



15th International Conference on Elastic
& Diffractive Scattering
(15th "Blois Workshop")

FINLAND (SAARISELKÄ)
September, 09-13

"Models of the hadron structure and
Data of the TOTEM Collaboration

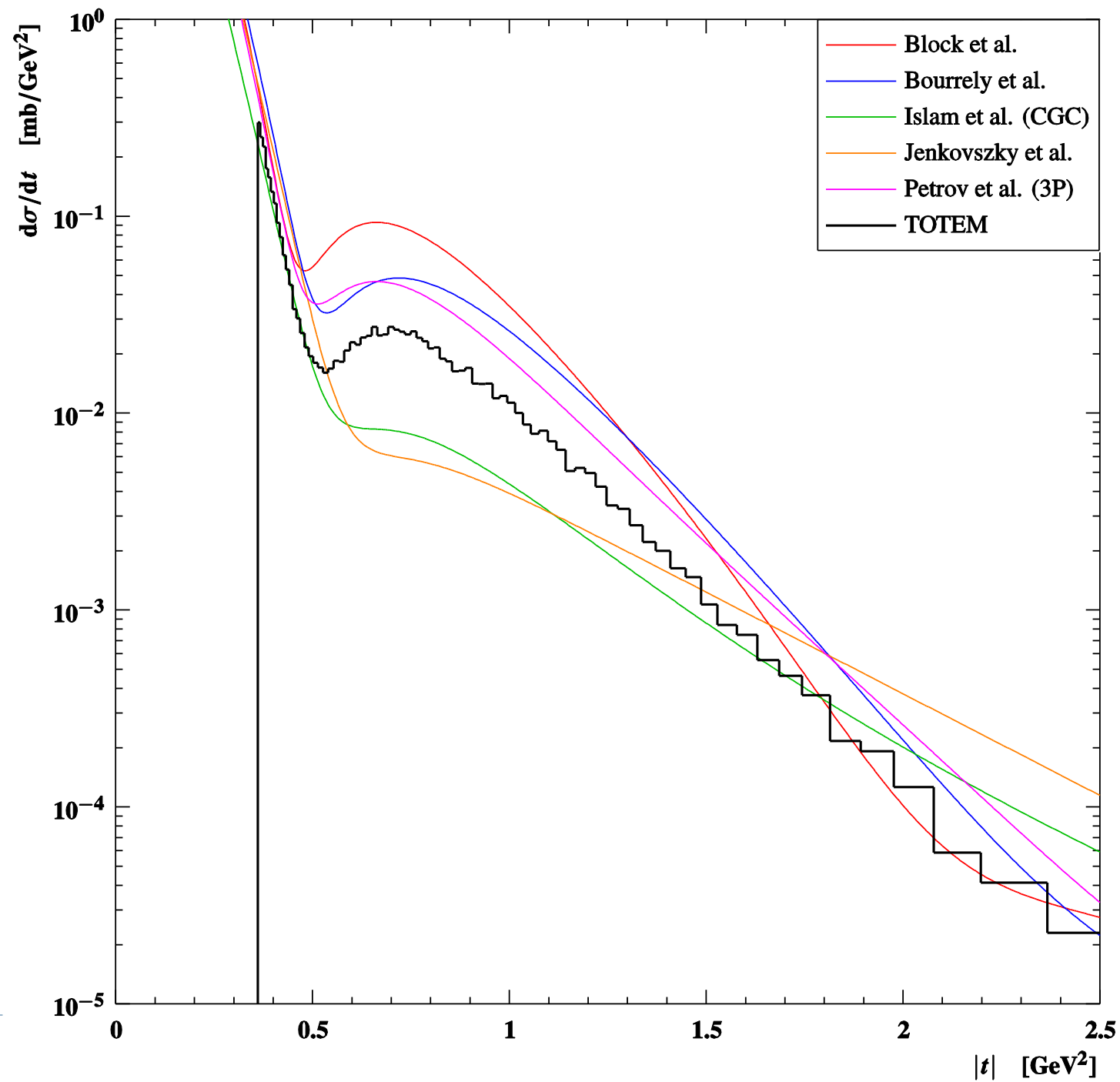
O.V. Selyugin
BLTPh, JINR



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- * Introduction
 - * Models of the high energy hadron elastic scattering
 - * GPDs and hadrons form-factors
 - * High Energy Generalized structure model (HEGS)
 - * The hard pomeron
 - * Expansion of HEGS (odderon, spin-flip)
 - * Conclusion
8. The differential cross sections

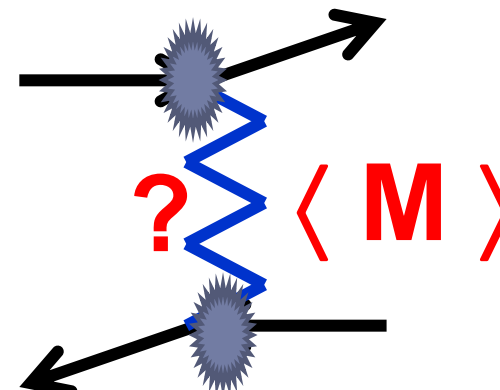




Scattering process described in terms of **Helicity Amplitudes** ϕ_i

All dynamics contained in the **Scattering Matrix** M

(Spin) Cross Sections expressed in terms of

<p>observables: 3 x-sections 5 spin asymmetries</p>	}	spin non-flip	$\phi_1(s,t) = \langle ++ M ++ \rangle$	
		double spin flip	$\phi_2(s,t) = \langle ++ M -- \rangle$	
		spin non-flip	$\phi_3(s,t) = \langle +- M +- \rangle$	
		double spin flip	$\phi_4(s,t) = \langle +- M -+ \rangle$	
		single spin flip	$\phi_5(s,t) = \langle ++ M +- \rangle = -\langle ++ M -+ \rangle$	

identical spin 1/2 particles



Pomerons

Regge theory

$$\sigma_{\text{tot}}(s) \sim$$

Landshoff 1984 – soft Π

Simple pole – $(s/s_0)^\alpha$

$$\alpha = 0.08$$

Double pole – $\text{Ln}(s/s_0)$

Triple pole – $\text{Ln}^2(s/s_0) + C$

1976 BFKL (LO) - $\Pi - \alpha_2 = 0.4$

1988 HERA data (Landshoff) – hard Π

$$\alpha_2 = 0.45$$

2005 – Kovner – $7 \Pi - \alpha_i = 0. \dots 0.4. \dots 0.8$



Impact parameters dependence

$$T(s, t) = is \int_0^{\infty} b db J_0(bq) (1 - \exp[i \chi(s, b)])$$

Saturation and non-linear equation

J.-R. Cudell, E.Predazzi, O.V. S., Phys.Rev.D79:034033,2009

J.-R. Cudell, O.S., Phys.Rev.Lett.102:032003,2009

$$\frac{dN}{dy} = (-Ln[1 - N]) (1 - N);$$

Factorization The Froissart bound

$$\chi(s, b) = h(s) f(b); \quad h(s) \approx s^{\Delta} \quad \sigma_{tot}(s) \leq a \log^2(s)$$

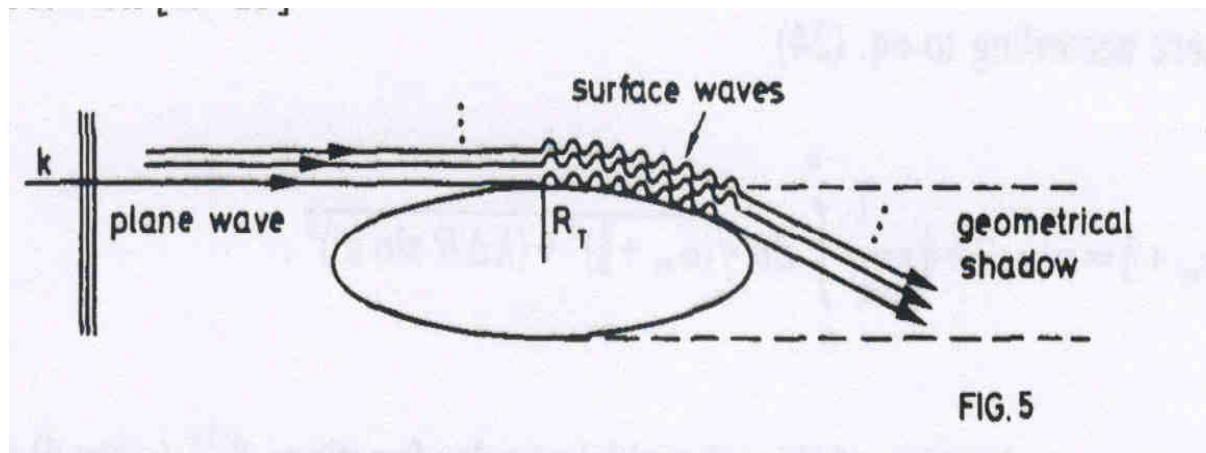
High energy elastic hadron scattering models

I) Hadron as whole

T.T.Wu, B. McCoy, “Theory “bags” with scattering”, P.R. D9 (1974);

B. Schrempp, F. Schrempp

“Strong interaction - a tunnelling phenomenon?”, N.Phys. B163 (1980))



II) Composite hadron

L. Van Hove, "The diffraction pattern of pp elastic scattering and the composite structure of the proton", N.Phys. B122 (1977)

M.Block, R. Fletcher, F. Halzen, B. Margolis, P. Valin:" Theoretical implications of Fermilab Tevatron total and elastic differential-cross –section measurements", Phys.Rev., D41 (1990).

$$\chi^{even}(s, b) = \chi_{qq}(s, b) + \chi_{qg}(s, b) + \chi_{gg}(s, b)$$

D.A. Faqundes, E.G.S. Luna, M.J. Menon, A.A. Natale, "Testing parameters in an eikonalized dynamical gluon mass model" in Nucl.Phys. A , arXiv:[1308.1206]



III) **Factorization** [Regions + Form Factors (FF)]

Chow-Yang model (1968) - hadron form factor -
Charge distributions - electromagnetic form factors

T.T. Chou, C.N. Yang, Phys.Rev.Lett. 20 (1968)

E. Leader, U. Maor, P.G. Williams, J. Kasman, Phys.Rev. D14 (1976)
“The Chou-Yang Hypothesis – A Critical Assessment”,



Form factors

Mittinen (1973) – “matter distribution”

Bourelly-Soffer-Wu (1978) -
$$G(t) = \left(\frac{1}{1 - t/m_2^2} \right) \left(\frac{1}{1 - t/m_2^2} \right) \left(\frac{a^2 + t}{a^2 - t} \right);$$

$G(t)$ – “stands for the proton “nuclear form factor” parametrized like the e-m form factor, as a two poles, the slowly varying function reflect the approximate proportionality between the charge density and hadronic matter distribution inside a proton.”

Broniowski – Arriola (2008)

“..The gravitation form factors, related to the matrix elements of the energy-momentum tensor [1] in a hadronic state and thus providing the distribution of matter within the hadron...”



General Parton Distributions (GPDs)

Sanielevici-Valin (1984) – [Valon model](#) – [Phys.Rev.D29 \(1984\)](#).

“matter form factor” (MFF) measures the interaction of a gluonic probe with the excited matter of the overlapping hadrons and should incorporate the static matter distributions of the participating hadrons...”

$$M_{AB}(s, t) = K_A(q^2) K_B(q^2) V(s, q^2);$$

$$K_p(q^2) = \frac{1}{3} \int_0^1 dx [2L_p^U(x) T_p^U(\vec{k}) + L_p^D(x) T_p^D(\vec{k})]; \quad \vec{k} = (1-x)\vec{q}.$$

$$L_p^U(x) = 7.98x^{0.65}(1-x)^2; \quad L_p^D(x) = 6.01x^{0.35}(1-x)^{2.3};$$

$$T_p^U(\vec{k}) = e^{-6.1k^2}; \quad T_p^D(\vec{k}) = e^{-3k^2}; \quad k^2 = (1-x)^2 q^2.$$



GPDs

Following to A. Radyushkin

Phys.Rev. D58, (1998) 114008

limit $Q_\gamma^2 = 0$, and $\xi = 0$

X.Ji [Sum Rules \(1997\)](#)

$$\Phi_{\xi=0}(x;t) = \Phi(x;t)$$

$$F_1^q(t) = \int_{-1}^1 dx H^q(x, \xi, t); \quad F_2^q(t) = \int_{-1}^1 dx E^q(x, \xi, t);$$

$$H^q(x;t) = H^q(x,0,t) + H^q(-x,0,t)$$

$$E^q(x;t) = E^q(x,0,t) + E^q(-x,0,t)$$

$$F_1^q(t) = \int_0^1 dx H^q(x, \xi, t); \quad F_2^q(t) = \int_0^1 dx \mathcal{E}^q(x, \xi, t);$$



H. Pagels, Phys.Rev., v.144, (1966)

“Energy-Momentum Structure Form Factors of Particles”

$$\langle p_1 | \theta_{\mu\nu}(0) | p_2 \rangle = \left(\frac{m^2}{p_0^1 p_0^2} \right)^2 \left[\frac{\bar{u}(p_1, s_1)}{4m} \right] \left[G_1(q^2)(l_\mu \gamma_\nu + l_\nu \gamma_\mu) + \frac{G_2(q^2) l_\mu l_\nu}{m} + \frac{G_3(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu)}{m} \right] u(p_2, s_2).$$

" $G_i(q^2)$ are the mechanical form factors describing the mechanical structure of the spin 1/2 system."

$$G_1(0) + G_2(0) = m \quad \text{“in analogy to the condition} \quad F_1(0) = e$$



M. Polyakov, Phys.Lett., B555 (2003);

K. Goeke, ... M. Polyakov, Phys.Rev. , D78 (2008)

“Nucleon form-factors of the energy momentum tensor in CQSM model

$$\int_{-1}^1 dx x \sum_f H^f(x, \xi, t) = M_2^{\mathcal{O}}(t) + \frac{4}{5} d_1^{\mathcal{O}}(t) \xi^2;$$

$$M_2(0) = \frac{1}{m_N} \int d^3r T_{00}(\vec{r}, \vec{s}) = 1;$$



Gravimagnetic form factor

$$\int_{-1}^1 dx \, x [H^q(x, \xi, t) + E^q(x, \xi, t)] = A_q(\Delta^2) + B_q(\Delta^2);$$

$$A^q(t) = \int_0^1 dx \, x H^q(x, t); \quad B^q(t) = \int_0^1 dx \, x \mathcal{E}^q(x, t);$$



