From
Color Glass Condensate
to
Pomerons, Odderons
and more...

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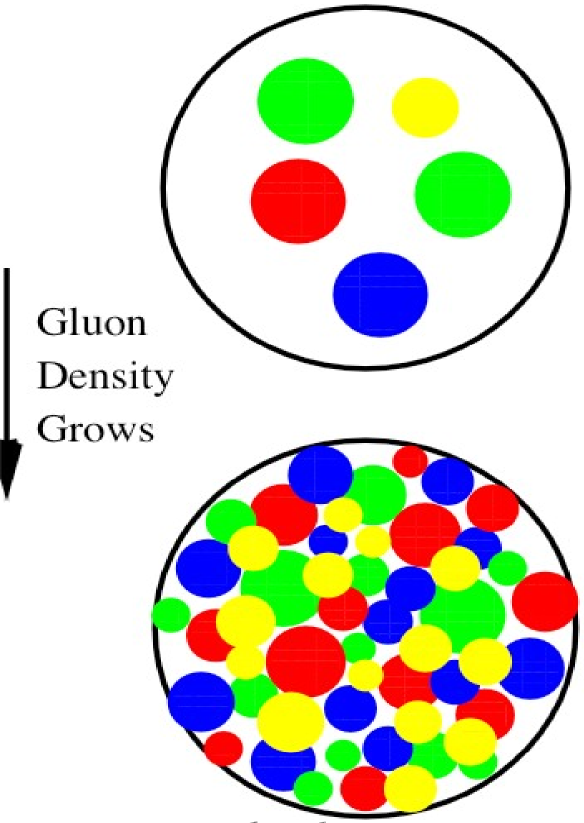
Gluon saturation/CGC

$$\mathbf{S \to \infty, \quad Q^2 \;\; fixed, \quad x_{Bj} \equiv \frac{Q^2}{S} \to 0}$$

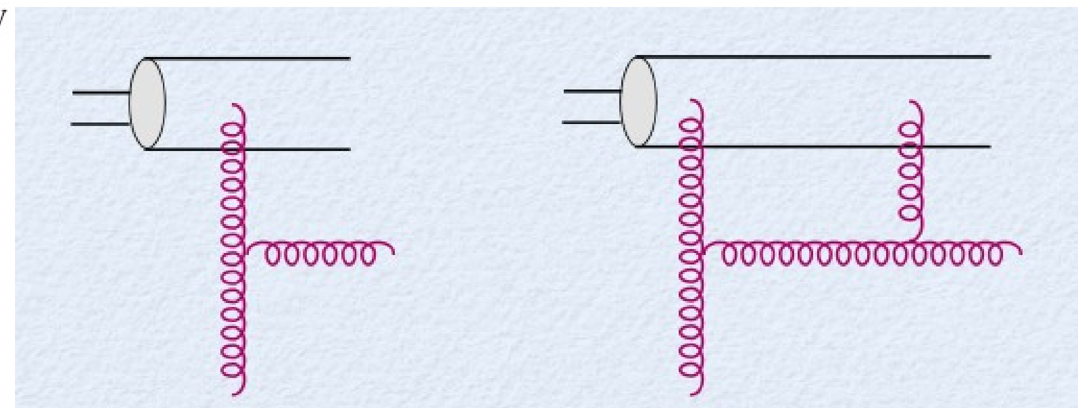
Gribov-Levin-Ryskin

“attractive” bremsstrahlung
vs. “repulsive” recombination



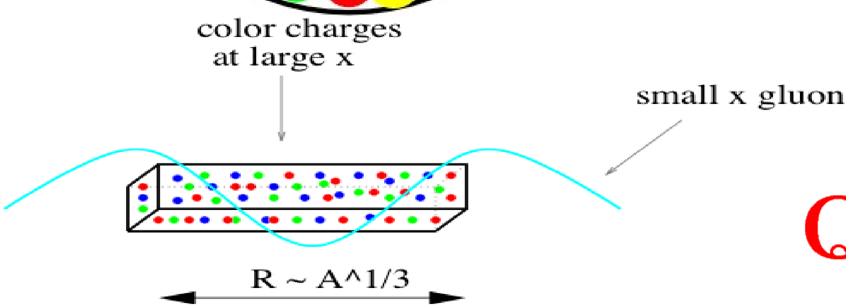
Low Energy

$$\frac{1}{x}$$



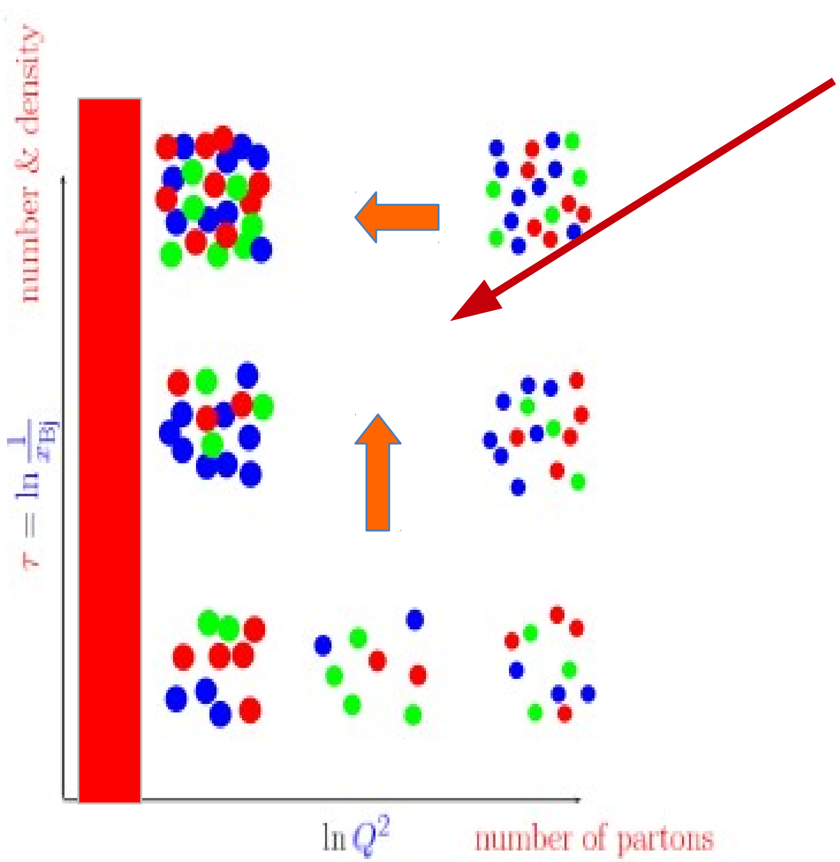
High Energy

$$\frac{\alpha_s \, xG(x, b_t, Q^2)}{S_\perp Q^2} \sim 1$$



$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

Many-body dynamics of universal gluonic matter



How does this happen ?

