Wilson loop correlators on the lattice and the asymptotic high-energy behaviour of hadronic total cross sections

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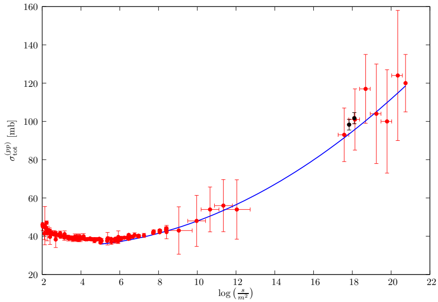
Based on work in collaboration with
E. Meggiolaro and N. Moretti

Rising Total Cross Sections



Experimental data support

$$\sigma_{tot}^{(hh)}(s) \sim B \log^2 s$$

with **universal** $B \simeq 0.3\,\mathrm{mb}$, independent of the colliding hadrons

(fit from [PDG 2012])

- $\longrightarrow \quad \sqrt{s} = 7, 8\,\mathrm{TeV}$ [TOTEM 2013] talk by J. Kašpar

Consistent with Froissart bound [Froissart (1961)] (unitarity + mass gap)

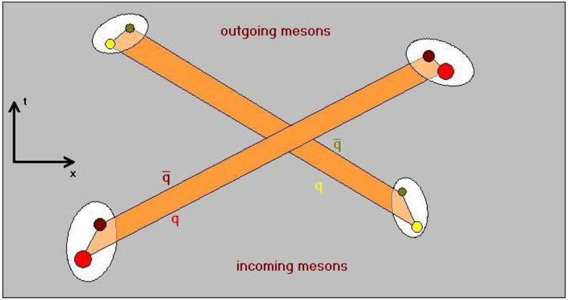
$$\sigma_{tot}^{(hh)}(s) \leq \frac{\pi}{m_\pi^2} \log^2\left(\frac{s}{s_0}\right)$$

Derivation of $\sigma_{tot}^{(hh)}(s)$ from first principles of QCD still lacking

Soft High-Energy Scattering and Non-Perturbative QCD

Two different energy scales, $\sqrt{s} \to \infty$ and $\sqrt{|t|} \lesssim 1\mathrm{GeV}$: NP approach

- Partonic scattering amplitudes from the correlation function of infinite lightlike Wilson lines, hadronic amplitudes obtained after folding with hadronic wave functions [Nachtmann (1991)]
- Partonic amplitudes are IR divergent $\to$ hadronic amplitudes: mesons as wave packets of transverse colourless dipoles [Dosch *et al.* (1996)]



Meson-Meson (Dipole-Dipole) Scattering

Elastic meson-meson from dipole-dipole scattering [Dosch *et al.* (1996)]

$$\mathcal{M}^{(hh)}(s,t) = \langle\!\langle \mathcal{M}^{(dd)}(s,t;\nu_1,\nu_2)\rangle\!\rangle$$

$\nu_i = (f_i, \vec{R}_{i\perp})$, f_i longitudinal mom. frac., $\vec{R}_{i\perp}$ transverse size

$$\langle\!\langle f \rangle\!\rangle = \int_0^1 df_1 \int d^2\vec{R}_{1\perp}\, |\psi_1(\nu_1)|^2 \int_0^1 df_2 \int d^2\vec{R}_{2\perp}\, |\psi_2(\nu_2)|^2\, f(\nu_1,\nu_2)$$

Dipole-dipole scattering amplitude

$$\mathcal{M}^{(dd)}(s,t;\nu_1,\nu_2) = \lim_{\chi\to\infty} -i\; 2s \int d^2\vec{b}_\perp e^{i\vec{q}_\perp\cdot\vec{b}_\perp} \mathcal{C}_M(\chi;\vec{b}_\perp,\nu_1,\nu_2)$$

$$\chi \underset{s\to\infty}{\simeq} \log\frac{s}{m^2},\quad t = -\vec{q}_\perp^{\,2}$$