Our ansatz

O.S., O. Terayev, Phys.Rev.D79:033003,2009
Found.Phys.40:1042,2010

1. Simplest;
2. Not far from Gaussian representation
3. Satisfy the $(1-x)^n$ ($n \geq 2$)
4. Valid for large t

$$H^q(x, t) \approx q(x) \exp\left[a_+ \frac{(1-x)^2}{x^{0.4}} t\right];$$

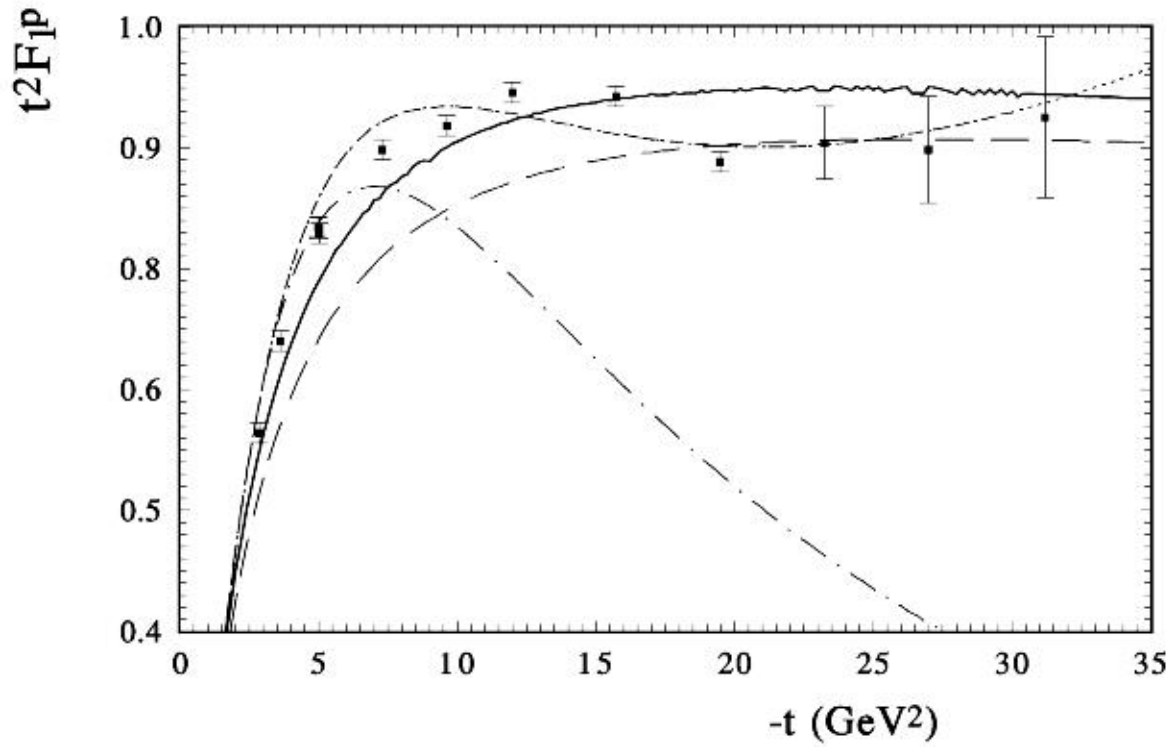
$q(x)$ is based on the MRST2002

$$u(x) = 0.262 x^{-0.69} (1-x)^{3.50} (1 + 3.83 x^{0.5} + 37.65 x);$$

$$d(x) = 0.061 x^{-0.65} (1-x)^{4.03} (1 + 49.05 x^{0.5} + 8.65 x);$$



$$F_{1p} * t^2$$



Martin – 2002 // H. KHANPOUR..-2012 (1205.5194)

$$x q_v(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} (1 + A_3 x^{A_4} + A_5 x^{A_6})$$

Pumplin et al. 2002 (CTEQ6M)

$$x q_v(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2 x} e^{A_3} (1 + e^{A_4 x} x)^{A_5}$$

Martin et al. 2002 (MRST02) .– Martin -2009 (LO, NLO, NNLO)

$$x q_v(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} (1 + A_3 x^{0.5} + A_4 x)$$

Gluck-Pisano 2008

$$x q_v(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} (1 + A_3 x^{0.5} + A_4 x + A_5 x^{1.5})$$



Alekhin et al. 2012 (ABM12)

$$x q_v(x, Q_0) = \frac{2\delta_{qu} + \delta_{qd}}{N_q^v} x^{a_q} (1-x)^{b_q} x^{\gamma_{1,q}x + \gamma_{2,q}x^2 + \gamma_{3,q}x^3}$$

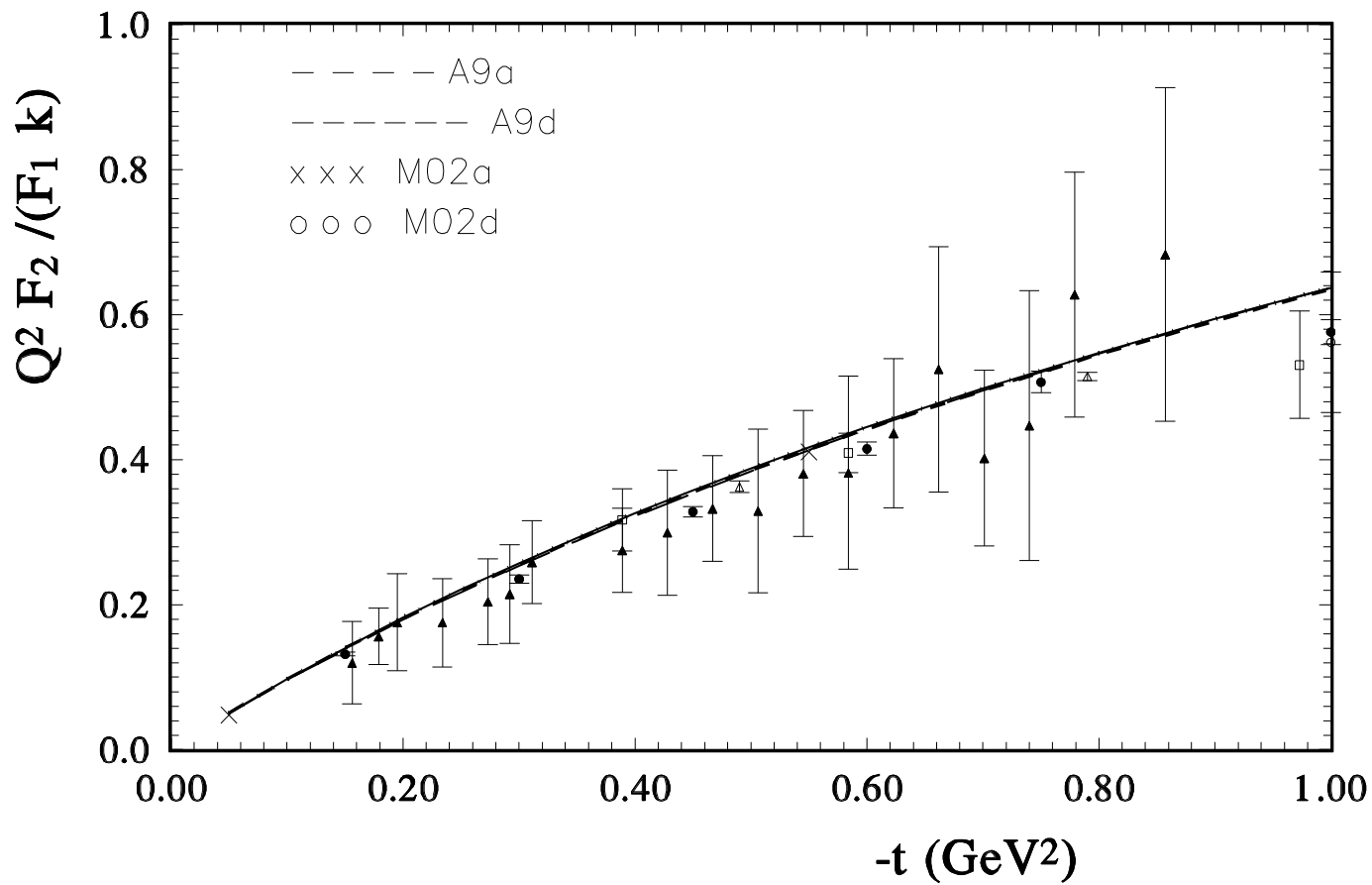
$$H^u(x, t) \square u(x) \exp\left[2\alpha_1 \left(\frac{(1-x)^{p_1}}{(x_0+x)^{p_2}} t\right)\right];$$

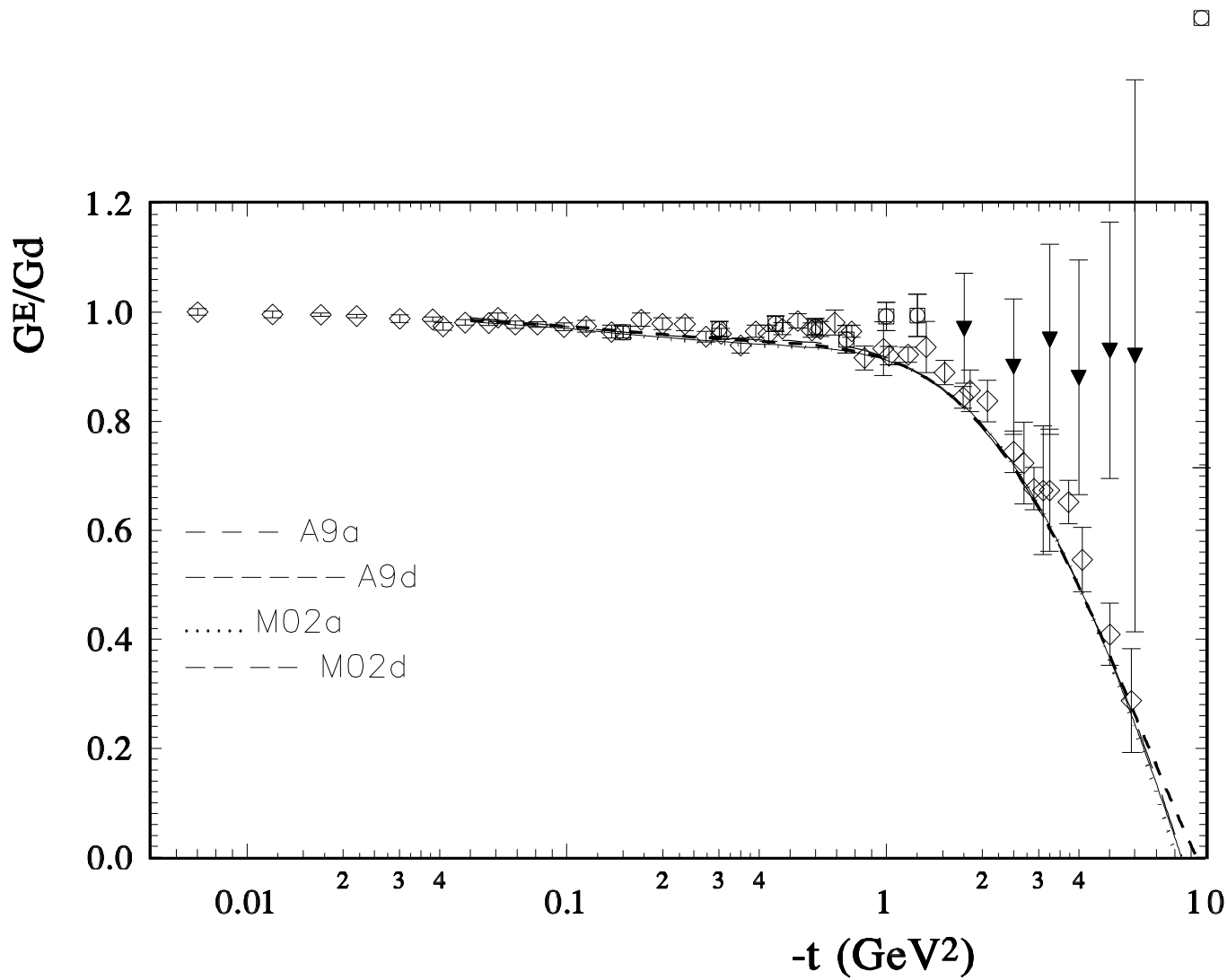
O.S., talk on Intern. Conf.
 "SPIN in High Energy Physics"
 (2012)

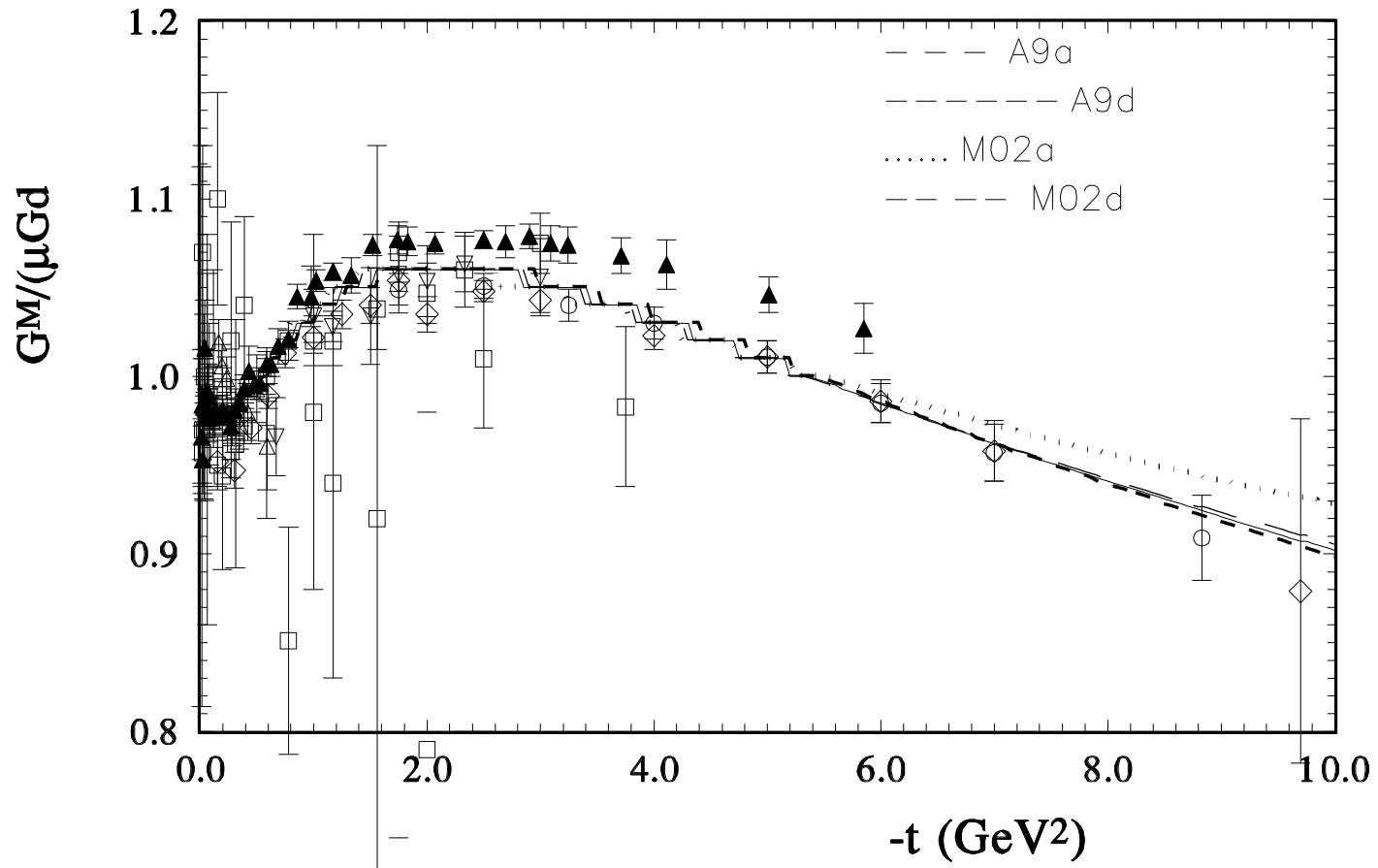
$$H^d(x, t) \square d(x) \exp\left[2\alpha_1 \left(\frac{(1-x)^{p_1(*k_d)}}{(x_0+x)^{p_2}} + d * x * (1-x)t\right)\right]$$

