Flowing Gluon Fields

Collective Phenomena in Classical QCD

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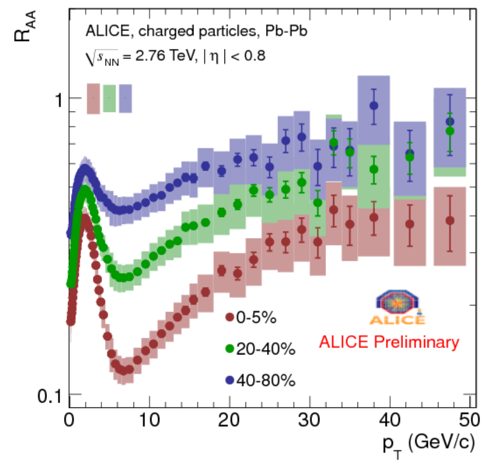
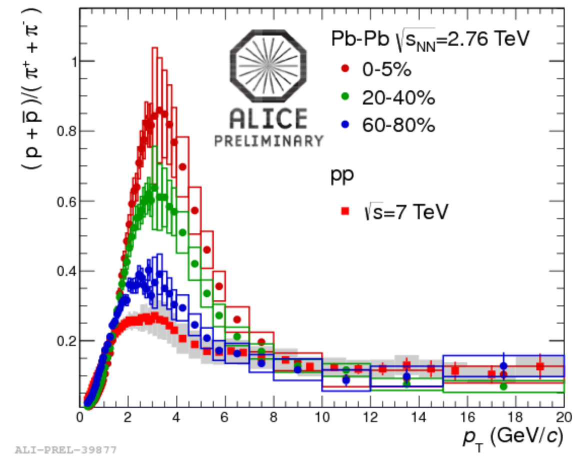
Office of Science
U.S. Department of Energy

JET Collaboration

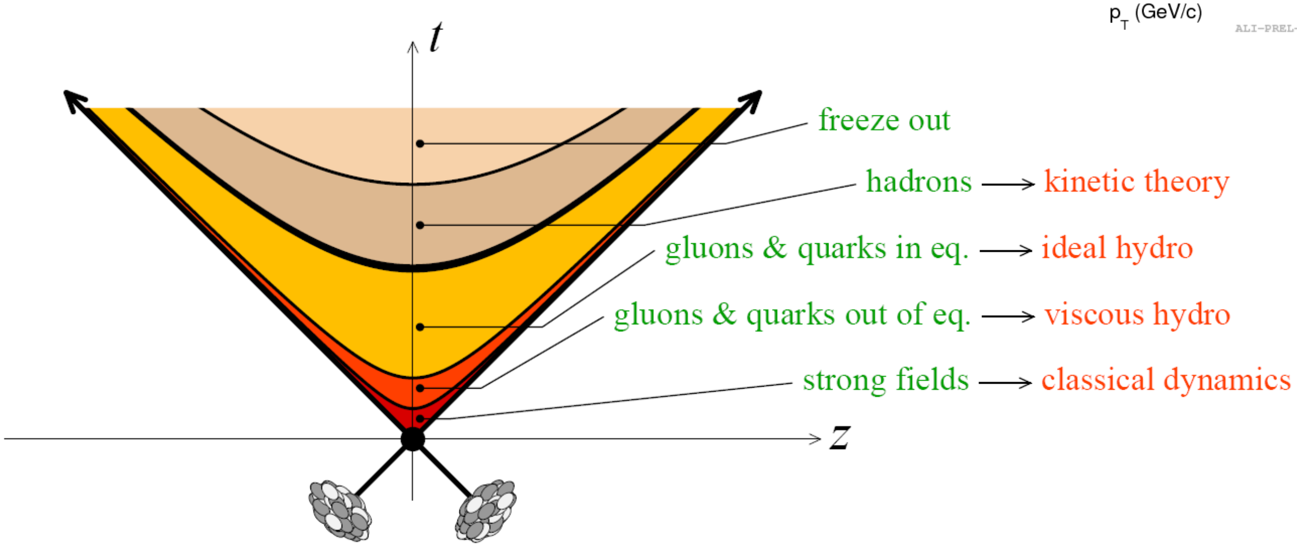
A Short Primer on URHICs

- Momentum Spectra
  - Hard: jets/hard probes
  - Intermediate: ??
  - Soft (bulk): color glass (?), npQCD, thermalization + hydro



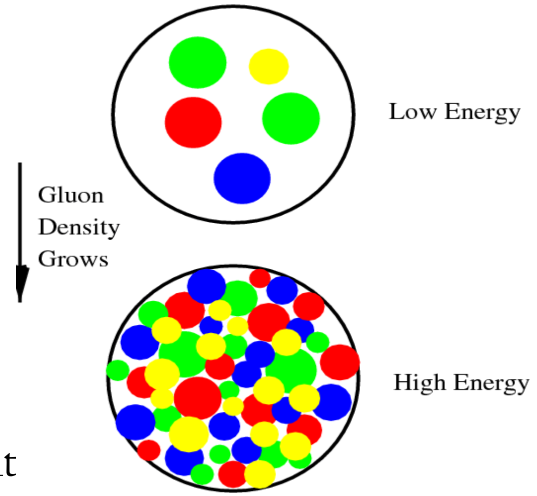


- Bulk (soft) time evolution



Color Glass

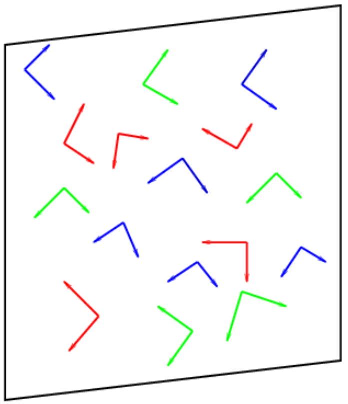
- Nuclei/hadrons at asymptotically high energy:
  - Saturated gluon density ~ $Q_s^{-2} \rightarrow$ scale $Q_s \gg \Lambda_{QCD}$
  - Probes interact with many quarks + gluons coherently.
  - Large occupation numbers → quasi-classical fields.
  - Large nuclei are better: $Q_s \sim A^{1/3}$



- Effective Theory a la McLerran & Venugopalan
  - Solve Yang-Mills equations for gluon field $A^\mu(\rho)$. Sources = light cone currents J (given by SU(3) charge distributions ρ = large-x partons).

$$\left[D_\mu, F^{\mu\nu}\right] = J^\nu$$

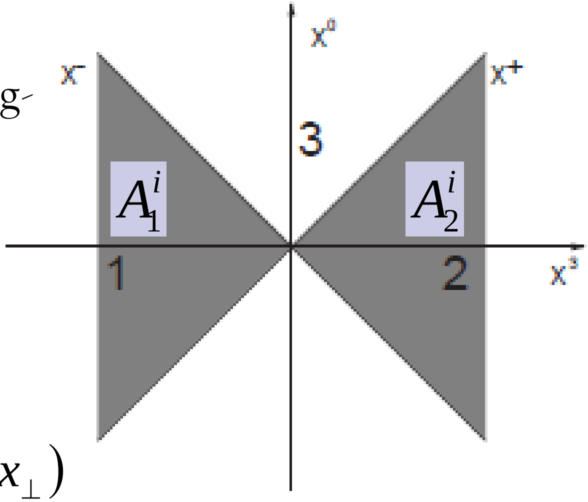
  - Calculate observables $O(\rho)$ from the gluon field $A^\mu(\rho)$.
  - Compute O event by event or the expectation value of O by sampling or averaging over ρ. ρ given by arbitrary frozen fluctuations of a color-neutral object.
  - MV weight: $$\langle O \rangle_\rho = \int [d\rho] O(\rho) W(\rho)$$



[L. McLerran, R. Venugopalan]

Colliding Nuclei

- Yang-Mills equations: two sources ρ_1, ρ_2
  - Intersecting light cone currents J_1, J_2 (given by ρ_1, ρ_2) solve Yang-Mills equations for gluon field $A^\mu(\rho_1, \rho_2)$.



