pp Elastic Scattering Amplitudes in t and b Spaces and Extrapolations to Cosmic Ray Energies

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Summary

- Differential cross sections and t - space amplitudes
- Regular behaviour with the energy
- b - space analytical forms and eikonals
- Unitarity conditions
- High energy extrapolations
- Conclusions

Differential cross section and t space amplitudes

$$\frac{d\sigma}{dt} = (\hbar c)^2 \ \ |T_R(s,t) + iT_I(s,t)|^2$$

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$$T_I(s,t) = T_I^N(s,t) + \sqrt{\pi}\; F^C(t)\sin(\alpha\Phi)$$

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$$F^C(s,t)e^{i\alpha\ \Phi(s,t)} = (-/+)\ \frac{2\alpha}{|t|} e^{i\alpha\ \Phi(s,t)}\ F^2_{\text{proton}}(t)$$

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$$F^C(s,t)e^{i\alpha\,\Phi(s,t)} = (-/+)\;\frac{2\alpha}{|t|}e^{i\alpha\;\Phi(s,t)}\,F_{\text{proton}}^2(t)$$

Coulomb Phase

$$F_{\text{proton}}(t) = [0.71/(0.71+|t|)]^2$$

$(\hbar c)^2 = 0.3894$ mb GeV^2

$$T_K^N(s,t) = \alpha_K(s)\mathrm{e}^{-\beta_K(s)|t|} + \lambda_K(s)\Psi_K(\gamma_K(s),t)$$

Nuclear amplitudes

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Shape functions

where $K=R$, $K=I$ for real and imaginary amplitudes

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Shape functions

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For |t|=0 we have the forward quantities...

Quantities in forward scattering

$$\sigma(s) = 4\sqrt{\pi}\,(\hbar c)^2 \;\; (\alpha_I(s) + \lambda_I(s))$$

Total cross section

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$$\rho(s) = \frac{T_R^N(s, t=0)}{T_I^N(s, t=0)} = \frac{\alpha_R(s) + \lambda_R(s)}{\alpha_I(s) + \lambda_I(s)}$$

Real/Imaginary

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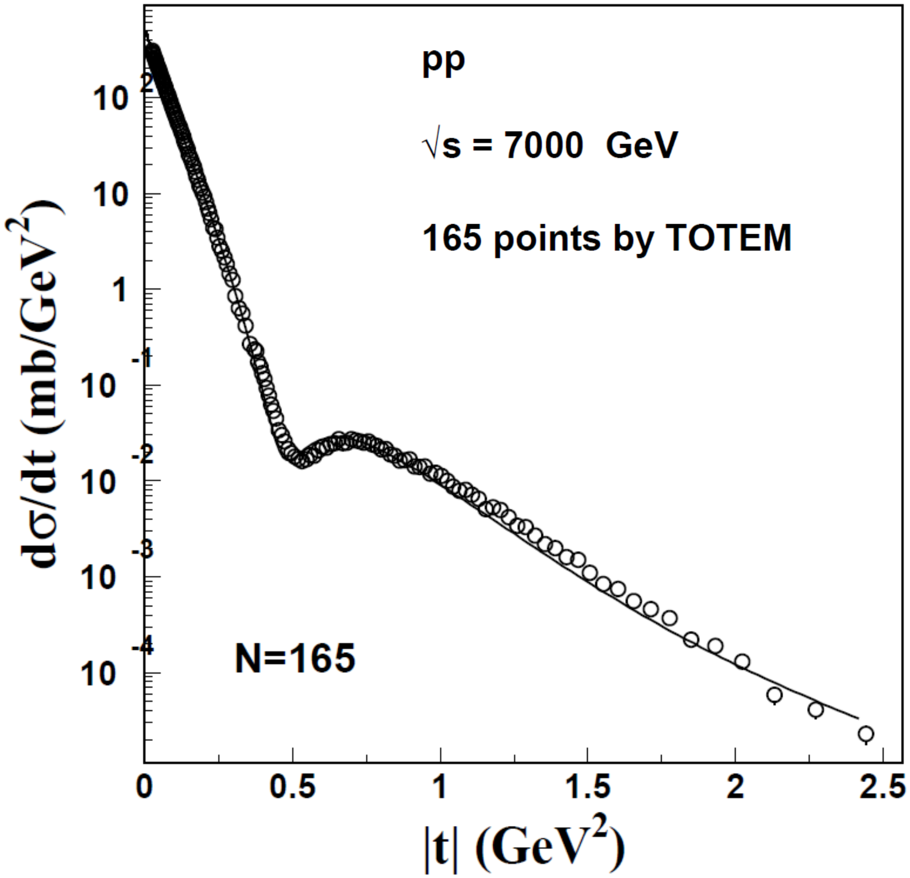
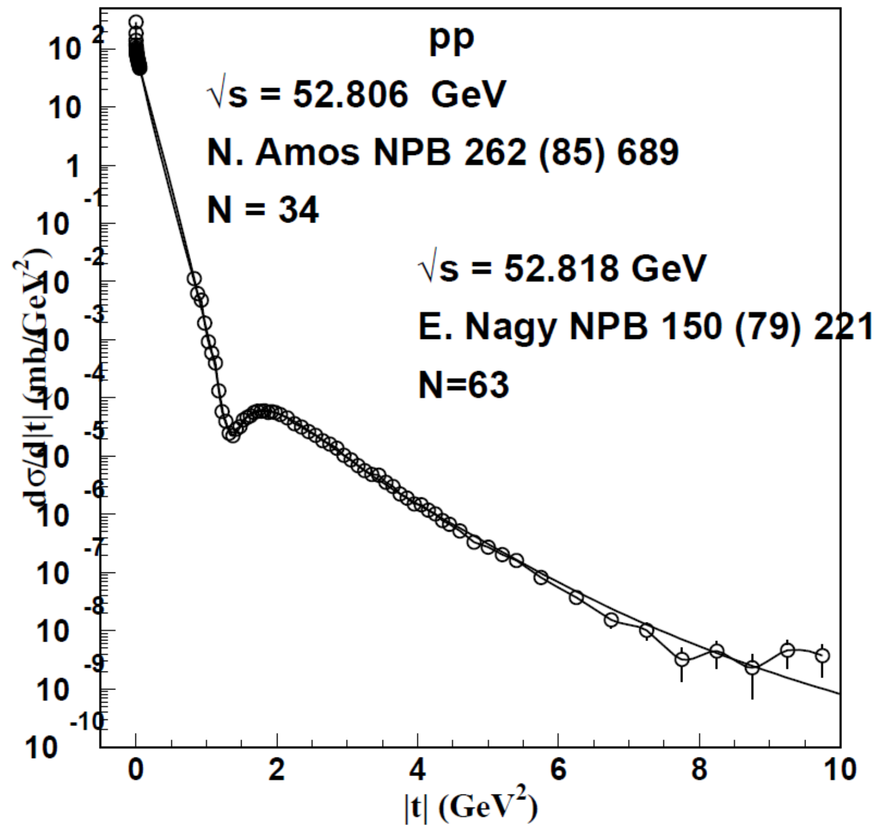
Real/Imaginary

$$B_K(s) = \frac{2}{T_K^N(s,t)} \frac{dT_K^N(s,t)}{dt}\Big|_{t=0}$$
$$= \frac{1}{\alpha_K(s) + \lambda_K(s)} \left[\alpha_K(s)\beta_K(s) + \frac{1}{8}\lambda_K(s) a_0 \Big(6\gamma_K(s) + 7\Big)\right]$$