How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f

Can this provide a first-principles understanding of the initial Conditions-thermalization in Heavy ion collisions?

MV effective Action + RGE

$$\mathbf{S}[\mathbf{A},\rho] = -\frac{\mathbf{1}}{\mathbf{4}}\int \mathbf{d^4x\,F^2_{\mu\nu}} + \frac{\mathbf{i}}{\mathbf{N_c}}\int \mathbf{d^2x_t dx^-}\,\delta(\mathbf{x^-})\mathbf{Tr}[\rho(\mathbf{x_t})\mathbf{U}(\mathbf{A^-})]$$

Large x: color source ρ small x: gluon field $\mathbf{A}^\mu$

$$\mathbf{U}(\mathbf{A^-}) = \hat{\mathbf{P}}\,\mathbf{Exp}\left[\mathbf{ig}\int \mathbf{dx^+\,A^-_a\,T_a}\right]$$

$$\mathbf{Z}[\mathbf{j}] = \int [\mathbf{D}\rho]\,\mathbf{W}_{\mathbf{\Lambda^+}}[\rho]\left[\frac{\int^{\mathbf{\Lambda^+}}[\mathbf{DA}]\delta(\mathbf{A^+})\mathbf{e}^{\mathbf{iS}[\mathbf{A},\rho]-\int \mathbf{j\cdot A}}}{\int^{\mathbf{\Lambda^+}}[\mathbf{DA}]\delta(\mathbf{A^+})\mathbf{e}^{\mathbf{iS}[\mathbf{A},\rho]}}\right]$$

weight functional:

$\mathbf{W}_{\mathbf{\Lambda^+}}[\rho]$ probability distribution of color source ρ at longitudinal scale $\mathbf{\Lambda}^+$

invariance under change of $\mathbf{\Lambda}^+ \longrightarrow$ RGE for $\mathbf{W}_{\mathbf{\Lambda^+}}[\rho]$

The classical field

saddle point of effective action-> Yang-Mills equations

$$\mathbf{D}_\mu \, \mathbf{F}_\mathbf{a}^{\mu\nu} = \delta^{\nu +} \delta(\mathbf{x}^-) \, \rho_\mathbf{a}(\mathbf{x_t})$$

solutions are non-Abelian
Weizsäcker-Williams fields

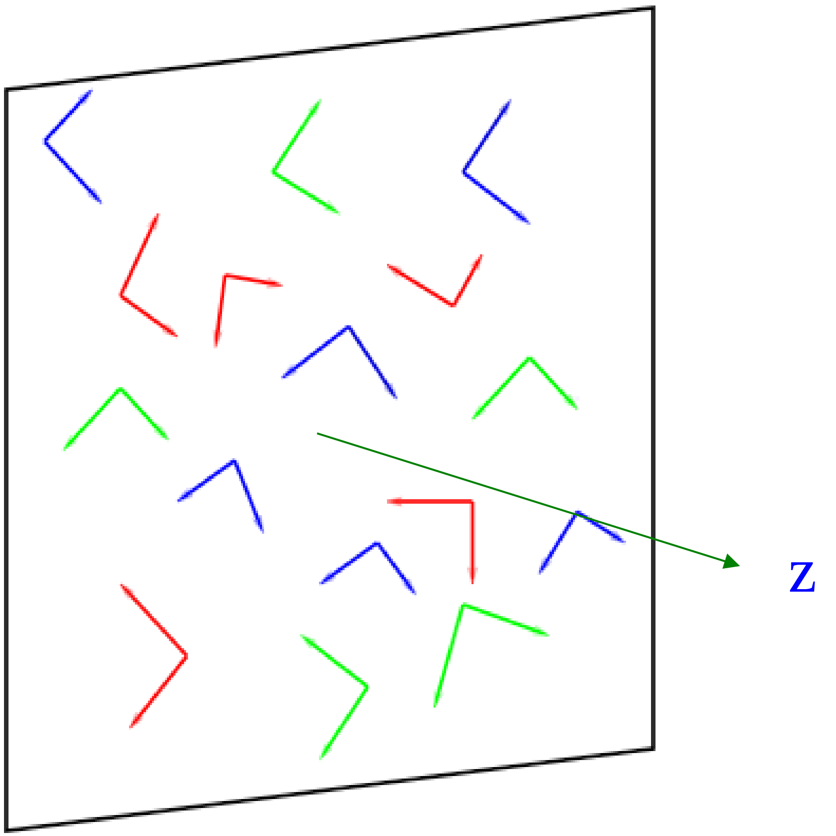
$$\mathbf{A}^+ = \mathbf{0}$$

$$\mathbf{A}^- = \mathbf{0}$$

$$\mathbf{A}_\mathbf{a}^\mathbf{i} = \theta(\mathbf{x}^-) \, \alpha_\mathbf{a}^\mathbf{i}(\mathbf{x_t})$$

$$\partial^\mathbf{i} \, \alpha_\mathbf{a}^\mathbf{i} = \mathbf{g} \, \rho_\mathbf{a}$$

pure (2d) gauge



$$\mathbf{E}_\perp \, , \, \mathbf{B}_\perp$$

Quantum corrections: JIMWLK evolution equation

$$\frac{d}{d\,ln 1/x}\langle O\rangle = \frac{1}{2}\left\langle\int d^2x\, d^2y\, \frac{\delta}{\delta\alpha_x^b}\, \eta_{xy}^{bd}\, \frac{\delta}{\delta\alpha_y^d}\, O\right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi}\int \frac{d^2z}{(2\pi)^2}\frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2}\left[\underbrace{1+U_x^\dagger U_y}_{\text{virtual}} - \underbrace{U_x^\dagger U_z - U_z^\dagger U_y}_{\text{real}}\right]^{bd}$$