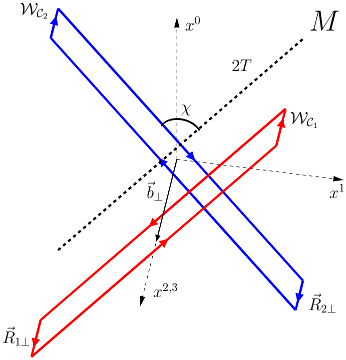
Wilson-loop correlation function ($\approx \mathcal{M}^{(dd)}$ in impact-parameter space)

$$\mathcal{G}_M(\chi;T;\vec{b}_\perp,\nu_1,\nu_2) \equiv \frac{\langle \mathcal{W}_{\mathcal{C}_1}\mathcal{W}_{\mathcal{C}_2}\rangle}{\langle \mathcal{W}_{\mathcal{C}_1}\rangle\langle \mathcal{W}_{\mathcal{C}_2}\rangle} - 1\,, \qquad \mathcal{C}_M \equiv \lim_{T\to\infty} \mathcal{G}_M$$

Analytic Continuation to Euclidean Space

NP techniques available in Euclidean space $\Rightarrow$ Euclidean formulation
[Meggiolaro (1997), Meggiolaro (2005)]

$$\mathcal{G}_E(\theta; T; \vec{b}_\perp, \nu_1, \nu_2) \equiv \frac{\langle \mathcal{W}_{\mathcal{C}_1} \mathcal{W}_{\mathcal{C}_2} \rangle}{\langle \mathcal{W}_{\mathcal{C}_1} \rangle \langle \mathcal{W}_{\mathcal{C}_2} \rangle} - 1 \,, \qquad \mathcal{C}_E \equiv \lim_{T\to\infty} \mathcal{G}_E$$



Analytic continuation relations
[Meggiolaro (2005), MG, Meggiolaro (2009)]

$$\mathcal{C}_M(\chi) = \mathcal{C}_E(\theta \to -i\chi)$$

AC + Euclidean symmetries $\Rightarrow$
crossing relations [MG, Meggiolaro (2006)]

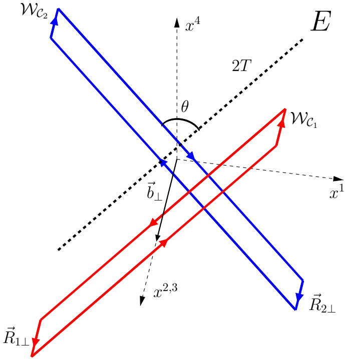
$$\mathcal{C}_M(i\pi - \chi; \nu_1, \nu_2) = \mathcal{C}_M(\chi; \nu_1, \bar{\nu}_2) = \mathcal{C}_M(\chi; \bar{\nu}_1, \nu_2)$$

$$\bar{\nu}_i = (1 - f_i, -\vec{R}_{i\perp})$$

Analytic Continuation to Euclidean Space

NP techniques available in Euclidean space ⇒ Euclidean formulation
[Meggiolaro (1997), Meggiolaro (2005)]

$$\mathcal{G}_E(\theta; T; \vec{b}_\perp, \nu_1, \nu_2) \equiv \frac{\langle \mathcal{W}_{\mathcal{C}_1} \mathcal{W}_{\mathcal{C}_2} \rangle}{\langle \mathcal{W}_{\mathcal{C}_1} \rangle \langle \mathcal{W}_{\mathcal{C}_2} \rangle} - 1 \,, \qquad \mathcal{C}_E \equiv \lim_{T\to\infty} \mathcal{G}_E$$



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$$\mathcal{C}_M(\chi; \bar{\nu}_1, \nu_2)$$
$$\bar{\nu}_i = (1 - f_i, -\vec{R}_{i\perp})$$

Wilson Loop Correlator on the Lattice

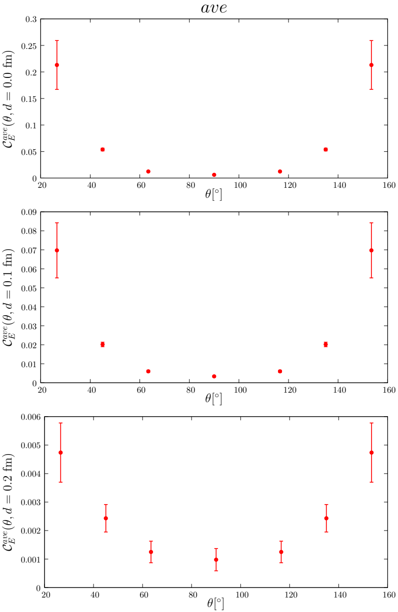
Euclidean formulation opens the way to NP techniques:

- Stochastic Vacuum Model [Berger, Nachtmann (1999), Shoshi *et al.* (2003)]
- Instanton Liquid Model [Shuryak, Zahed (2000), MG, Meggiolaro (2010)]
- AdS/CFT Correspondence [Janik, Peschanski (2000a,b), MG, Peschanski (2010)]
- Lattice Gauge Theory [MG, Meggiolaro (2008), MG, Meggiolaro (2010)]

Lattice calculation of the correlator gives first-principles "true" prediction of QCD (within errors) $\Rightarrow$ analytic NP calculations have to be compared to lattice results, in order to test the goodness of the approximations involved

- Compare data with numerical predictions of the various models
- Fit lattice data with model functions

Lattice Calculations: Setup and (Some) Results



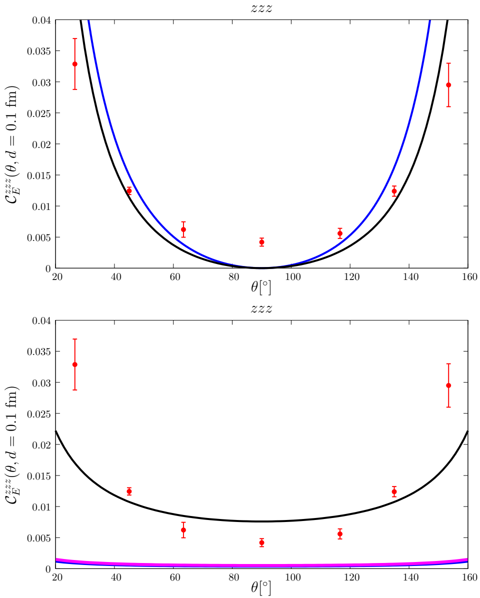
Simulations

- Wilson action for $SU(3)$ gauge theory (*quenched* QCD)
- 16^4 hypercubic lattice, periodic bc
- $\beta = 6.0 \to a \simeq 0.1\,\mathrm{fm}$
- 30000 measurements

Wilson loop configurations

- longitudinal plane
  - angles: $\cot\theta = 0, \pm 1/2, \pm 1, \pm 2$
- transverse plane
  - transverse size $= 1a$
  - transverse distance $= 0, 1, 2a$
  - "zzz": $\vec{d}_\perp \parallel \vec{r}_{1\perp} \parallel \vec{r}_{2\perp}$
  - "zyy": $\vec{d}_\perp \perp \vec{r}_{1\perp} \parallel \vec{r}_{2\perp}$
  - "ave": average over orientations ($\to$ meson-meson scattering)
- limit $T \to \infty$ $\to$ longest loops

NP models and Lattice Results



Are the analytic NP calculations compatible with the lattice results?

Comparison of numerical predictions/fits from SVM/ILM to lattice data not satisfactory

$$\mathcal{C}_E^{(\mathrm{SVM})} = \frac{2}{3} e^{-\frac{1}{3}\cot\theta\, K_{\mathrm{SVM}}} + \frac{1}{3} e^{\frac{2}{3}\cot\theta\, K_{\mathrm{SVM}}} - 1$$

$$\mathcal{C}_E^{(\mathrm{ILM})} = \frac{K_{\mathrm{ILM}}}{\sin\theta}$$

SVM, ILM do not lead to rising total cross sections:

$$\sigma_{tot} \underset{s\to\infty}{\longrightarrow} \mathrm{const.}$$

Lattice Results and Rising Total Cross Sections

Are the lattice results compatible with rising total cross sections?

- Fits to more general functions can be performed, but care is needed because of the analytic continuation
- Admissible fitting functions are constrained by physical requirements (unitarity, crossing symmetry,...)

Look for a parameterisation of the lattice data that

1. fits well the numerical results
2. satisfies unitarity after analytic continuation

   unitarity constraint: $|A(s,|\vec{b}_\perp|)+1| = |\langle\!\langle \mathcal{C}_M(\chi;\vec{b}_\perp,\nu_1,\nu_2)+1\rangle\!\rangle| \le 1$

   sufficient condition: $|\mathcal{C}_M(\chi;\vec{b}_\perp,\nu_1,\nu_2)+1| \le 1 \qquad \forall \vec{b}_\perp,\nu_1,\nu_2$

3. leads to rising total cross sections at high energy

Exponential Form of the Correlator

Assumption: $\mathcal{C}_E = \exp\{K_E\} - 1$

$K_E \in \mathbb{R}$ since $\mathcal{C}_E \in \mathbb{R}$

Well justified assumption: true at large-N_c, satisfied by known models, true at large impact parameter, confirmed by lattice data

In QCD we expect $\mathcal{C}_E \sim (\sum) e^{-\mu|\vec{b}_\perp|}$ at large $|\vec{b}_\perp| \Rightarrow K_E \sim (\sum) e^{-\mu|\vec{b}_\perp|}$

Various "natural" possibilities for the mass scale μ: glueball masses, inverse vacuum correlation length...

