Proton structure and elastic scattering amplitudes

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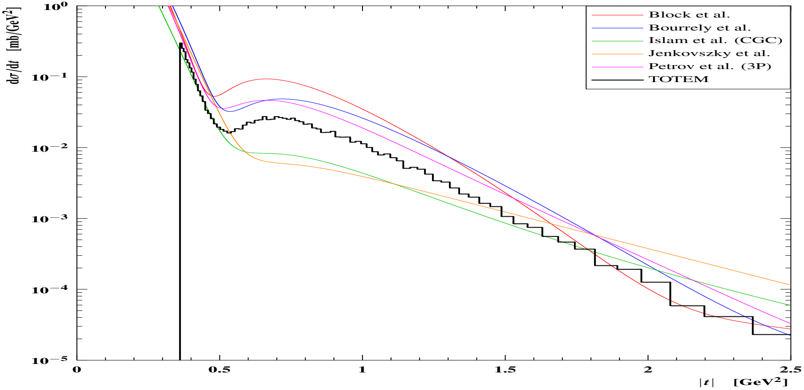
Based on arXiv:**1206.5474 (review)**+
1202.2016, 1204.1914, 1204.4866, 1208.3073,
1209.1935, 1212.3313, 1304.5345, 1306.5384
All published already.

Kinematically simplest process with two variables $s = 4E^2$ and $t = -2p^2(1 - \cos\theta) \approx -p^2\theta^2$ at $\theta \ll 1$

The only measurable characteristics

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}|A|^2 = \frac{1}{16\pi s^2}[(\mathrm{Im}A(s,t))^2 + (\mathrm{Re}A(s,t))^2]$$

Two functions $\mathrm{Im}A(s,t)$, $\mathrm{Re}A(s,t)$ as parts of a single analytic function $A(s,t)$



Other **FOUR** characteristics: $\sigma_t(s)$, $\sigma_{el}(s)$, $\rho(s,t)$, $B(s,t)$

$$\sigma_t(s) = \frac{\mathrm{Im} A(p, \theta = 0)}{s} - \text{optical theorem}$$

$$\sigma_{el}(s) = \int_{t_{min}}^{0} dt \frac{d\sigma}{dt}(s,t)$$

$$\rho(s,t) = \frac{\mathrm{Re} A(s,t)}{\mathrm{Im} A(s,t)}$$

The diffraction cone

$$\frac{d\sigma}{dt} / \left(\frac{d\sigma}{dt} \right)_{t=0} = e^{Bt} \approx e^{-Bp^2\theta^2} \qquad (B \approx \mathrm{const}(t))$$

The amplitude in the diffraction cone (Gaussian, imaginary)

$$A(s,t) \approx is\sigma_t e^{Bt/2} \approx 4ip^2 \sigma_t e^{-Bp^2\theta^2/2}$$

Coulomb-nuclear interference – $\rho(s,0)$.

Theory

The local dispersion relation

$$\rho(s,0) \approx \frac{1}{\sigma_t}\left[\tan\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)\right]\sigma_t \approx \frac{\pi}{2}\frac{d\ln\sigma_t}{d\ln s}$$

The unitarity relation

$$\mathrm{Im}A(p,\theta) = I_2(p,\theta) + F(p,\theta) =$$

$$\frac{1}{32\pi^2}\int\int d\theta_1 d\theta_2 \frac{\sin\theta_1\sin\theta_2 A(p,\theta_1)A^*(p,\theta_2)}{\sqrt{[\cos\theta-\cos(\theta_1+\theta_2)][\cos(\theta_1-\theta_2)-\cos\theta]}} + F(p,\theta$$

The region of integration

$$|\theta_1-\theta_2| \le \theta, \qquad \theta \le \theta_1+\theta_2 \le 2\pi-\theta.$$

$$\mathrm{Im}a_l(s) = |a_l(s)|^2 + F_l(s) - \textit{partial wave representation}$$