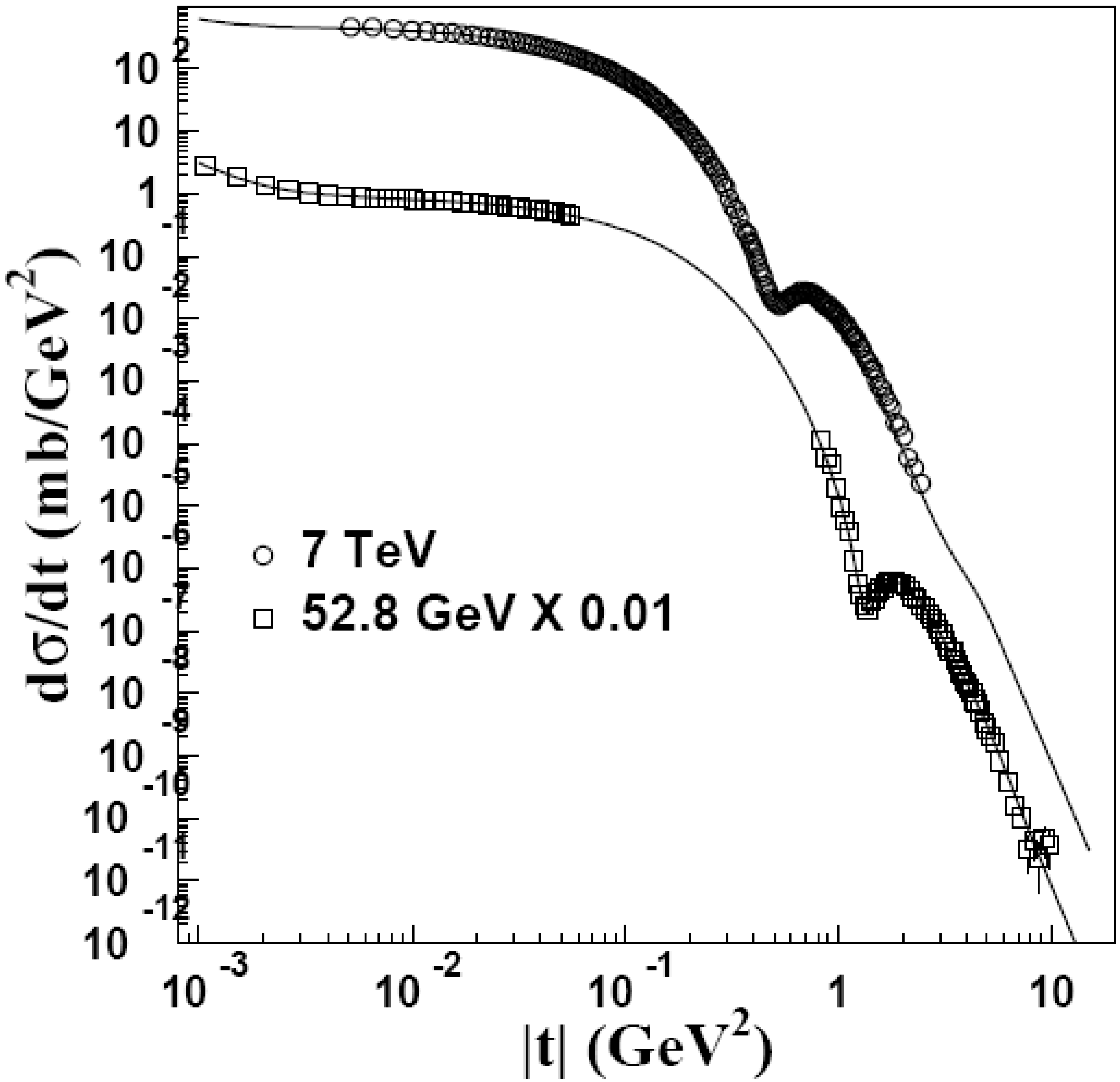
Real and Imaginary slopes

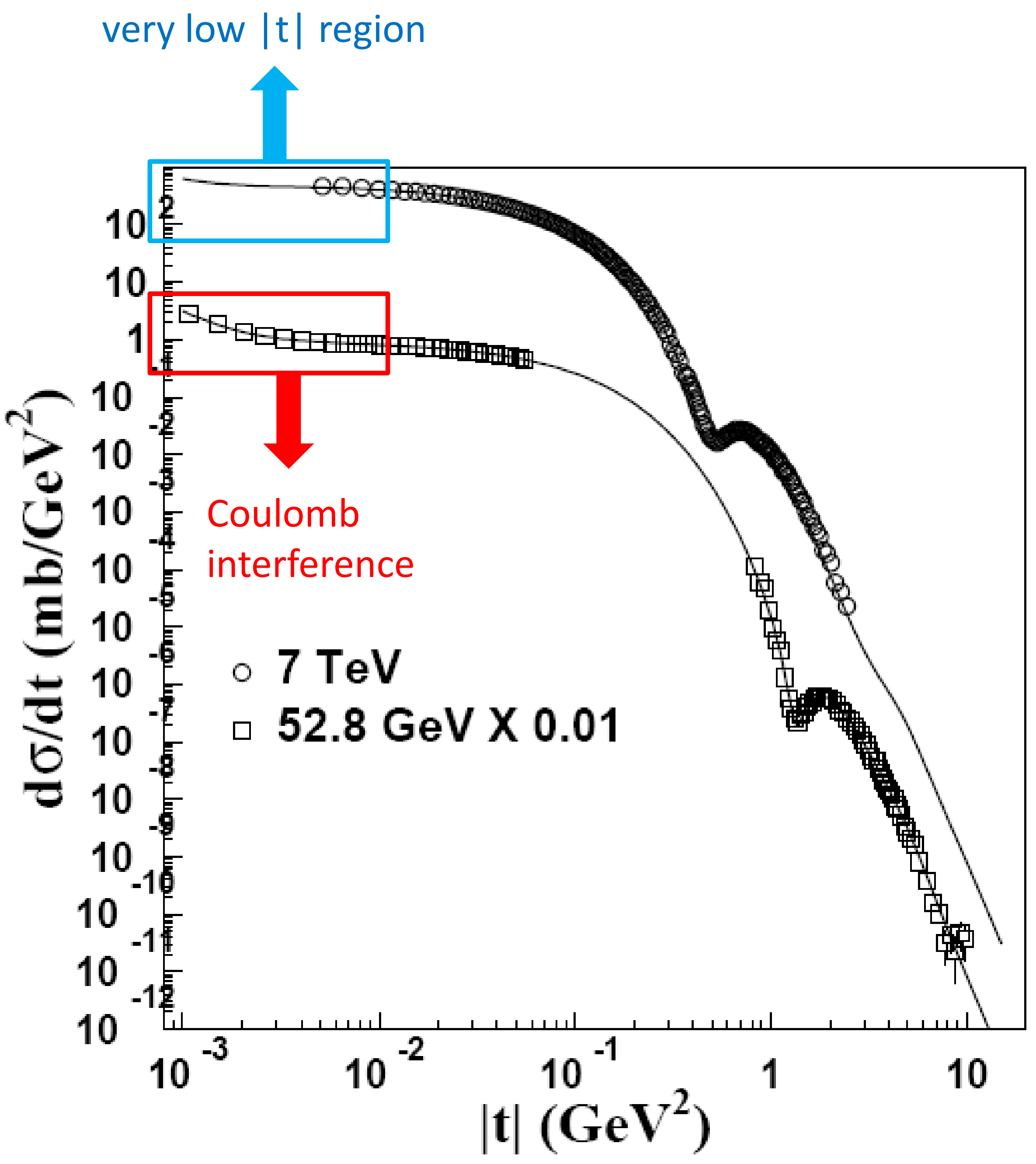
Differential cross sections





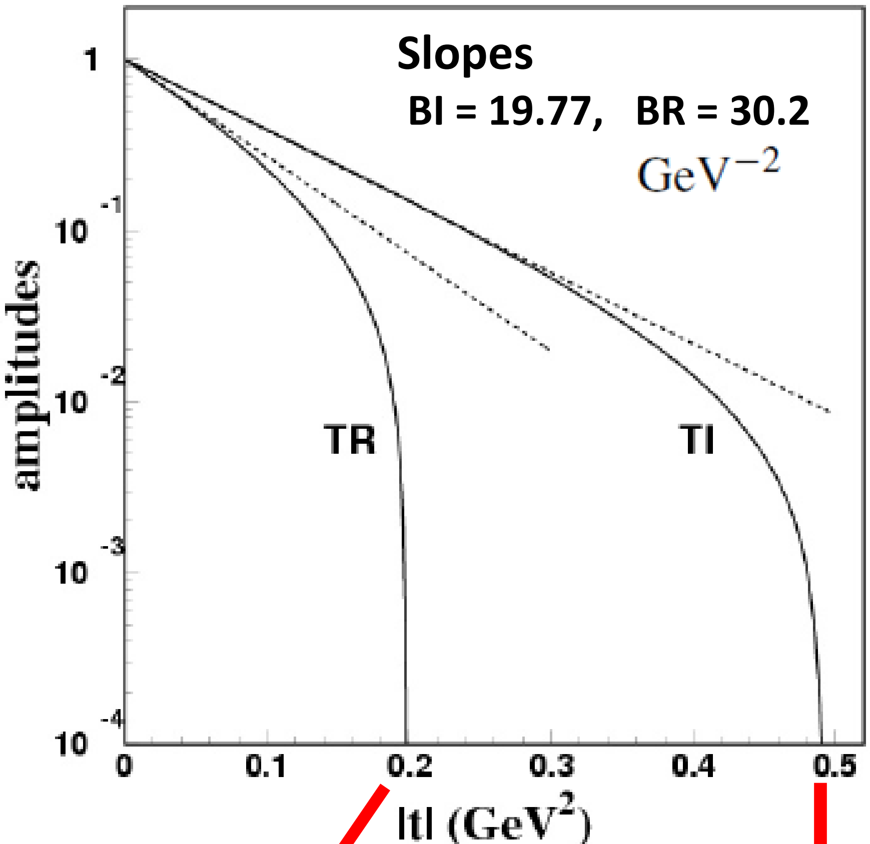
Differential cross-sections comparison of 52.8 GeV and 7 TeV

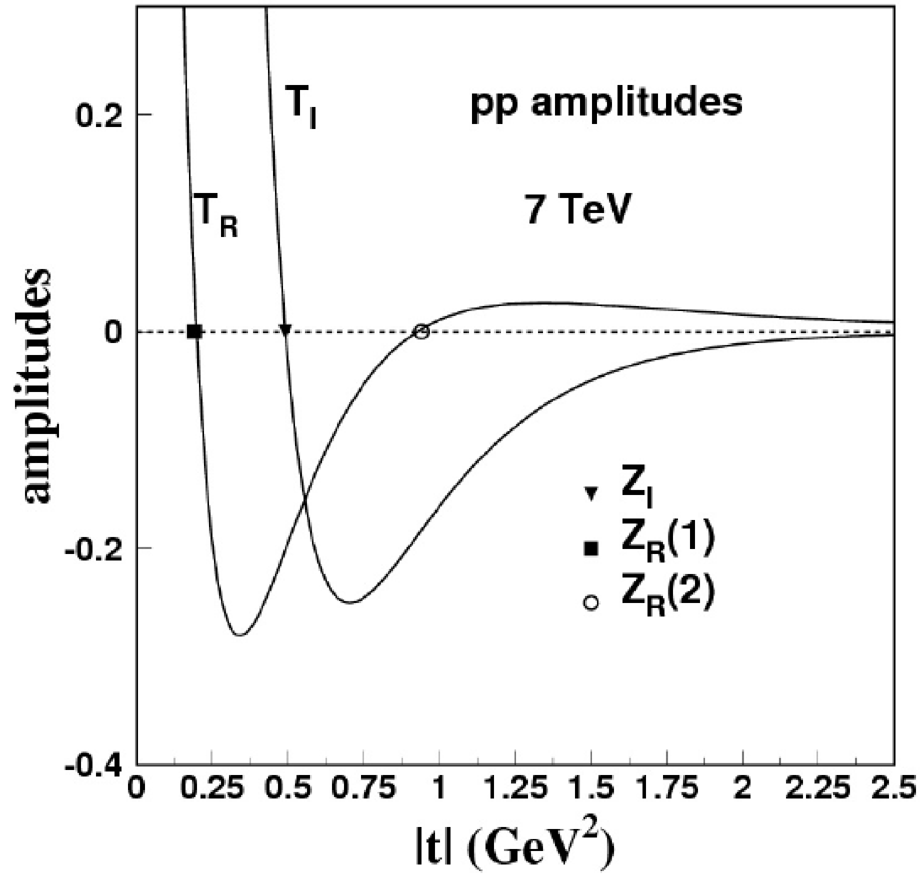



very low |t| region
Coulomb interference
○ 7 TeV
□ 52.8 GeV X 0.01
dσ/dt (mb/GeV^2)
|t| (GeV^2)