U is a Wilson line in adjoint representation

QCD at low x: CGC
(a high gluon density environment)

two main effects: *"multiple scatterings"*
evolution with ln (1/x)

CGC observables: $< \mathrm{Tr}\, V \cdots\cdots V^\dagger >$ with $\mathbf{V(x_t) = \hat{P} e^{ig\int dx^- A_a^+ t_a}}$

$$\mathbf{A_a^\mu(x_t, x^-) \sim \delta^{\mu +}\, \delta(x^-)\, \alpha_a(x_t)} \qquad \mathbf{\alpha^a(k_t) = g\, \rho^a(k_t)/k_t^2}$$

gluon distribution: $\mathbf{xG(x, Q^2) \sim \int^{Q^2} \frac{d^2k_t}{k_t^2}\, \phi(x, k_t)}$ with $\mathbf{\phi(x, k_t^2) \sim < \rho_a^\star(k_t)\, \rho_a(k_t) >}$

pQCD with collinear factorization: *single scattering*
evolution with ln Q^2

Observables

DIS:

structure functions (diffraction)

particle production

dilute-dense (pA, forward pp) collisions:

multiplicities

p_t *spectra*

di-hadron angular correlations

dense-dense (AA, pp) collisions:

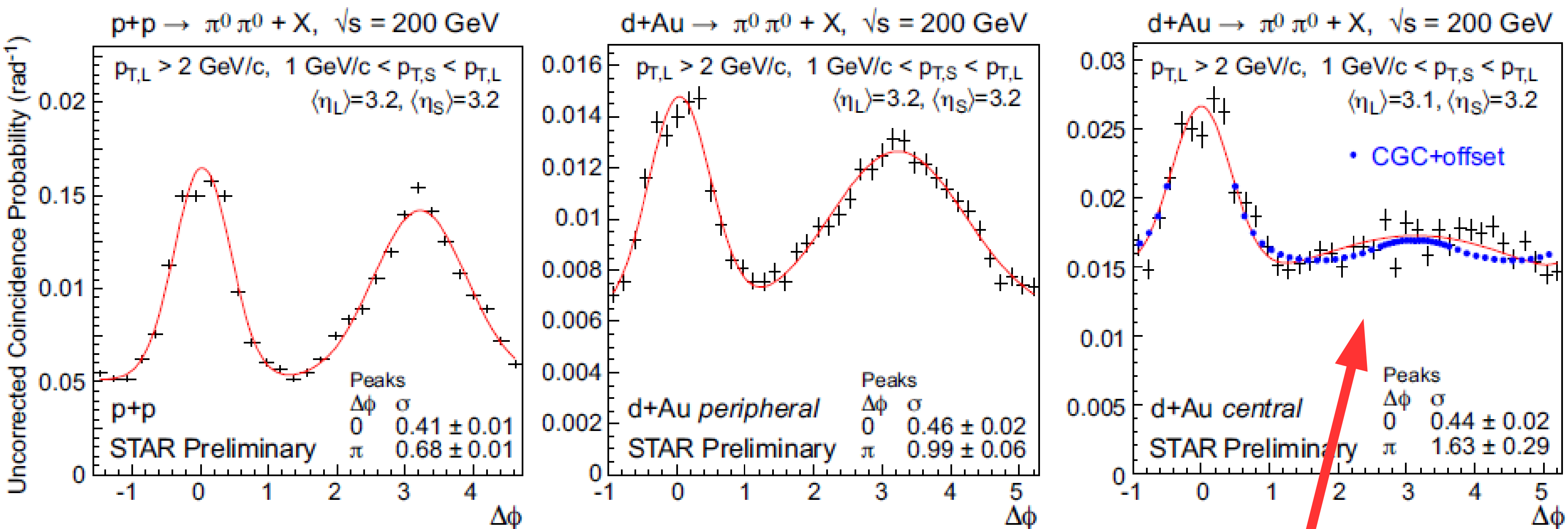
multiplicities, spectra

long range rapidity correlations

Spin asymmetries

disappearance of back to back hadrons

Recent STAR measurement (arXiv:1008.3989v1):