- Forward light cone (3): free Yang-Mills equations for fields A, $A^i_\perp$

$$\frac{1}{\tau^3}\partial_\tau \tau^3 \partial_\tau A - \left[D^i,\left[D^i,A\right]\right] = 0$$
$$\frac{1}{\tau}\left[D^i,\partial_\tau A^i_\perp\right] - ig\tau\left[A,\partial_\tau A\right] = 0$$
$$\frac{1}{\tau}\partial_\tau \tau \partial_\tau A^i_\perp - ig\tau^2\left[A,\left[D^i,A\right]\right] - \left[D^j,F^{ji}\right] = 0$$

$$A^\pm = \pm x^\pm A(\tau, x_\perp)$$
$$A^i = A^i_\perp(\tau, x_\perp)$$

[A. Kovner, L. McLerran, H. Weigert]

- Boundary conditions on the forward light cone:

$$A^i_\perp(\tau = 0, x_\perp) = A^i_1(x_\perp) + A^i_2(x_\perp)$$
$$A(\tau = 0, x_\perp) = -\frac{ig}{2}\left[A^i_1(x_\perp), A^i_2(x_\perp)\right]$$

- MV setup is boost-invariant, but not symmetric between + and – direction.

Gluon Fields in The Forward Lightcone

- Goals:
  - Calculate fields and energy momentum tensor of *early time* gluon field as a function of space-time coordinates.
  - Analyze energy density and flow field.
  - Derive constraints for further hydrodynamic evolution of equilibrating QGP.
- Small-time expansion

$$A(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(x_\perp)$$

$$A_\perp^i(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{\perp(n)}^i(x_\perp)$$

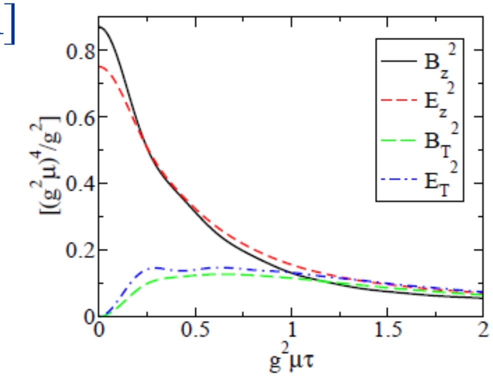
- Results: recursive solution for gluon field:

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D_{(k)}^i, \left[D_{(l)}^i, A_{(m)}\right]\right]$$

$$A_{\perp(n)}^i = \frac{1}{n^2}\left(\sum_{k+l=n-2} \left[D_{(k)}^j, F_{(l)}^{ji}\right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, \left[D_{(l)}^i, A_{(m)}\right]\right]\right)$$

Numerical solution

[T. Lappi]



$$A_{\perp(0)}^i(x_\perp) = A_1^i(x_\perp) + A_2^i(x_\perp)$$

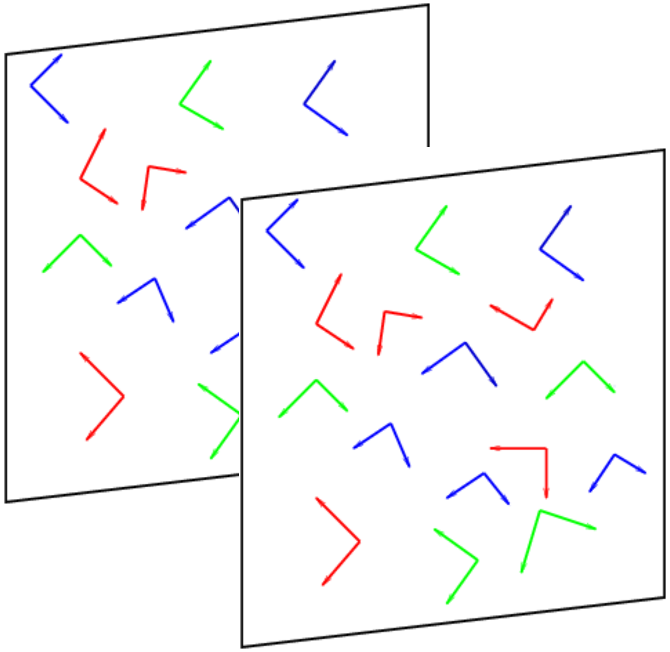
$$A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A_1^i(x_\perp), A_2^i(x_\perp)\right]$$

[RJF, J. Kapusta, Y. Li, 2006]
[Fujii, Fukushima, Hidaka, 2009]

Result: Fields

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.

$$F_1^{i+} = \delta\left(x^-\right)A_1^i$$



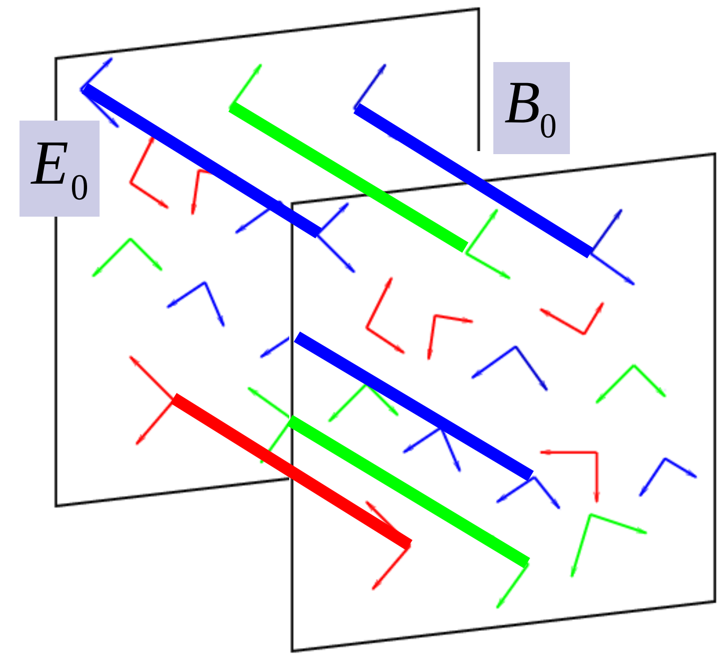
$$F_2^{i-} = \delta\left(x^+\right)A_2^i$$

Result: Fields

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.
- Immediately after overlap (forward light cone, $\tau \to 0$): strong *longitudinal* electric & magnetic fields. Non-abelian effect!

