{\mu\nu}(q_0,\vec{q}) = \frac{-g_{\mu\nu}+q_\mu q_\nu/q^2}{q^2-m_\rho^2-\mathrm{Re}\,\Pi(q_0,\vec{q})+i\,\mathrm{Im}\,\Pi(q_0,\vec{q})}$$

The in medium ρ propagator in terms of real and imaginary part of self energy.

- **ππ, πω, πa$_1$,πh$_1$ loop has been considered in the self energy function.**

- **Folding with πρ/3π decay width, ω,a$_1$,h$_1$ mesons results in suppression of ππ cross section.**

Calculating the one loop ρ self energy function



$$\Pi_{\mu\nu}(q)=i\int\frac{d^4k}{(2\pi)^4}N_{\mu\nu}(q,k)\Delta_\pi(k)\Delta_h(q-k)$$

vacuum propagator for ***π***

vacuum propagator For h

one loop self-energy in vacuum with internal lines for pion and a hadron h.

The N’s are resulting from the vertices appearing from the interaction Lagrangian.

Ref : S. Ghosh et al, Eur. Phys. J C 70, 251 (2010)

$$\Pi_{\mu\nu}(q)=i\int\frac{d^4k}{(2\pi)^4}N_{\mu\nu}(q,k)D_\pi(k)D_h(q-k)$$

thermal propagator for ***π***

thermal propagator for h

The ρ self energy at finite temperature determined in the real time formalism of thermal field theory.

$$D(k)=\Delta(k)+2in\pi\delta\left(k^2-m^2\right)$$

Temperature dependence enters in self energy through this term.

Thermal propagator in terms of vacuum propagator and distribution function n at a non-zero temperature.

Complete self energy function at finite temperature

$$\Pi_{\mu\nu}(q)=i\int\frac{d^4k}{(2\pi)^4}N_{\mu\nu}(q,k)\Delta_\pi(k)\Delta_h(q-k)$$

the first term refers to vacuum

$$-\int\frac{d^4k}{(2\pi)^3}N_{\mu\nu}(q,k)\left\{\mathit{\Delta}_h(q-k)n(\omega)\delta\left(k^2-m_\pi^2\right)+\mathit{\Delta}_\pi(k)n(\omega')\delta\left((q-k)^2-m_h^2\right)\right\}$$

$$-i\int\frac{d^4k}{(2\pi)^2}N_{\mu\nu}(q,k)n(\omega)n(\omega')\delta\left(k^2-m_\pi^2\right)\delta\left((q-k)^2-m_h^2\right)$$

second and third term are medium dependent

$$\omega=\sqrt{m_\pi^2+\vec{k}^2}$$

energies of pions

$$\omega'=\sqrt{m_\pi^2+\left(\vec{q}-\vec{k}\right)^2}$$

energies of hadrons

- The real part of the self energy function modifies the mass term in the propagator .

- The imaginary part of self energy is related to the decay width by the relation **$\mathrm{Im}\Pi(q_0,\vec{q})=-q_0\Gamma(q_0,\vec{q})$.**

Imaginary part of ρ self energy function

$$\operatorname{Im}\Pi^{\mu\nu}(q_0,\vec{q}) = -\pi\int\frac{d^3k}{(2\pi)^3}\frac{1}{4\omega_\pi\omega_h}\left[\begin{array}{l} N^{\mu\nu}\left(k^0=\omega_\pi\right)\left\{\begin{array}{l}[1+f(\omega_\pi)+f(\omega_h)]\delta(q_0-\omega_\pi-\omega_h)\\ +[f(\omega_\pi)-f(\omega_h)]\delta(q_0-\omega_\pi+\omega_h)\end{array}\right\}+ \\ N^{\mu\nu}\left(k^0=-\omega_\pi\right)\left\{\begin{array}{l}[f(\omega_h)-f(\omega_\pi)]\delta(q_0+\omega_\pi-\omega_h)\\ -[1+f(\omega_\pi)+f(\omega_h)]\delta(q_0+\omega_\pi+\omega_h)\end{array}\right\}\end{array}\right]$$

The different scattering and decays processes mentioned above lead to loss or gain of ρ meson within the medium.