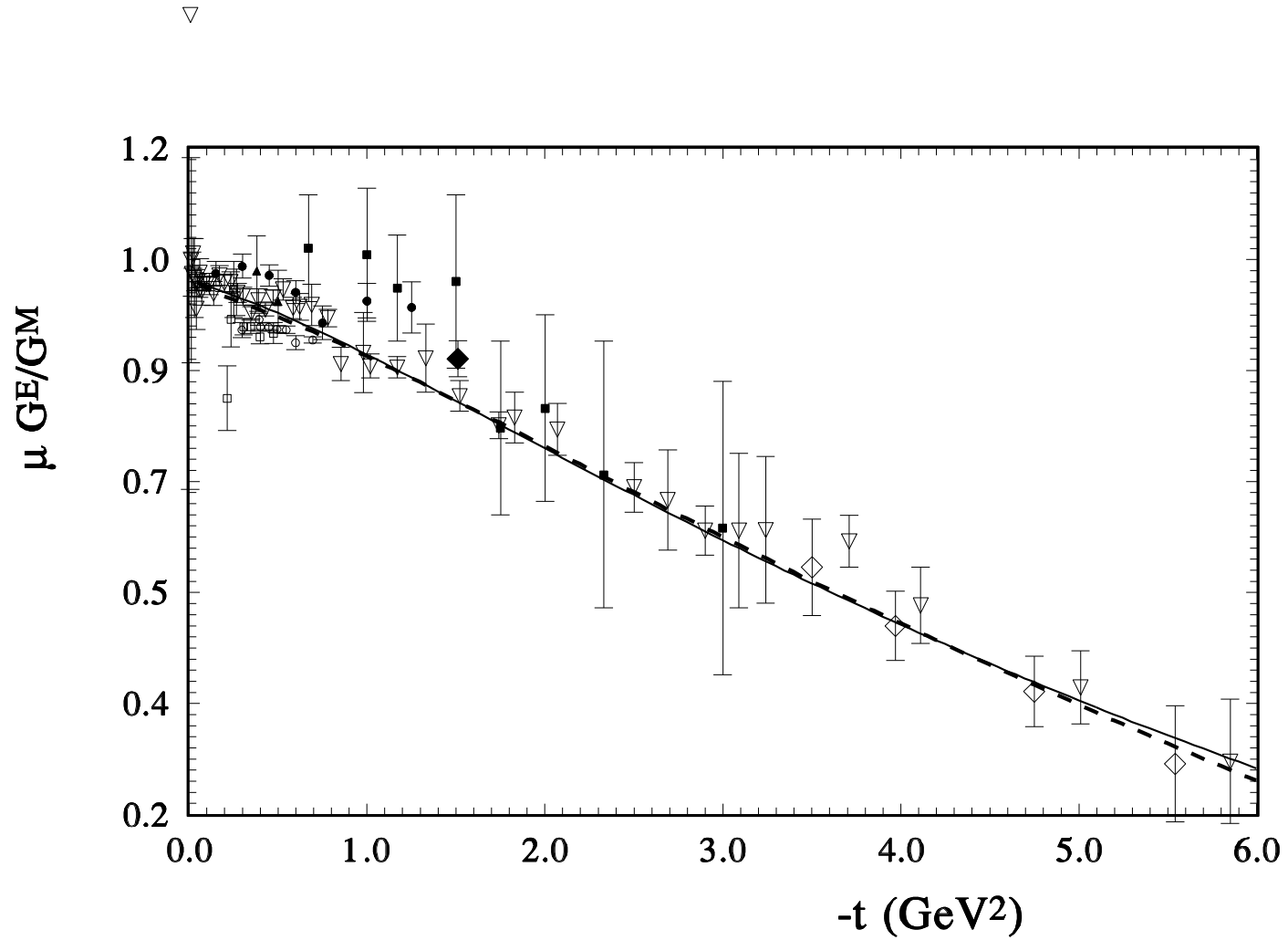
	a1	a2	a3	a4	a5	a6	a7	a8	a9
	p1	p2			du	dd	x0	k	d
AI-09	2.03	0.5	0.4	0.39	2.4	-0.14	0.006	0.41	-1.61
AI-09-0	2.-fix	0.5	0.38	0.39	2.6	0.03	0.007	1.-fix	-----
G-07-a	1.88	0.43	0.6	0.6	1.7	-0.14	0.013	0.54	-1.
G-07-a0	2.-fix	0.34	0.68	0.73	1.7	-0.14	0.003	1 - fix	-----
R4	1.89	0.51	0.5	0.43	2.6	0.5	0.01	0.35	-2.
R4-0	2.- fix	0.42	0.57	0.56	2.2	0.15	0.005	1 - fix	-----
M-09NN	1.92	0.4	0.57	0.66	0.93	-1.1	0.008	0.58	-0.61
M-09NN	2- fix	0.35	0.6	0.7	1.5	-0.5	0.003	1. fix	-----
G07NL	1.49	0.35	0.6	0.71	0.58	-1.05	0.007	0.89	0.46

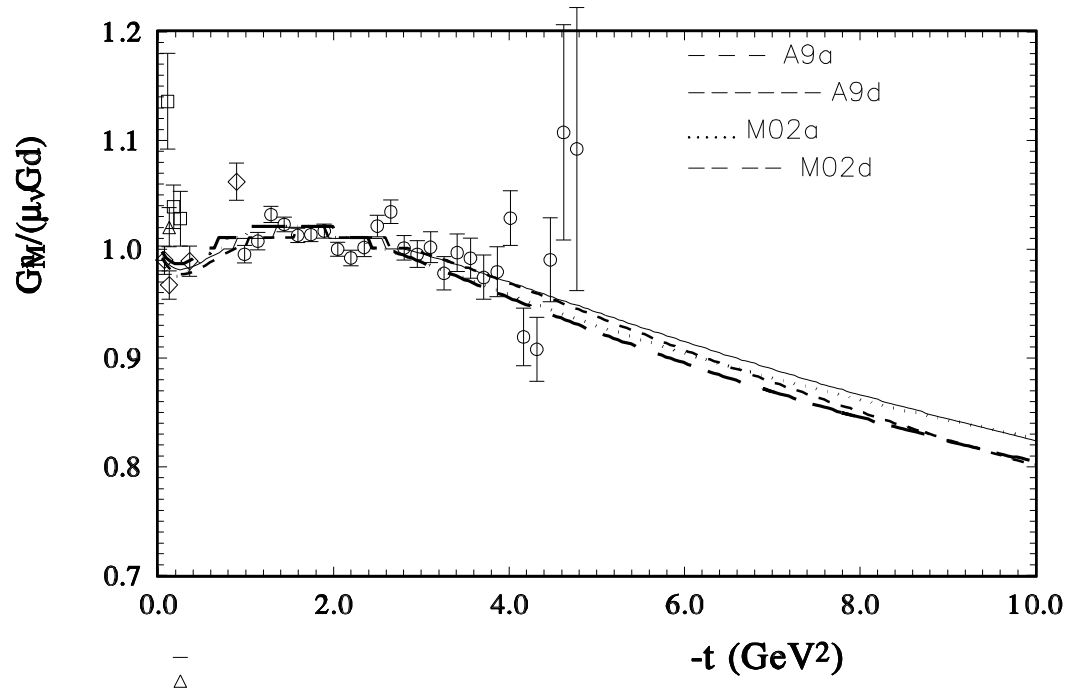
	469b	AI a8 a9	AI a8	a8	basic
1	AL-09	1152	1236	1274	1349
2	AL-12	1183	1325	1359	1384
3	KK12A	1248	1442	1497	1500
4	KK12B	1242	1424	1424	1436
5	R4	1245	1293	1306	1419
6	M09-Lo	1304	1384	1878	2197
7	M09-NLo	1322	1333	1425	1836
8	M09-NNLo	1227	1242	1285	1731
9	<u>GP-08 NLo</u>	<u>1305</u>	<u>1434</u>	<u>1987</u>	<u>3064</u>
10	<u>GP-08 NNLo</u>	<u>3504</u>	<u>4180</u>	<u>5680</u>	<u>5689</u>
11	G07-Lo	1198	1211	1836	2168
12	<u>G-07-NLo</u>	1303	1321	<u>7686</u>	<u>9077</u>
13	G-07-MS-a	1187	1222	1258	1558
14	G-07-MS-b	1254	1344	1365	1624
15	Pum-02	1247	1330	1395	1766

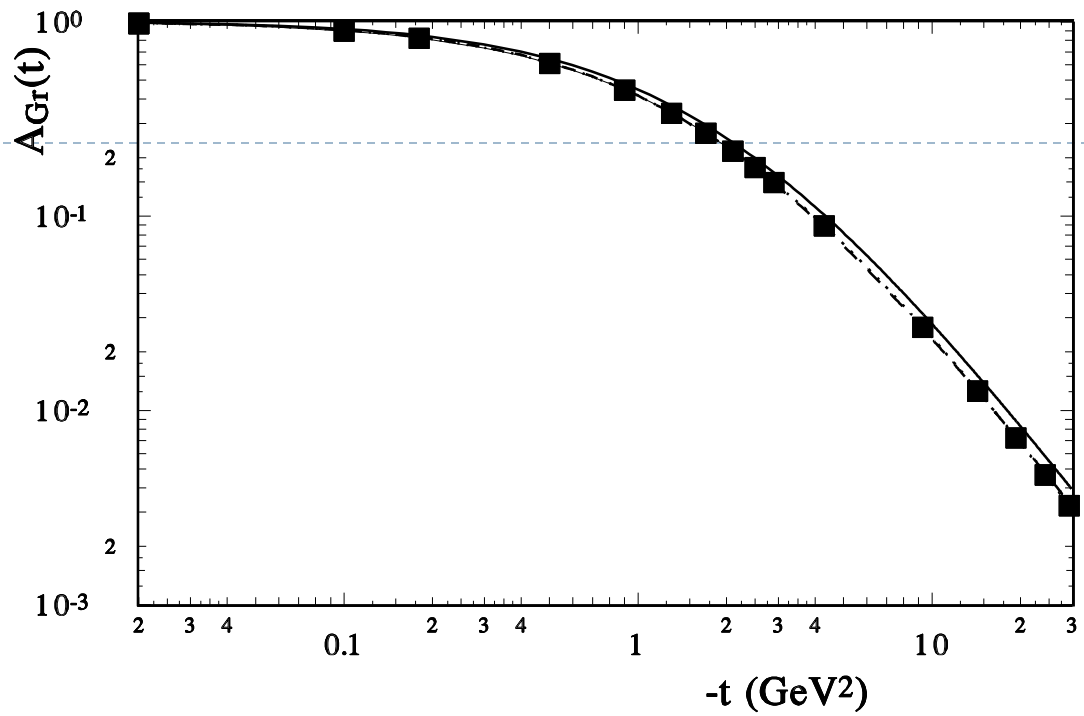












$$G_A(t) = \frac{\Lambda_A^4}{(\Lambda_A^2 - t)^2};$$

$$\Lambda_A^2 = 2 \text{ (1.8) } \text{GeV}^2;$$



Elastic nucleon scattering

General Parton Distributions -GPDs

Electromagnetic
form factors
(charge
distribution)

Gravitation
form factors
(matter distribution)

$$F_1^D(t) = \frac{4M_p^2 - t \mu_p}{4M_p^2 - t} G_D(t);$$

$$G_D(t) = \frac{\Lambda^4}{(\Lambda^2 - t)^2};$$

$$G_A(t) = \frac{\Lambda_A^4}{(\Lambda_A^2 - t)^2};$$



$$\hat{s} = s e^{i\pi/2};$$

$$F_1^B(s, t) = h_1 G_{em}(t) (\hat{s})^{\Delta_1} e^{\alpha_1 t \ln(\hat{s})}; \quad F_2^B(s, t) = h_2 G_A(t)^2 (\hat{s})^{\Delta_1} e^{\alpha_1/4 t \ln(\hat{s})};$$

$$F^B(s, t) = F_1^B(s, t) (1 + r_1 / \sqrt{\hat{s}}) + F_2^B(s, t) (1 + r_2 / \sqrt{\hat{s}});$$

$$\chi(s, b) = 2\pi \int_0^\infty q e^{i\vec{q}\vec{b}} J_0(bq) F^B(s, q) dq$$

$$F(s, t) = \frac{i}{2\pi} \int_0^\infty b e^{i\vec{q}\vec{b}} J_0(bq) [1 - e^{-\chi(s, b)}] db$$

$$h_1 = 0.74; \quad h_2 = 0.23; \quad \Delta_1 = 0.11;$$

$$n = 980; \quad 0.00075 < |t| < 10 \text{ GeV}^2; \quad 52.6 \leq \sqrt{s} \leq 1960 \text{ GeV}; \quad \sum \chi^2 / n = 2.$$



$$F_1^{em}(t) = \alpha f_1^2(t) \frac{s - 2m^2}{t}; \quad F_3^{em}(t) = F_1^{em};$$

and for spin-flip amplitudes:

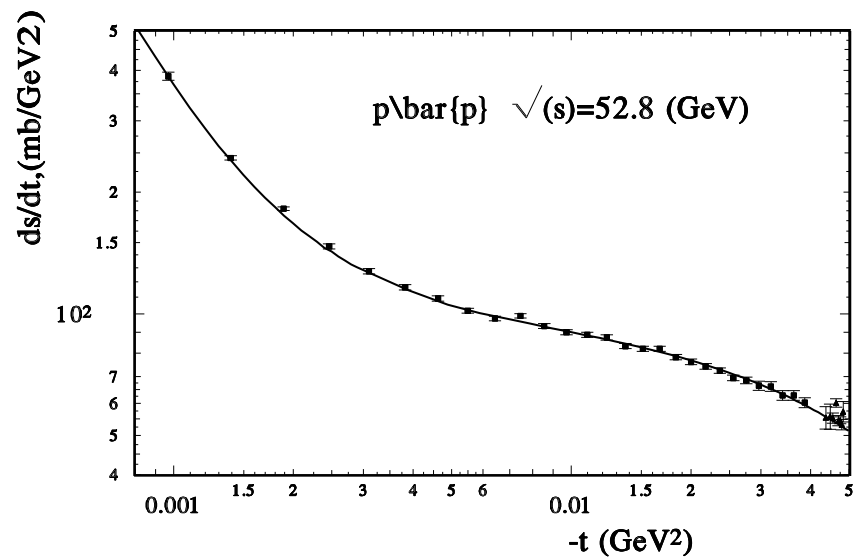
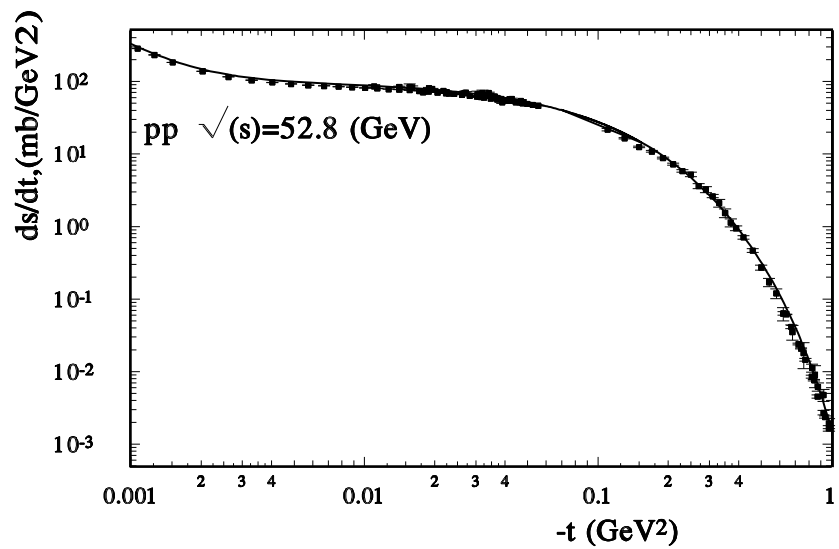
$$F_2^{em}(t) = \alpha \frac{f_2^2(t)}{4m^2} s; \quad F_4^{em}(t) = -F_2^{em}(t),$$

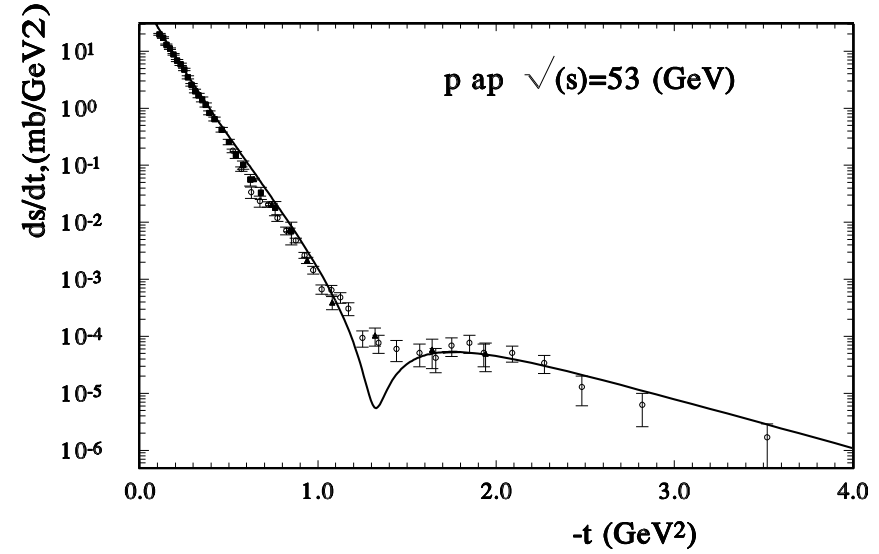
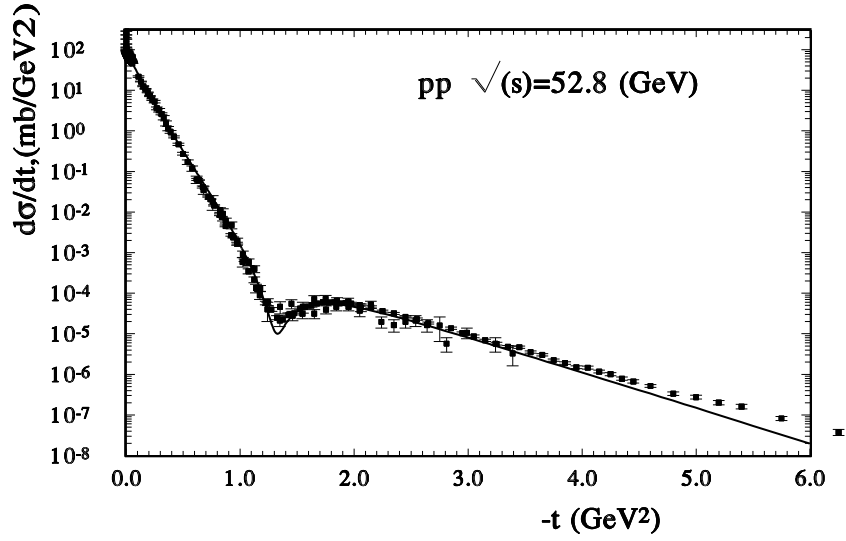
$$F_5^{em}(t) = \alpha \frac{s}{2m\sqrt{|t|}} f_1(t) f_2(t),$$