$$F_{(0)}^{+-} = ig\left[A_1^i, A_2^i\right] \quad \leftarrow \quad E_0$$

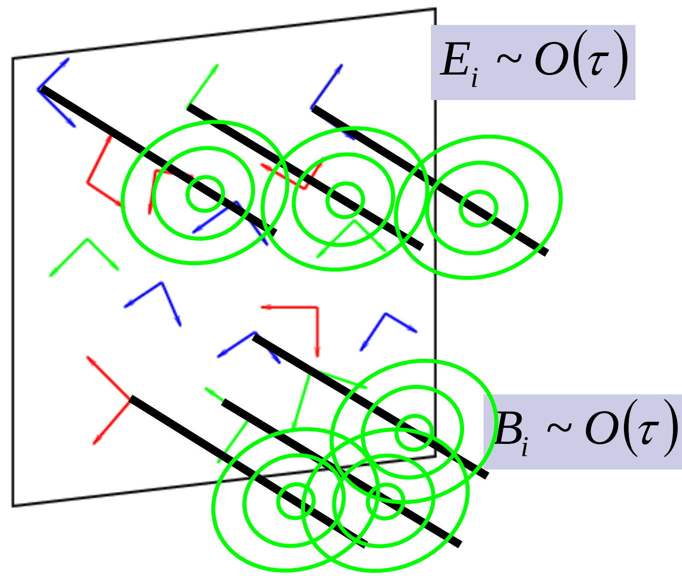
$$F_{(0)}^{21} = ig\varepsilon^{ij}\left[A_1^i, A_2^j\right] \quad \leftarrow \quad B_0$$



[L. McLerran, T. Lappi, 2006]
[RJF, J.I. Kapusta, Y. Li, 2006]

Result: Fields

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.
- Immediately after overlap (forward light cone, $\tau \to 0$): strong *longitudinal* electric & magnetic fields. Non-abelian effect!
- *Transverse* E, B fields start linearly in time τ



$$F^{i\pm}_{(1)} = -\frac{e^{\pm\eta}}{2\sqrt{2}}\left(\varepsilon^{ij}\left[D^j_{(0)}, B_0\right] \pm \left[D^i_{(0)}, E_0\right]\right)$$

[RJF, J.I. Kapusta, Y. Li, 2006]
[G. Chen, RJF, 2013]

Energy-Momentum Tensor

- Initial ($\tau = 0$) structure of the energy-momentum tensor:

Gauge fields:

$$T_{\mathrm{f}}^{\mu\nu} = 2\,\mathrm{Tr}\left[F^{\mu\lambda}F_{\lambda}^{\nu}\right] + \frac{1}{2}g^{\mu\nu}\,\mathrm{Tr}\left[F^{\kappa\lambda}F_{\kappa\lambda}\right]$$

Particle distributions f:

$$T_{\mathrm{p}}^{\mu\nu} = \int \frac{d^3 p}{p^0} p^{\mu} p^{\nu} f(\vec{p})$$

$$T_{\mathrm{f}(0)}^{\mu\nu} = \begin{pmatrix} \varepsilon_0 & & & \\ & \varepsilon_0 & & \\ & & \varepsilon_0 & \\ & & & -\varepsilon_0 \end{pmatrix} \rightarrow T_{\mathrm{equ}}^{\mu\nu} = \begin{pmatrix} e & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

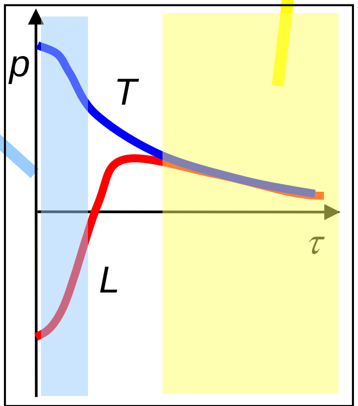
Transverse pressure
$P_T = \varepsilon_0$

Longitudinal pressure
$P_L = -\varepsilon_0$

Later: Ideal plasma
(local rest frame)

$$\varepsilon_0 = \frac{1}{2}\left(E_0^2 + B_0^2\right)$$

- Toward equilibrium: pressure isotropization (in local rest frame)



Energy Momentum Tensor

- General structure up to order τ^2:

[RJF, J.I. Kapusta, Y. Li, (2006)]
[G. Chen, RJF (2013)]

$$\boxed{\frac{1}{2}\left(E^2+B^2\right)} \qquad \boxed{\vec{S}=\vec{E}\times\vec{B}}$$

$$T_{\mathrm{f}}^{\mu\nu}=\begin{pmatrix} \varepsilon_0+O\left(\tau^2\right) & \alpha^1\cosh\eta+\beta^1\sinh\eta & \alpha^2\cosh\eta+\beta^2\sinh\eta & O\left(\tau^2\right) \\ \alpha^1\cosh\eta+\beta^1\sinh\eta & \varepsilon_0+O\left(\tau^2\right) & O\left(\tau^2\right) & \alpha^1\sinh\eta+\beta^1\cosh\eta \\ \alpha^2\cosh\eta+\beta^2\sinh\eta & O\left(\tau^2\right) & \varepsilon_0+O\left(\tau^2\right) & \alpha^2\sinh\eta+\beta^2\cosh\eta \\ O\left(\tau^2\right) & \alpha^1\sinh\eta+\beta^1\cosh\eta & \alpha^2\sinh\eta+\beta^2\cosh\eta & -\varepsilon_0+O\left(\tau^2\right) \end{pmatrix}$$

- Transverse Poynting vector gives transverse flow.

$$\alpha^i=-\frac{\tau}{2}\nabla^i\varepsilon_0$$

Like hydrodynamic flow, determined by gradient of transverse pressure $P_T=\varepsilon_0$; even in rapidity.

$$\beta^i=\frac{\tau}{2}\varepsilon^{ij}\left(\left[D^j,B_0\right]E_0-\left[D^j,E_0\right]B_0\right)$$

Non-hydro like; odd in rapidity ??

- Example for second order:

Depletion/increase of energy density due to transverse flow

$$T^{00}=\varepsilon_0-\frac{\tau^2}{8}\left[2\nabla^i\alpha^i+\sinh 2\eta\,\nabla^i\beta^i+\left(2-\cosh 2\eta\right)\delta\right]$$

Due to longitudinal flow

Modelling Color Charges

- So far everything written in terms of color charge densities ρ_1, ρ_2
- MV: Gaussian distribution around color-neutral average

$$\left\langle \rho_i^a(x_1)\rho_j^b(x_2)\right\rangle_\rho = \frac{g^2}{N_c^2-1}\delta_{ij}\delta^{ab}\lambda\left(x_1^\mp\right)\delta\left(x_1^\mp - x_2^\mp\right)\delta^2\left(\mathbf{x}_{1T}-\mathbf{x}_{2T}\right)$$

$$\mu_i = \int dx^\mp \lambda\left(x^\mp\right)$$

  - Sample distribution to obtain event-by-event observables.
  - Here: calculating expectation value as function of average color charge densities μ_1, μ_2.