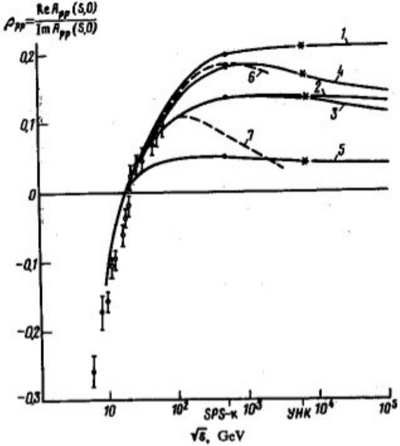
$$\mathrm{Im}h(s,b) \approx |h(s,b)|^2 + F(s,b) - \textit{spatial (impact parameter) view}$$

Froissart bound

$$\sigma_t \le \frac{\pi}{2m_\pi^2}\ln^2(s/s_0)$$

I.M. Dremin, M.T. Nazirov, Pis'ma ZhETF 37 (**1983**) 163
(JETP Lett. 37 (1983) 198) (see also arXiv:1304.5345)

Predictions according to the integral dispersion relations



The ratio of real to imaginary part of elastic pp-scattering amplitude at $t=0$ according to dispersion relations with different assumptions about high energy behavior of the total cross section (different signs at low and high energies!).

Previous (J. Caspar) talk

$\rho(s, 0) = 0.107 \pm 0.030$

Local dispersion relations with $\sigma_t = 98.6 \pm 2.8$ **mb at 7 TeV and** $\sigma_t = 101.7 \pm 2.9$ **mb at 8 TeV predict**
$-0.2 < \rho(s, 0) < 0.66$ **for** $\pm 1\sigma$
and
$\rho(s, 0) = 0.23$ **for the central values of** σ_t**.**

$\rho(s, 0) = 0.107$ **would ask for smaller difference of cross-section values** $\Delta\sigma_t \approx 1.5$ **mb in place of 3.1 mb and for a deeper minimum at** $|t| \approx 0.52$ **(see fit below). Still within the error bars. Higher accuracy needed!**

NB! In the model fit below it would give rise to larger (of the order of 1!) positive values of $\rho(t)$ **in Orear region.**

OUR GUESSES ABOUT ASYMPTOTICS

$$\sigma_t(s) \le \frac{\pi}{2m_\pi^2} \ln^2(s/s_0)$$

THE BLACK DISK: $\sigma_t = 2\pi R^2$; $R = R_0 \ln s$; $\frac{\sigma_{el}}{\sigma_t} = \frac{\sigma_{in}}{\sigma_t} = 0.5$

$B(s) = \frac{R^2}{4}$; $\rho(s, t=0) = \frac{\pi}{\ln s}$ None observed in experiment!

THE GRAY DISKS: two parameters - radius+opacity

Gray and Gaussian disks $(X = \sigma_{el}/\sigma_t;\ Z = 4\pi B/\sigma_t;\ \alpha \le 1)$

Model	$1 - e^{-\Omega}$	σ_t	σ_{el}	B	Z	XZ	X/Z
Gray	$\alpha\theta(R-b)$	$2\pi\alpha R^2$	$\pi\alpha^2 R^2$	$R^2/4$	$1/2\alpha$	$1/4$	α^2
Gauss	$\alpha e^{-b^2/R^2}$	$2\pi\alpha R^2$	$\pi\alpha^2 R^2/2$	$R^2/2$	$1/\alpha$	$1/4$	$\alpha^2/4$

The energy behavior

$\sqrt{s}$, GeV	2.70	4.74	6.27	7.62	13.8	62.5	546	1800	7000
X	0.42	0.27	0.24	0.22	0.18	0.18	0.21	0.23	0.25
Z	0.64	1.09	1.26	1.34	1.45	1.50	1.20	1.08	1.00
XZ	0.27	0.29	0.30	0.30	0.26	0.25	0.26	0.25	0.25
X/Z	0.66	0.25	0.21	0.17	0.16	0.12	0.18	0.21	0.25