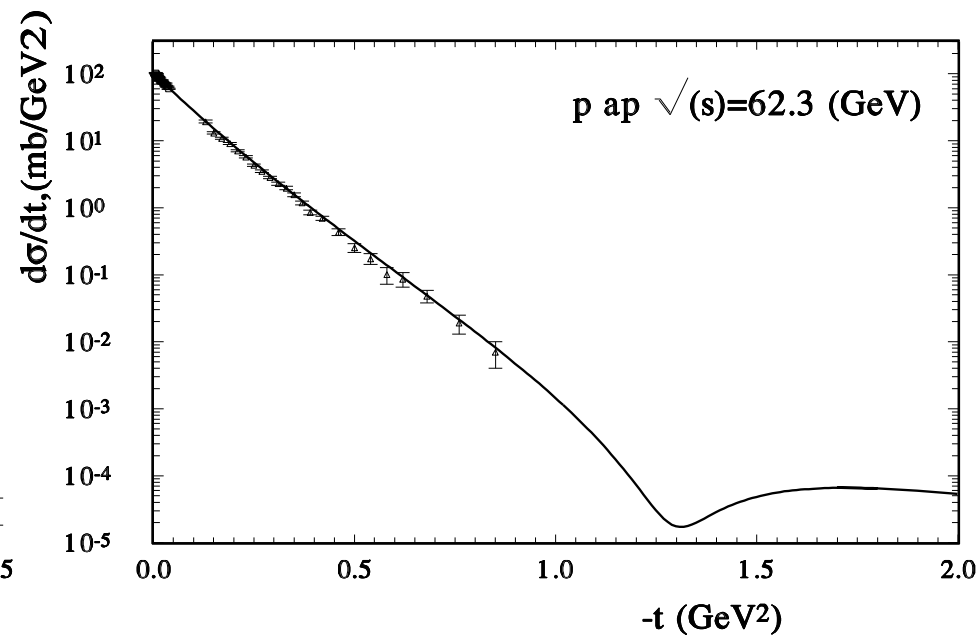
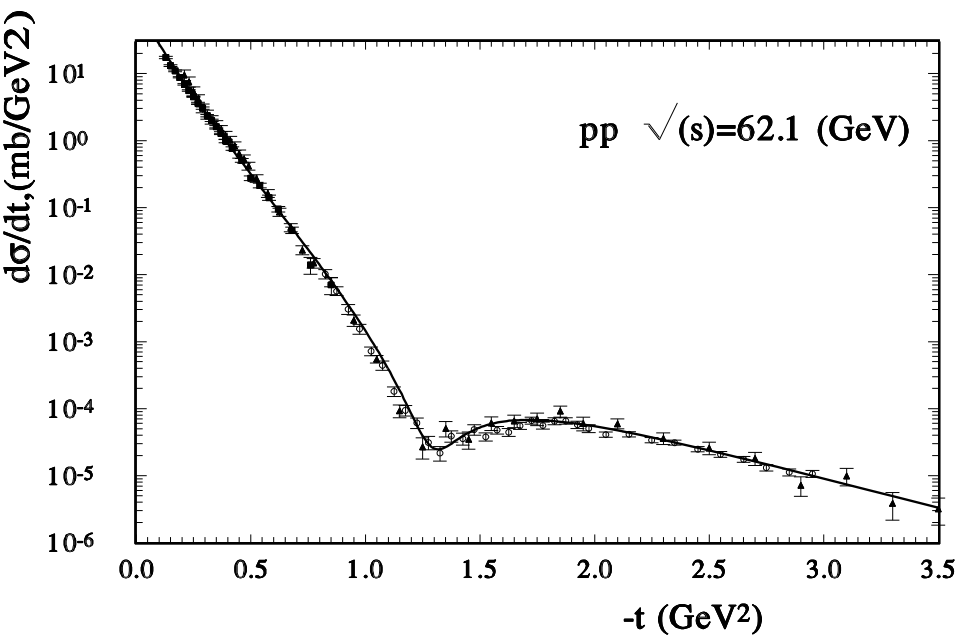
where the form factors are:

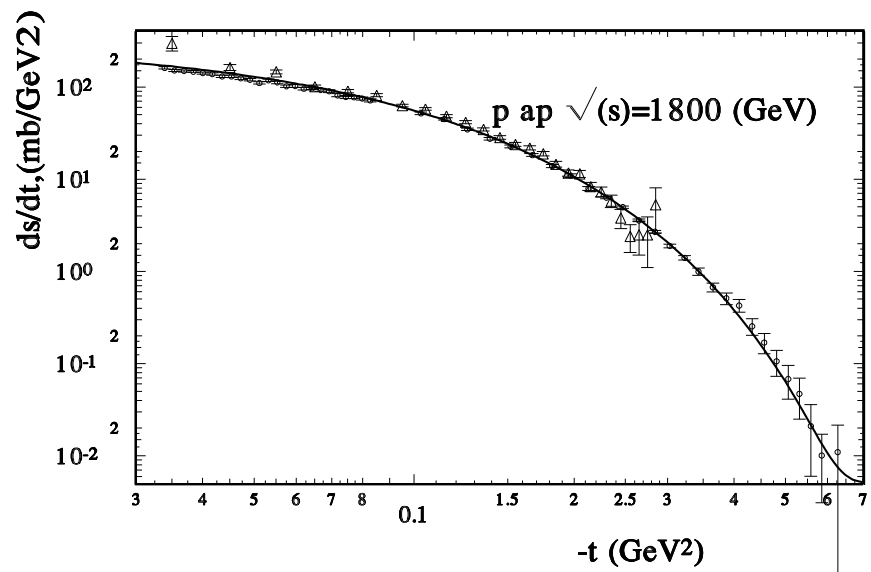
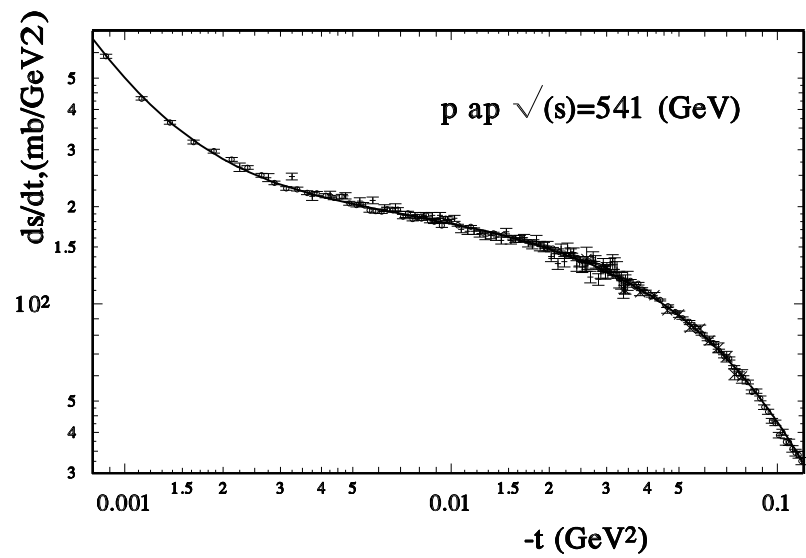
$$f_1(t) = \frac{4m_p^2 - (1+k)t}{4m_p^2 - t} G_d(t);$$

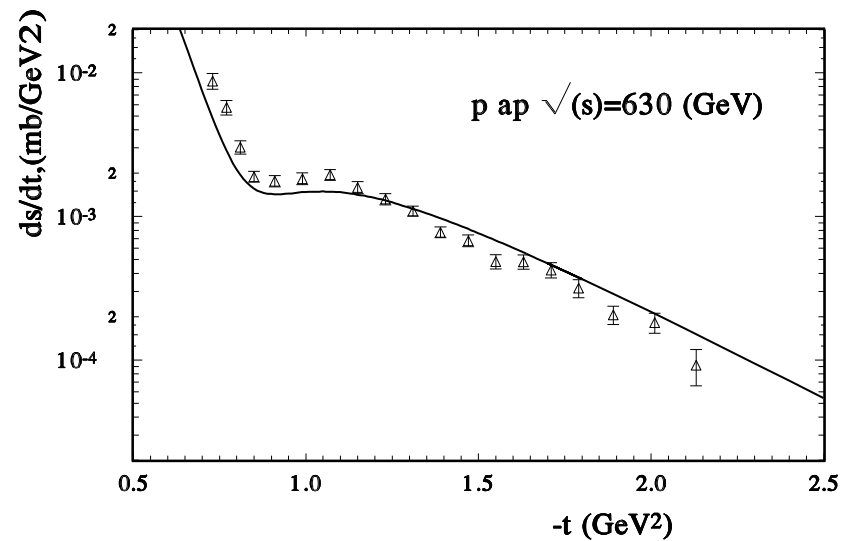
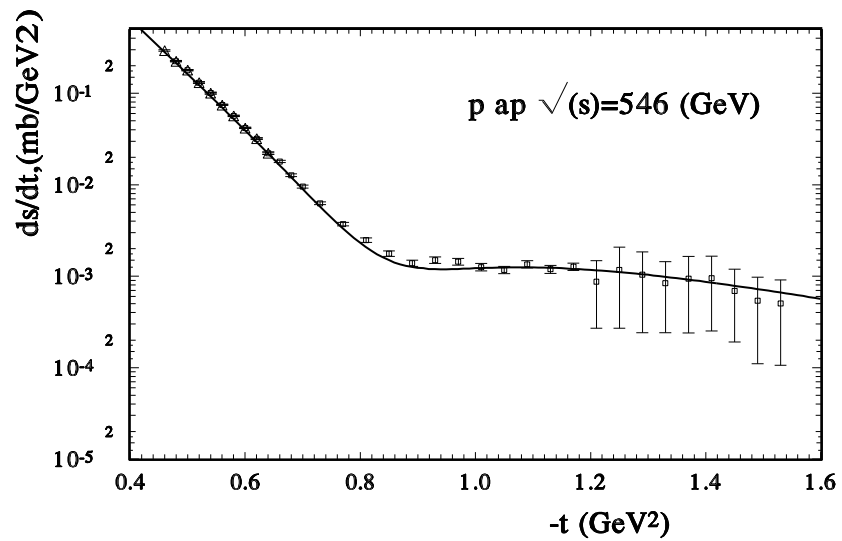
$$f_2(t) = \frac{4m_p^2 k}{4m_p^2 - t} G_d(t);$$

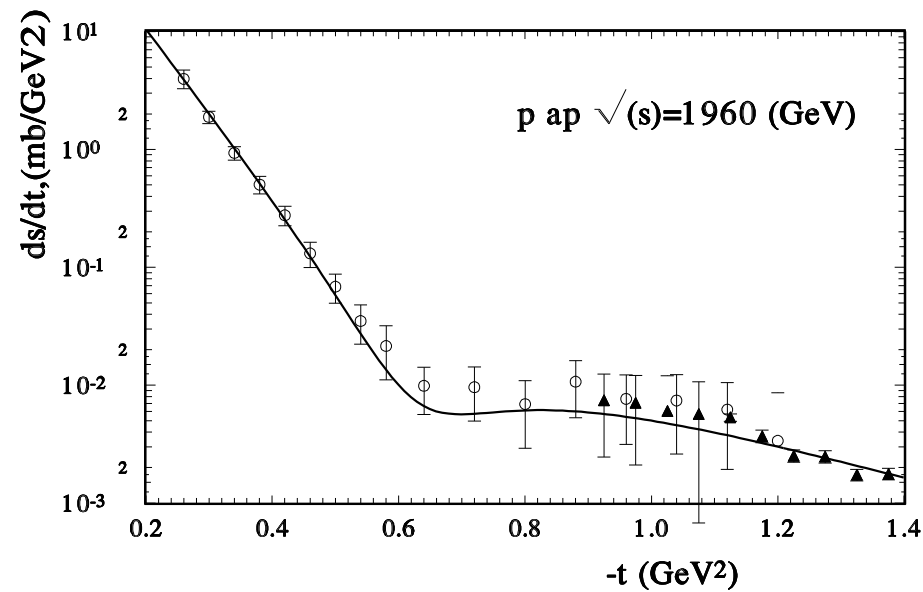
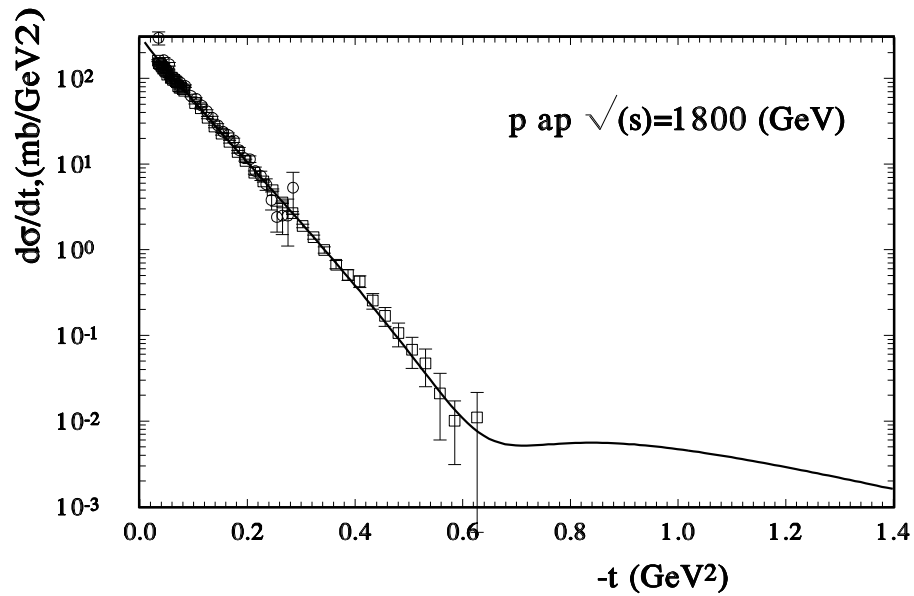