- MV: μ = *const*. But flow comes from gradients in nuclear profiles!
  - Keep μ approximately constant on length scales $1/Q_s$, allow variations on larger length scales: $\mu^2(\mathbf{x}_T) >> m^{-1}\left|\nabla^i\mu^2(\mathbf{x}_T)\right| >> m^{-2}\left|\nabla^i\nabla^j\mu^2(\mathbf{x}_T)\right|$ where m is an infrared mass scale $m \ll Q_s$.
  - MV well-behaved: typical cancellation of singularities still go through: ($\gamma \sim \langle A^+_{\text{cov}} A^+_{\text{cov}}\rangle$)

$$\Gamma(\mathbf{x}_T,\mathbf{y}_T) = 2\gamma(\mathbf{x}_T,\mathbf{y}_T) - \gamma(\mathbf{x}_T,\mathbf{x}_T) - \gamma(\mathbf{y}_T,\mathbf{y}_T) = \mu^2\frac{r^2}{8\pi}\ln m^2r^2 \;\rightarrow\; \Gamma(\mathbf{x}_T,\mathbf{y}_T) = \mu^2(\mathbf{R})\frac{r^2}{8\pi}\ln m^2r^2 + O\left(\frac{\nabla^i\nabla^j\mu^2(\mathbf{R})}{m^2}\right)$$

  - Applicability: typically everywhere ~ 1 fm or more away from the surface of a large nucleus.

[G. Chen, RJF, 2013]
[G. Chen et al., in preparation]

Averaging

- Take expectation values.
- Energy density ~ product of nuclear gluon distributions ~ product of color source densities

$$\varepsilon_0 = \frac{g^6 N_c\left(N_c^2-1\right)}{8\pi}\mu_1\mu_2 \ln^2\frac{Q^2}{m^2}$$

[T. Lappi, 2006]
[RJF, Kapusta, Li, 2006]
[Fujii, Fukushima, Hidaka, 2009]

- "Hydro" flow:

$$\alpha^i = -\tau\frac{g^6 N_c\left(N_c^2-1\right)}{64\pi^2}\nabla^i\left(\mu_1\mu_2\right)\ln^2\frac{Q^2}{m^2}$$

[G. Chen, RJF, 2013]
[G. Chen et al., in preparation]

- Odd flow term:

$$\beta^i = -\tau\frac{g^6 N_c\left(N_c^2-1\right)}{64\pi^2}\left(\mu_2\nabla^i\mu_1-\mu_1\nabla^i\mu_2\right)\ln^2\frac{Q^2}{m^2}$$

Transverse Field: Abelian Arguments

- Once the (non-abelian) longitudinal fields E_0, B_0 are seeded, the *averaged* transverse flow field is an abelian effect.
- Can be understood in terms of Ampere's, Faraday's and Gauss' Law.
  - Longitudinal fields E_0, B_0 decrease in both *z* and *t* away from the light cone
- Gauss at fixed time *t*:
  - Long. flux imbalance compensated by transverse flux
  - Gauss: **rapidity-odd radial** field
- Ampere/Faraday as function of *t*:
  - Decreasing long. flux induces transverse field
  - Ampere/Faraday: **rapidity-even curling** field
- Full classical QCD:

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\, \varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\, \varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$

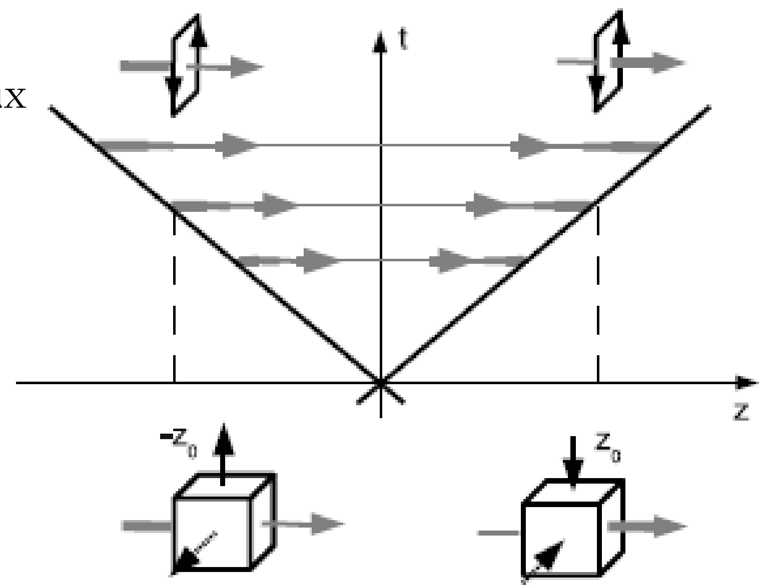


Figure 1: Two observers at $z = z_0$ and $z = -z_0$ test Ampère's and Faraday's Laws with areas a^2 in the transverse plane and Gauss' Law with a cube of volume a^3. The transverse fields from Ampère's and Faraday's Laws (black solid arrows) are the same in both cases, while the transverse fields from Gauss' Law (black dashed arrows) are observed with opposite signs. Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength.

Averaging: From Fields To Flow

- Calculating the Poynting vector to first order in τ:

$$S^i_{\text{even}} = \frac{\tau}{2}\cosh\eta\left(E_0\left[D^i, E_0\right] + B_0\left[D^i, B_0\right]\right) = \alpha^i \cosh\eta$$

$$S^i_{\text{odd}} = \frac{\tau}{2}\sinh\eta\, \varepsilon^{ij}\left(E_0\left[D^j, B_0\right] - B_0\left[D^j, E_0\right]\right) = \beta^i \sinh\eta$$

- Averaging over source configurations: what are $\left\langle E_0 \nabla^i B_0 \right\rangle$ and $\left\langle B_0 \nabla^i E_0 \right\rangle$?

- With $E_0 = ig\left[A_1^i, A_2^i\right], \quad B_0 = ig\varepsilon^{ij}\left[A_1^i, A_2^j\right]$ we have $\left\langle E_0 \nabla^i B_0 \right\rangle = -\left\langle B_0 \nabla^i E_0 \right\rangle$

- Lorentz symmetry and dimensionality dictate $\beta^i \sim \mu_1 \nabla^i \mu_2, \ \mu_2 \nabla^i \mu_1$

[G. Chen, RJF, 2013]

Transverse Field: Visualization

- Transverse fields for randomly seeded longitudinal fields.