After AC, $\mathcal{C}_M = \exp\{K_M\} - 1 \Rightarrow$ unitarity condition $\mathrm{Re}\, K_M \le 0$

Does $|K_M| \to \infty$ for $\chi \to \infty$ lead to rising $\sigma_{\rm tot}$? $K_M \sim s^n\, e^{-\mu|\vec{b}_\perp|}$

MG, Meggiolaro, Moretti, *JHEP* **1209** (2012) 031

How a Froissart-like Total Cross Section Can Be Obtained

Assume for $\chi, |\vec{b}_\perp| \to \infty$

$$\mathcal{C}_M = \exp\{K_M\} - 1 \sim \exp\left(i\,\beta\,e^{\eta}\,e^{-\mu|\vec{b}_\perp|}\right) - 1$$

$\beta = \beta(\nu_1, \nu_2)$, $\mathrm{Im}\,\beta \geq 0$, $\eta(\chi) \in \mathbb{R}$, $\eta(\chi \to \infty) \to \infty$

Using the optical theorem ▸ details

$$\sigma_{\rm tot}^{(hh)} \simeq 4\pi \int_0^\infty dbb\,[1 - \langle\langle e^{i\beta e^{\eta-\mu b}}\rangle\rangle] \sim \frac{2\pi}{\mu^2}\langle\langle \eta^2 \rangle\rangle = \frac{2\pi}{\mu^2}\eta^2$$

For $e^{\eta} = \chi^p e^{n\chi} \sim (\log s)^p s^n \Rightarrow \quad \sigma_{\rm tot}^{(hh)} \sim B \log^2 s \qquad B = \frac{2\pi n^2}{\mu^2}$

- B is **universal**, independent of mesonic wave functions and masses
- B is unaffected by the small-$|\vec{b}_\perp|$ behaviour due to unitarity

New Analysis of the Lattice Data

Fit the data with functional forms that

- satisfy unitarity after analytic continuation
- lead to rising total cross sections

Use averaged correlators, "closer" to meson-meson amplitude in b-space

$$\mathcal{C}^{ave}_{E,M} = < \mathcal{C}_{E,M} > = < \exp\{K_{E,M}\} > \; - 1 = \exp\{K^{ave}_{E,M}\} - 1$$

$< \bullet > = \int d^2\hat{R}_{1\perp} \int d^2\hat{R}_{2\perp} \bullet$ is a positive and normalised measure

$K^{ave}_{M}(\chi) = K^{ave}_{E}(\theta = -i\chi)$

Constraints:

1. Unitarity: $\mathrm{Re}\, K^{ave}_{M} \leq 0$
2. $\mathcal{C}^{ave}_{E}(\pi - \theta) = \mathcal{C}^{ave}_{E}(\theta)$ by construction: only C-even (Pomeron) contributions to $\mathcal{M}^{(hh)}$

Parameterisation I

First strategy: combine known QCD results and variations thereof

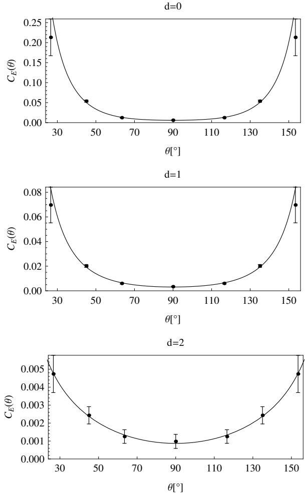
Example: exponentiate two-gluon exchange and one-instanton contribution, plus a term that can yield a rising $\sigma_{\rm tot}$

$$K_E = \frac{K_1}{\sin\theta} + K_2 \cot^2\theta + K_3 \cos\theta \cot\theta$$

$$K_M = i\frac{K_1}{\sinh\chi} + iK_3 \cosh\chi \coth\chi - K_2 \coth^2\chi$$

Unitarity condition: $K_2 \geq 0$ (satisfied within errors)