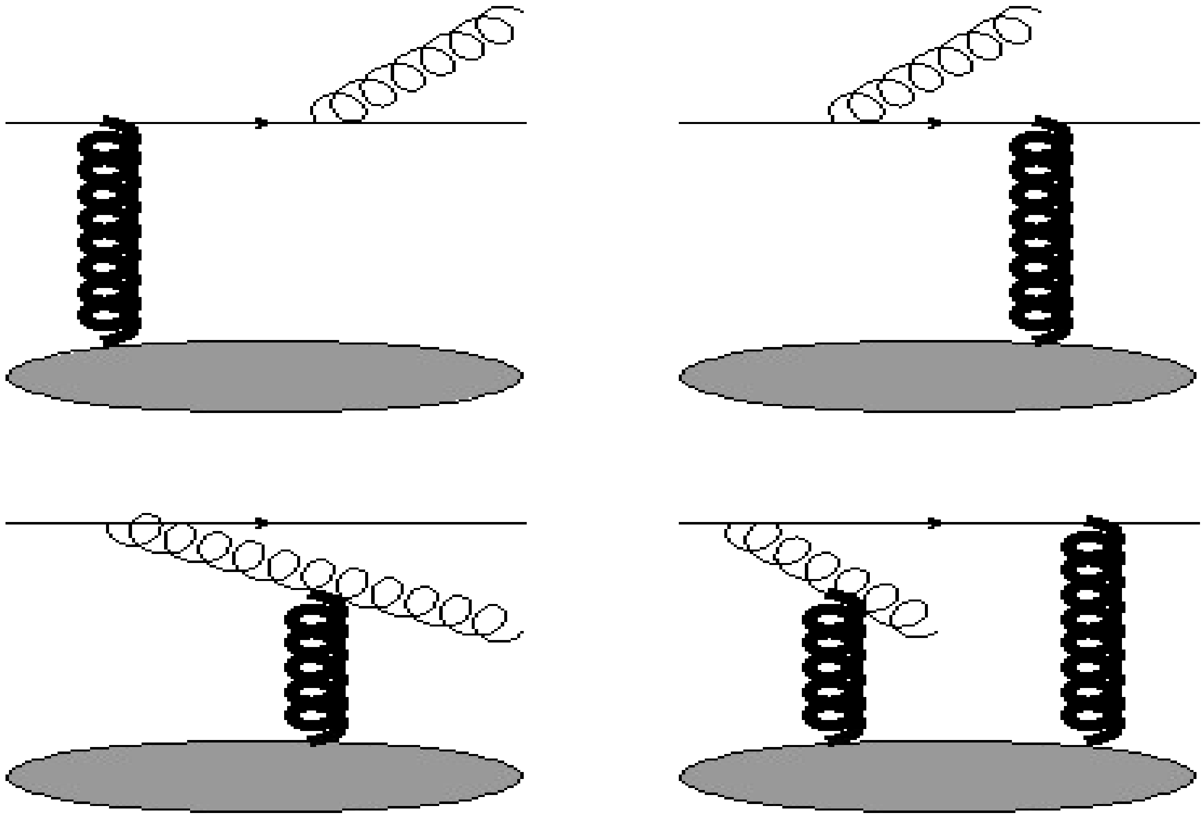


CGC fit from Albacete + Marquet, PRL (2010)
Tuchin, NPA846 (2010)
A. Stasto, B-W. Xiao, F. Yuan, PLB716 (2012)
T. Lappi, H. Mantysaari, NPA908 (2013)

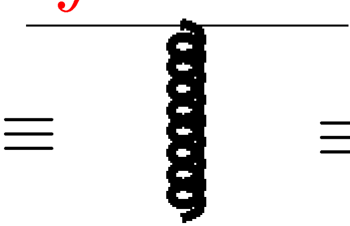
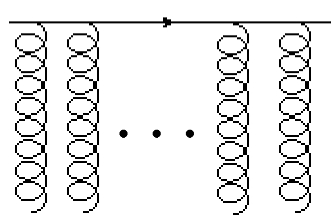
multiple scatterings de-correlate the hadrons

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

Di-jet production: pA $\mathbf{q(p)\,T \rightarrow q(q)\,g(k)}$

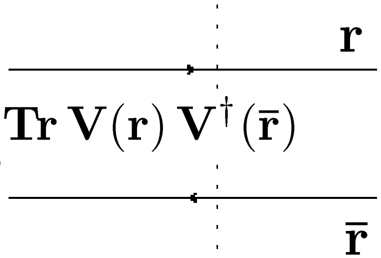


$$\mathbf{d}\sigma \sim \int \mathbf{K} \otimes \left[< \mathbf{Tr V\, V^\dagger} > + < \mathbf{Tr V\, V^\dagger\, V\, V^\dagger} > + \cdots \right]$$

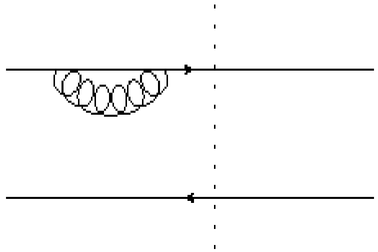
$\mathbf{V} \equiv$  $\equiv$  $\sim \mathbf{1} + \mathbf{O}(\mathbf{g}\,\rho) + \mathbf{O}(\mathbf{g^2}\,\rho^{\mathbf{2}})$

Evolution (energy dependence) of the 2-point function (dipole): DIS, single inclusive production

$$\frac{\mathbf{d}}{\mathbf{dy}}\mathbf{Tr\,V(r)\,V^\dagger(\bar{r})}$$



$=$



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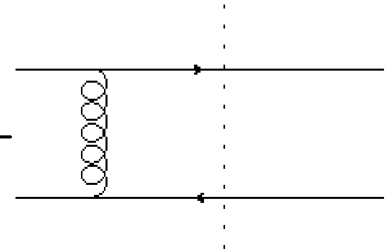
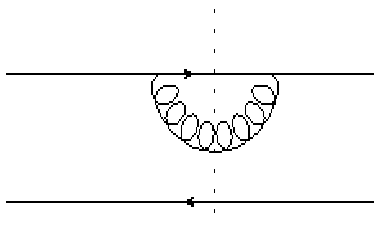


$$\mathbf{Tr\,V(r)\,V^\dagger(\bar{r})}$$

$+$



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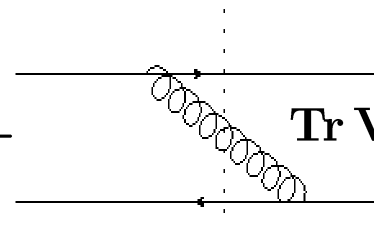


$$\mathbf{Tr\,V(r)\,V^\dagger(z)\,Tr\,V(z)\,V^\dagger(r)}$$

$$\frac{\mathbf{1}}{\mathbf{(r-z)^2}} \qquad \frac{\mathbf{(r-z)\cdot(\bar{r}-z)}}{\mathbf{(r-z)^2(\bar{r}-z)^2}}$$

$$\frac{\mathbf{d}}{\mathbf{dy}} < \mathbf{TrV(r)\,V^\dagger(\bar{r}} > = -\frac{\bar{\alpha}_{\mathbf{s}}}{\mathbf{2}\pi}\int \mathbf{d^2z}\,\frac{\mathbf{(r-\bar{r})^2}}{\mathbf{(r-z)^2(\bar{r}-z)^2}}\times$$