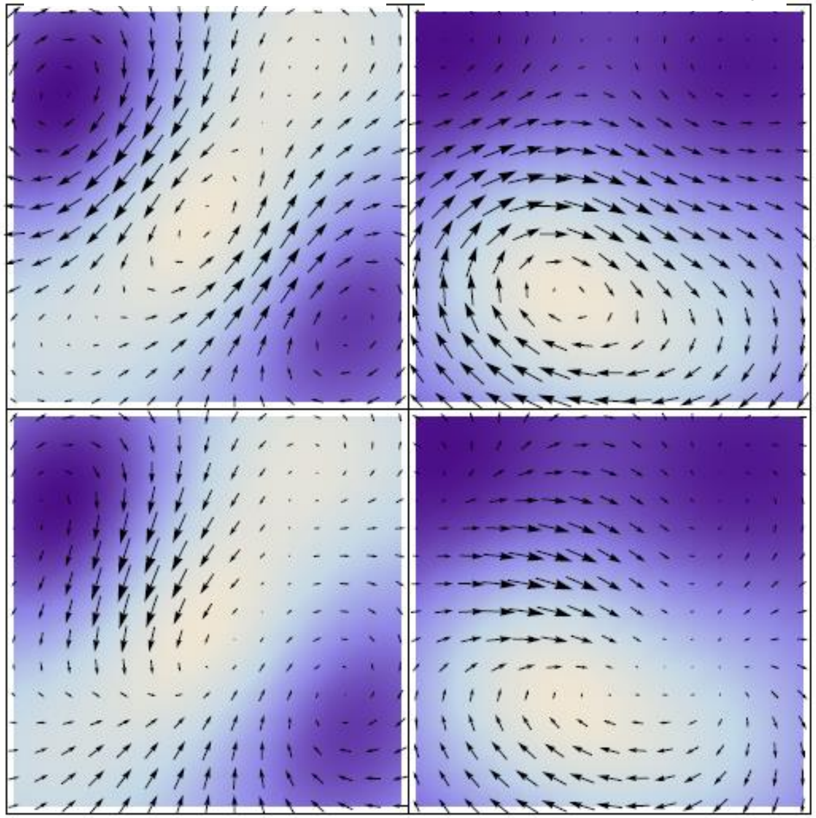


Figure 2: Transverse electric fields (left panels) and magnetic fields (right panels) at $\eta = 0$ (upper panels) and $\eta = 1$ (lower panels) in an abelian example for a random distribution of fields A_1^i, A_2^i. The initial longitudinal fields B_0 (left panels) and E_0 (right panels) are indicated through the density of the background (lighter color = larger values). At $\eta = 0$ the fields are divergence-free and clearly following Ampére's and Faraday's Laws, respectively.

Transverse Flow: Visualization

- Transverse Poynting vector for randomly seeded longitudinal fields.

(background $= \varepsilon_0$)

$\eta = 0$

(no odd flow)

$\eta = 1$

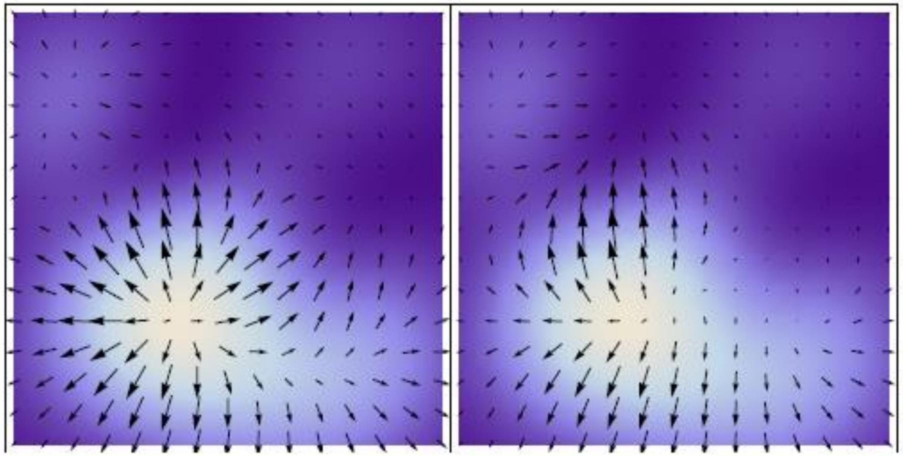
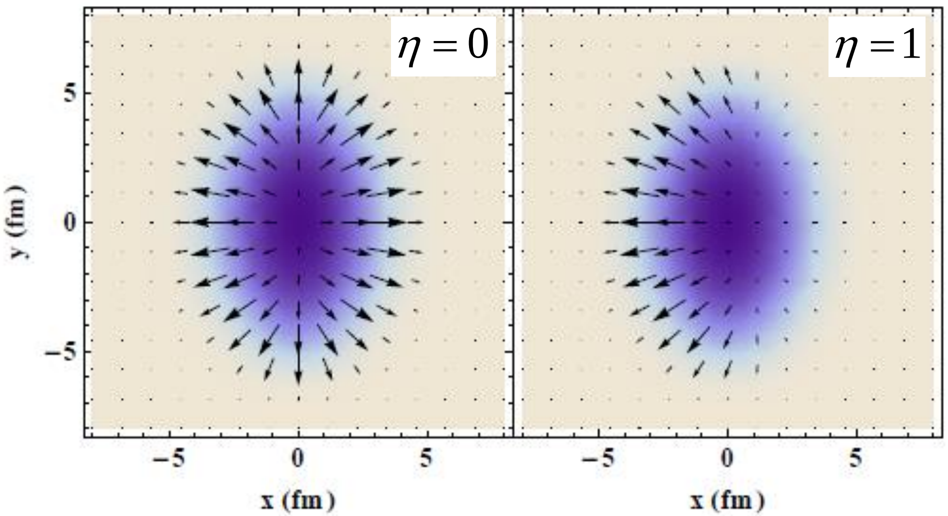


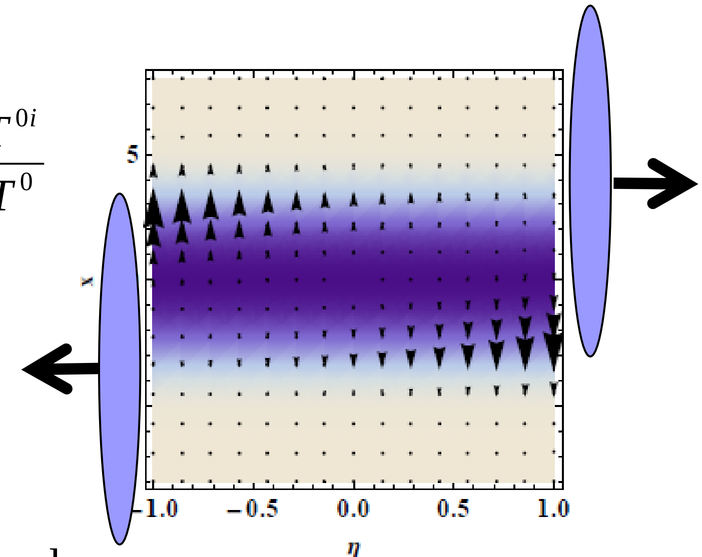
Figure 3: Example for transverse flow of energy for $\eta = 0$ (left panel) and $\eta = 1$ (right panel) in the abelian example for the same random distribution of fields A_1^i, A_2^i as in Fig. 2. The initial energy density T^{00} is shown through the density of the background (lighter color = larger values). At $\eta = 0$ the flow follows the gradient in the energy density in a hydro-like way while away from mid-rapidity energy flow gets quenched in some directions and amplified in others.

Phenomenology: $b \neq 0$

- Odd flow needs an asymmetry: e.g. finite impact parameter
- Flow field for Au+Au collision, b = 4 fm.