- **The first term can be interpreted as the probability of decay for the process ρ → πh with statistical weight factor {1+*f* (ω_π) }{1+*f*(ω_h)} for emission minus the probability of inverse decay πh → ρ with weight factor *f* (ω_π)*f*(ω_h) for absorption.**

- **The second term can be interpreted as the probability of decay for the process ρh→π with statistical weight factor *f* (ω_h){1+*f*(ω_π)} minus the probability for π → ρh with weight factor *f* (ω_π){1+*f*(ω_h)} .**

Introduction of temperature dependent chemical potential - effect of early chemical freeze-out

Pions get out of chemical equilibrium early , at T~ **170 MeV.**

⬇

Only elastic processes including the resonances dominate the dynamics of the system.

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At a lower temperature ~100 MeV momentum transfer ceases to give kinetic freeze out.

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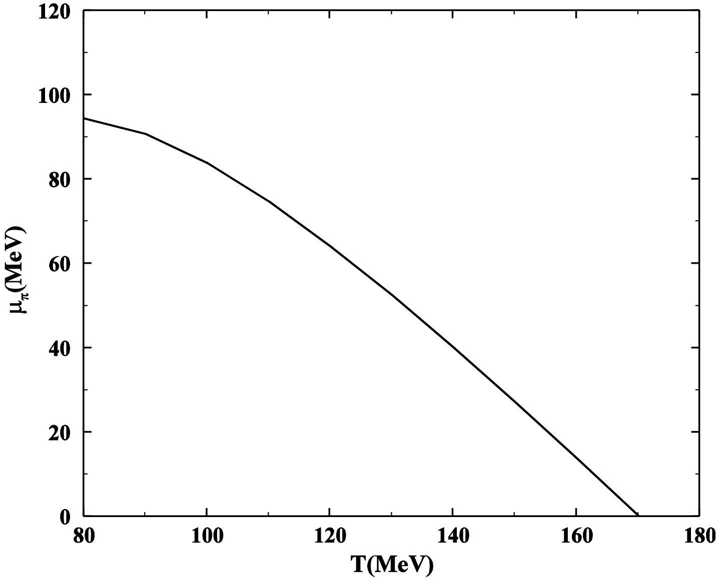
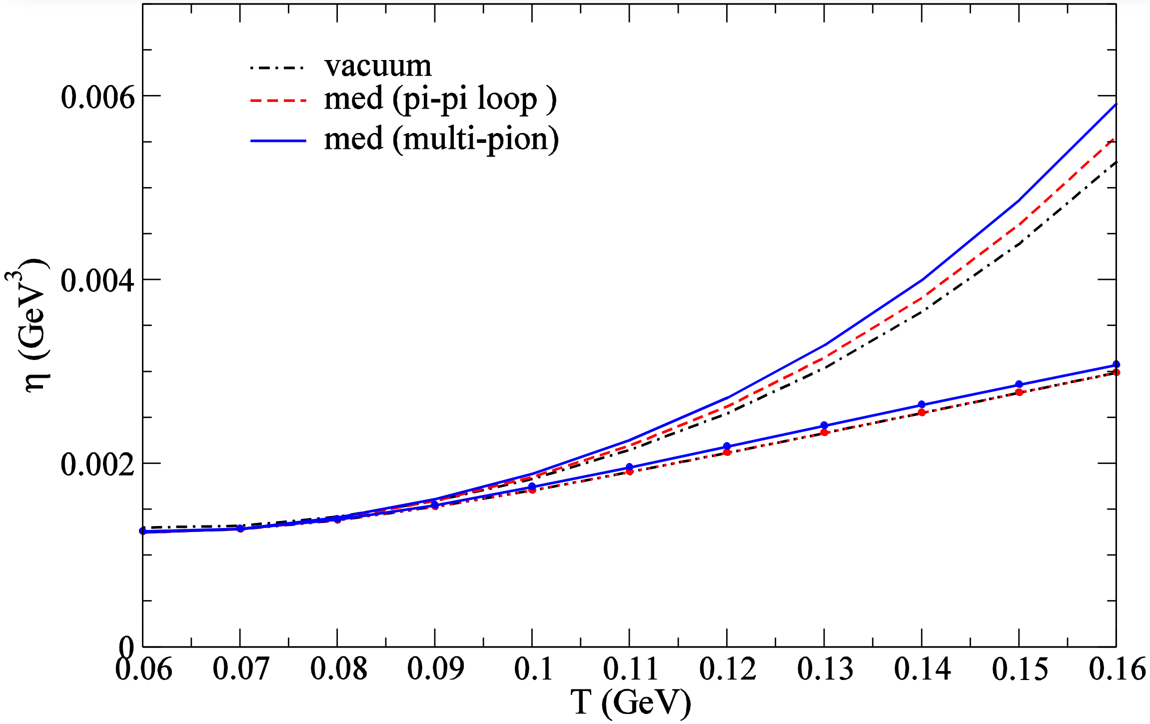
The chemical potential starts building up with decrease of temperature.