Amplitudes in t space

Attention : different imaginary and real slopes

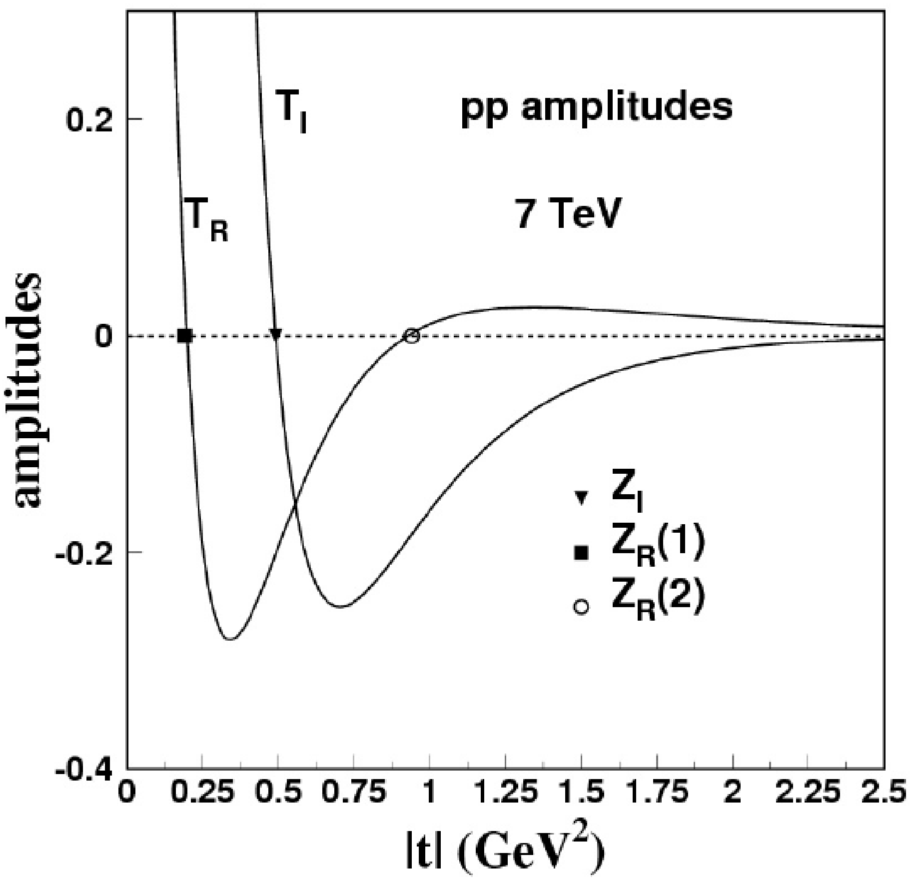


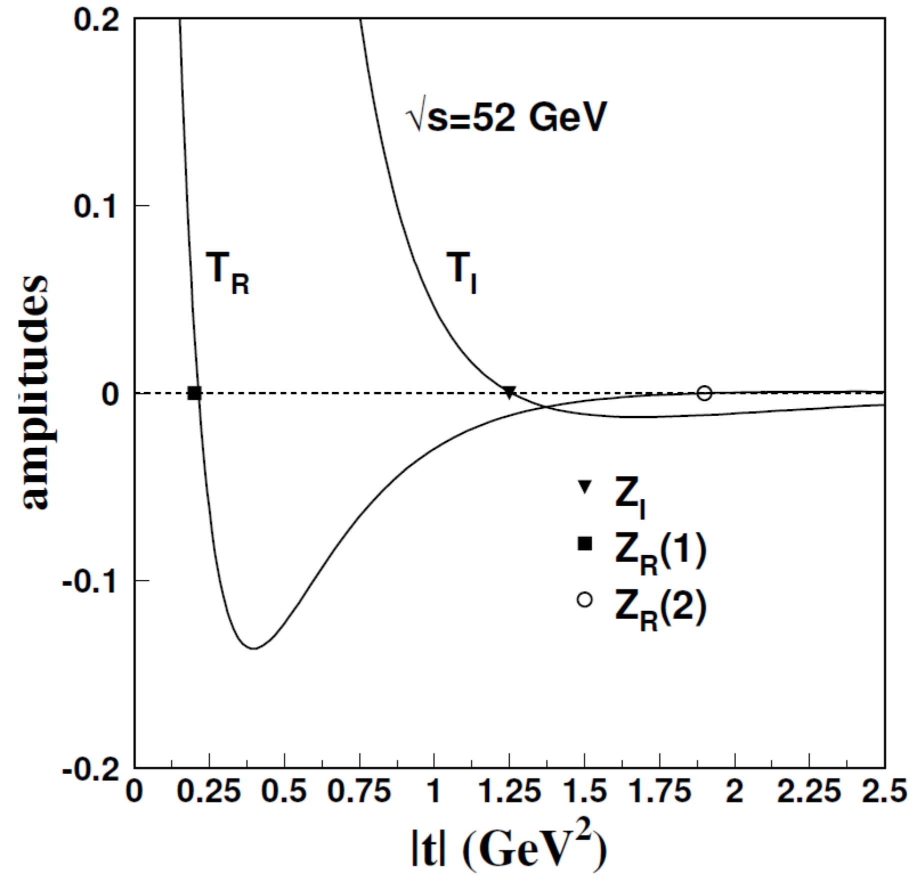


Real zero by Martin

Imaginary zero near the dip

comparison of pp 7 TeV with 0.0528 TeV amplitudes





Prediction for the large |t| region

The amplitudes for the data up to $|t| = 2.5\ \text{GeV}^2$ are

$$T_K^N(s,t) = \alpha_K(s)\mathrm{e}^{-\beta_K(s)|t|} + \lambda_K(s)\Psi_K(\gamma_K(s),t)$$ Nuclear amplitudes

$$\Psi_K(\gamma_K(s),t) = 2\ \mathrm{e}^{\gamma_K}\left[\frac{\mathrm{e}^{-\gamma_K\sqrt{1+a_0|t|}}}{\sqrt{1+a_0|t|}} - \mathrm{e}^{\gamma_K}\ \frac{e^{-\gamma_K\sqrt{4+a_0|t|}}}{\sqrt{4+a_0|t|}}\right]$$ Shape functions

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For large |t| we add a perturbative tri-gluon exchange Rggg

A. Donnachie, P. V. Landshoff, *Zeit. Phys. C* **2**, 55 (1979)

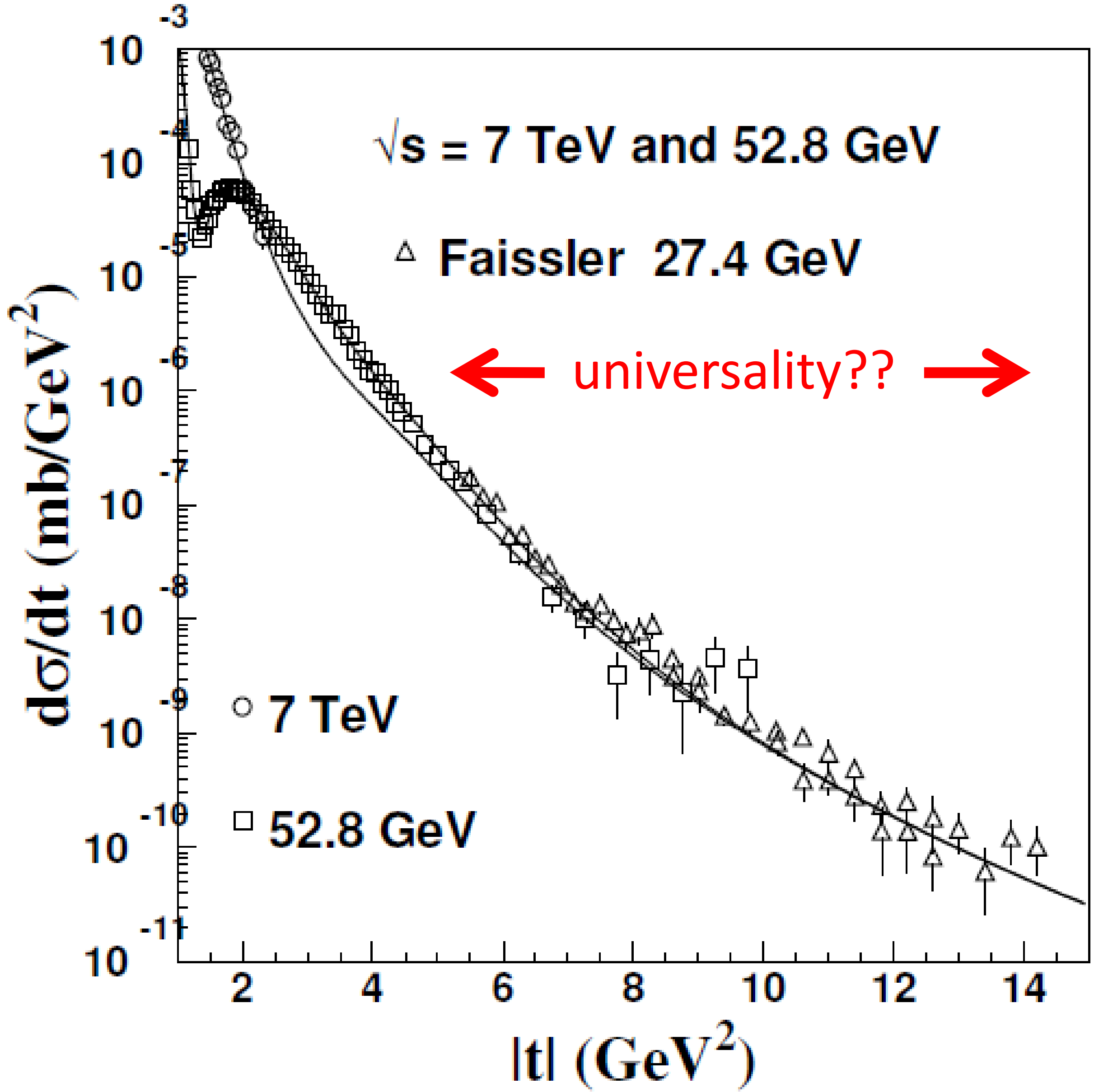
$$T_{R(\mathrm{tail})}(s,t) = T_K^N(s,t) + R_{ggg}(t)$$