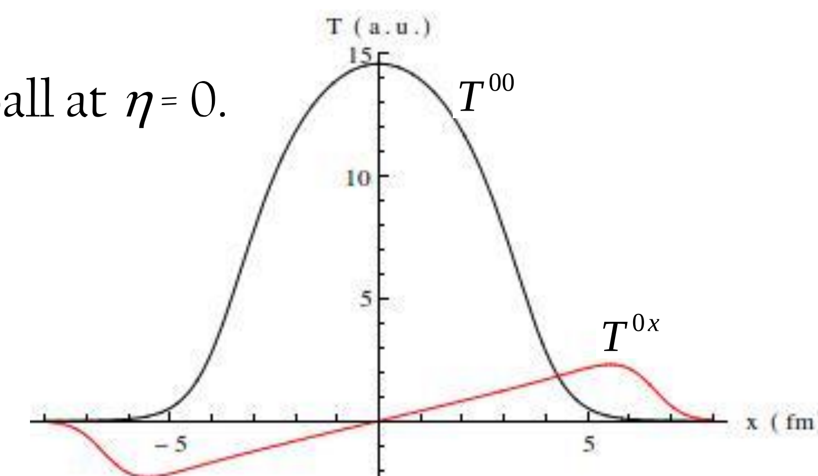


$$V^i = \frac{T^{0i}}{T^0}$$



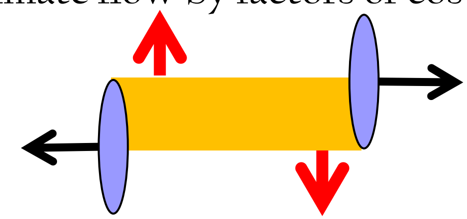
$\text{Background} = \varepsilon_0$

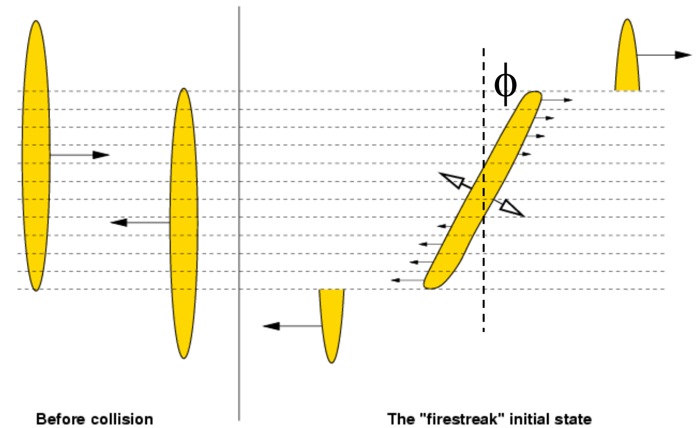
- Radial flow following gradients in the fireball at η = 0.
- Clearly: directed flow away from η = 0.
- Fireball tilted, angular momentum.
- Careful: time τ ~ 0.1-0.2 fm/c



Phenomenology: $b \neq 0$

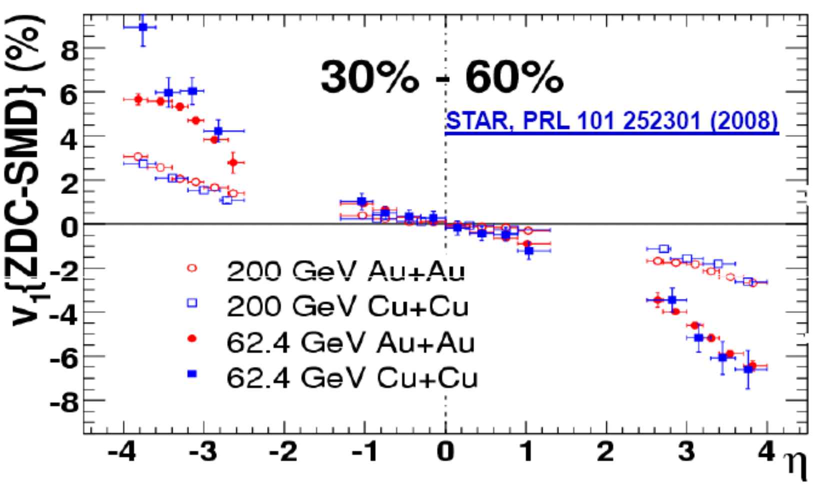
- Angular momentum is natural: some old models have it, most modern hydro calculations don't.
  - Do we underestimate flow by factors of cos ϕ?



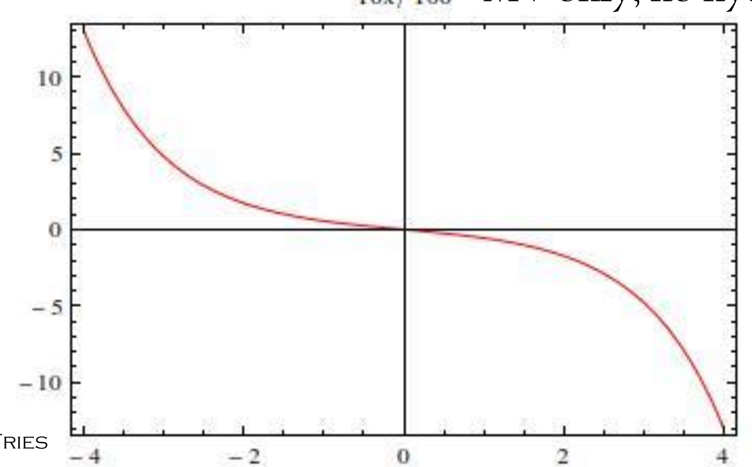


[Gosset, Kapusta, Westfall (1978)]

- Note that boost-invariance is not broken.
- Directed flow v_1:
  - Hydro needs tilted initial conditions or initial flow.