I. The energy evolution of the proton's shape

Purely phenomenological fit (see arXiv:1306.5384)

$$A(s,t) = s\sqrt{16\pi} f(s,t).$$

Fit at ISR in U. Amaldi, K. Schubert Nucl. Phys. B166 (1980) 301

$$f(s,t) = i\alpha[A_1 \exp(\frac{1}{2}b_1\alpha t) + A_2 \exp(\frac{1}{2}b_2\alpha t)] - iA_3 \exp(\frac{1}{2}b_3 t),$$

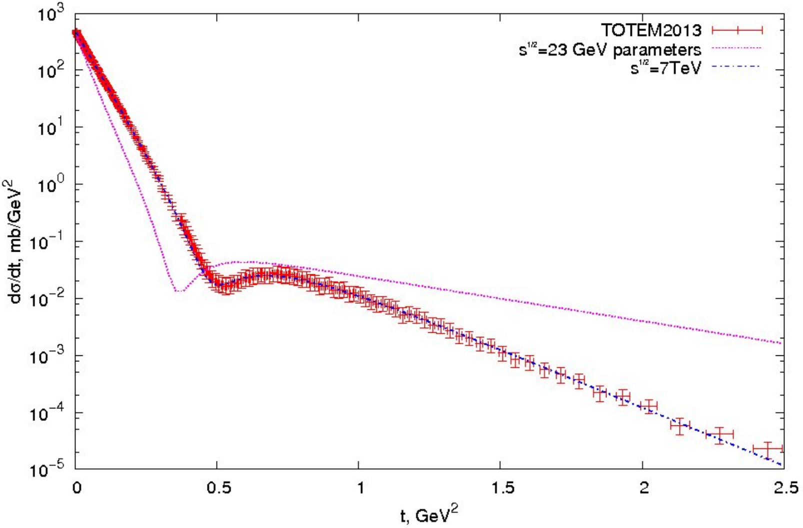
where $\alpha(s)$ is complex and is given by

$$\alpha(s) = [\sigma_t(s)/\sigma_t(23.5\,\mathrm{GeV})](1 - i\rho_0(s)).$$

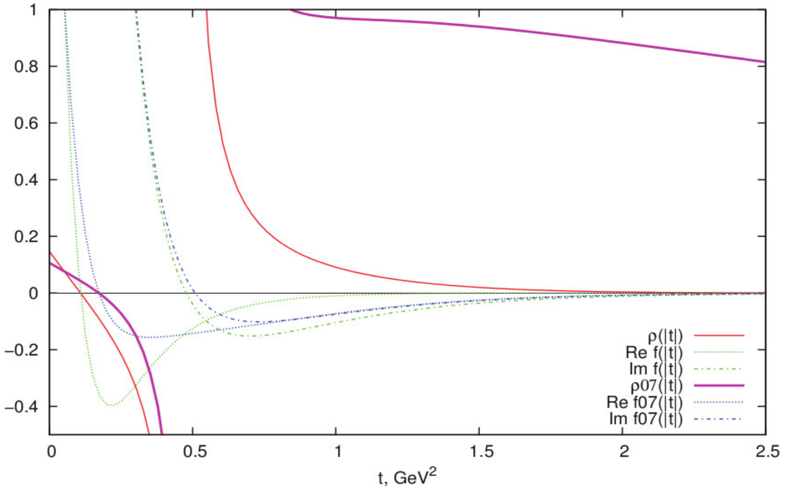
$\mathrm{Re} f \propto \rho_0$; $\rho(s,0) \approx 1.05\rho_0$

6 parameters at given s (A_i, b_i, ρ_0 minus norm. at t=0)

ALAN!