Leading term: $K_3 \cos\theta \cot\theta \to iK_3 \frac{e^\chi}{2}$



▸ fit parameters

Parameterisation II

Second strategy: adapt to QCD results obtained in related models

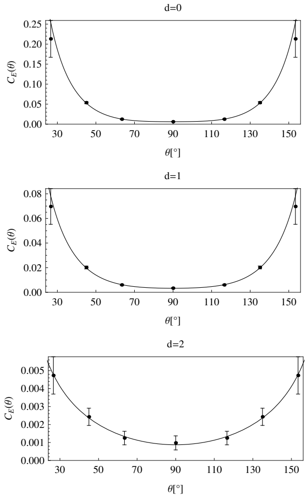
Example: AdS/CFT expression, plus $\theta \cot\theta$ term to make the expression crossing-even

$$K_E = \frac{K_1}{\sin\theta} + K_2\left(\frac{\pi}{2} - \theta\right)\cot\theta + K_3\cos\theta\cot\theta$$

$$K_M = i\frac{K_1}{\sinh\chi} + iK_2\frac{\pi}{2}\coth\chi + iK_3\cosh\chi\coth\chi - \chi K_2\coth\chi$$

Unitarity condition: $K_2 \geq 0$ (satisfied within errors)

Leading term: $K_3\cos\theta\cot\theta \to iK_3\frac{e^{\chi}}{2}$



fit parameters

Parameterisation III

Our best parameterisation (out of ~ 70):

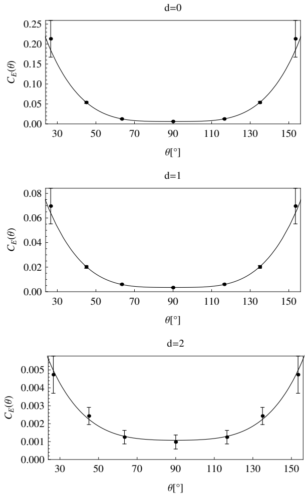
Exponentiate one-instanton contribution, plus a term that can yield a rising cross section

$$K_E = \frac{K_1}{\sin\theta} + K_2\left(\tfrac{\pi}{2} - \theta\right)^3 \cos\theta$$

$$K_M = i\,\frac{K_1}{\sinh\chi} + iK_2 \cosh\chi\left(\tfrac{3}{4}\pi^2\chi - \chi^3\right) + K_2 \cosh\chi\left(\tfrac{\pi^3}{8} - \tfrac{3}{2}\pi\chi^2\right)$$

Unitarity condition: $K_2 \geq 0$ (satisfied within errors)

Leading term: $K_2(\frac{\pi}{2} - \theta)^3 \cos\theta \to -iK_2\chi^3 \frac{e^\chi}{2}$



fit parameters

Remarks

- **Universal** total cross section $\sigma_{\rm tot}^{(hh)} \sim B \log^2 s$ in the three cases
- Estimate of B through a fit of the coefficient of the leading term with an exponential: fair agreement with experimental value $B_{\rm exp} \simeq 0.3\mathrm{mb}$

	μ (GeV)	$\lambda = \frac{1}{\mu}$ (fm)	$B = \frac{2\pi}{\mu^2}$ (mb)
Corr 1	4.64(2.38)	$0.042^{+0.045}_{-0.014}$	$0.113^{+0.364}_{-0.037}$
Corr 2	3.79(1.46)	$0.052^{+0.032}_{-0.014}$	$0.170^{+0.277}_{-0.081}$
Corr 3	3.18(0.98)	$0.062^{+0.028}_{-0.015}$	$0.245^{+0.263}_{-0.100}$

- Experimental data available for baryon-baryon and meson-baryon: Wilson-loop approach extends to baryons adopting a quark-diquark picture, analysis carries over unchanged
- "Quenched" data: does $\sigma_{\rm tot}^{(hh)}$ change including dynamical fermions?

Conclusions and Outlook

- We have provided a framework to investigate the issue of total cross sections on the lattice by means of numerical simulations
- We have found parameterisations of lattice data yielding rising $\sigma_{\rm tot}$
- Rather good comparison of our results with experiments, even though errors are quite large

Open issues:

- Analytical: better understanding of the parameterisations, identification of the relevant mass scale μ from QCD?
- Numerical: inclusion of fermion effects, larger distances, more angles,. . .