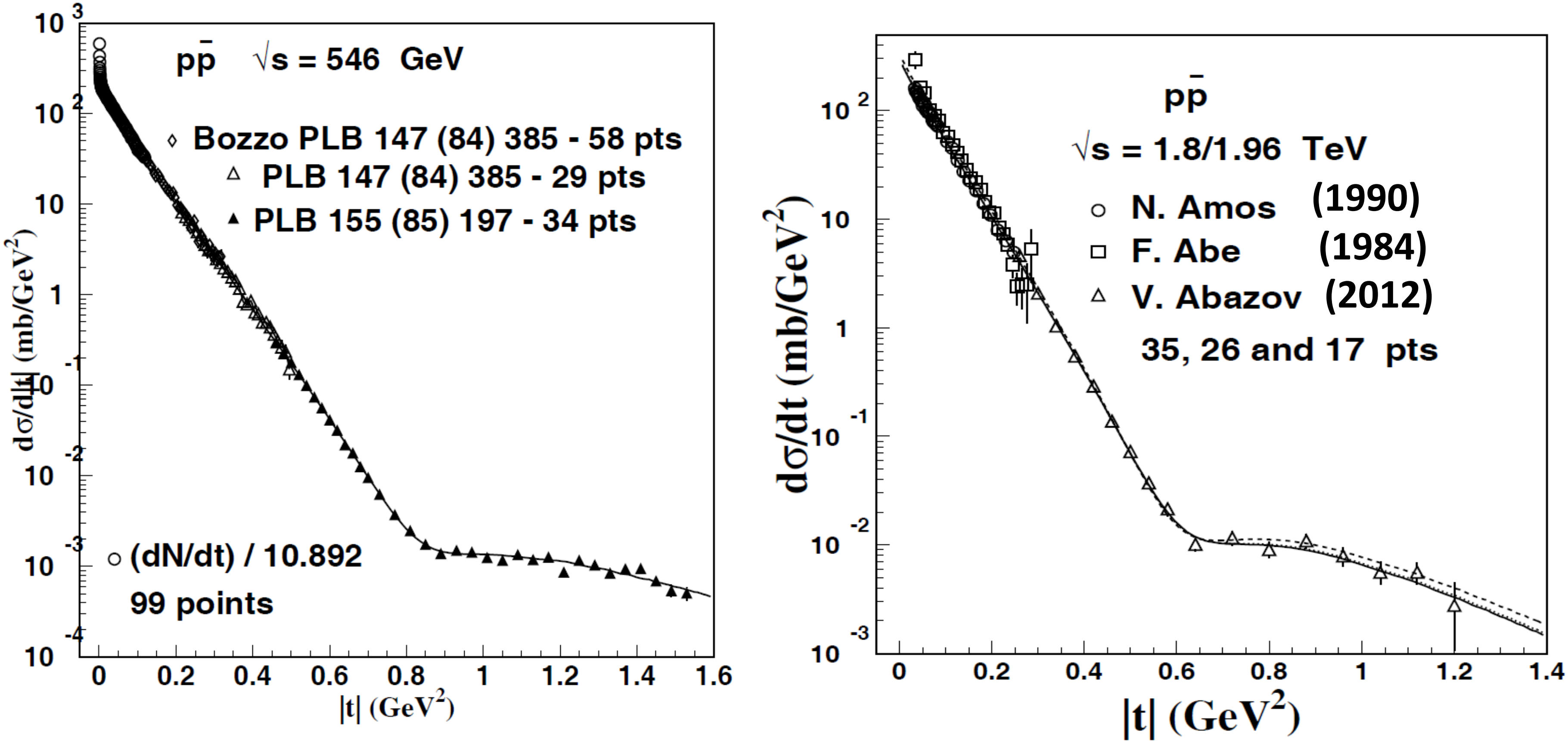
$$R_{ggg}(t) \equiv \pm 0.45\ t^{-4}(1 - e^{-0.005|t|^4})(1 - e^{-0.1|t|^2})$$

For large |t|

$$d\sigma/dt \approx (\hbar c)^2\ [T_{R(\mathrm{tail})}(t)]^2 \approx 0.08\ t^{-8}\ \ (\mathrm{mb/GeV^2})$$ energy independent !!!!

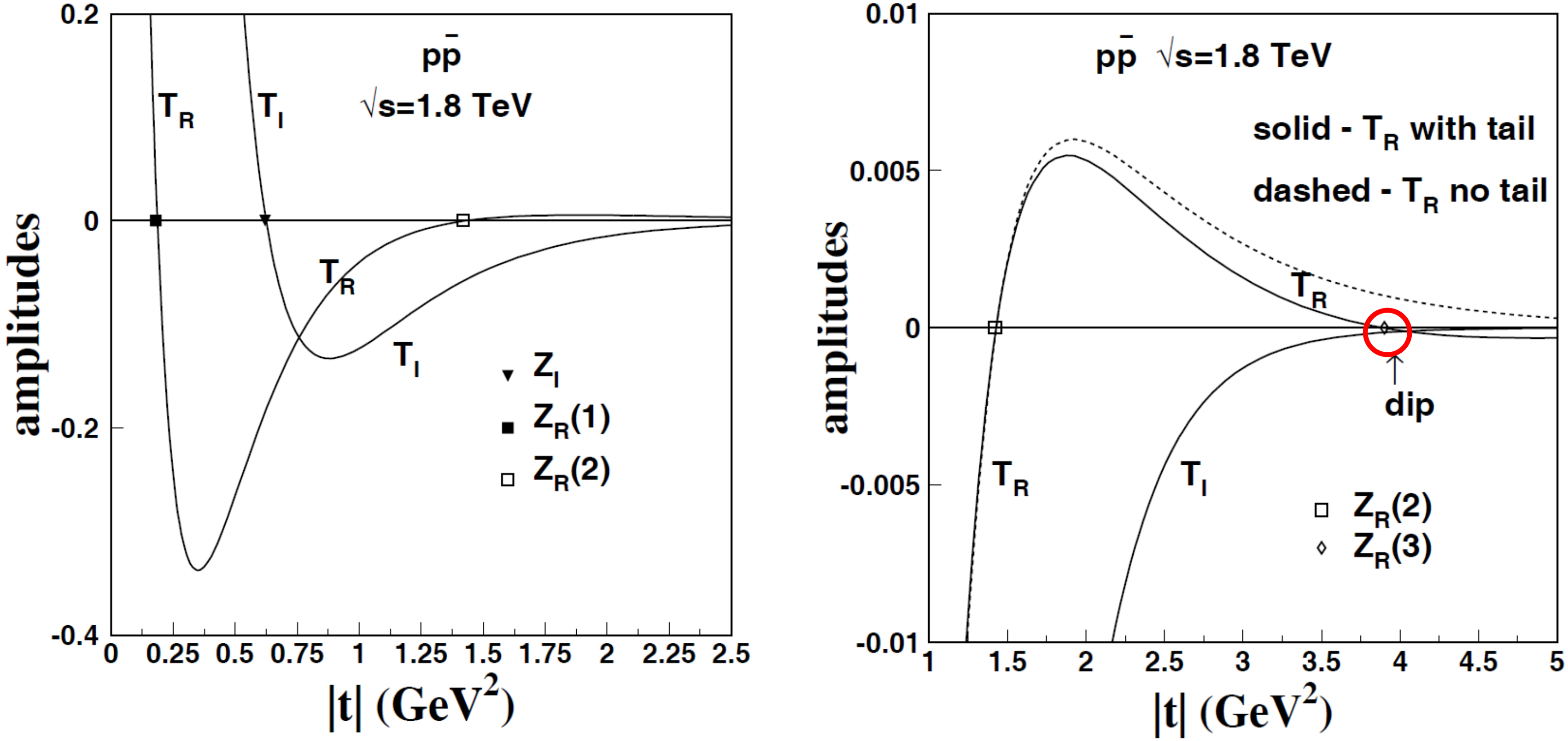
Long t region (compare with 52.8 GeV)





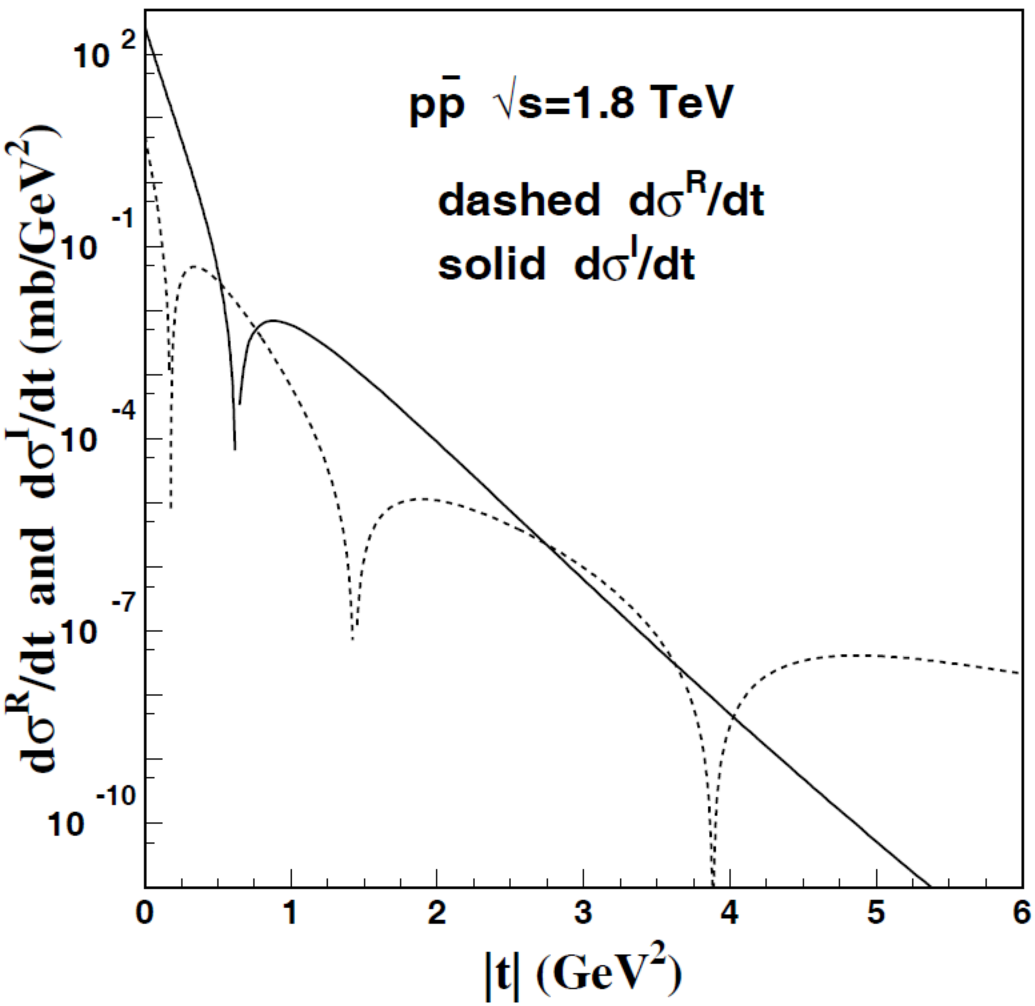
$p\bar{p}$ $\sqrt{s}$ = 546 GeV
◇ Bozzo PLB 147 (84) 385 - 58 pts
△ PLB 147 (84) 385 - 29 pts
▲ PLB 155 (85) 197 - 34 pts
○ (dN/dt) / 10.892
99 points
dσ/d|t| (mb/GeV^2)
|t| (GeV^2)
$p\bar{p}$
$\sqrt{s}$ = 1.8/1.96 TeV
○ N. Amos (1990)
□ F. Abe (1984)
△ V. Abazov (2012)
35, 26 and 17 pts
dσ/dt (mb/GeV^2)
|t| (GeV^2)