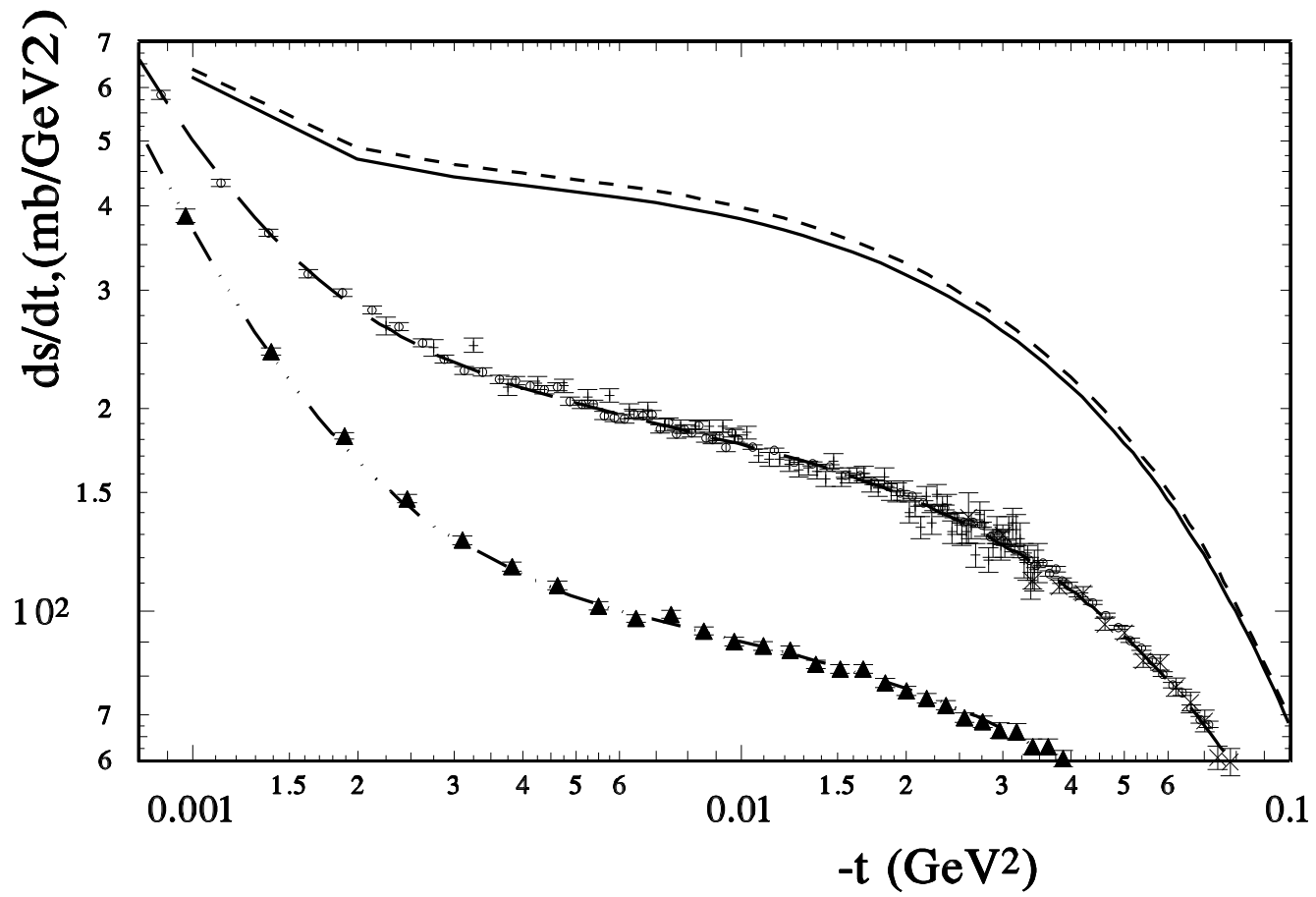








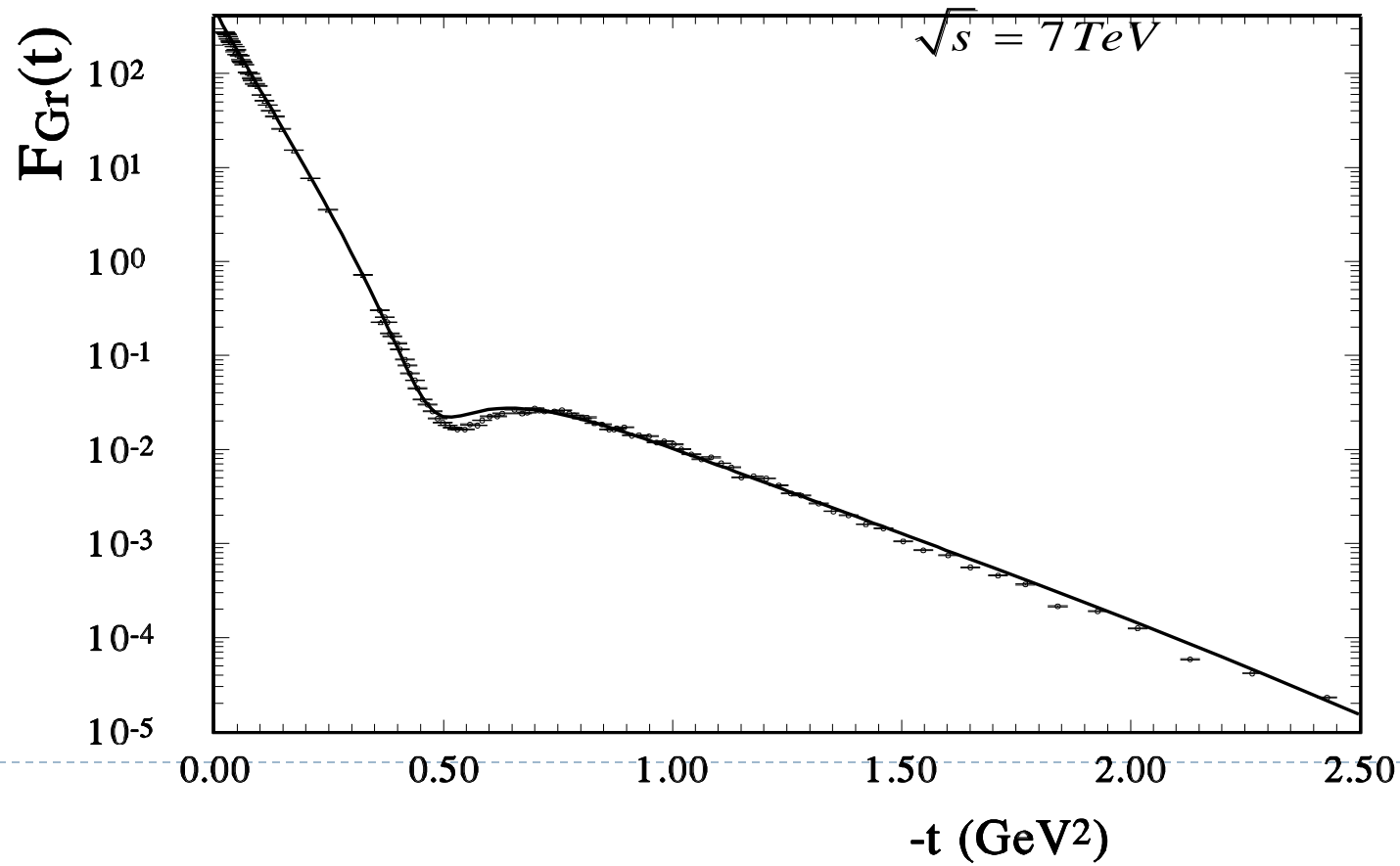


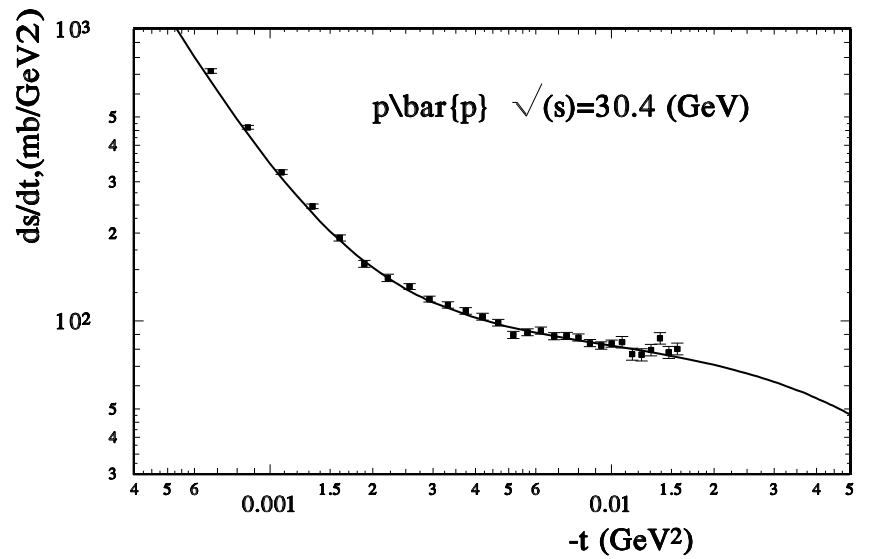
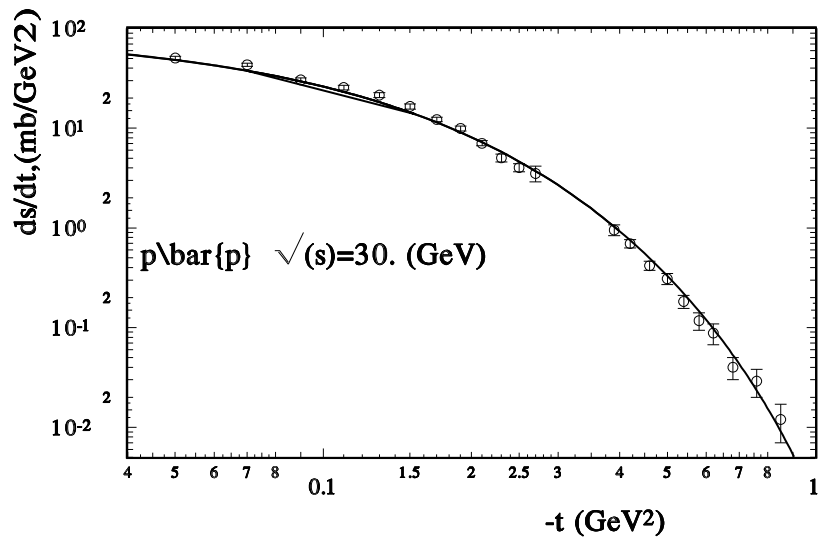


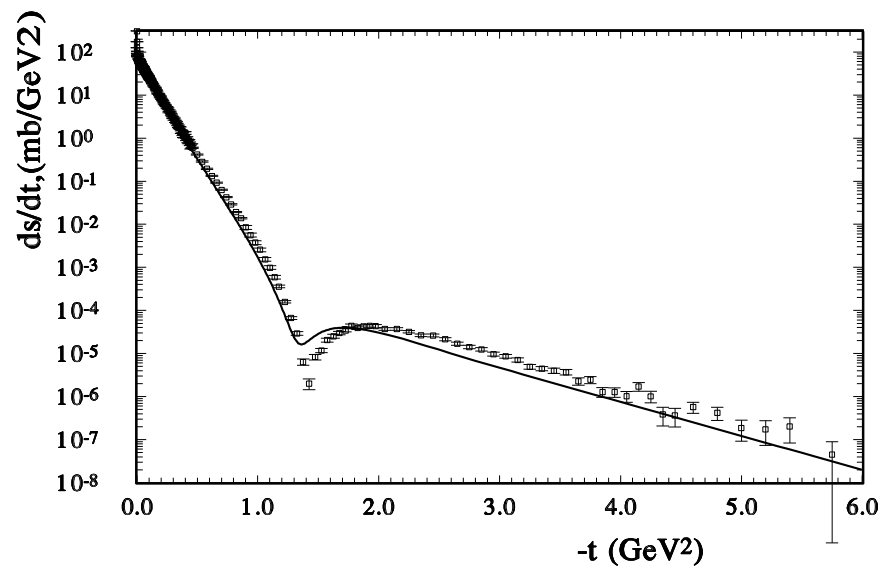
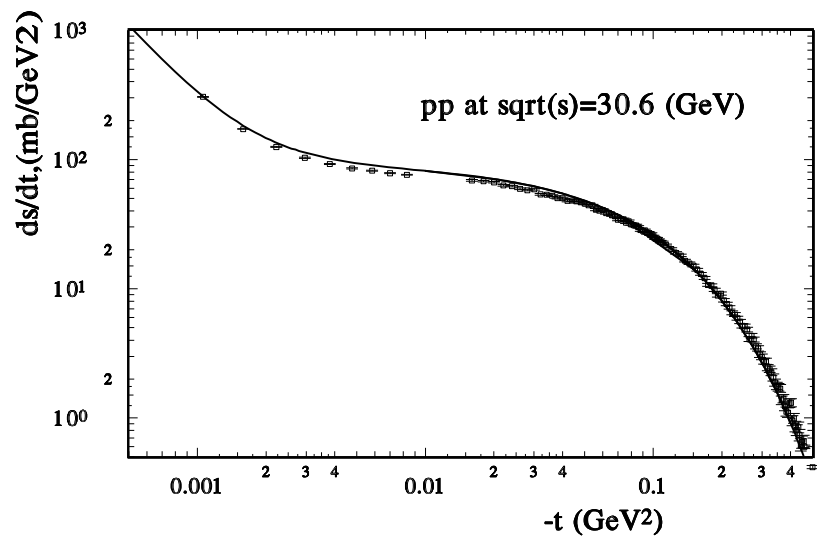
Fit exp. data $52.8 \text{ GeV} \leq \sqrt{s} \leq 1.8 \text{ TeV}$

O.S. EPJC (2012); arXiv: 1204.4418

predictions





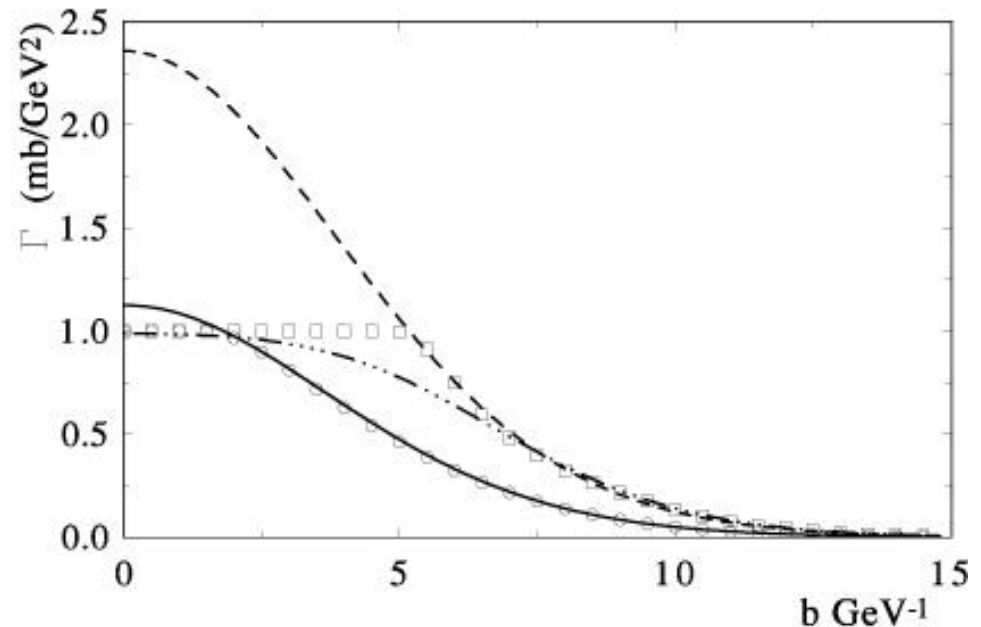


Soft and hard Pomeron

Donnachie-Landshoff model;

Schuler-Sjostrand model

$$T_1(s, t) = \left[h_1 \left(\frac{s}{s_0} \right)^{\Delta_1} e^{\alpha_1 t \ln(s/s_0)} + h_2 \left(\frac{s}{s_0} \right)^{\Delta_2} e^{\alpha_2 t \ln(s/s_0)} \right] F^2(t)$$



Hard pomeron

O.S., [Nucl.Phys. A 903 \(2013\) 54-64](#); [arxiv\[1205.5867\]](#)

$$T_1(s, t) = \left[h_1 \left(\frac{s}{s_0} \right)^{\Delta_1} e^{\alpha_1 t \ln(s/s_0)} + h_2 \left(\frac{s}{s_0} \right)^{\Delta_2} e^{\alpha_2 t \ln(s/s_0)} \right] G_{em}^2(t)$$