Fit of the TOTEM data – dash-dotted curve. Dotted curve is calculated with parameters used at 23.5 GeV and with $\rho_0 = 0.14$

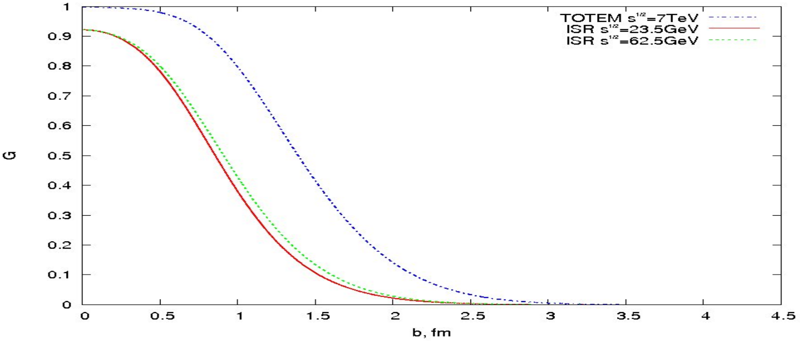


Real (dotted) and imaginary (dash-dotted) parts of the amplitude and their ratio (solid) for $\rho(s,0) = 0.14$. Bold lines are for $\rho(s,0) = 0.107$. Note the difference in Orear region. Similar curves are predicted by other models with **the dip as zero of $\mathrm{Im}A(t)$!**

The energy evolution of the impact parameter picture

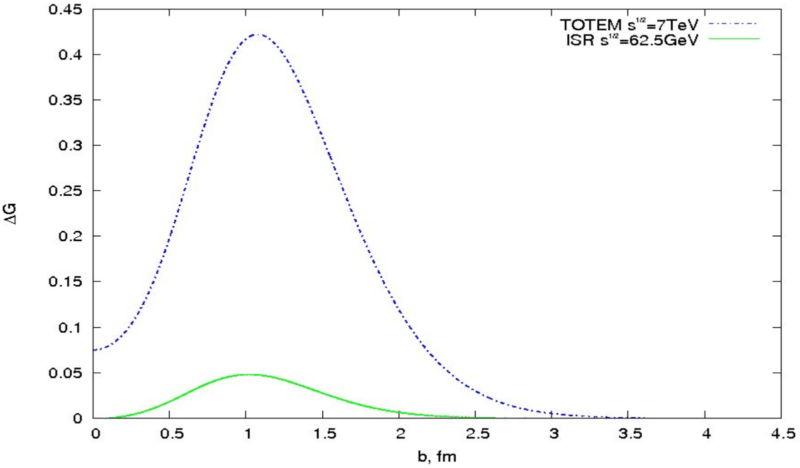
$$i\Gamma(s,b) = \frac{1}{\sqrt{\pi}} \int_0^\infty dqqf(s,t) J_0(qb). \qquad (1)$$

$$2\Re\Gamma(s,b) = |\Gamma(s,b)|^2 + G(s,b), \qquad (2)$$



The overlap functions at 23.5 GeV (solid curve), 62.5 GeV (dotted curve) and 7 TeV (dash-dotted curve)

$$\Delta G(b) = G(s_1, b) - G(s_2, b) \quad (\sqrt{s_1} = 7\,\mathrm{TeV},\ \sqrt{s_2} = 23.5\,\mathrm{GeV})$$



The difference between the overlap functions. Dash-dotted curve is for 7 TeV and 23.5 GeV energies, solid curve is for 62.5 GeV and 23.5 GeV energies.

Conclusion: The parton density at the periphery increases!

THEORETICAL APPROACHES (MODELS)

1. Geometric picture and eikonal
Islam talk

2. Electromagnetic analogies
Selugin talk

3. Reggeon exchanges
Jenkovszky talk

4. QCD-inspired approaches
Gluons and quarks as active partons. Similar form factors.

All approaches are comparatively successful in the diffraction cone but their predictions failed outside it!