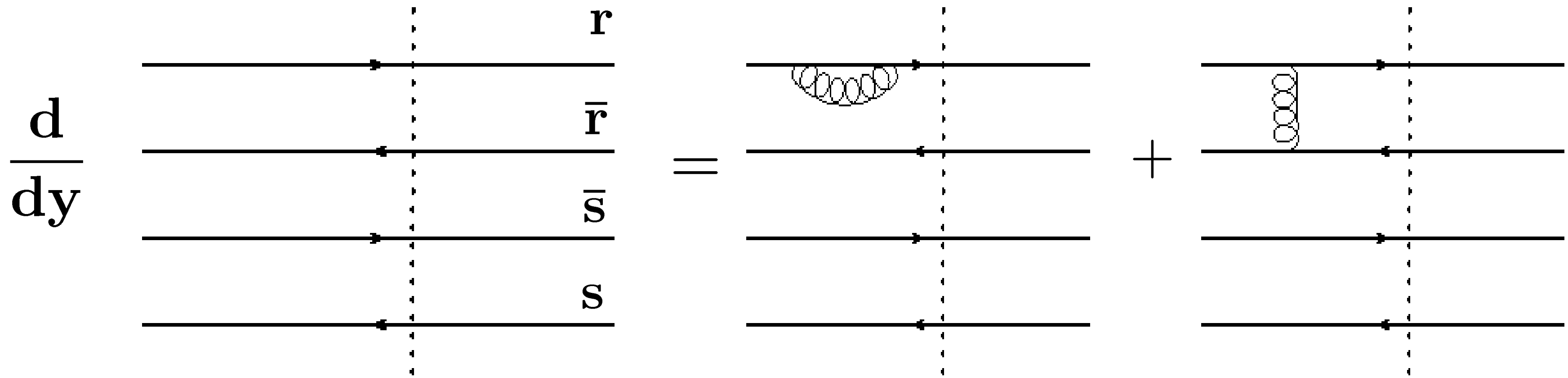
$$\left[< \mathbf{TrV(r)\,V^\dagger(\bar{r})} > -\frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{TrV(r)\,V^\dagger(z)\,TrV(z)\,V^\dagger(\bar{r})} >\right]$$

BK equation (known to NLO)

Evolution of quadrupole from JIMWLK

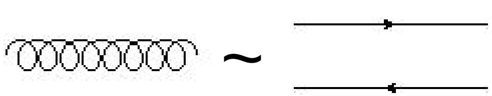
$$\mathbf{Q(r, \bar{r}, \bar{s}, s) \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >}$$

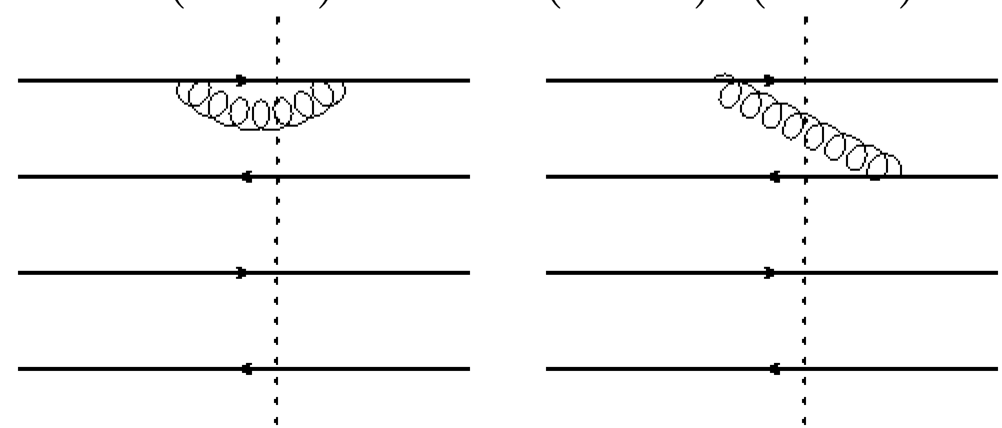
radiation kernels as in dipole



$$\mathbf{\frac{1}{(r-z)^2}} \qquad \mathbf{\frac{(r-z)\cdot(\bar{r}-z)}{(r-z)^2(\bar{r}-z)^2}}$$

large N_c





$+ \cdots$

Evolution of quadrupole from JIMWLK

$$
\begin{aligned}
&\frac{d}{dy}\,\langle Q(r,\bar{r},\bar{s},s)\rangle \\
= \quad & \frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z \Bigg\{ \Bigg\langle \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2} + \frac{(r-s)^2}{(r-z)^2(s-z)^2} - \frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\right] Q(z,\bar{r},\bar{s},s)\,S(r,z) \\
+ \quad & \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2} + \frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2} - \frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\right] Q(r,z,\bar{s},s)\,S(z,\bar{r}) \\
+ \quad & \left[\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2} + \frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2} - \frac{(\bar{r}-s)^2}{(s-z)^2(\bar{r}-z)^2}\right] Q(r,\bar{r},z,s)\,S(\bar{s},z) \\
+ \quad & \left[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2} - \frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\right] Q(r,\bar{r},\bar{s},z)\,S(z,s) \\
- \quad & \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2} + \frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2} + \frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}\right] Q(r,\bar{r},\bar{s},s) \\
- \quad & \left[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2} - \frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2} - \frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\right] S(r,s)\,S(\bar{r},\bar{s}) \\
- \quad & \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2} + \frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2} - \frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2} - \frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\right] S(r,\bar{r})\,S(\bar{s},s) \Bigg\rangle \Bigg\}
\end{aligned}
$$

$$\frac{d}{d\,y}Q = \int P_1\,[Q\,S] - P_2\,[Q] + P_3[S\,S]$$ with $$P_1 - P_2 + P_3 = 0$$

J. Jalilian-Marian, Y. Kovchegov: PRD70 (2004) 114017
Dominguez, Mueller, Munier, Xiao: PLB705 (2011) 106
J. Jalilian-Marian: Phys.Rev. D85 (2012) 014037

quadrupole evolution: models

$$< Q(r,\bar{r},\bar{s},s) > \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$$