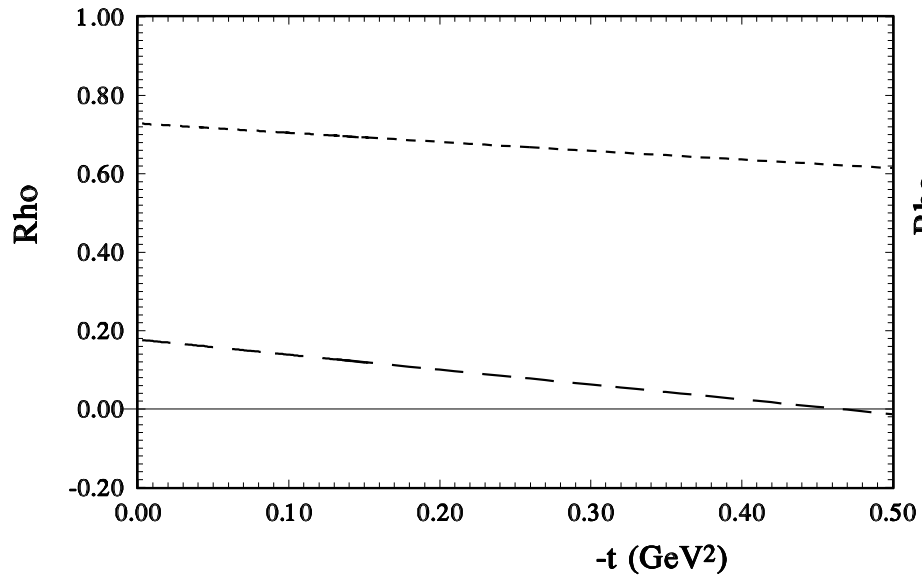
$$\alpha_1 = 0.25; \quad \Delta_1 = 0.1;$$

$$\alpha_2 = 0.1; \quad \Delta_2 = 0.4;$$

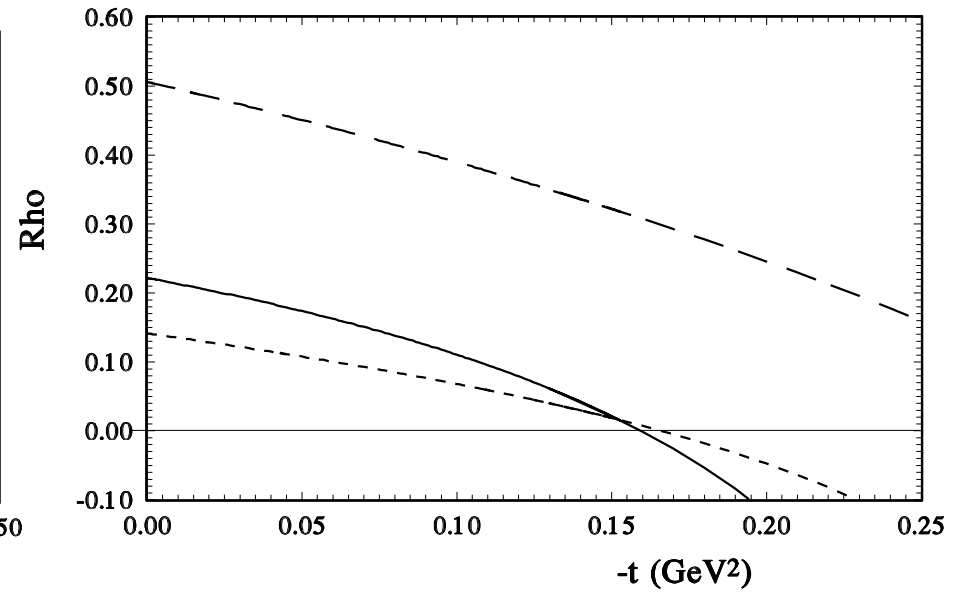


Soft and hard pomeron

Born amplitude



After the eikonalization



Hard Pomeron in the HEGS model calculation

O.S., Nucl.Phys. A 903 (2013) 54-64; [arxiv\[1205.5867\]](#)

$h_{hardPom.}, GeV^{-2}$	Δ_2	α'_{hp}, GeV^{-2}	$\sum_1^n \chi_i^2$
$0. \pm 0.01$	0.4_{fixed}	0.1_{fixed}	1783
-0.008 ± 0.001	0.4_{fixed}	$< 0.$	1713
-0.02 ± 0.01	0.33 ± 0.04	$< 0.$	1698
-0.012 ± 0.01	0.4_{fixed}	0.08 ± 0.01	1731

A. Donnachie, P.V. Landshoff- [arxiv\[1309.1292\]](#)



Expanding of model
(HEGS)

$$20 \leq \sqrt{s} \leq 7000 \text{ GeV};$$

$$n = 980 \rightarrow 2020; \quad 0.00075 < |t| < 10 \rightarrow 15 \text{ GeV}^2;$$

$$\hat{s} = s e^{i\pi/2};$$

$$F_1^B(s, t) = h_1 G_{em}(t) (\hat{s})^{\Delta_1} e^{\alpha_1 t \ln(\hat{s})}; \quad F_2^B(s, t) = h_2 G_A(t)^2 (\hat{s})^{\Delta_1} e^{\alpha_1/4 t \ln(\hat{s})};$$

$$F^B(s, t) = [F_1^B(s, t)(1 + r_1 / \sqrt{\hat{s}})] + [F_2^B(s, t)(1 + r_2 / \sqrt{\hat{s}})];$$

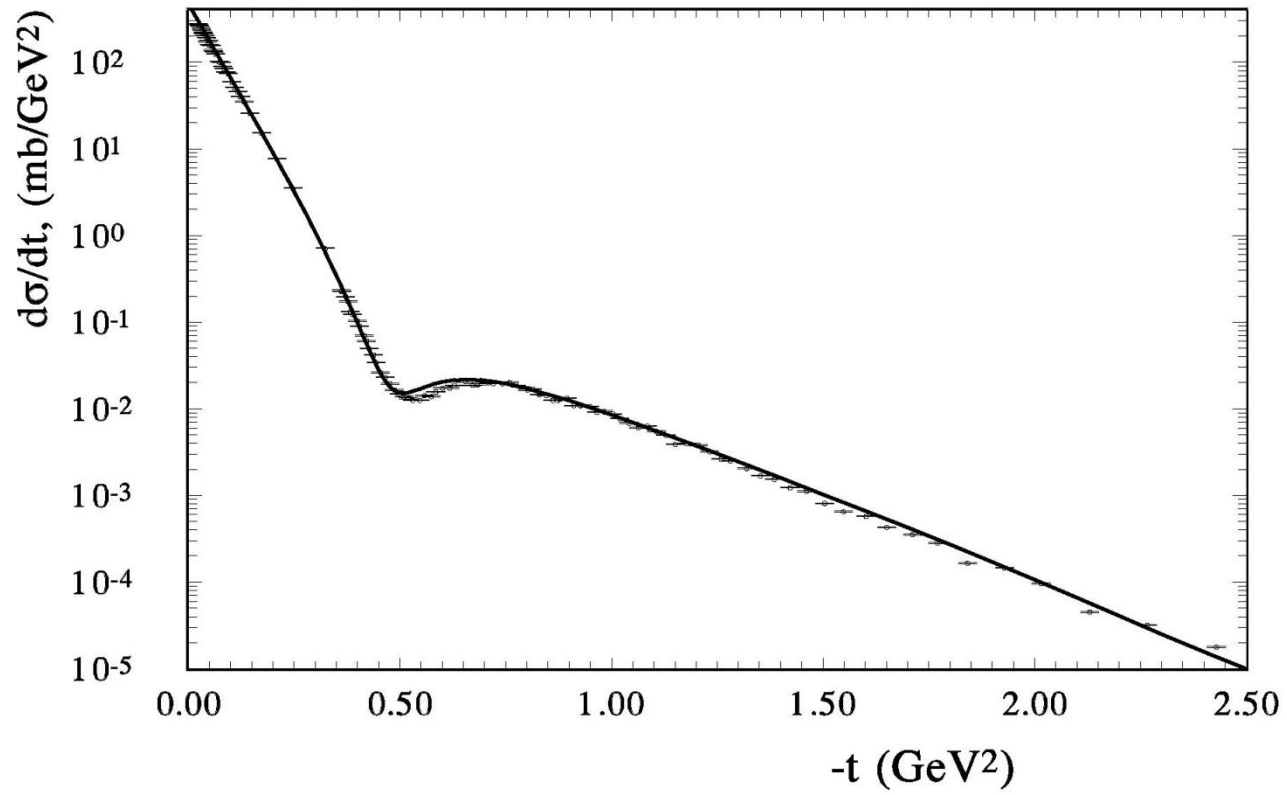
$$F_{Odd}^B(s, t) = h_{Odd} G_A(t)^2 (\hat{s})^{\Delta_1} \frac{t}{r_0^2 + t} e^{\alpha_1/4 t \ln(\hat{s})};$$

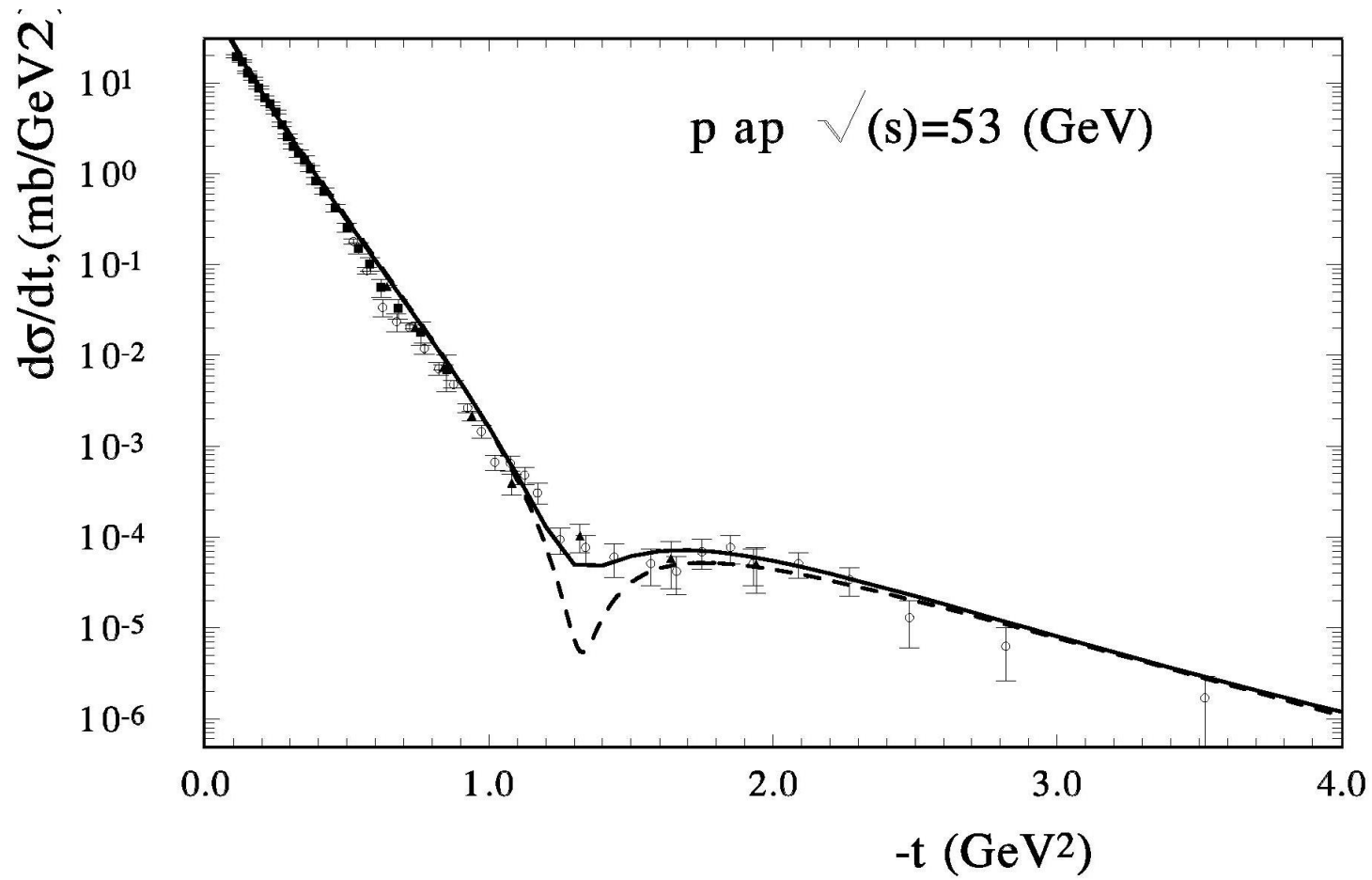
$$F^{+-}(s, t) = h_{sf} G_A(t)^2 e^{\mu t}; \quad h_{sf} = 0.02 \text{ GeV}^{-2}; \quad \mu = 0.5 \text{ GeV}^{-2}$$

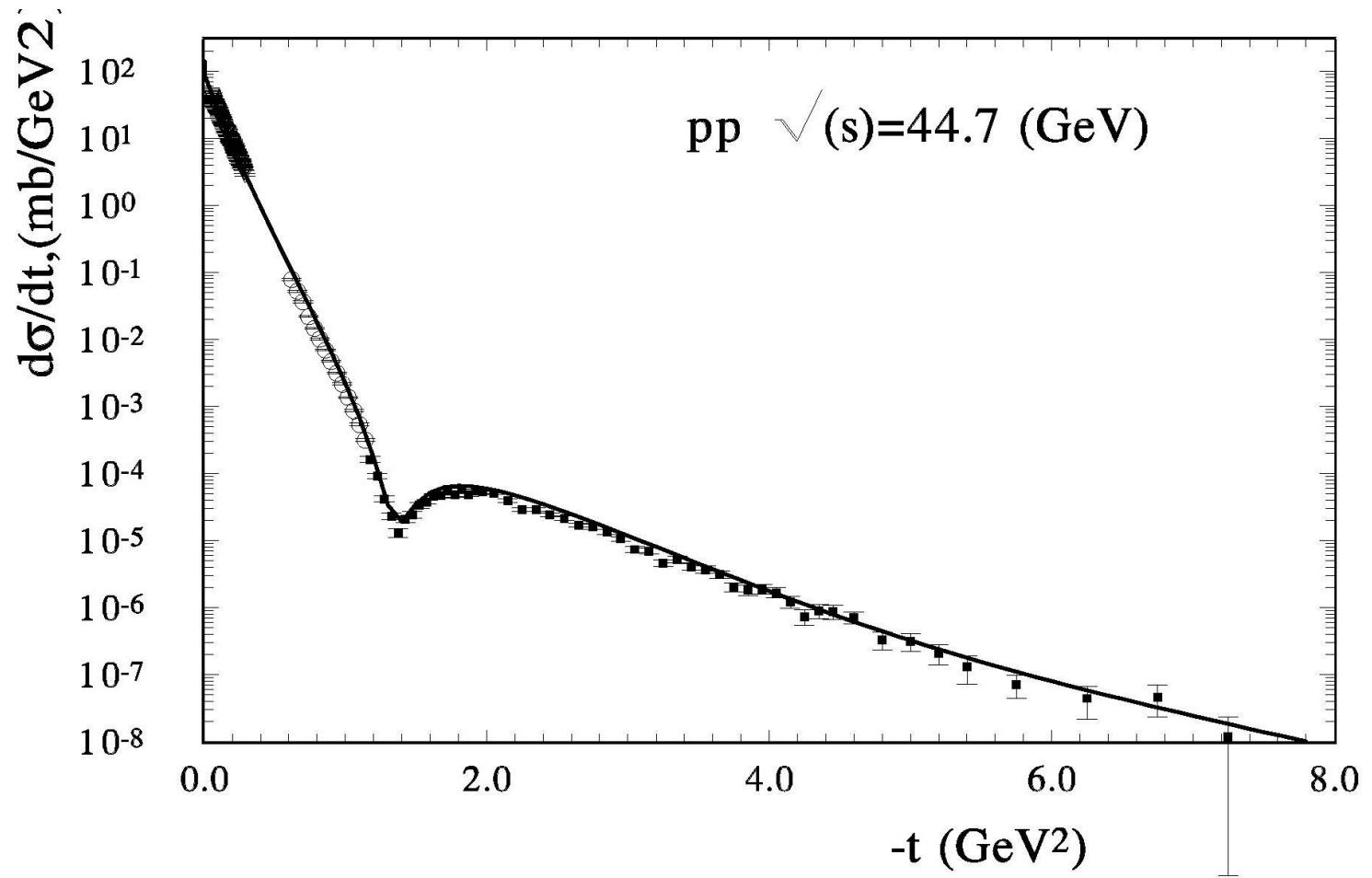
$$h_1 = 0.74; \quad h_2 = 0.23; \quad \Delta_1 = 0.11;$$