Ref: T. Hirano, and K. Tsuda Phys Rev. C 66, 054905 (2002)

Shear viscosity as a function of temperature in Chapman-Enskog approximation

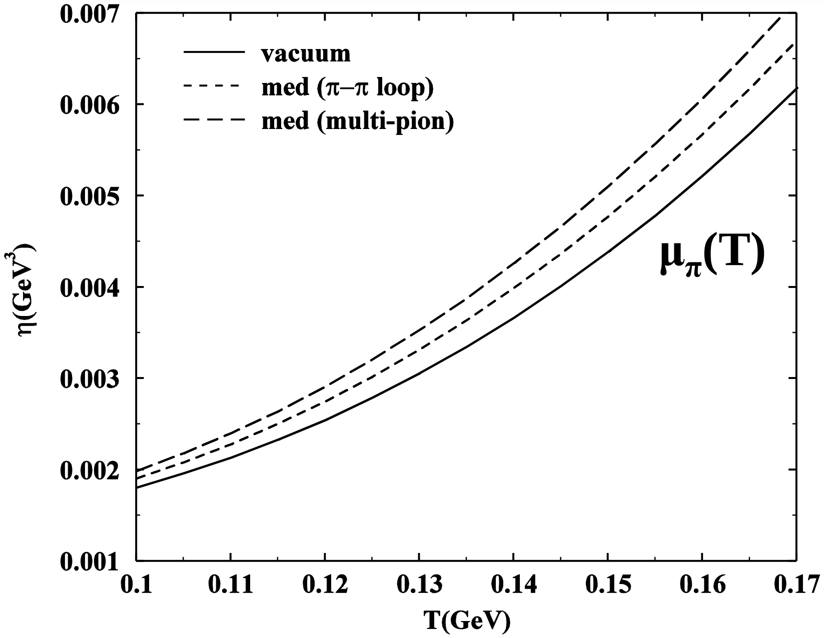


Ref: Sukanya Mitra , Sabyasachi Ghosh and Sourav Sarkar, Phys Rev. C 85, 064917 (2012)

We can see a clear difference in the temperature dependence of shear viscosity with and without medium modification of the ρ propagator and it is more prominent from the heavy meson loops which considered as the multipion contribution to ρ self-energy compare to ππ loop.