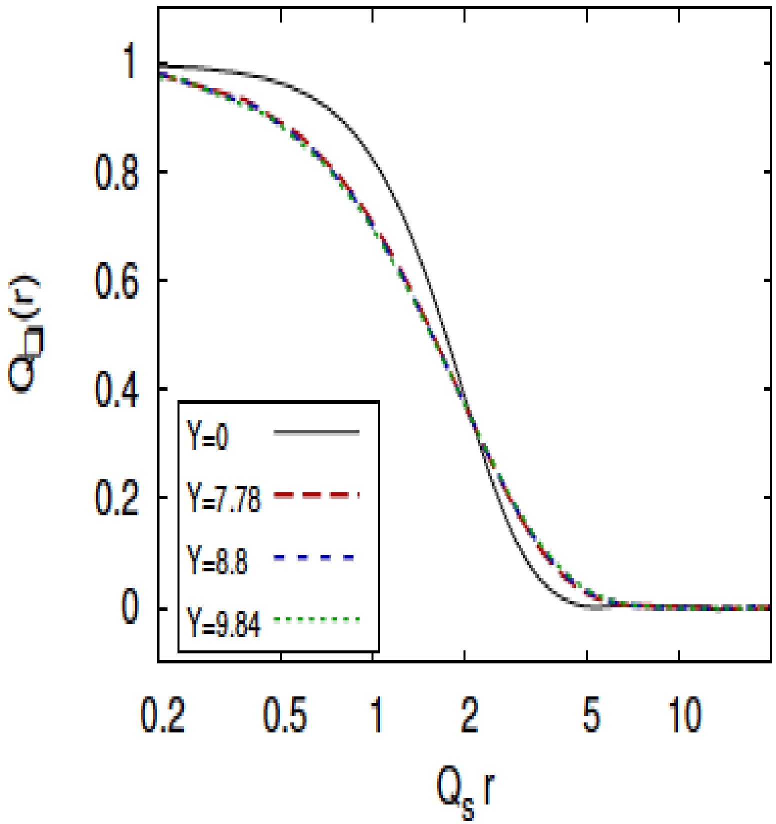
Gaussian model

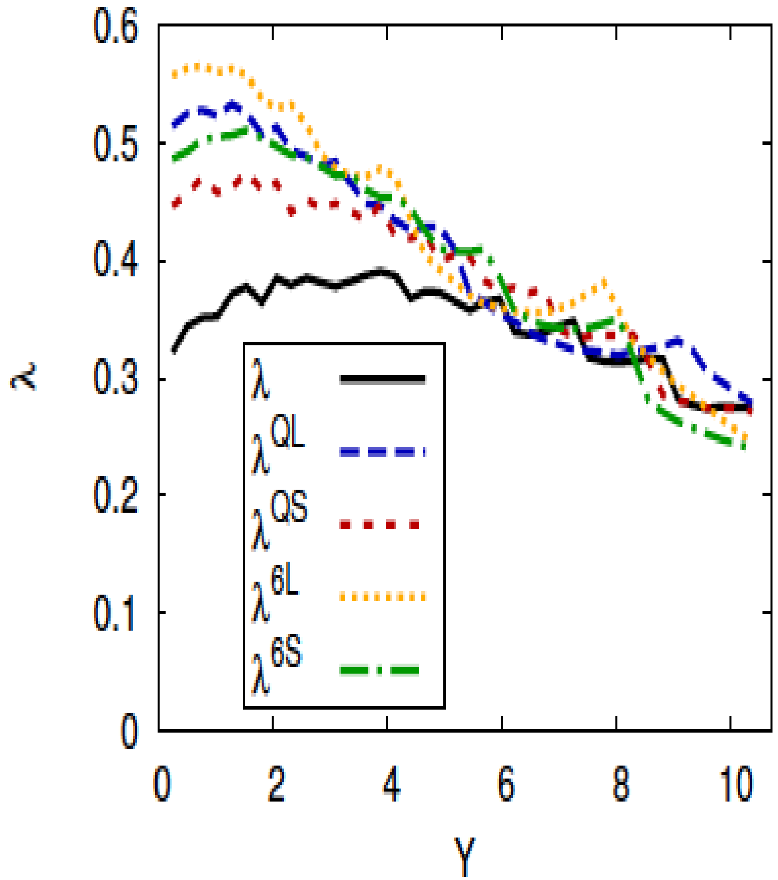
$$Q_{sq}(z) = [S(z)]^2 \left[\frac{N_c+1}{2} \left(\frac{S(z)}{S(\sqrt{2}z)} \right)^{\frac{2}{N_c+1}} - \frac{N_c-1}{2} \left(\frac{S(\sqrt{2}z)}{S(z)} \right)^{\frac{2}{N_c-1}} \right]$$

Gaussian + large N_c

$$Q_{sq}(z) = \left[1 + 2\ln \left(\frac{S(z)}{S(\sqrt{2}z)} \right) \right]$$

Quadrupole evolution





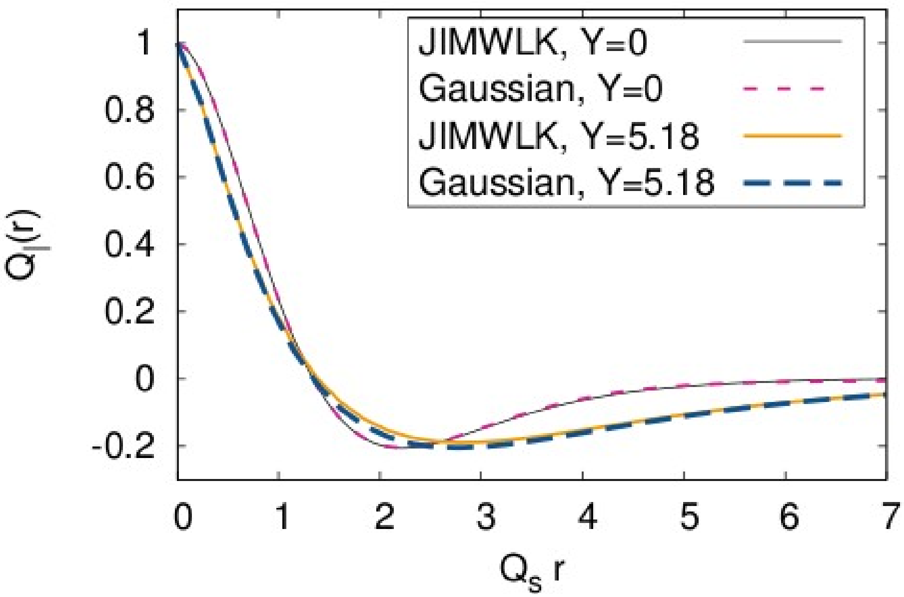
Geometric scaling also present in quadrupoles