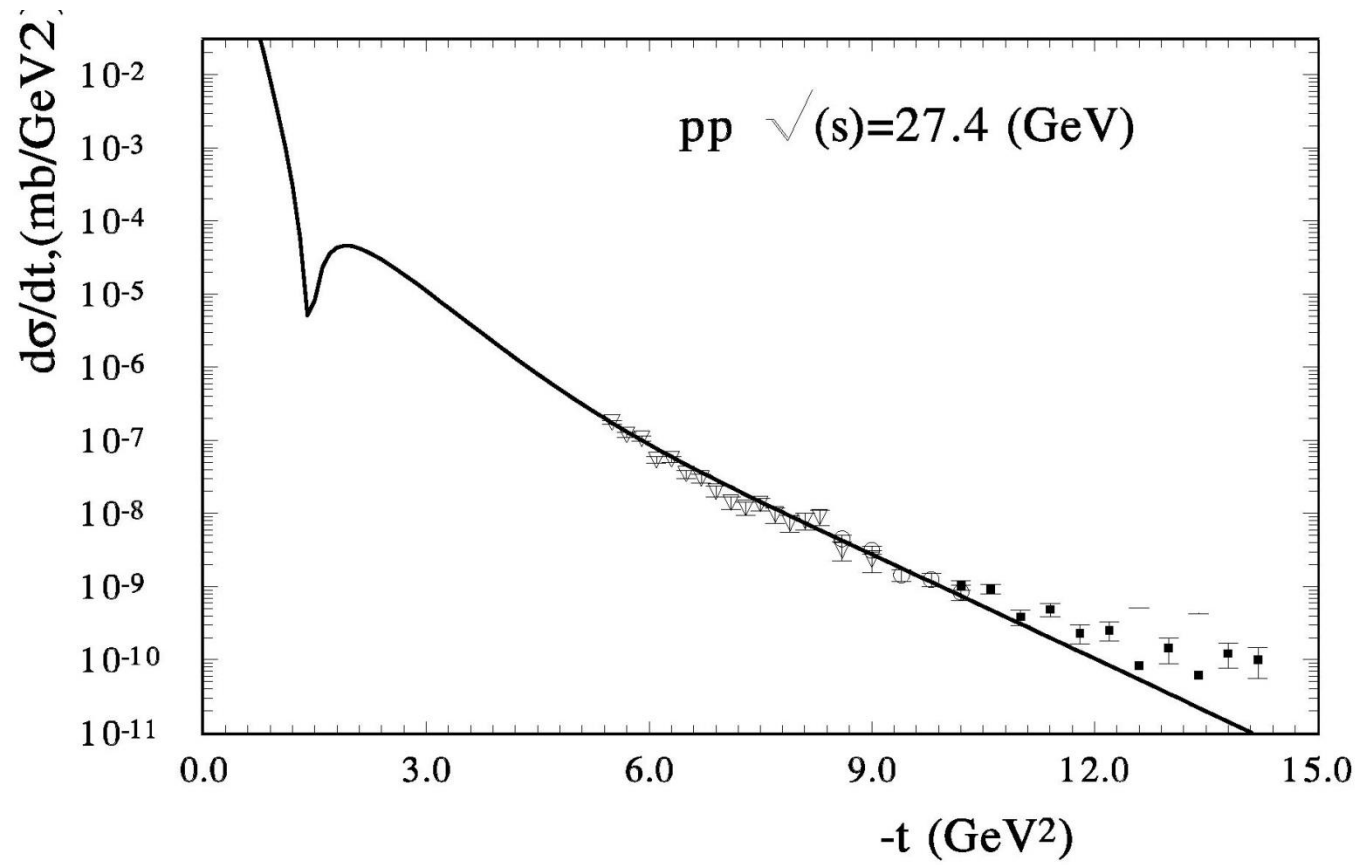
$$20 \leq \sqrt{s} \leq 7000 \text{ GeV}; \quad \sum \chi^2 / n = 1.6$$