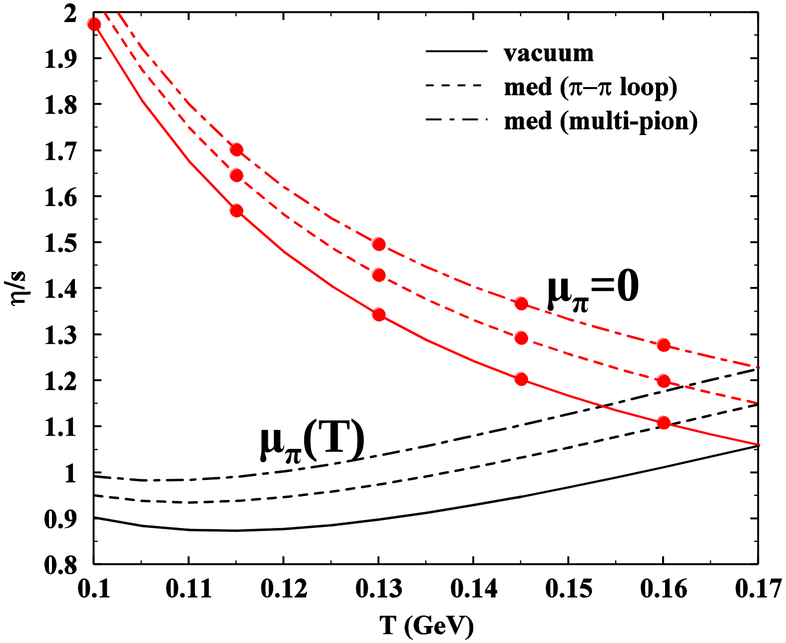
The upper set of curves uses the upper limit of integration over ψ~2 , which corresponds $E_{C.M.}$ =$2m_\pi \cosh\psi$ ~1 GeV for ππ scattering while the lower set denotes actual upper limit , i.e. ∞, the difference between two sets of curve indicates the uncertainties of result due to insufficient information of cross section at higher energies. ,

Shear viscosity as a function of temperature in Chapman-Enskog approximation



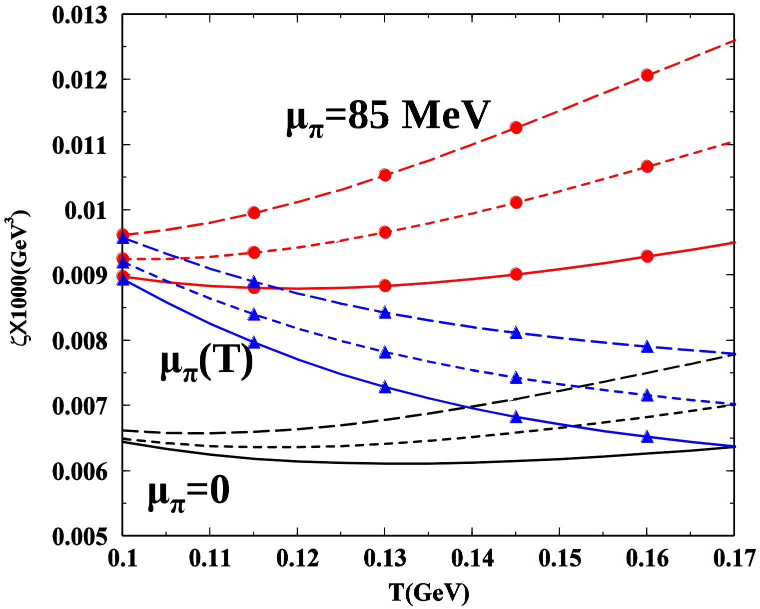
We can see a clear difference in the temperature dependence of shear viscosity with and without medium modification of the ρ propagator and it is more prominent from the heavy meson loops which considered as the multipion contribution to ρ self-energy compare to ππ loop.

Shear viscosity to entropy density ratio shows different trends for zero and temperature dependent chemical potential. η/s increase with T for $\mu_\pi = \mu_\pi(T)$. in contrast with the usual decreasing trend of η/s for $\mu_\pi=0$.