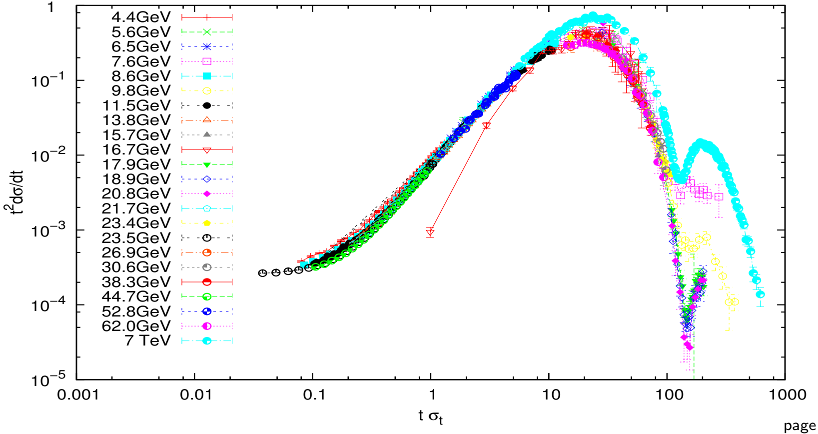
But - the next talk. Fit, not a prediction!

II. Scaling laws in the diffraction cone (arXiv:1212.3313)

$$\frac{\pi}{2}\left[\frac{\partial \ln \mathrm{Im}A(s,t)}{\partial \ln s}-1\right]=\rho(s,0)\left[1+\frac{\partial \ln \mathrm{Im}A(s,t)}{\partial \ln t}\right]$$

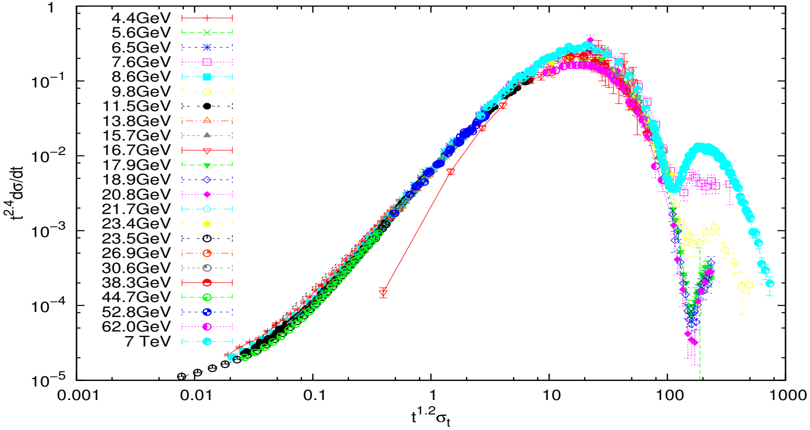
$$\frac{\partial \ln \mathrm{Im}A(s,t)}{\partial \ln \sigma_t}-\frac{\partial \ln \mathrm{Im}A(s,t)}{\partial \ln t}=1+\frac{d\ln s}{d\ln \sigma_t}.$$

$$t^2 d\sigma/dt=\Phi(t\sigma_t)-\textit{geometric scaling}$$



$$\rho(s,t) = \rho(s,0)\left[1 + \frac{1}{a}\frac{\partial \ln \mathrm{Im}A(s,t)}{\partial \ln |t|}\right].$$

$$t^{2a} d\sigma/dt = \omega(t^a \sigma_t), \qquad a = 1.2.$$



The value of a accounts for different energy behavior of B and σ_t.
That is the origin of the violation of geometric scaling!
Conclusion: NO GEOMETRIC SCALING

THREE REGIONS: the diffraction cone, the Orear regime, the hard parton scattering

$$\mathrm{Im}A(p,\theta) = I_2(p,\theta) + F(p,\theta) =$$

$$\frac{1}{32\pi^2}\int\int d\theta_1 d\theta_2 \frac{\sin\theta_1 \sin\theta_2 A(p,\theta_1)A^*(p,\theta_2)}{\sqrt{[\cos\theta - \cos(\theta_1+\theta_2)][\cos(\theta_1-\theta_2) - \cos\theta]}} + F(p,\theta$$

$$|\theta_1 - \theta_2| \le \theta, \qquad \theta \le \theta_1 + \theta_2 \le 2\pi - \theta$$

I_2 - two-particle intermediate states (σ_{el}), F - inelastic ones (σ_{inel}). For angles θ outside the diffraction cone one amplitude in I_2 is at small angles and another at large ones: **linear** integral equation

$$\mathrm{Im}A(p,\theta) = \frac{p\sigma_t}{4\pi\sqrt{2\pi B}}\int_{-\infty}^{+\infty} d\theta_1 f_\rho e^{-Bp^2(\theta-\theta_1)^2/2}\mathrm{Im}A(p,\theta_1) + F(p,\theta).$$

$f_\rho = 1 + \rho(s,0)\rho(\theta_1)$.