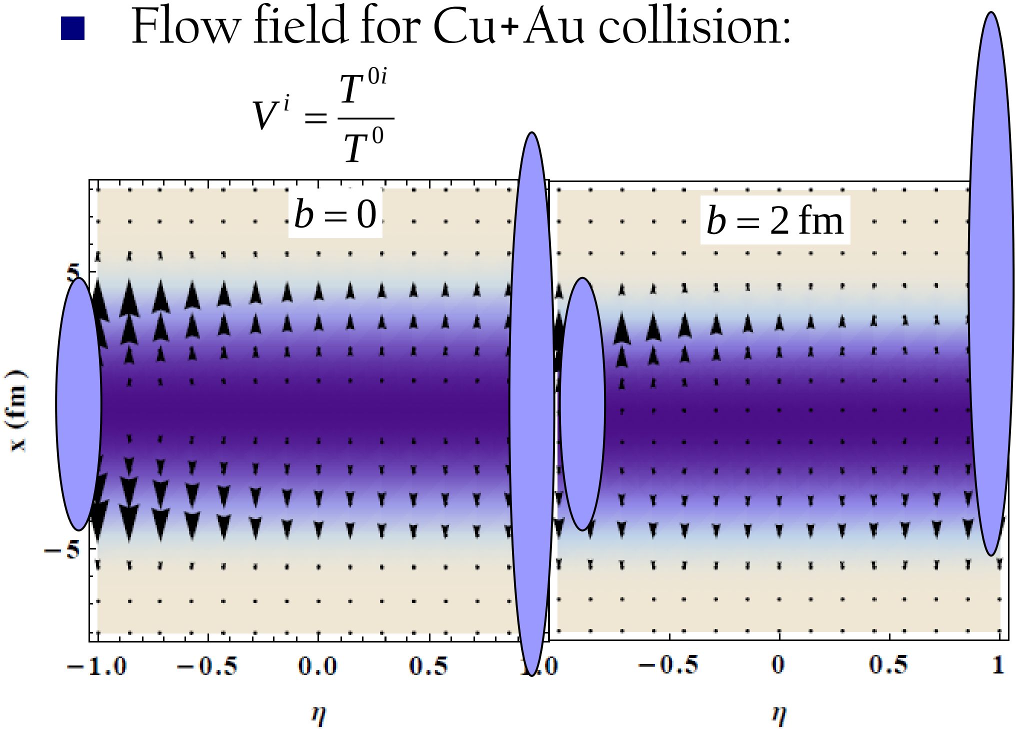
T0x / T00 MV only, no hydro



Phenomenology: A ≠ B

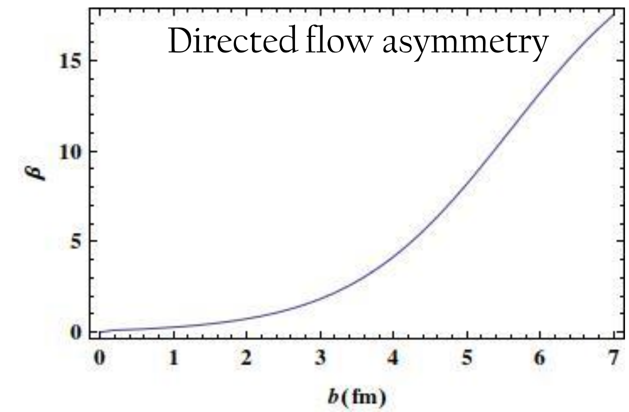
- Odd flow needs an asymmetry: e.g. asymmetric system
- Flow field for Cu+Au collision:

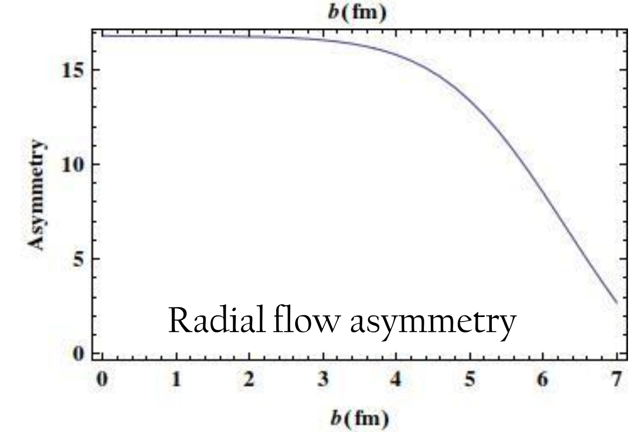
$$V^i = \frac{T^{0i}}{T^0}$$



Example: Forward-backward asymmetries. Here: p+Pb

$$\frac{\langle T^{0x}\rangle}{\langle T^{00}\rangle}(\eta=-2) - \frac{\langle T^{0x}\rangle}{\langle T^{00}\rangle}(\eta=+2)$$



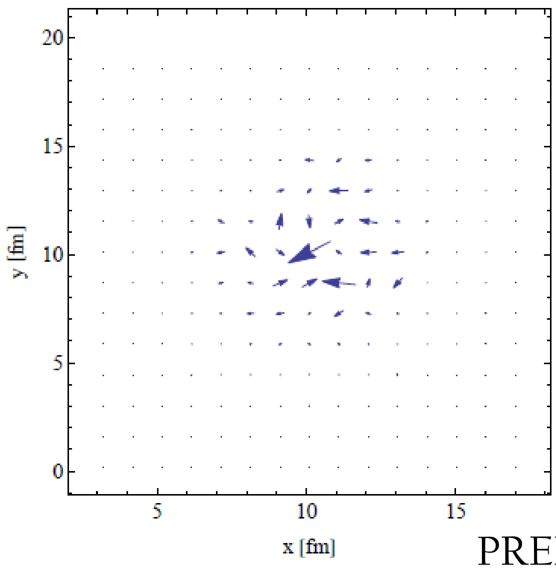


- Odd flow increases expansion in the wake of the larger nucleus, suppresses flow on the other side.
- Should lead to very characteristic flow patterns in asymmetric systems.

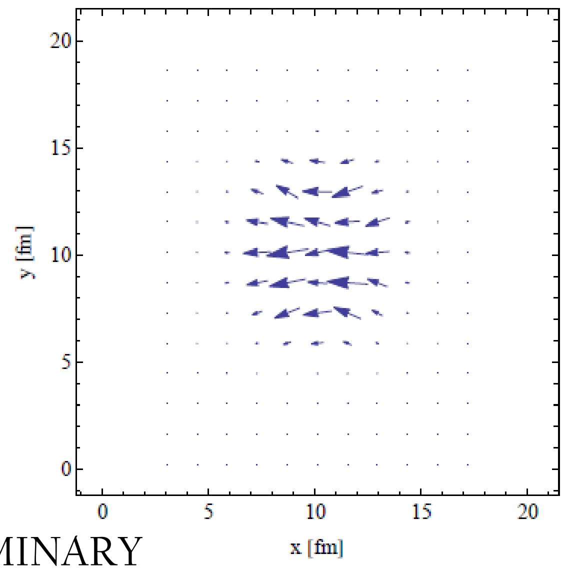
Event-By-Event Picture

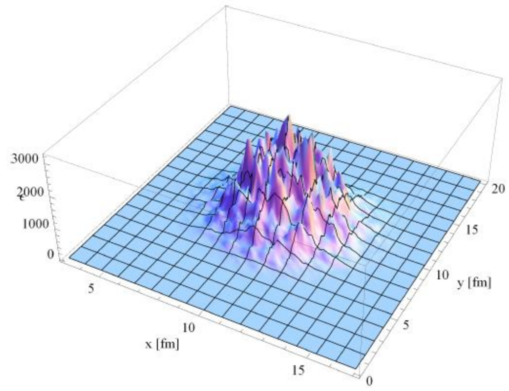
- Numerical simulation of β in Au+Au, sampling charge distributions in the nuclei.

$N = 1$



$N = 100$





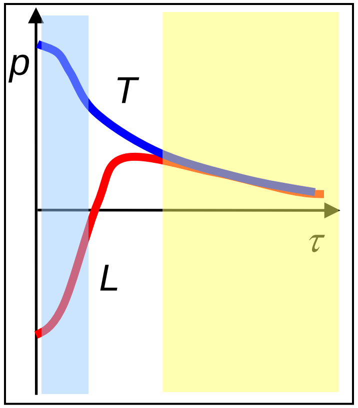
PRELIMINARY

- Individual events dominated by fluctuations.
- Averaging N › 100 events: recover directed flow.

Matching to Hydrodynamics

- No equilibration in clQCD; thermalization = difficult problem.
- Pragmatic solution: extrapolate from both sides ($r(\tau)$ = interpolating fct.)

  $T^{\mu\nu} = T_{\mathrm{f}}^{\mu\nu} r(\tau) + T_{\mathrm{pl}}^{\mu\nu}\left(1 - r(\tau)\right)$, enforce $\partial_\mu T^{\mu\nu} = 0$
  and other conservation laws.