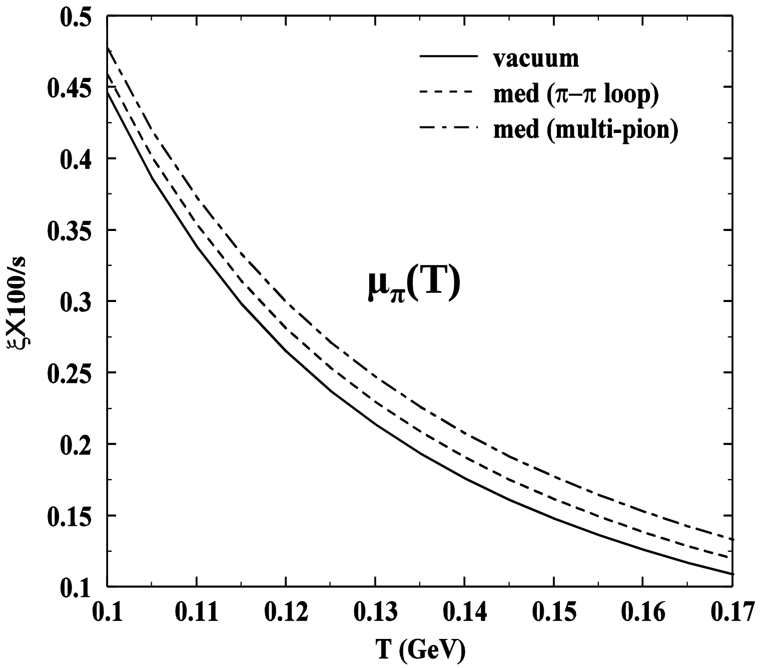
Ref: Sukanya Mitra and Sourav Sarkar, Phys Rev. D 87, 094026 (2013)

Bulk viscosity as a function of temperature in Chapman-Enskog approximation



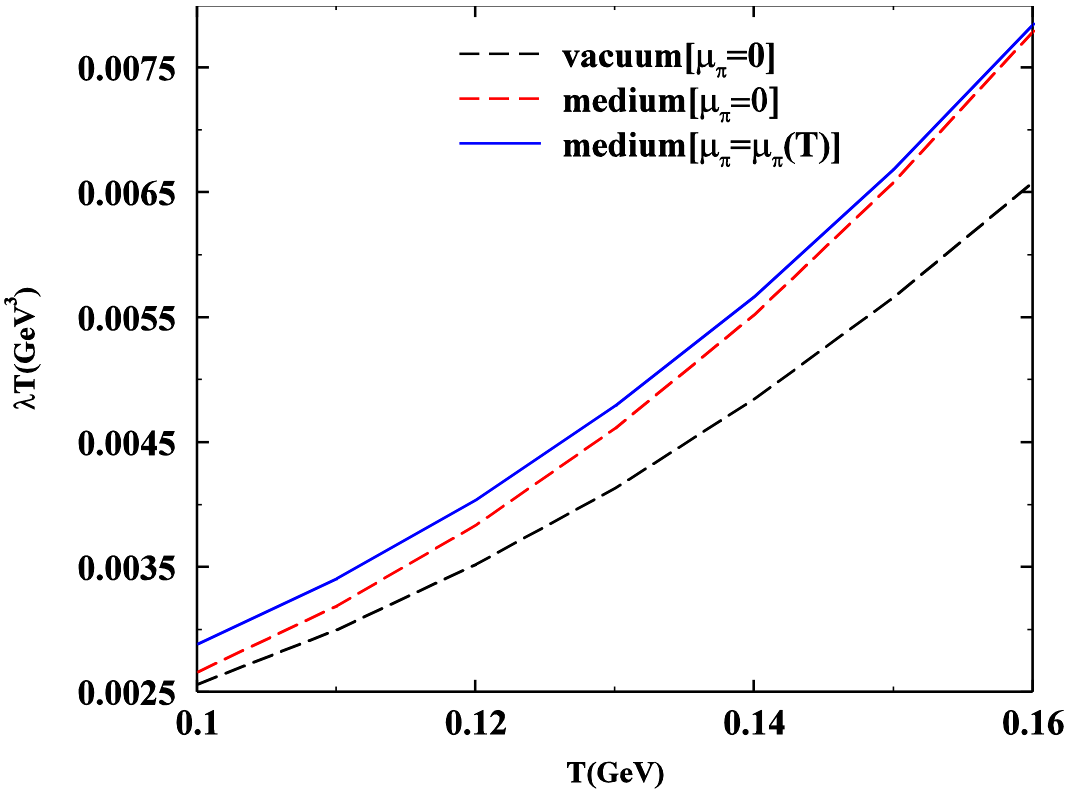
The three set of curves infer a large dependence on pion chemical potential, where in each set the effect of medium on $\pi\pi$ cross-section is clearly visible for both pion loop as well as heavy meson loop in ρ propagator.