Amplitudes

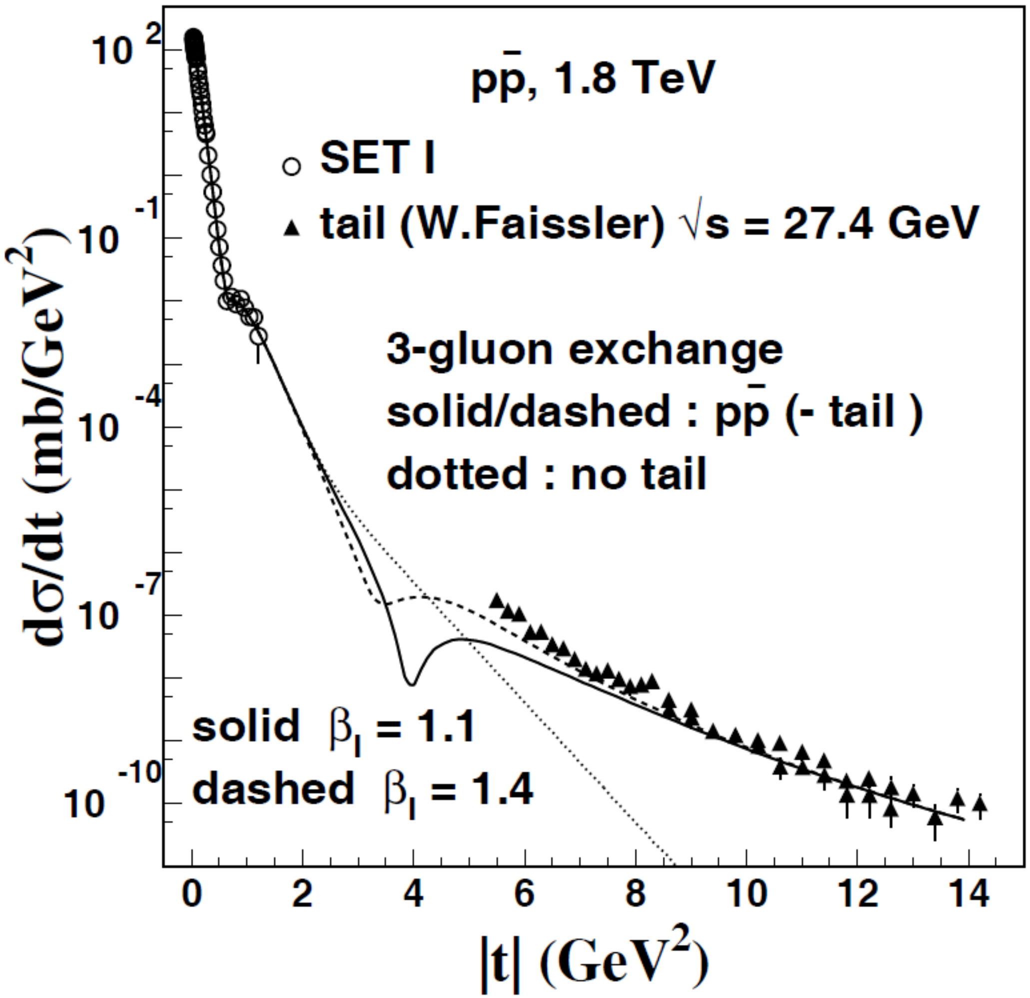


pp̄
√s=1.8 TeV
T_R
T_I
T_R
T_I
Z_I
Z_R(1)
Z_R(2)
amplitudes
|t| (GeV²)
pp̄ √s=1.8 TeV
solid - T_R with tail
dashed - T_R no tail
T_R
dip
T_R
T_I
Z_R(2)
Z_R(3)
amplitudes
|t| (GeV²)