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$\beta = \beta(\nu_1, \nu_2)$, $\mathrm{Im}\,\beta \geq 0$, $\eta(\chi) \in \mathbb{R}$, $\eta(\chi \to \infty) \to \infty$

Optical theorem $\quad \sigma_{\mathrm{tot}}^{(hh)} \sim \frac{4\pi}{\mu^2}\mathrm{Re}\,\langle\langle J(\eta,\beta)\rangle\rangle \sim \frac{2\pi}{\mu^2}\eta^2$

$$J(\eta,\beta) = \mu^2 \int_0^\infty dbb\,[1 - e^{i\beta e^{\eta-\mu b}}] = \int_0^{e^\eta} \frac{dz}{z} \log\left(\frac{e^\eta}{z}\right)[1 - e^{i\beta z}]$$

$$= \frac{1}{2}\eta^2 - \underbrace{\int_1^{e^\eta} \frac{dz}{z} \log\left(\frac{e^\eta}{z}\right) e^{i\beta z}}_{\mathcal{O}(\eta)} + \underbrace{\int_0^1 \frac{dz}{z} \log\left(\frac{e^\eta}{z}\right)[1 - e^{i\beta z}]}_{\mathcal{O}(\eta)}$$

back

Fit Parameters

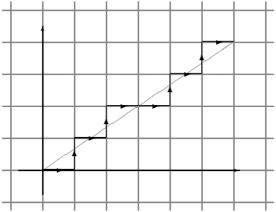
▸ back 1 ▸ back 2 ▸ back 3

Corr 1	$d=0$	$d=1$	$d=2$
K_1	$5.85(42)\cdot 10^{-3}$	$3.07(37)\cdot 10^{-3}$	$8.7(3.1)\cdot 10^{-4}$
K_2	$9.60(98)\cdot 10^{-2}$	$2.44(49)\cdot 10^{-2}$	$-5.3(84.5)\cdot 10^{-5}$
K_3	$-7.8(1.3)\cdot 10^{-2}$	$-1.37(72)\cdot 10^{-2}$	$1.7(1.9)\cdot 10^{-3}$
$\chi^2_{\text{d.o.f.}}$	2.81	1.25	0.05
Corr 2	$d=0$	$d=1$	$d=2$
K_1	$6.03(42)\cdot 10^{-3}$	$3.26(38)\cdot 10^{-3}$	$8.7(3.2)\cdot 10^{-4}$
K_2	$4.63(46)\cdot 10^{-1}$	$1.33(25)\cdot 10^{-1}$	$-1.2(54.2)\cdot 10^{-4}$
K_3	$-4.54(50)\cdot 10^{-1}$	$-1.26(28)\cdot 10^{-1}$	$1.7(6.7)\cdot 10^{-3}$
$\chi^2_{\text{d.o.f.}}$	0.55	0.31	0.05
Corr 3	$d=0$	$d=1$	$d=2$
K_1	$6.02(36)\cdot 10^{-3}$	$3.46(29)\cdot 10^{-3}$	$1.07(20)\cdot 10^{-3}$
K_2	$1.29(5)\cdot 10^{-1}$	$4.47(27)\cdot 10^{-2}$	$2.11(73)\cdot 10^{-3}$
$\chi^2_{\text{d.o.f.}}$	0.17	0.11	0.10

Table: Parameters (with their errors) for the Correlators 1, 2, and 3, obtained from best fits to the averaged lattice data, and the corresponding $\chi^2_{\text{d.o.f.}}$, for the transverse distances $d = 0, 1, 2$.

Loop Construction

Rotation invariance breaking $\rightarrow$ approximation for tilted Wilson loops



Bresenham prescription: lattice path that minimizes the distance from the continuum path

$\mathcal{W}_L(\vec{l}_{\parallel}, \vec{r}_{\perp}; n)$: center in n, sides l ($\parallel$ plane) and r ($\perp$ plane)

Lattice Wilson-loop correlators

$$\mathcal{G}_L(\vec{l}_{1\parallel}, \vec{l}_{2\parallel}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp}) = \frac{\langle \mathcal{W}_L(\vec{l}_{1\parallel}, \vec{r}_{1\perp}; d)\mathcal{W}_L(\vec{l}_{2\parallel}, \vec{r}_{2\perp}; 0)\rangle}{\langle \mathcal{W}_L(\vec{l}_{1\parallel}, \vec{r}_{1\perp}; d)\rangle\langle \mathcal{W}_L(\vec{l}_{2\parallel}, \vec{r}_{2\perp}; 0)\rangle} - 1$$