Analytic solution if $F(p,\theta) \ll \mathrm{Im}A(p,\theta)$ and $f_\rho \approx$const outside the diffraction cone!

(I.V. Andreev, I.M. Dremin JETP Lett. 6 (**1967**) 262; Wien,1968)

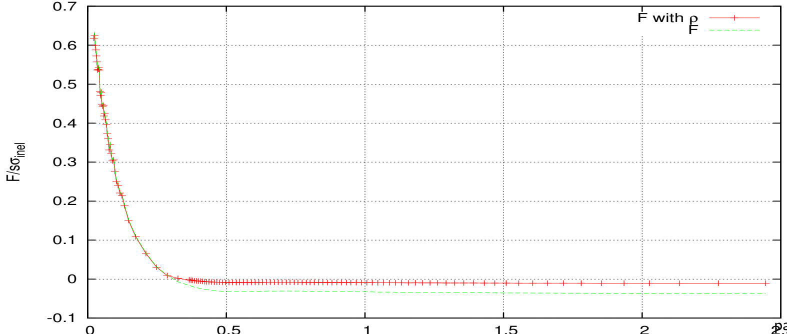
The proof of the assumption about the small overlap function.

$F(p,\theta)$ computed from experimental data is negligible outside cone:

$$F(p,\theta) = \; 16p^2 \left(\pi \frac{d\sigma}{dt}/(1+\rho^2)\right)^{1/2} -$$

$$\frac{8p^4 f_\rho}{\pi} \int_{-1}^{1} dz_2 \int_{z_1^-}^{z_1^+} dz_1 \left[\frac{d\sigma}{dt_1} \cdot \frac{d\sigma}{dt_2}\right]^{1/2} K^{-1/2}(z,z_1,z_2),$$

$z_i = \cos\theta_i; \quad K(z,z_1,z_2) = 1 - z^2 - z_1^2 - z_2^2 + 2zz_1z_2,$
$z_1^{\pm} = zz_2 \pm [(1-z^2)(1-z_2^2)]^{1/2}$



The elastic diff. x-section **outside** the diffraction cone contains the exponentially decreasing with θ (or $\sqrt{|t|}$) term (Orear regime!) with imposed on it damped oscillations. **No zero of $\mathrm{Im}A(t)$!**:

$$\frac{d\sigma}{p_1 dt} = \left(e^{-\sqrt{2B|t| \ln \frac{4\pi B}{\sigma_t f_\rho}}} + p_2 e^{-\sqrt{2\pi B|t|}} \cos(\sqrt{2\pi B|t|} - \phi) \right)^2 .$$

The experimental values of the diffraction cone slope B and the total cross section σ_t determine mostly the shape of the differential cross section in the Orear region. The value of $Z = 4\pi B/\sigma_t$ is so close to 1 that the fit is extremely sensitive to $f_\rho = 1 + \rho(s,0)\rho(t)$. Thus, it becomes possible for the first time to estimate the ratio ρ outside the diffraction cone from fits of experimental data.