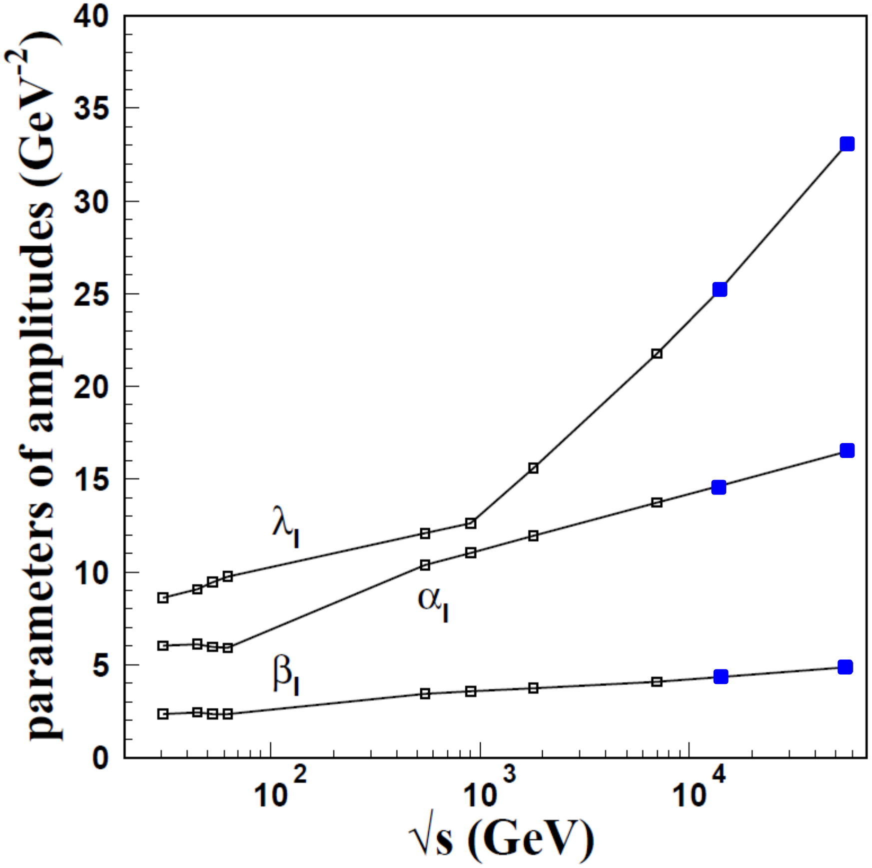
Partial cross sections : dip formation



Prediction of dip for large |t|

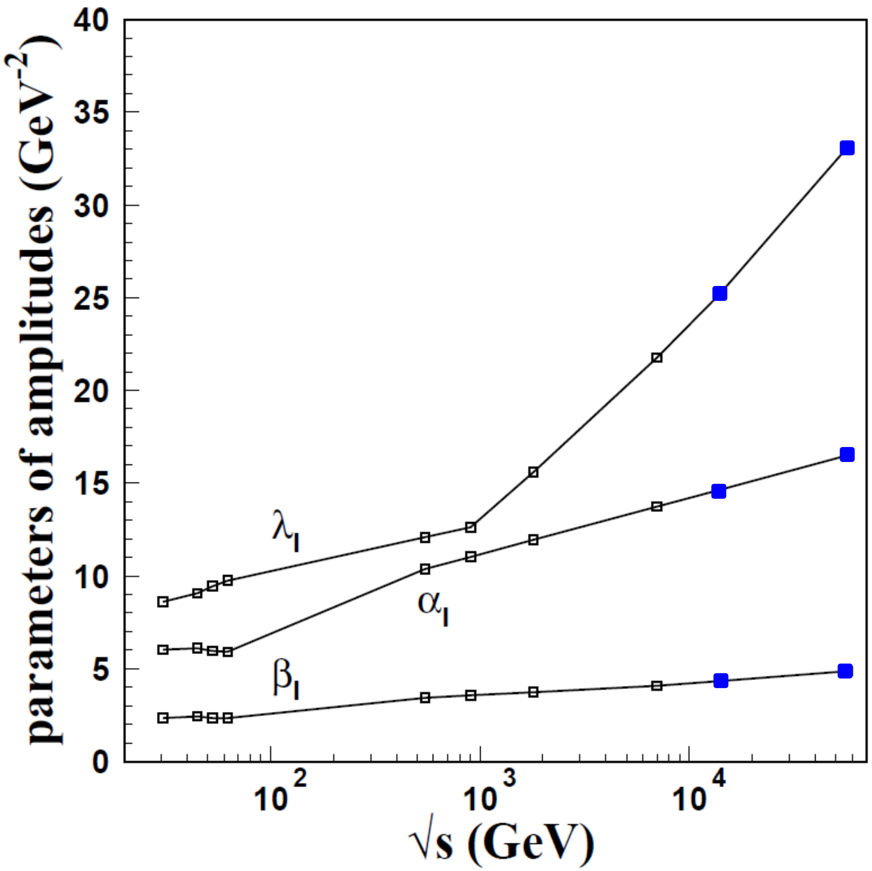


Energy dependence of the parameters



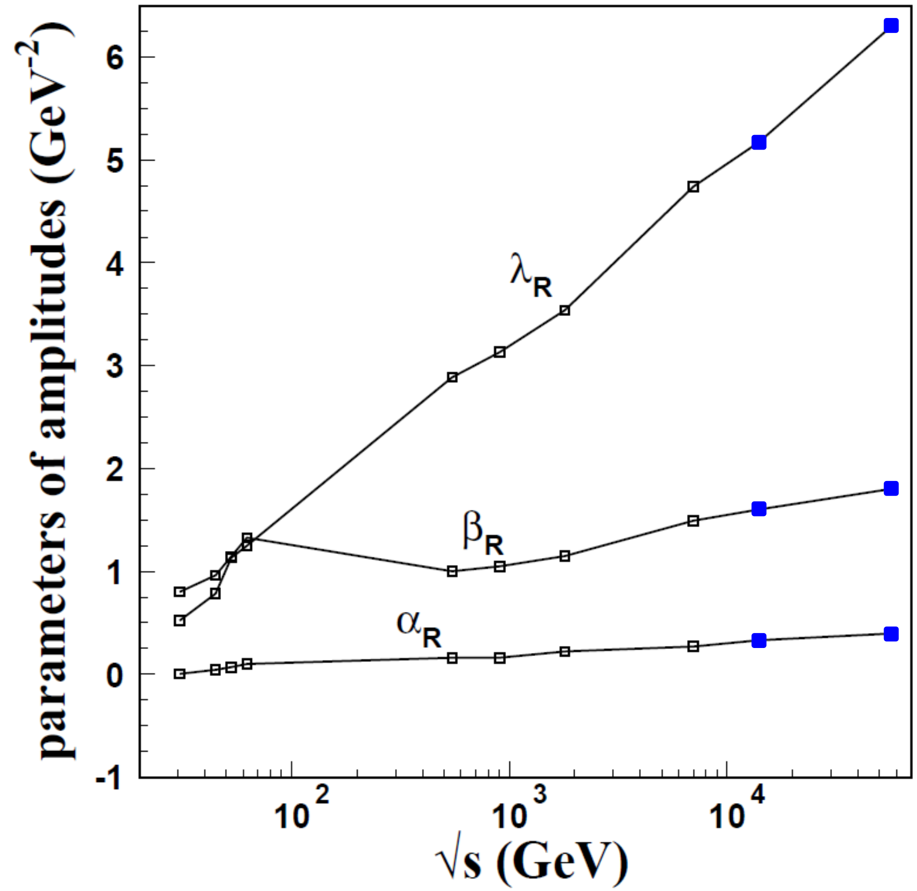
Imaginary quantities

Energy dependence of the parameters



Imaginary quantities

Real quantities



b space amplitudes

Fourier Transform

$$\tilde{T}_K(s,b) = \frac{1}{2\pi}\int d^2\vec{q}\; e^{-i\vec{q}.\vec{b}}\; T_K^N(s,t=-q^2)$$

we have analytical forms

$$\tilde{T}_K(s,b) = \frac{\alpha_K}{2\;\beta_K} e^{-\frac{b^2}{4\beta_K}} + \lambda_K\;\tilde{\psi}_K(s,b)$$

b space amplitudes

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With the shape functions

$$\tilde{\psi}_K(s,b) = \frac{2\,e^{\gamma_K}}{a_0} \frac{e^{-\sqrt{\gamma_K^2+\frac{b^2}{a_0}}}}{\sqrt{\gamma_K^2+\frac{b^2}{a_0}}} \left[1 - e^{\gamma_K}\, e^{-\sqrt{\gamma_K^2+\frac{b^2}{a_0}}}\right]$$

b space amplitudes

Fourier Transform

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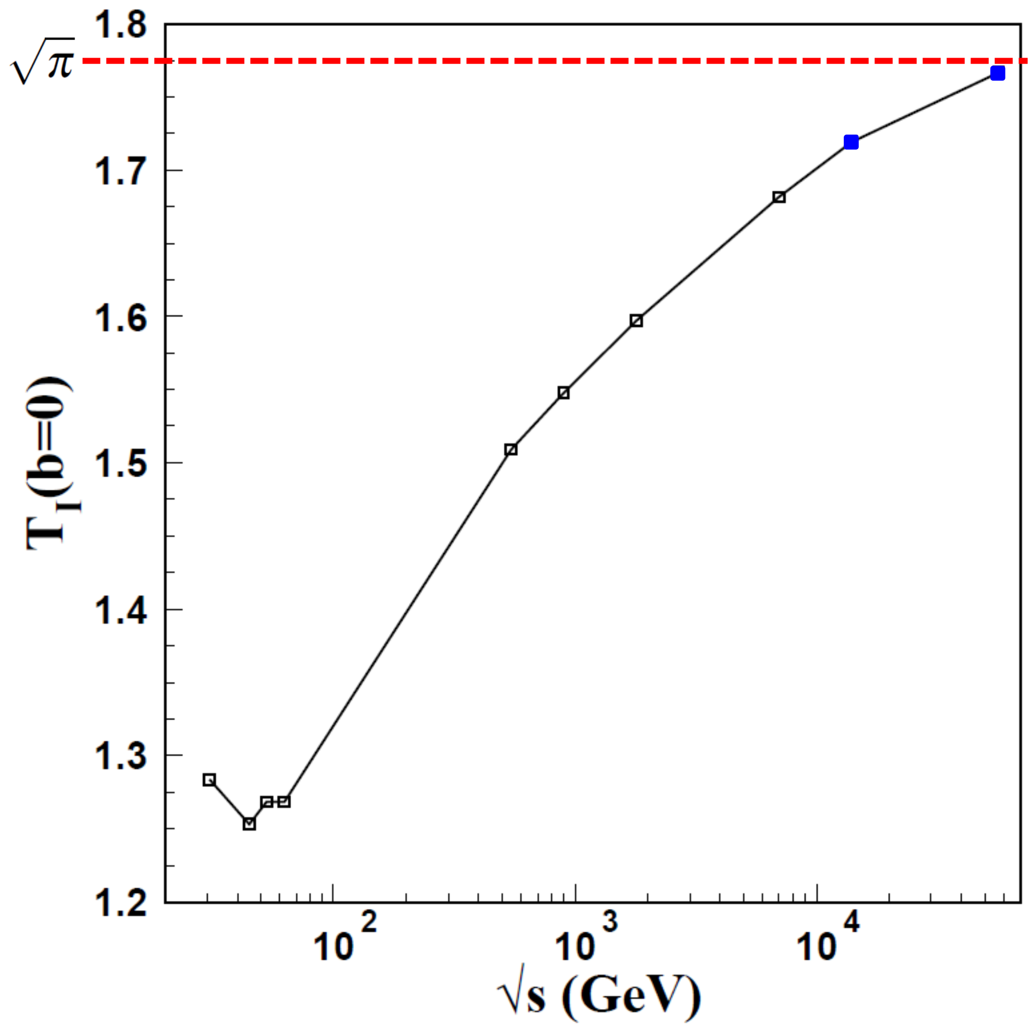
$$e^{-\gamma b}/b$$

Yukawa like

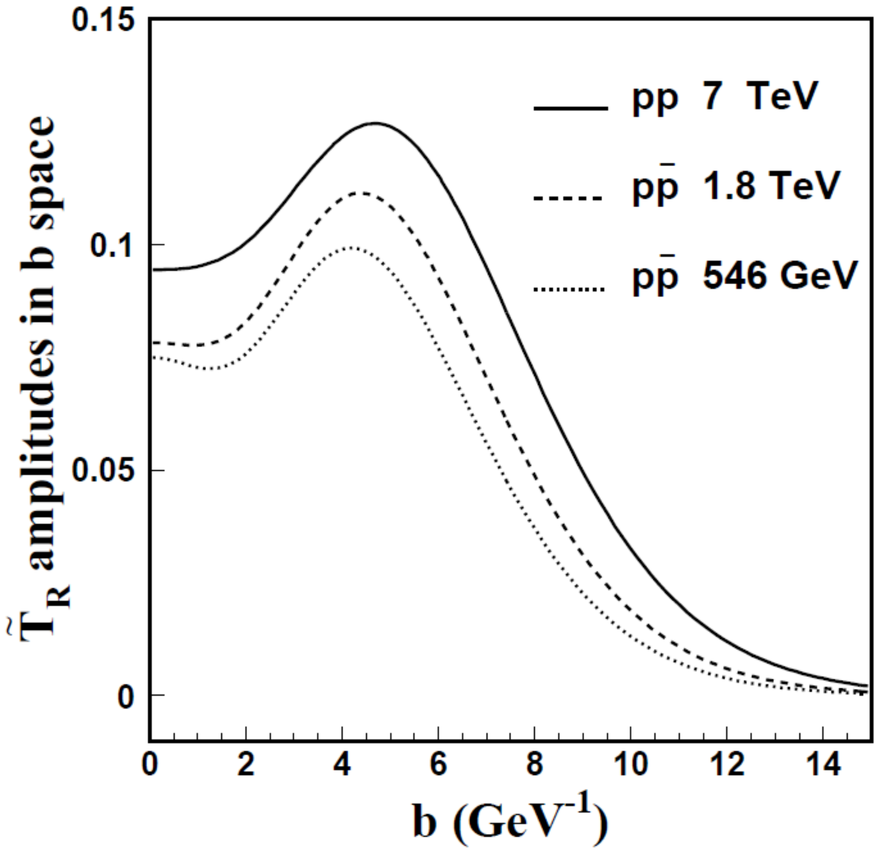
At b=0

$$\tilde{T}_K(s,0) = \frac{\alpha_K}{2\;\beta_K}$$

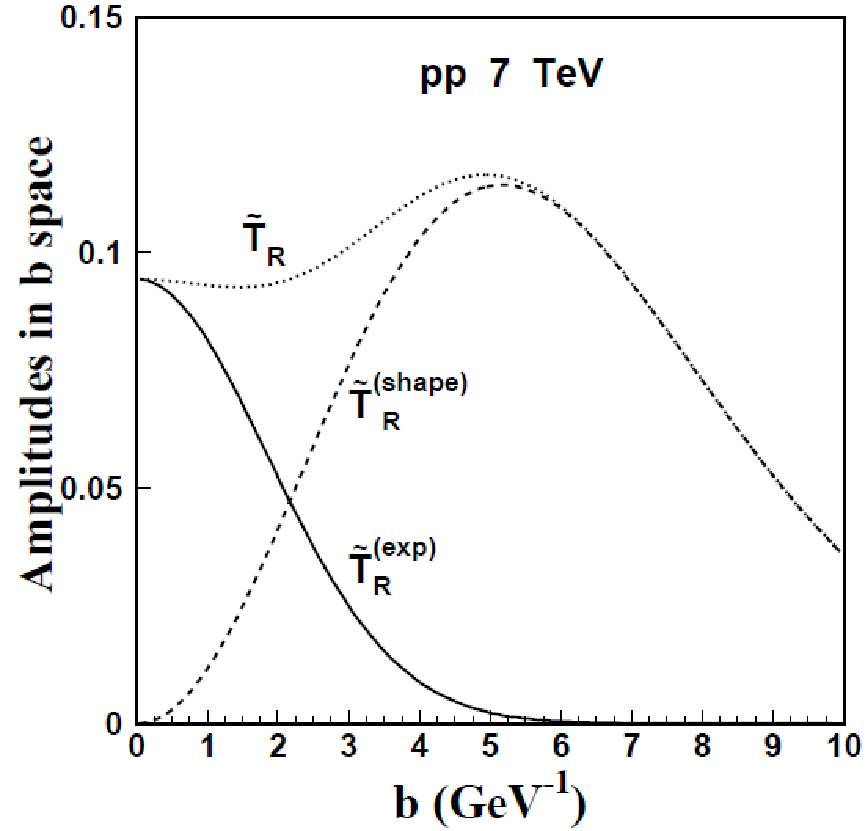


$\sqrt{\pi}$
1.8
1.7
1.6
1.5
1.4
1.3
1.2
$T_I(b=0)$
10^2
10^3
10^4
√s (GeV)