The medium dependence is also observed for bulk viscosity to entropy density ratio as a function of temperature using a temperature dependent pion chemical potential .



Ref: Sukanya Mitra and Sourav Sarkar, Phys Rev. D 87, 094026 (2013)

Thermal conductivity as a function of temperature in Chapman-Enskog approximation



In case of thermal conductivity also the effect of medium is clearly seen through the self energy via π-π and π-meson loop. The zero and temperature dependent pion chemical potential gives small but distinctly different results.

Submitted in PRD

Summary……

- We have evaluated the invariant amplitude for $\pi\pi$ scattering from effective field theory using ρ meson exchange which agrees reasonably with the experimental data. To describe the scattering at low energies σ–exchange diagrams are also included.

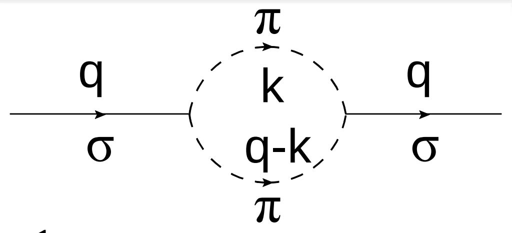
- Medium effects on ρ and σ propagation is introduced through one loop self energy to obtain modified cross section at finite temperature . This shows a suppression in cross section at finite temperature.

- Shear and bulk viscosity coOefficiet along with thermal conductivity is evaluated in Chapman-Enskog approximation with those cross sections and with a temperature dependent chemical potential resulting from early chemical freezeout. The temperature dependence of η with and without medium effects shows noticeable difference.

- The shear and bulk viscosity tO entropy density ratio is evaluated which is also modified due to the finite temperature medium effects.

Thank You

In medium self energy of σ meson



$$D_{\mu\nu}(q_0,\vec{q})=\frac{-1}{q^2-m_\sigma^2-\mathrm{Re}\,\Pi(q_0,\vec{q})+i\,\mathrm{Im}\,\Pi(q_0,\vec{q})}$$

The in medium σ propagator in terms of real and imaginary part of self energy.

$$\Pi(q^0,\vec{q})=N\int\frac{d^3k}{(2\pi)^3}\frac{1}{4\omega_\pi\omega'_\pi}\left[\frac{1+f^{(0)}(\omega_\pi)+f^{(0)}(\omega'_\pi)}{q^0-\omega_\pi-\omega'_\pi+i\eta\varepsilon(q^0)}+\frac{f^{(0)}(\omega'_\pi)-f^{(0)}(\omega_\pi)}{q^0-\omega_\pi+\omega'_\pi+i\eta\varepsilon(q^0)}+\frac{f^{(0)}(\omega_\pi)-f^{(0)}(\omega'_\pi)}{q^0+\omega_\pi-\omega'_\pi+i\eta\varepsilon(q^0)}-\frac{1+f^{(0)}(\omega_\pi)+f^{(0)}(\omega'_\pi)}{q^0+\omega_\pi+\omega'_\pi+i\eta\varepsilon(q^0)}\right]$$

In medium self energy of σ meson.

$$f^{(0)}(\omega)=\frac{1}{e^{(\omega-\mu_\pi)/T-1}}$$

The in medium distribution function, containing the temperature dependence explicitly.