$d = (0, 0, \vec{d}_{\perp})$: transverse distance

Rotation invariance restored in the continuum limit

$$\mathcal{G}_L(\vec{l}_{1\parallel}, \vec{l}_{2\parallel}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp}) \underset{a\to 0}{\simeq} \mathcal{G}_E(\theta; aL_1, aL_2; a\vec{d}_{\perp}, a\vec{r}_{1\perp}, a\vec{r}_{2\perp}) + \mathcal{O}(a)$$

$2L_i = |\vec{l}_{i\parallel}|$: length

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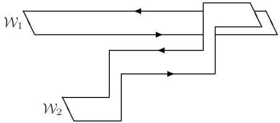
Rotation invariance breaking → approximation for tilted Wilson loops



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Rotation invariance restored in the continuum limit

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$2L_i = |\vec{l}_{i\parallel}|$: length

NP models and Lattice Results

Analytic results

Stochastic Vacuum Model (SVM)	$\mathcal{C}_E = \frac{2}{3} e^{-\frac{1}{3}\cot\theta\, K_{\rm SVM}} + \frac{1}{3} e^{\frac{2}{3}\cot\theta\, K_{\rm SVM}} - 1$
Perturbation Theory (PT)	$\mathcal{C}_E = K_{\rm PT}\cot^2\theta$
Instanton Liquid Model (ILM)	$\mathcal{C}_E = \frac{K_{\rm ILM}}{\sin\theta}$
ILM + PT (ILMp)	$\mathcal{C}_E = \frac{K_{\rm ILMp}}{\sin\theta} + K'_{\rm ILMp}(\cot\theta)^2$
AdS/CFT correspondence	$\mathcal{C}_E = e^{\frac{K_{\rm AdS}}{\sin\theta} + K'_{\rm AdS}\cot\theta + K''_{\rm AdS}\cos\theta\cot\theta} - 1$

Are the analytic NP calculations compatible with the lattice results?

- Comparison of SVM and ILM to lattice data is not satisfactory
- SVM, ILM do not lead to rising total cross sections: $\sigma_{tot} \underset{s\to\infty}{\longrightarrow} \mathrm{const.}$
- ILMp gives improved best fits, but still does not give a rising σ_{tot}
- AdS/CFT: $\sigma_{tot} \propto s^{\frac{1}{3}}$ for onium-onium scattering in $\mathcal{N} = 4$ SYM

[MG, Peschanski (2010)]

A Nontrivial Example: Onium Scattering in $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM: replace mesons with "onia", wave packets of colourless dipoles, and describe "onium-onium" scattering in terms of dipoles

Large N_c, strong coupling: AdS/CFT correspondence [Maldacena (1998)]

$\mathcal{C}_E$ at large $b = |\vec{b}_\perp|$ from a supergravity calculation [Janik, Peschanski (2000a)]

$$\mathcal{C}_E^{(\mathrm{AdS/CFT})} = \exp\left\{ [K_S + K_D] \frac{1}{\sin\theta} + K_B \cot\theta + K_G \frac{(\cos\theta)^2}{\sin\theta} \right\} - 1$$

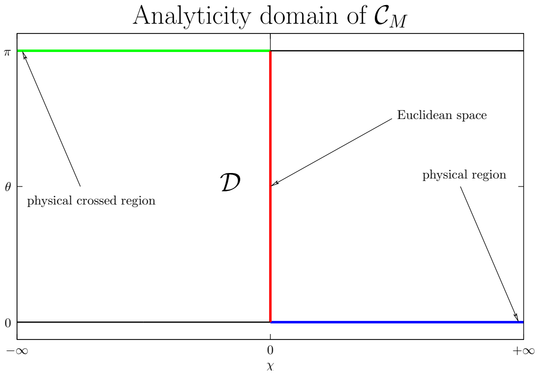
$K_X = K_X(b) \sim$ exchange of supergravity field X between the string worldsheets

At large b, $K_G(b) \sim \frac{\bar{K}_G}{b^6}$; after $\theta \to -i\chi$, $\chi \to \infty$,

$$\mathcal{C}_M^{(\mathrm{AdS/CFT})} \sim \exp\left\{ \frac{i\bar{K}_G}{b^6} \frac{e^\chi}{2} \right\} - 1 \to \sigma_{tot} \propto s^{\frac{1}{3}}$$

Rising total cross section for "onium-onium" scattering [MG, Peschanski (2010)]