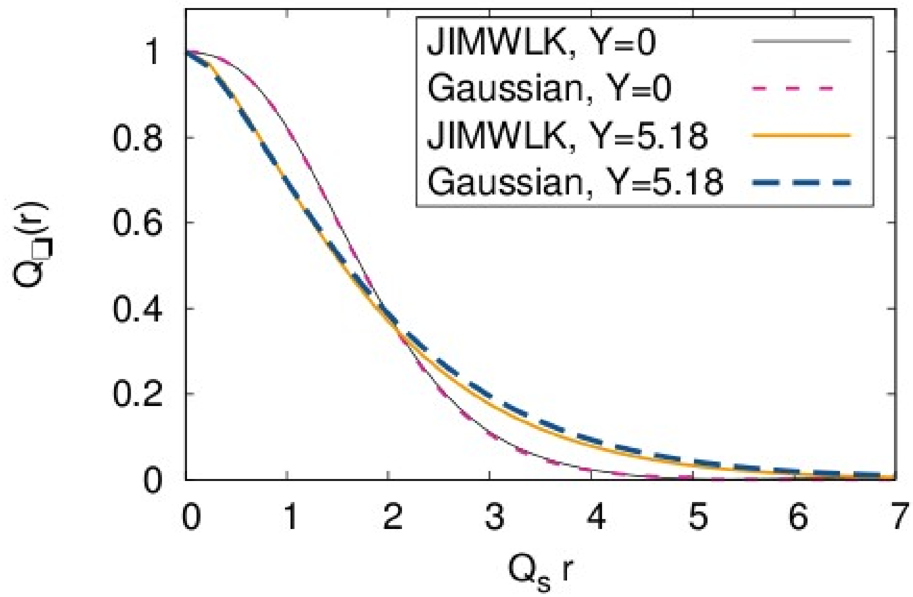
Growth of the saturation scale

Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:PLB706 (2011) 219

Quadrupole evolution

comparing with Gaussian model





quadrupole evolution: <u>linear regime</u>

define $\mathbf{T(r,\bar{r}) \equiv 1 - S(r,\bar{r})}$ $\quad$ $\mathbf{T_Q(r,\bar{r},\bar{s},s) \equiv 1 - Q(r,\bar{r},\bar{s},s)}$

<u>**expand in powers of gauge fields (or color charges)**</u>

ignore contribution of non-linear terms: T T and T_Q T

$\mathbf{O}(\alpha^{\mathbf{2}})$ $\quad$ $\mathbf{T_Q(r,\bar{r},\bar{s},s) \rightarrow T(r,\bar{r}) + T(r,s) + \cdots}$

with $\quad \mathbf{T(r,\bar{r})} \sim \alpha^{\mathbf{2}}\mathbf{(r,\bar{r})}$

<u>quadrupole evolution reduces to a sum of BFKL evolution eqs</u>

Dominguez, Mueller, Munier, Xiao: PLB705 (2011) 106
J. Jalilian-Marian: Phys.Rev. D85 (2012) 014037
D. Triantafyllopoulos

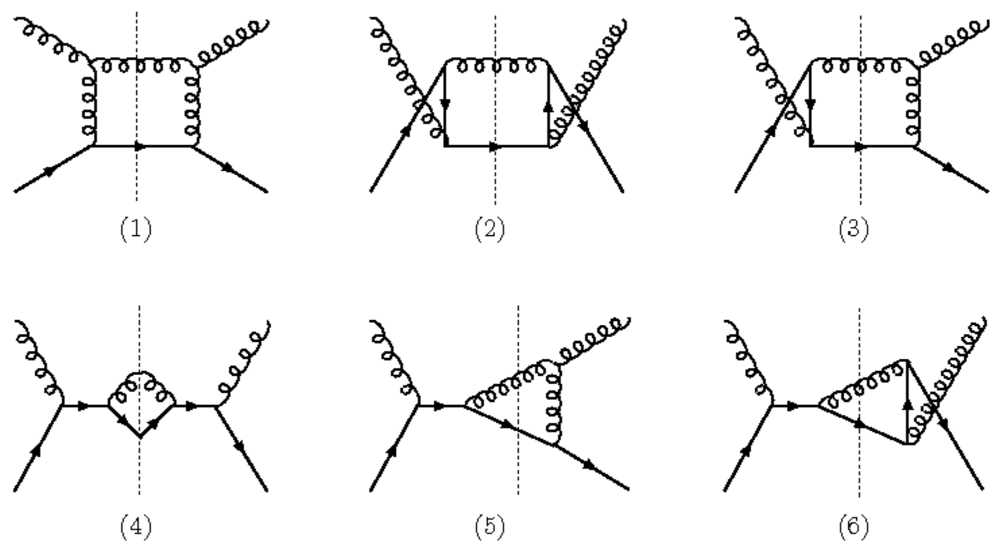
di-hadron correlations: high p_t limit

$\mathbf{O}(\alpha^{\mathbf{2}})$

Dominguez, Marquet, Xiao, Yuan (2011)
Dominguez, Xiao, Yuan (2011)

factorization of target distribution functions and hard scattering matrix element

$$\mathbf{d}\sigma \sim \mathbf{\Phi} \otimes \frac{\mathbf{d}\sigma}{\mathbf{dt}}^{\mathbf{2}\to\mathbf{2}}$$



$$\frac{d\sigma}{dt}^{q\,g\to q\,g} \sim \frac{1}{s^2}\left[\frac{4}{9}\frac{s^2+u^2}{-s\,u} + \frac{s^2+u^2}{t^2}\right]$$