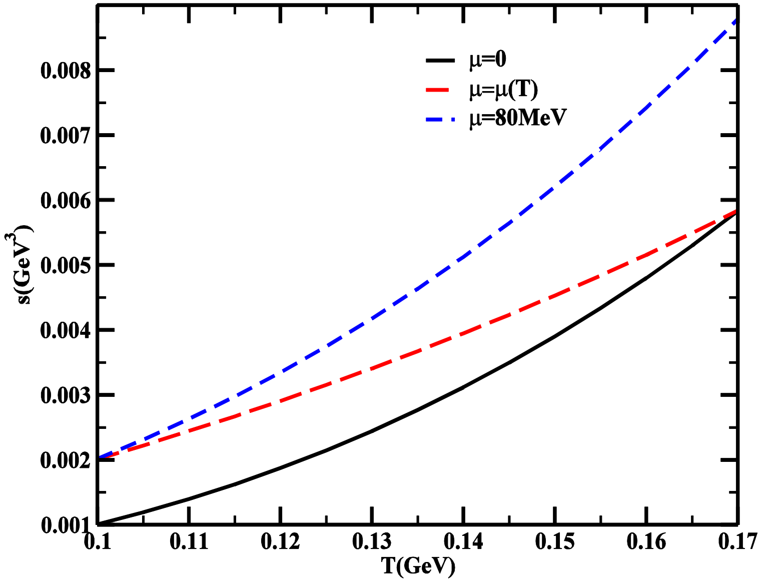
Entropy for an interacting pion gas

$$s = \frac{(\varepsilon + P)}{T} - n\frac{\mu_\pi}{T}$$

Definition of entropy from the thermodynamic laws.

$$= \frac{g_\pi}{2\pi^2} m_\pi^2 \left[m_\pi S_3^1(z) - \mu_\pi S_2^1(z) \right]$$

Entropy for free pion gas.



$$\Delta s = -\frac{3m_\pi^4}{16\pi^4 f_\pi^2} S_1^1(z)\left[m_\pi S_2^0(z) - \mu_\pi S_1^0(z) \right]$$

Correction in entropy due to interaction between pions up to order O(T^6).

Shear Viscosity and its evidence in the matter created at heavy ion collision

Shear viscosity is the measure of stress due to the velocity gradient between different layers of the fluid.

$$\frac{F}{A} = \eta \frac{\partial u_x}{\partial y}$$

Shear viscosity

According to Kinetic Theory of Gases

$$\eta = \frac{1}{3} n \langle p \rangle \lambda = \frac{1}{3} \frac{\langle p \rangle}{\sigma}$$