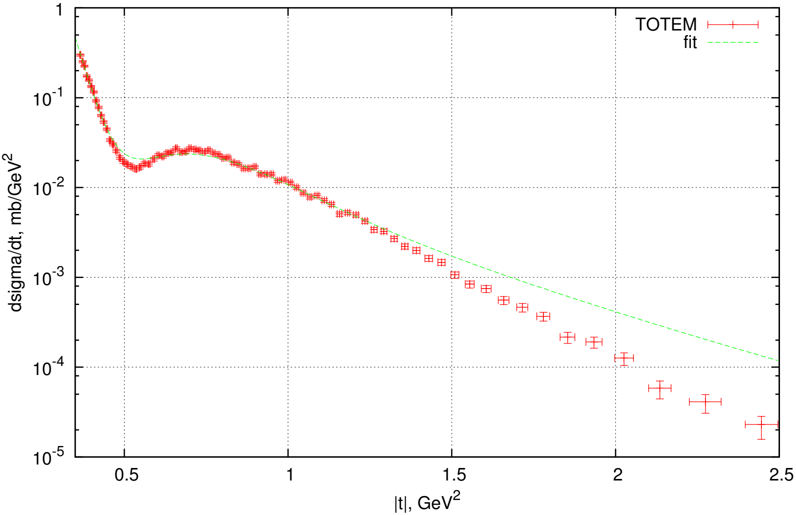
At the LHC, its average value is negative and equal to -2.1!
Does it contradict to the above fit?

Do we approach the black disk limit $Z \to 0.5$?

In Orear slope the decrease of Z must be compensated by the decrease of f_ρ but $\rho(s,0) \propto \ln^{-1} s$! Is it possible that $\rho(t)$ in Orear region increases in modulus being negative? **YES!** - next talk.

The "unitarity fit" at 7 TeV (dip+Orear in $0.3 < |t| < 1.5$ GeV^2)

(arXiv:1202.2016)



III. PROBLEM: The ratio $\rho(t)$ outside the diffraction cone. Conclusion: the problem is unsolved yet!

At $t = 0$, it is known from Coulomb-nuclear interference experimentally and from dispersion relations theoretically.
No experimental results for $\rho(t)$ are available. However, it can be calculated (Martin formula) if the imaginary part is known:

$$\rho(s,t) = \rho(s,0)\left[1 + \frac{t(d\,\mathrm{Im}A(t)/dt)}{\mathrm{Im}A(t)}\right]$$

Then the equation for $\rho(t)$ follows from the unitarity condition

$$\frac{dv}{dx} = -\frac{v}{x} - \frac{2}{x^2}\left(\frac{Ze^{-v^2} - 1}{\rho^2(t=0)} - 1\right)$$

$x = \sqrt{2B|t|}, \;\; v = \sqrt{\ln(Z/f_\rho)}, \;\; \rho(t) = (Ze^{-v^2} - 1)/\rho(t=0)$
where v is the solution of the equation.
Asymptotics at $|t| \to \infty \;\; \rho \to (Z-1)/\rho(t=0)$.
Then $f_\rho \to Z$ and $\ln(Z/f_\rho) \to 0$!
Prediction: the drastic changes can be expected in this region of $|t|$! (arXiv:1204.4866)

Conclusions

- The black disk limit is still far away (see the Table).
- The evolution of the impact parameter overlap function with energy shows steady increase of the contribution of the peripheral regions of protons (ISR $\to$ S$p\bar{p}$S $\to$ LHC).
  The parton density at the periphery increases! (see ΔG)
  Inelastic diffraction? Model dependence?
- Most theoretical models describe the diffraction peak but fail outside it.
- Scaling laws in the diffraction cone are predicted by the local dispersion relations + Martin formula but comparison with experiment requires some modification of the latter because it shows that geometric scaling is not valid.

- At intermediate angles between the diffraction cone and hard parton scattering region the unitarity condition predicts the Orear regime with exponential decrease in angles and imposed on it damped oscillations.
- The experimental data on elastic *pp* differential cross section at low and high ($\sqrt{s}$=7 TeV) energies have been fitted in this region with well described position of the dip and Orear slope.
- The fit by the "unitarity formula" allows for the first time at 7 TeV to estimate the average value of $\rho(s,t)$ far from forward direction t=0. It happens to be about -2.
- All model fits ask for the pole of $\rho(s,t)$ at the dip! The unitarity condition does not require the pole! The estimate of $\rho(s,t)$ in the unitarity condition is attempted.
- The overlap function is small and negative in the Orear region. That confirms the assumption used in solving the unitarity equation. Important corollary: the phases of inelastic amplitudes are crucial in any model of inelastic processes.