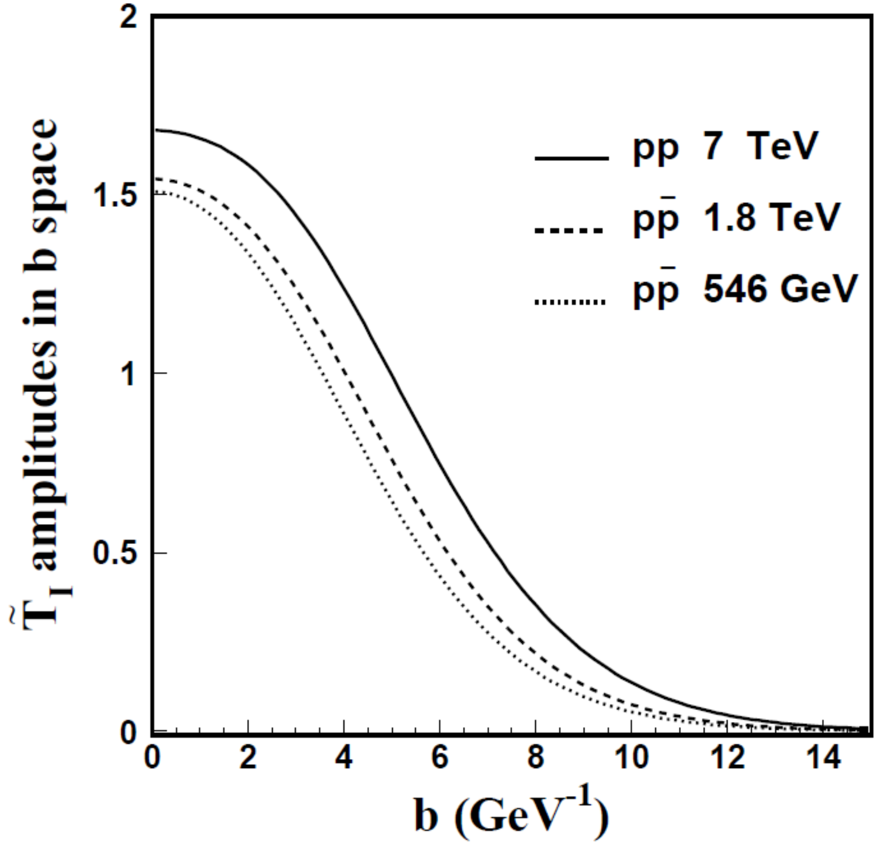
Real amplitudes in b space



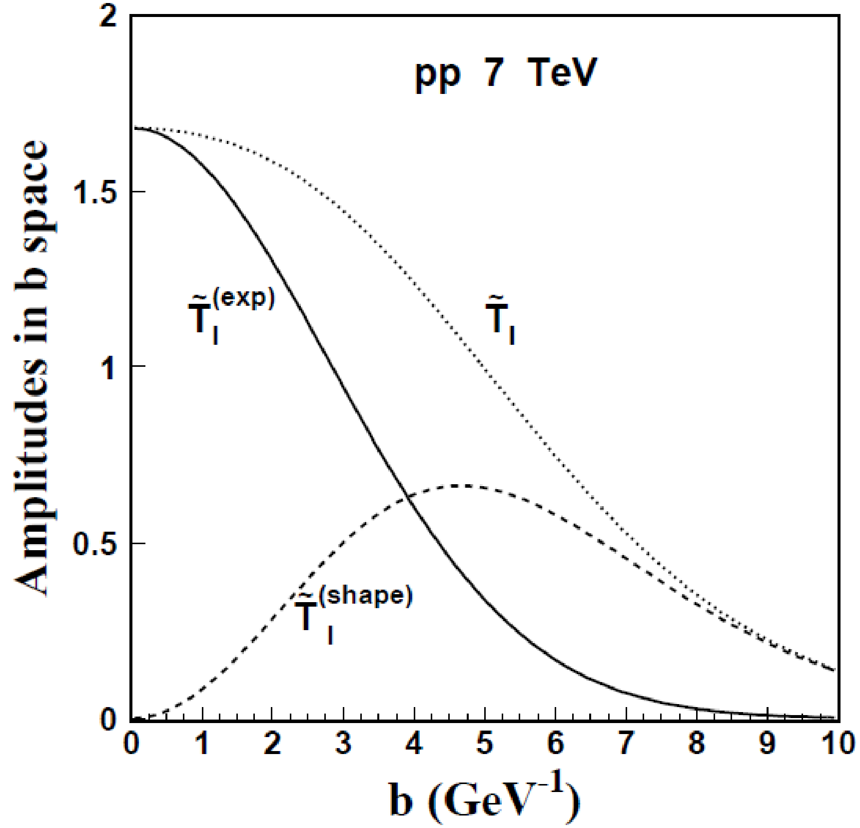
Parts of the real amplitude for 7 TeV



Imaginary amplitudes in b space



Parts of the imaginary amplitudes for 7 TeV



Eikonal representation

The amplitudes in the b space are

$$\tilde{T}_R\left(s,\vec{b}\right) = \tilde{T}_R\left(s,\vec{b}\right) + i\tilde{T}_I\left(s,\vec{b}\right) \equiv i\sqrt{\pi}\left(1 - e^{i\chi(s,b)}\right)$$

with the complex eikonal $\chi = \chi_R + i\chi_I$

Eikonal representation

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with the complex eikonal $\chi=\chi_R+i\chi_I$

and the real and imaginary parts are

$$\chi_R=\tan^{-1}\left(\frac{\tilde{T}_R}{\sqrt{\pi}-\tilde{T}_I}\right)$$

$$\chi_I=-\ln\sqrt{\left(\frac{1}{\sqrt{\pi}}\tilde{T}_R\right)^2+\left(1-\frac{1}{\sqrt{\pi}}\tilde{T}_I\right)^2}$$

Eikonal representation

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with the complex eikonal $\chi = \chi_R + i\chi_I$

and the real and imaginary parts are

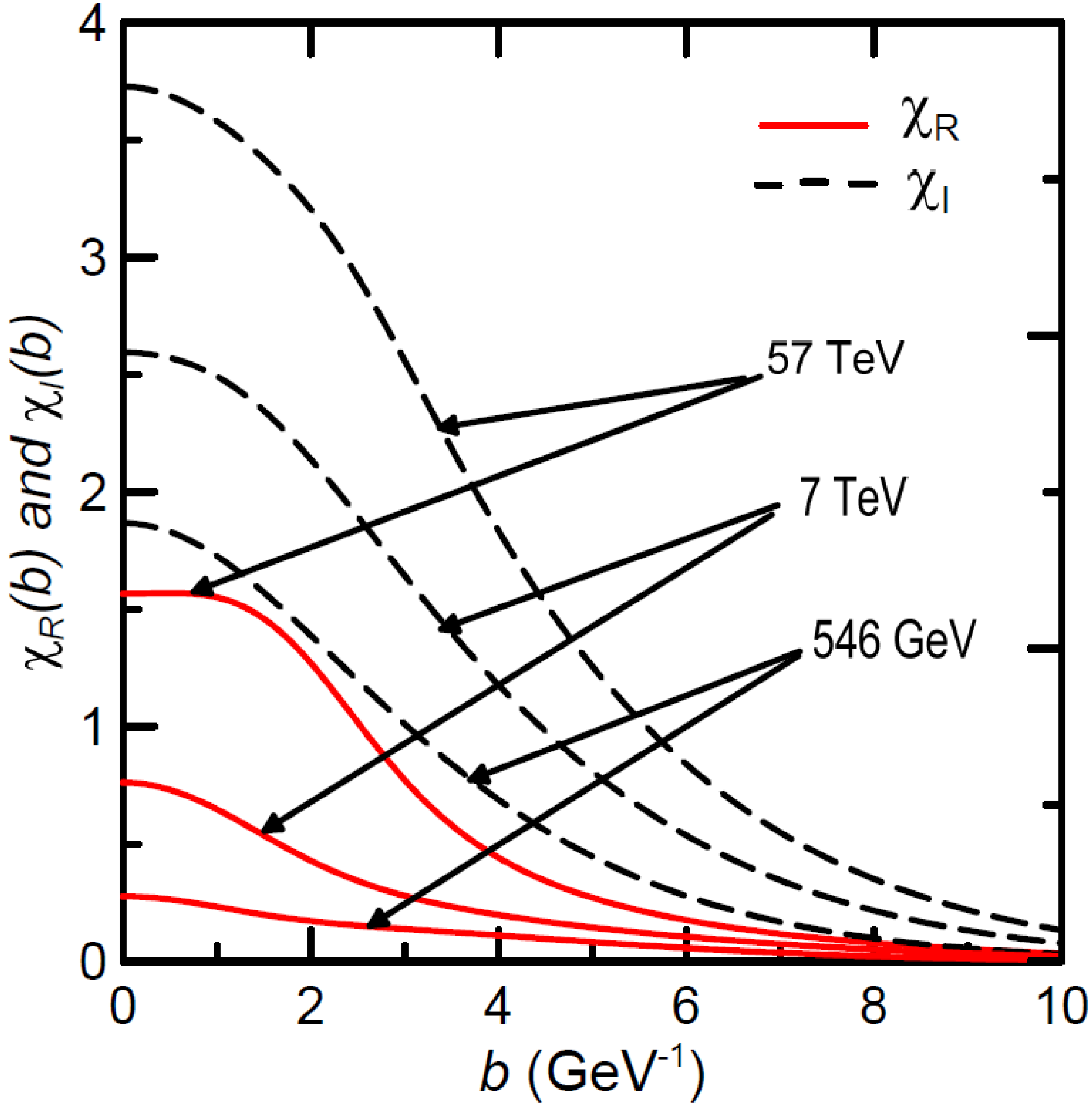
$$\chi_R = \tan^{-1}\left(\frac{\tilde{T}_R}{\sqrt{\pi} - \tilde{T}_I}\right)$$

$$\chi_I = -\ln\sqrt{\left(\frac{1}{\sqrt{\pi}}\tilde{T}_R\right)^2 + \left(1 - \frac{1}{\sqrt{\pi}}\tilde{T}_I\right)^2}$$

To avoid this pole we expect

$$\tilde{T}_I(s,b) < \sqrt{\pi}$$

Eikonal representation



Unitarity

The unitarity condition of the scattering amplitude can be expressed as

$$\left| e^{i\chi(s,b)} \right| \leq 1$$

which is equivalent to say $\chi_I(s,b) \geq 0$.