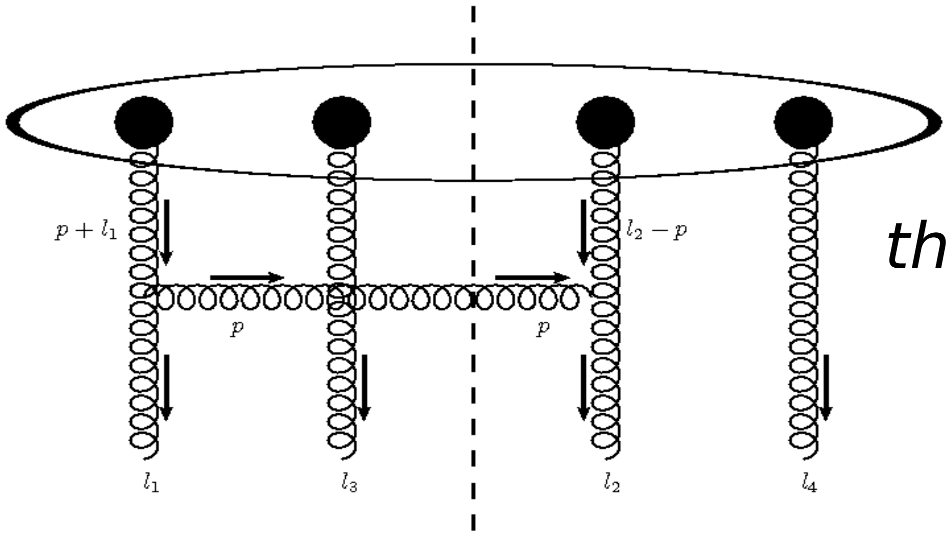
partons are back to back

quadrupole evolution: linear regime

BJKP equation

$\mathbf{O}(\alpha^4)$: 4-gluon exchange

J. Jalilian-Marian, PRD85 (2012) 014037



the color structure is identical on both sides of this eq. (independent of color averaging)

$$\frac{d}{dy}\hat{T}_4(l_1,l_2,l_3,l_4) = \frac{N_c\,\alpha_s}{\pi^2}\int d^2p_t\left[\frac{p^i}{p_t^2}-\frac{(p^i-l_1^i)}{(p_t+l_1)^2}\right]\cdot\left[\frac{p^i}{p_t^2}-\frac{(p^i-l_2^i)}{(p_t+l_2)^2}\right]$$
$$\hat{T}_4(p_t+l_1,l_2-p_t,l_3,l_4)+\cdots$$
$$-\frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2p_t\left[\frac{l_1^2}{p_t^2(l_1-p_t)^2}+\{l_1\to l_2,l_3,l_4\}\right]\hat{T}_4(l_1,l_2,l_3,l_4)$$

this will <u>de-correlate</u> the produced partons at high $p_t > Q_s$

color structure

$$\mathbf{\hat{T}_4(l_1, l_2, l_3, l_4) \equiv \frac{1}{N_c} Tr\, \rho(l_1)\, \rho(l_2)\, \rho(l_3)\, \rho(l_4) = Tr\, (t^a\, t^b\, t^c\, t^d)\, \rho^a(l_1)\, \rho^b(l_2)\, \rho^c(l_3)\, \rho^d(l_4)}$$

$$\begin{aligned} Tr\,(t^a\, t^b\, t^c\, t^d) \quad &= \quad \frac{1}{4N_c} \left[\delta^{ab}\delta^{cd} - \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right] \\ &+ \quad \frac{1}{8} \left[d^{abr} d^{cdr} - d^{acr} d^{bdr} + d^{adr} d^{bcr}\right] \\ &+ \quad \frac{i}{8} \left[d^{abr} f^{cdr} - d^{acr} f^{bdr} + d^{adr} f^{bcr}\right] \end{aligned}$$

overall state is a singlet, how about pairwise?

for N_c = 3

$$\left[\delta^{\mathbf{ab}}\delta^{\mathbf{cd}} + \delta^{\mathbf{ac}}\delta^{\mathbf{bd}} + \delta^{\mathbf{ad}}\delta^{\mathbf{bc}}\right] = \mathbf{3} \left[\mathbf{d^{abr}d^{cdr} + d^{acr}d^{bdr} + d^{adr}d^{bcr}}\right]$$

the linear regime

$\mathbf{O}(\alpha^{\mathbf{3}})$: 3-gluon (odderon) exchange

Dipole odderon: *Kovchegov, Szymanowski, Wallon*

$\mathbf{Tr\, V\, V^{\dagger}\, V}$ *Hatta, Iancu, Itakura, McLerran* **BJKP equation**

BJKP equation describes evolution of n-Reggeized gluons in a singlet state

JIMWLK (linear) and BJKP eqs. agree for n=2,3,4

non-linear interactions:

1) Non-linear JIMWLK evolution
2) Triple pomeron vertex: *Chirilli, Szymanowski, Wallon (2010)*

"$\mathrm{n} \to \mathrm{n}+1$ vertices" **Work in Progress**

(weak coupling) QCD at high energy

Two distinct approaches:

1) CGC

McLerran-Venugopalan effective action
JIMWLK-BK evolution

2) Reggeized-gluon exchange

BJKP equation
triple,... pomeron vertex

Conjecture: CGC contains BJKP + multi-pomeron vertices

Goal/wish: <u>hard diffraction in pp</u>

The role of initial conditions

McLerran-Venugopalan (93) $$< \mathbf{O}(\rho) > \equiv \int \mathbf{D}[\rho]\, \mathbf{O}(\rho)\, \mathbf{W}[\rho]$$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}}} \qquad \mu^{\mathbf{2}} \equiv \frac{\mathbf{g^2\, A}}{\mathbf{S_\perp}}$$

$$\mathbf{T(r_t)} \equiv \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr}\,[\mathbf{1} - \mathbf{V(r_t)^\dagger\, V(0)}] > \sim \mathbf{1} - \mathbf{e}^{-[\mathbf{r_t^2\, Q_s^2}]^\gamma \mathbf{\log(e} + \frac{\mathbf{1}}{\mathbf{r_t}\,\Lambda_{\mathbf{QCD}}})}$$

with $\gamma = \mathbf{1.119}$

how about higher order terms in ρ *?*

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \left[\frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}} - \frac{\mathbf{d^{abc}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})}{\kappa_{\mathbf{3}}} + \frac{\mathbf{F^{abcd}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})\rho^{\mathbf{d}}(\mathbf{x_t})}{\kappa_{\mathbf{4}}} \right]}$$

these higher order terms make the single inclusive spectra steeper and give <u>leading N_c</u> correlations (ridge)

Dumitru-Jalilian-Marian-Petreska, PRD84 (2011) 014018
Dumitru-Petreska, NPA9 (2012) 59