n : density of the particle

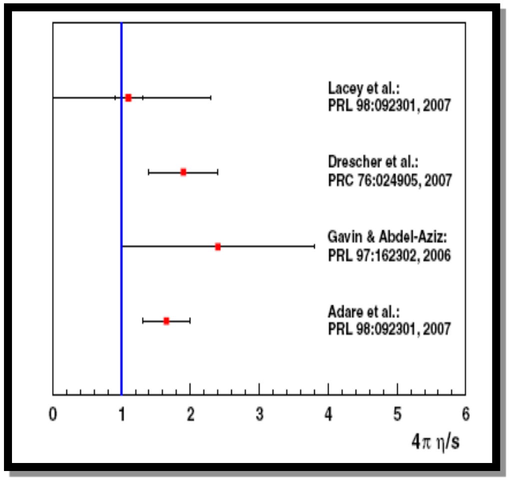
< p> : average particle momentum

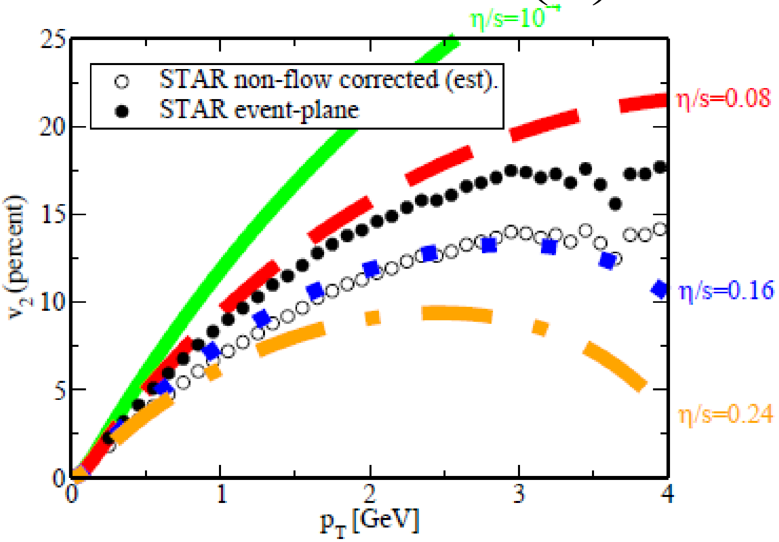
λ : mean free path

$$\lambda = \frac{1}{n\sigma}$$

According to Uncertainty principle;

$$p\lambda \geq \hbar \quad \Rightarrow \frac{\eta}{s} \geq \frac{1}{3}\left(\frac{n}{s}\right)\hbar \quad \Rightarrow \frac{\eta}{s} \geq \frac{1}{4\pi} \qquad s \approx 3.6n$$





The ideal hydrodynamics over predicts the charged hadron elliptic flow , viscous hydrodynamics with a small value of η/s matches the experimental data well, which is a strong evidence of non-zero shear viscosity in the medium created in heavy ion collision.

Transport coefficients using Chapman-Enskog Approximation

Bulk viscosity

Key equation $\Re[A] = -\frac{1}{T} f^{(0)}(p)\{1+f^{(0)}(p)\}Q$

Inserting $L_n^{1/2}(\tau)$ on both sides and integrating over dΓ_p

$[A(\tau), L_n^{1/2}(\tau)] = \frac{\alpha_n}{n}$ ➡ **Bracket expression for transport coefficient**

$$\alpha_n = -\frac{1}{nT}\int d\Gamma_p f^{(0)}(p)\{1+f^{(0)}(p)\}Q L_n^{1/2}(\tau)$$

$$[F,G] = \frac{1}{4n^2}\int d\Gamma_p d\Gamma_{p'} d\Gamma_k d\Gamma_{k'} f^{(0)}(p) f^{(0)}(k)\{1+f^{(0)}(p')\}\{1+f^{(0)}(k')\}\delta(F)\delta(G)W$$

$$\delta(F) = F(p)+F(k)-F(p')-F(k')$$

Expanding the unknown coefficient A in terms of Laugurre polynomial

$$\xi = -\int \frac{d^3p}{p^0} f^0(p)\left(1+Af^0(p)\right)QA$$

$$\left[A(\tau), L_n^{1/2}(\tau)\right] = \frac{\alpha_n}{n}$$

$$A(\tau) = \sum_{m=0}^{\infty} a_m L_m^{1/2}(\tau)$$

→ **Expanding in term of Laugurre Polynomial of order 1/2**

$$\boxed{\xi = T\frac{\alpha_2^2}{a_{22}}}$$

→ **First approximation of bulk viscosity coefficient**

$$\alpha_2 = -\frac{1}{nT}\int d\Gamma_p f^{(0)}(p)\left\{1+f^{(0)}(p)\right\}QL_2^{1/2}(\tau)$$

$$a_{22} = \left[L_2^{1/2}(\tau), L_2^{1/2}(\tau)\right]$$

→ **The bracket quantity is 12 dimensional integral needed to be solved by proper choicec of geometry.**