In terms of amplitudes this corresponds to

$$e^{-2\Im\chi} = \left(1 - \frac{1}{\sqrt{\pi}}\tilde{T}_I\right)^2 + \left(\frac{1}{\sqrt{\pi}}\tilde{T}_R\right)^2 \leq 1$$

for all s and b.

These are satisfied by our representations.

We write elastic, total and inelastic differential cross sections in b space

$$\sigma_{el} = \int d^2\vec{b}\, \frac{d\bar{\sigma}_{el}}{d^2\vec{b}}\;, \quad \sigma_{Tot} = \int d^2\vec{b}\, \frac{d\bar{\sigma}_{Tot}}{d^2\vec{b}}\;, \quad \sigma_{inel} = \int d^2\vec{b}\, \frac{d\bar{\sigma}_{inel}}{d^2\vec{b}}$$

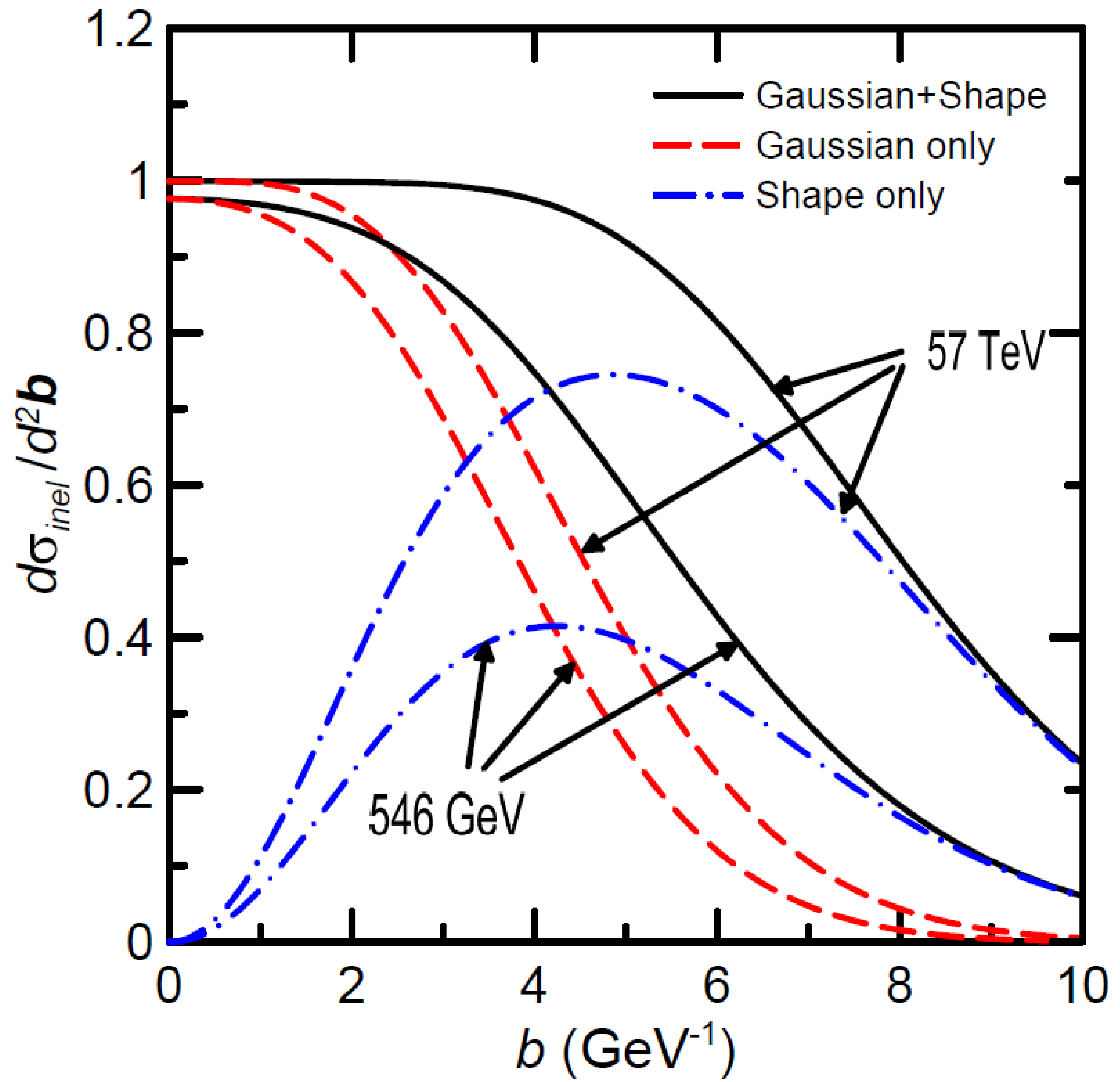
and identify adimensional differential cross sections

$$\frac{d\bar{\sigma}_{el}}{d^2\vec{b}} = 1 - 2\cos(\chi_R)\, e^{-\chi_I} + e^{-2\chi_I}$$

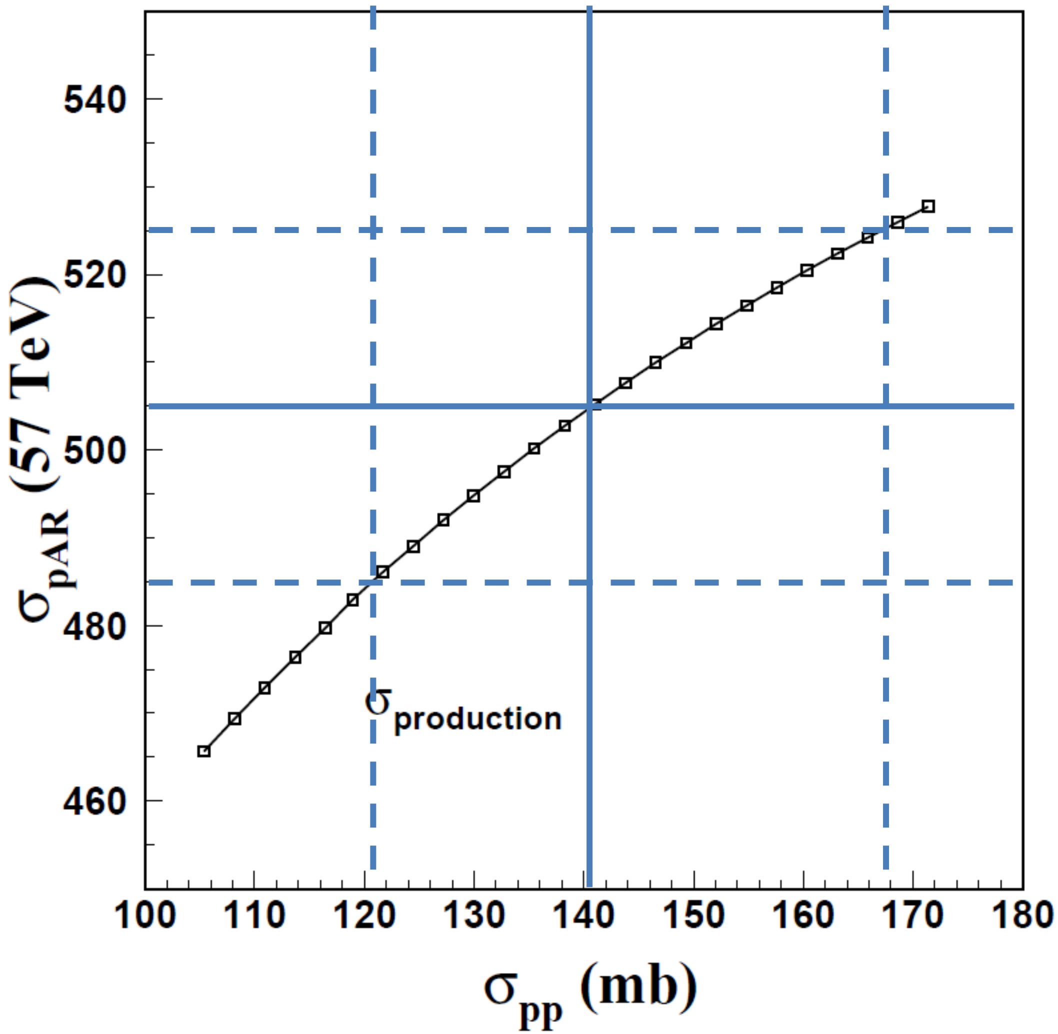
$$\frac{d\bar{\sigma}_{Tot}}{d^2\vec{b}} = 2\left\{1 - \cos(\chi_R)\, e^{-\chi_I}\right\}$$

$$\frac{d\bar{\sigma}_{inel}}{d^2\vec{b}} = 1 - e^{-2\chi_I}$$

Inelastic cross sections



Cosmic ray



σ_{pAR} (57 TeV)
540
520
500
480
460
$\sigma_{production}$
100
110
120
130
140
150
160
170
180
σ_{pp} (mb)

Conclusion

At low energies the growth of the total cross section is dominated by the gaussian distribution, on the other hand for large energies the increase is due to the shape function of the form ~ $e^{-\gamma b}/b$. A physical interpretation is that when the energy increases the partonic density starts to saturate in the center of the proton but increases in the periphery due the shape function.

Thanks