- Fast equilibration: $r(\tau) = \Theta(\tau_0 - \tau)$
- Analytic solution available for matching ideal hydro.
  - 4 equations + EOS to determine 5 fields in ideal hydro.

$$\vec{v}_\perp = \frac{1}{\cosh\eta} \frac{\vec{\alpha}}{\epsilon_0 - \frac{\tau_{th}^2}{8}(-2\triangle\epsilon_0 + \delta) + p},$$

$$v_L = \tanh\eta,$$

$$e + p = \left(\epsilon_0 - \frac{\tau_{th}^2}{8}(-2\triangle\epsilon_0 + \delta) + p\right)\left(1 - \frac{\vec{\alpha}^2}{\left(\epsilon_0 - \frac{\tau_{th}^2}{8}(-2\triangle\epsilon_0 + \delta) + p\right)^2}\right)$$

  - Odd flow β drops out: we are missing angular momentum.

Matching to Hydrodynamics

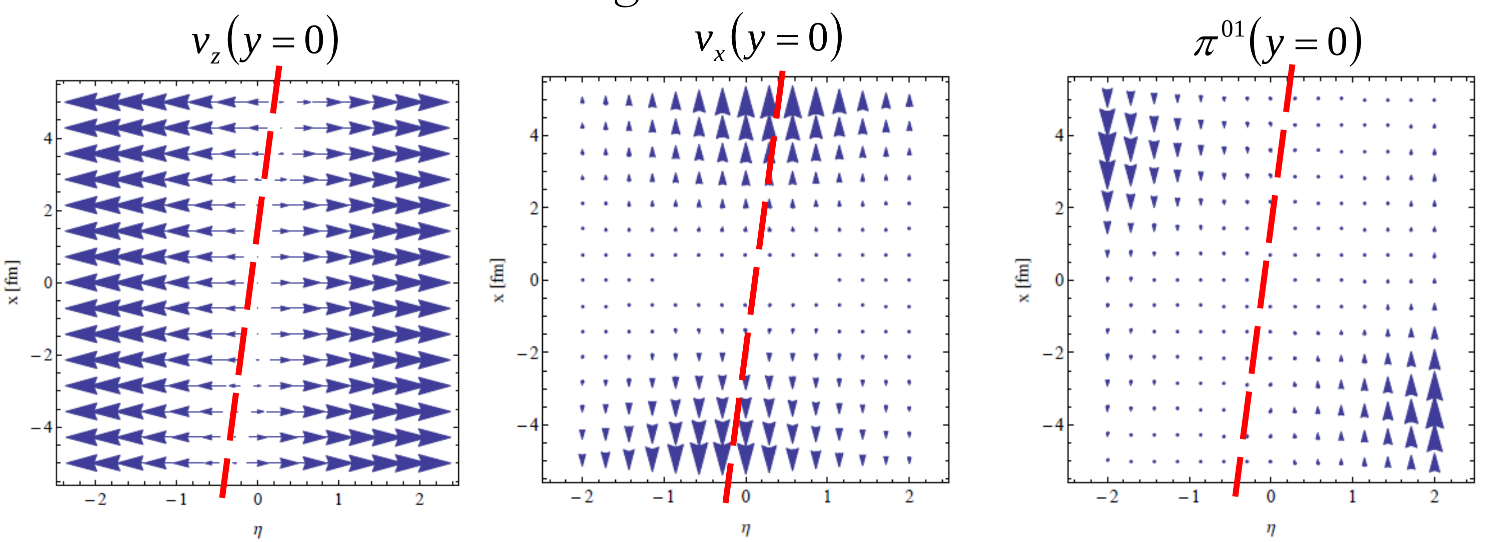
- Instantaneous matching to viscous hydrodynamics using in addition

$$\partial_\mu M^{\mu\nu\lambda} = 0 \qquad M^{\mu\nu\lambda} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}$$

$$T^{\mu\nu}_{viscous} = (e + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

  - Mathematically equivalent to imposing smoothness condition on all components of $T_{\mu\nu}$.

- Numerical solution of the matching:



- Tilting and odd flow terms translate into hydrodynamics fields.

Effect on Particle Spectra

- Need to run viscous 3+1-D hydro with large viscous corrections.
- Viscous freeze-out.

- Nothing to show yet, but work in progress.

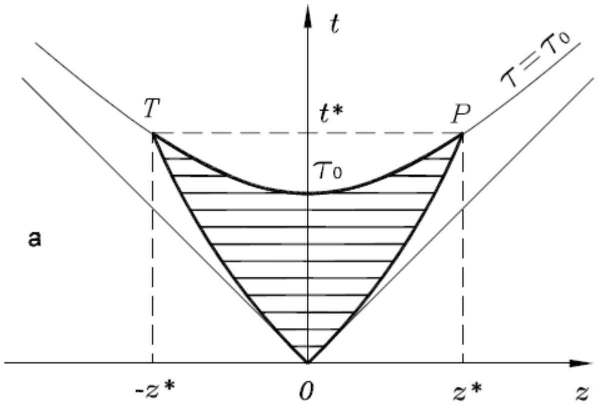
Summary

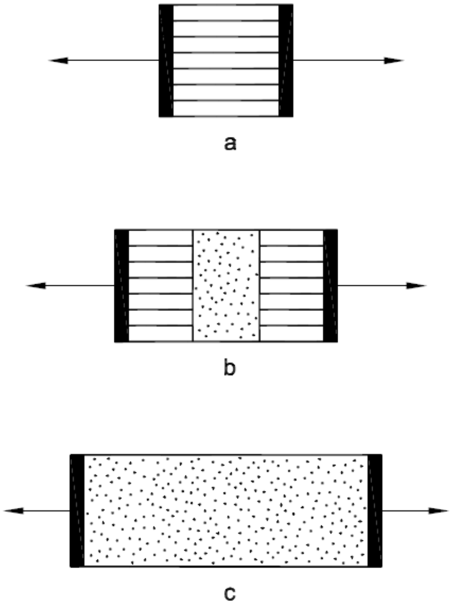
- We can calculate the fields and energy momentum tensor in the clQCD approximation for the early stage of high energy nuclear collisions.
- Transverse energy flow shows interesting and unique (?) features: directed flow, A+B asymmetries.
- These features are usually not included in hydrodynamic simulations. We find that key features easily translate into hydrodynamic fields in simple thermalization models.
- Phenomenology needs hydrodynamics with large dissipative corrections.

Backup

Space-Time Picture

- Finally: field has decayed into plasma at $\tau = \tau_0$





- Energy is taken from deceleration of the nuclei in the color field.
- Full energy momentum conservation: $f^{\nu} = -\partial_{\mu} T_{\mathrm{f}}^{\mu\nu}$

Space-Time Picture

- Deceleration: obtain positions η* and rapidities y* of the baryons at $\tau = \tau_0$

$$\cosh y^* = \cosh y_0 \left[1 - 2a\left(v_0\sqrt{1+a} - a\right)\right]$$

$$a = \frac{\tau_0 \langle \varepsilon_f \rangle}{\rho_m}$$

  - For given initial beam rapidity y_0, mass area density ρ_m.

[Kapusta, Mishustin]

- BRAHMS:
  - $dy = 2.0 \pm 0.4$
  - Nucleon: 100 GeV → 27 GeV
  - We conclude:

$$\tau_0 \langle \varepsilon_f \rangle \approx 9\,\mathrm{GeV/fm}^2$$

[RF]