Analytic Continuation to Euclidean Space



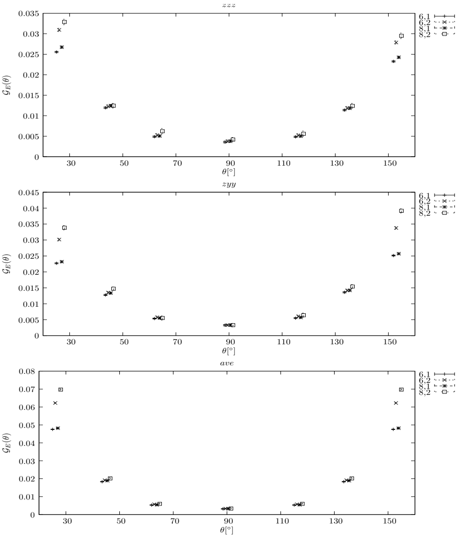
Analytic continuation relations [Meggiolaro (2005), MG, Meggiolaro (2009)]

$$\mathcal{G}_M(\chi; T) = \mathcal{G}_E(\theta \to -i\chi; T \to iT)\,, \quad \mathcal{C}_M(\chi) = \mathcal{C}_E(\theta \to -i\chi)$$

AC + Euclidean symmetries $\Rightarrow$ crossing relations [MG, Meggiolaro (2006)]

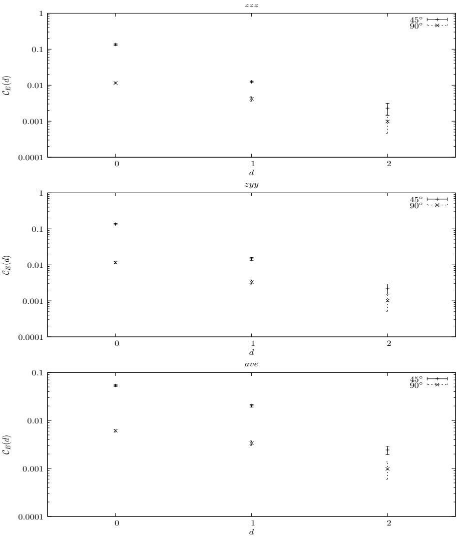
$$\mathcal{G}_M(i\pi - \chi; \vec{R}_{1\perp}, \vec{R}_{2\perp}) = \mathcal{G}_M(\chi; \vec{R}_{1\perp}, -\vec{R}_{2\perp}) = \mathcal{G}_M(\chi; -\vec{R}_{1\perp}, \vec{R}_{2\perp})$$

Angular Dependence



- $\mathcal{G}_L$ against $\theta[°]$ for various L_1, L_2 and for the various configurations at $d = 1$
- Stable vs. loop lengths, stabilisation slower near $\theta = 0, \pi$ due to the relation with the static $d-d$ potential [Appelquist, Fischler (1978)]
- $\mathcal{G}_L^{ave}$ symmetric with respect to $\pi/2 \rightarrow$ sensitive only to C–even contributions
- C-odd component (Odderon) in "zzz" / "zyy" (possibly relevant to baryon-baryon scattering)

Distance Dependence



- $\mathcal{C}_L$ against d (lattice units) for $\theta = 45^\circ, 90^\circ$ for the various configurations (logarithmic scale)
- $\mathcal{C}_L$: correlator for the largest loops available ($\approx \lim_{L_{1,2}\to\infty} \mathcal{G}_L$)
- Rapid (exponential) decrease with distance
- Errors become large at $d = 2$ $\to$ "brute force" approach not viable at larger distances

Work in Progress

$$\mathcal{C}_M = \exp\{K_M\} - 1$$

- for $K_M = \sum_k K_M^{(k)} = i \sum_k \beta_k \chi^{p_k} e^{n_k \chi} \, |\vec{b}_\perp|^{\alpha_k} \, e^{-\mu_k |\vec{b}_\perp|} \Rightarrow$

$$\sigma_{\rm tot}^{(hh)} \sim B \log^2 s \,, \qquad B \to 2\pi \max_k \left(\frac{n_k}{\mu_k}\right)^2$$

- particle of mass M and spin J expected to contribute to the sum with

$$K_M^{(k)} \sim s^{J-1} e^{-M|\vec{b}_\perp|} \quad \longrightarrow \quad \mu_k = M, \quad n_k = J - 1$$