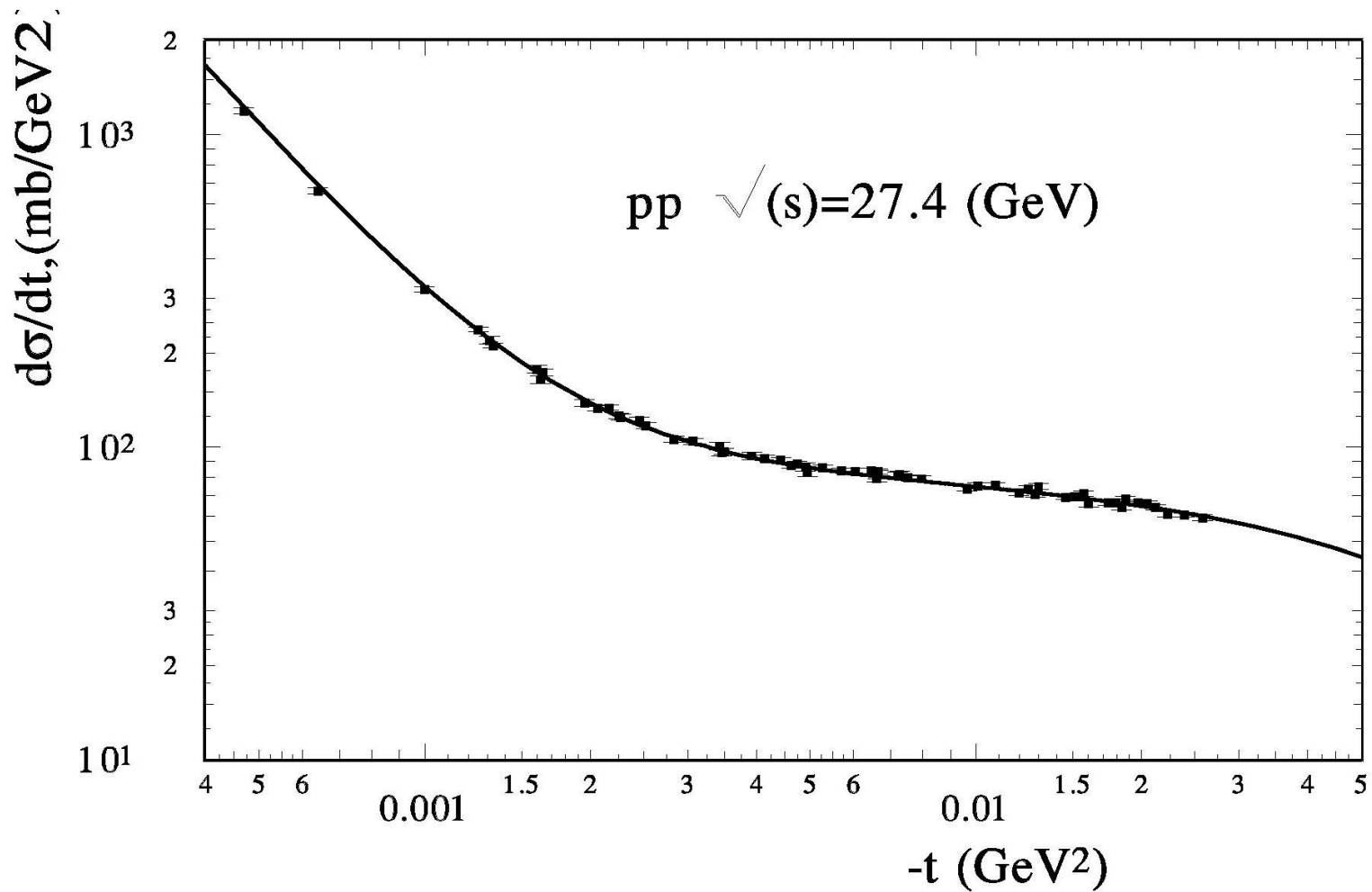


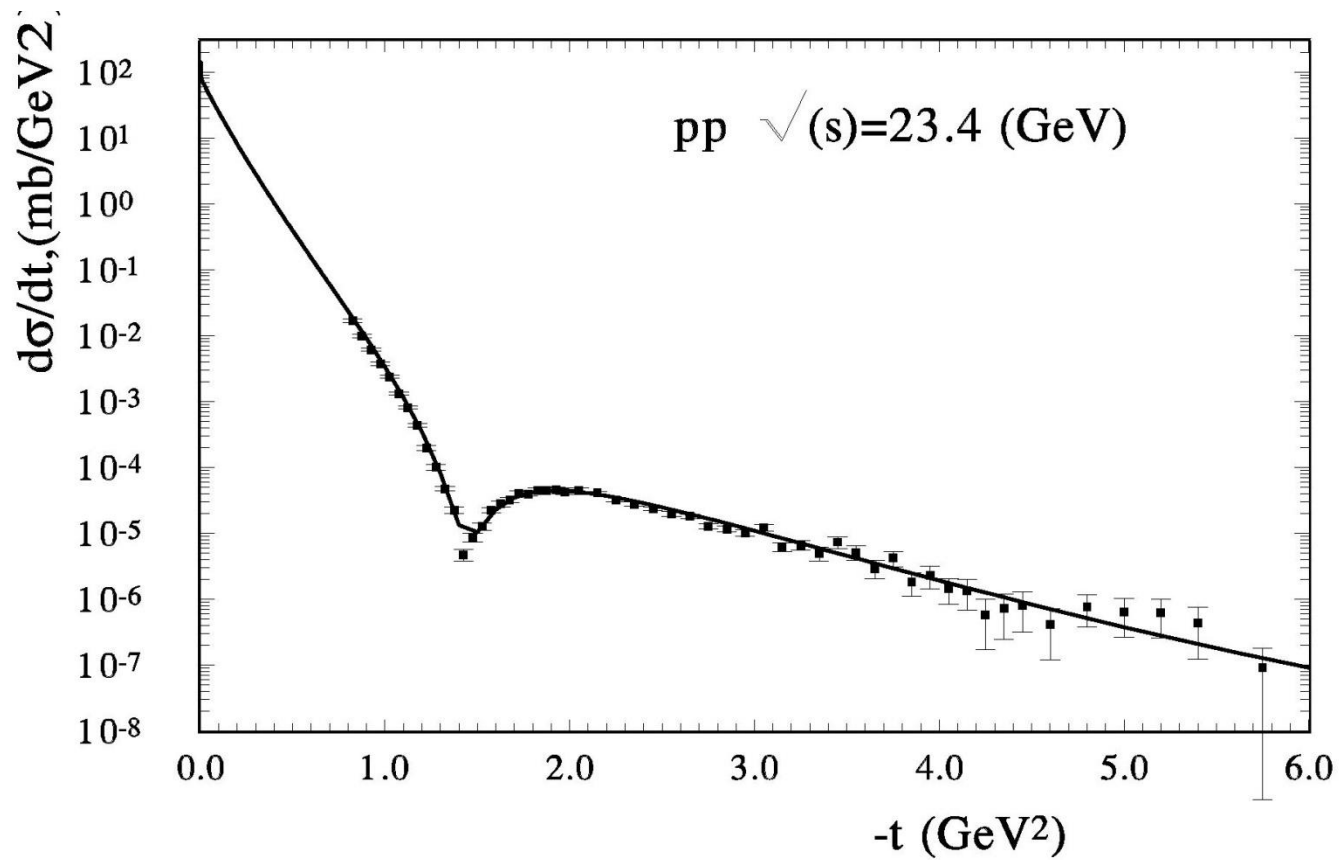


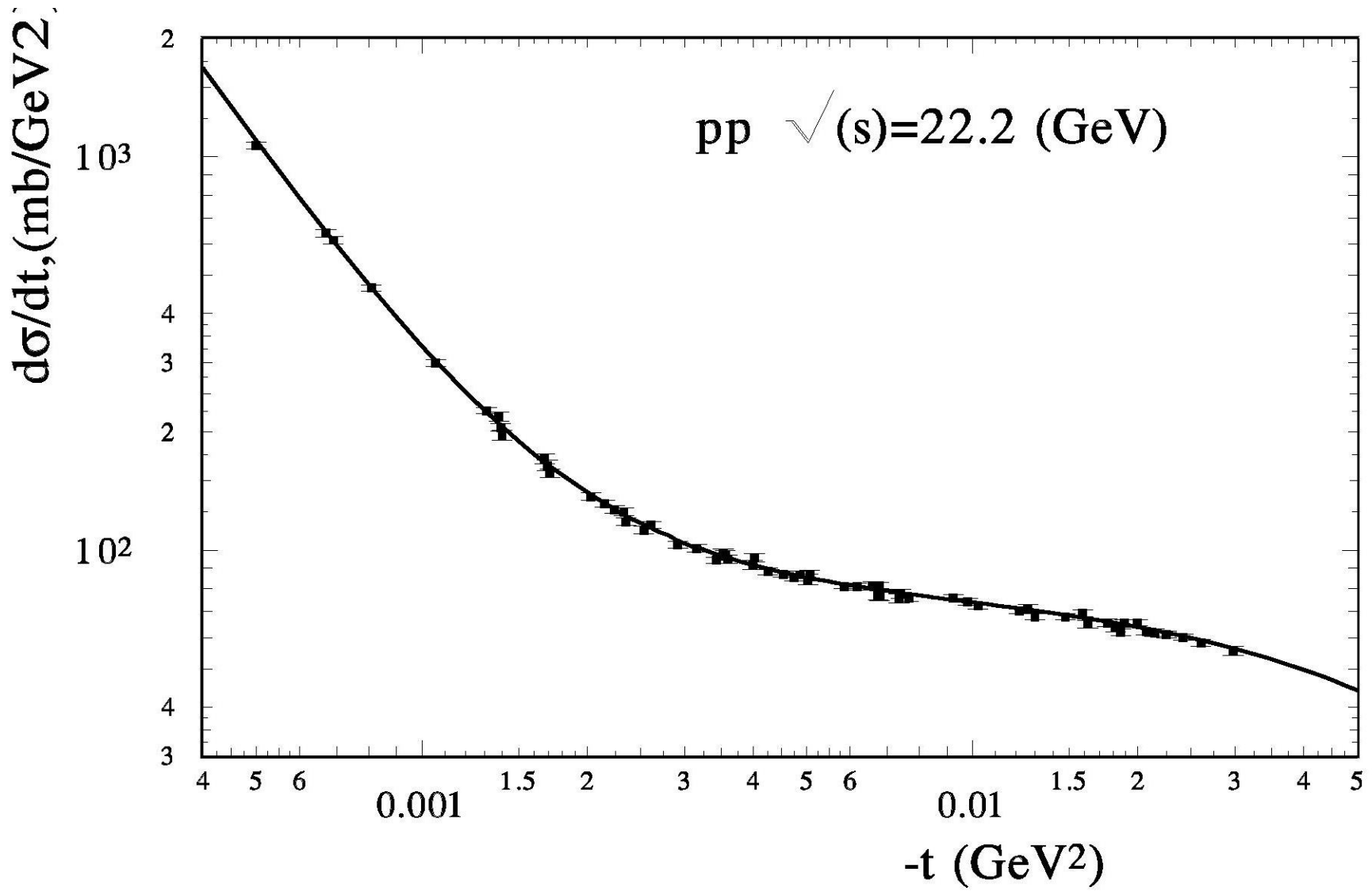


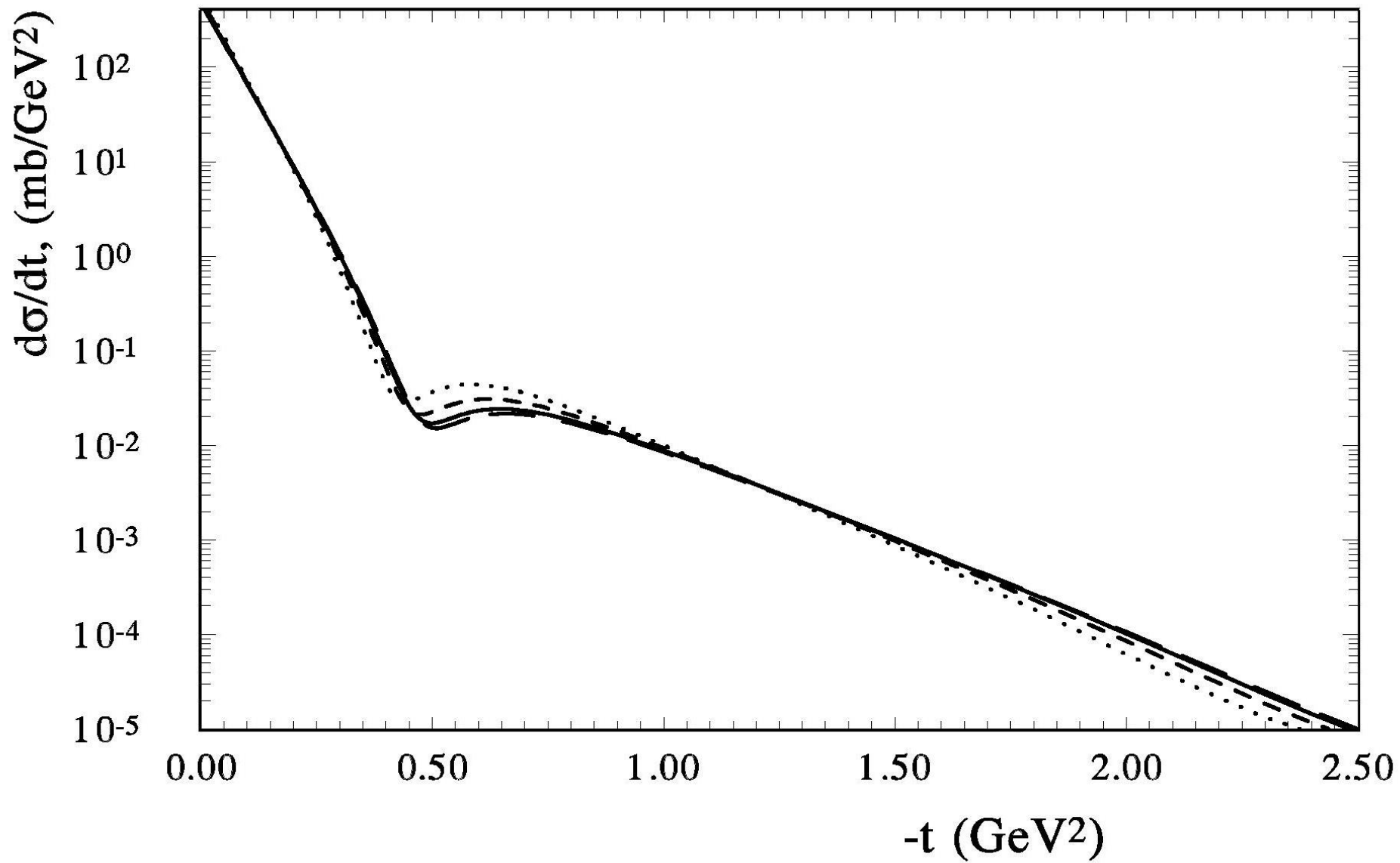












$\sqrt{s} \text{ GeV}$ $\sigma_{tot}(s) \text{ mb}$ $\rho(s, t = 0)$

22.2	39.85	0.0013
52.8	42.85	0.076
541	61.91	0.128
1800	76.25	0.127
7000	95.9	0.121
8000	98	0.12
10000	101.6	0.119
140000	107.3	0.117



Summary

1. The elastic scattering reflects the generalized structure of the hadron.
2. The our model GPDs leads to the well description of the proton and neutron electromagnetic form factors and its elastic scattering simultaneously.
3. The new High Energy Generalized structure model (HEGS) gives the quantitatively description of the elastic nucleon scattering at high energy with only 3 fitting high energy parameters.
4. The model leads to the good coincides the model calculations with the preliminary data at 7 TeV.



Summary

5. In the framework of the model the contribution of the hard pomeron do not feeling.
6. The extending variant of the model show the contribution of the odderon and the spin-flip amplitude with the small energy dependence.
7. The model open the new way to determine the true form of the GPDs and standard parton distributions.



END

THANKS
FOR YOUR
ATTENTION



Hard Pomeron

$$f(s) \propto s^{\Delta} \quad (\Delta_h = 0.4)$$

Odderon

$$f(s) \propto ? \left[1/\sqrt{s}; \text{const.}; s^{\Delta} \quad (\Delta_s = 0.1) \right]$$

$$F(s, t) \propto t / (r^2 - t) s^{\Delta_s} \exp[Bt] G_{gr.}^2(t)$$

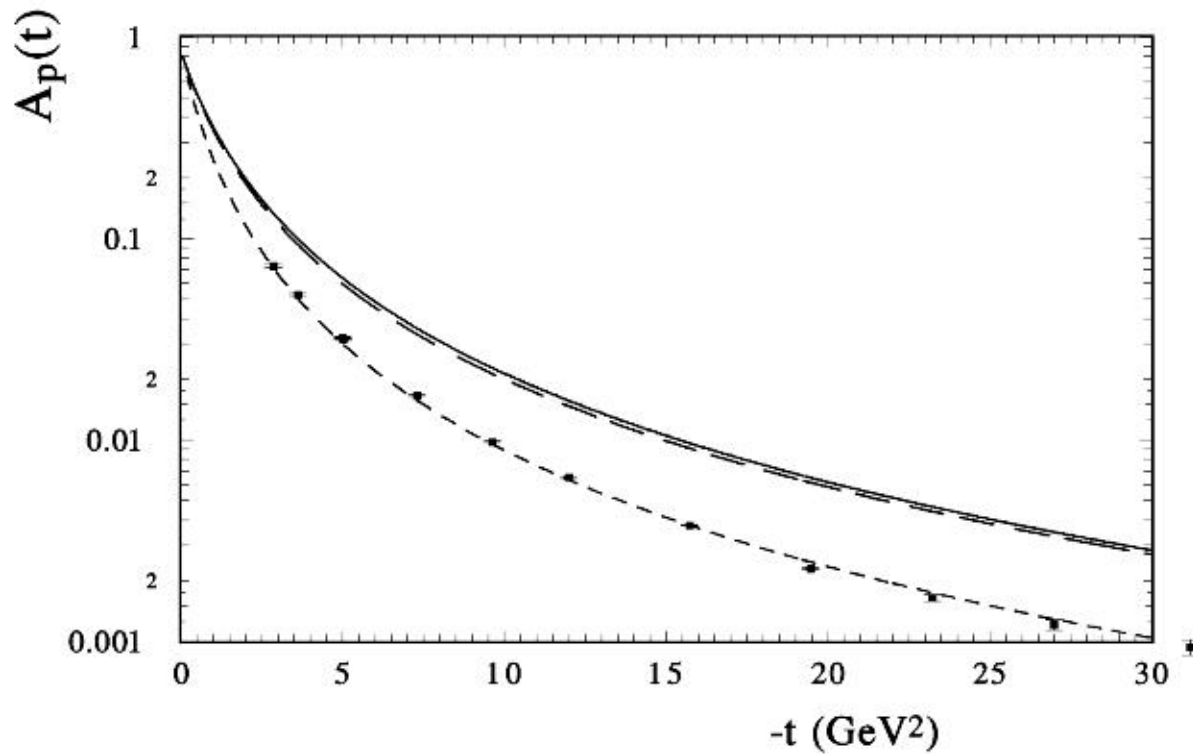
Spin-flip

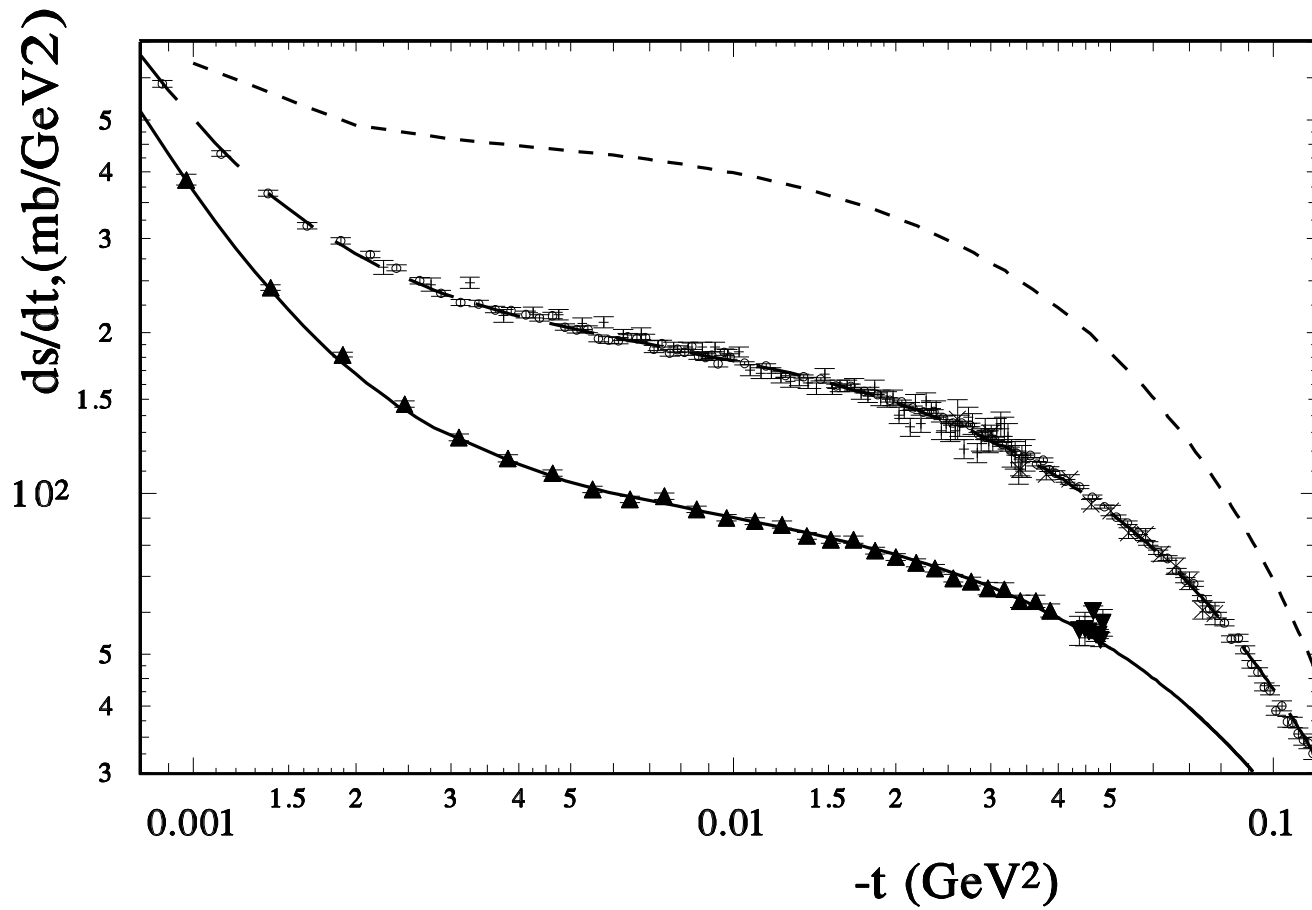
$$f(s) \propto ? \left(1/\sqrt{s}; \text{cons}; \text{Ln}(s) \right)$$

$$F^{+-}(s, t) \propto q^3 \exp[B_{sf} t] G_{gr}^2$$

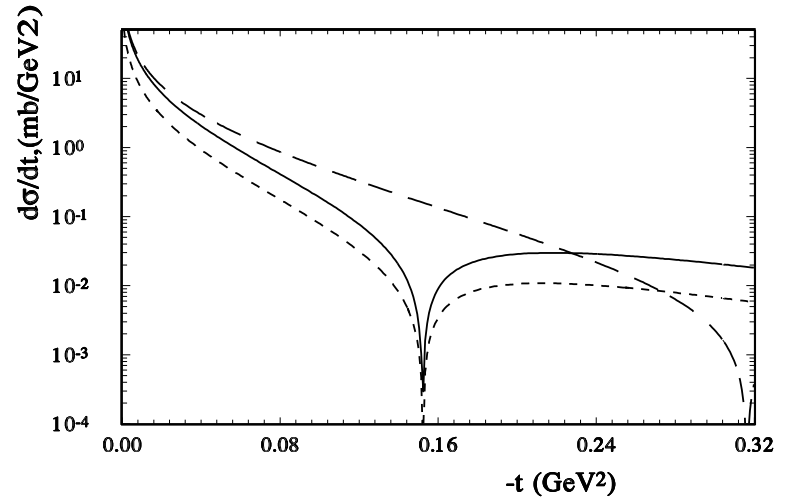
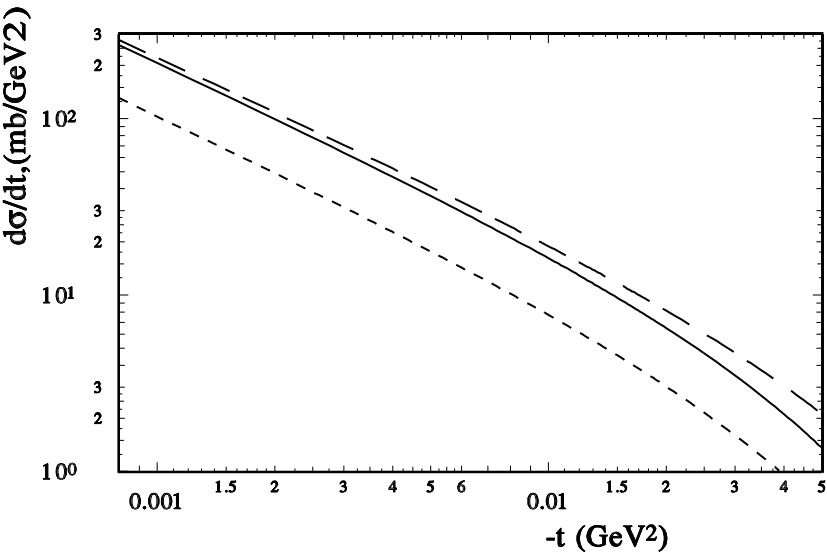


$A_p(t)$ Gravitation form factor A_p and proton Dirac FF



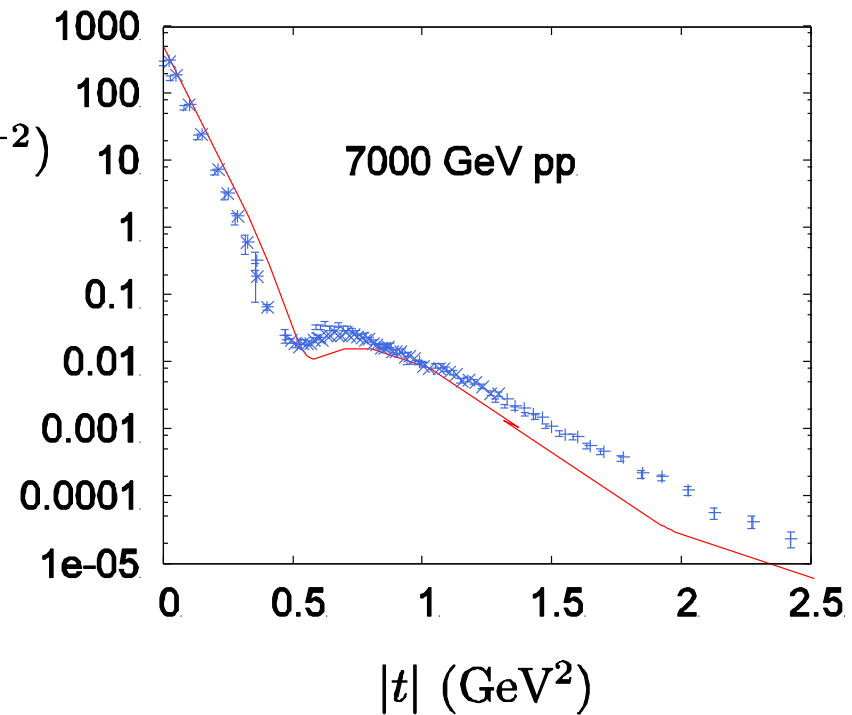
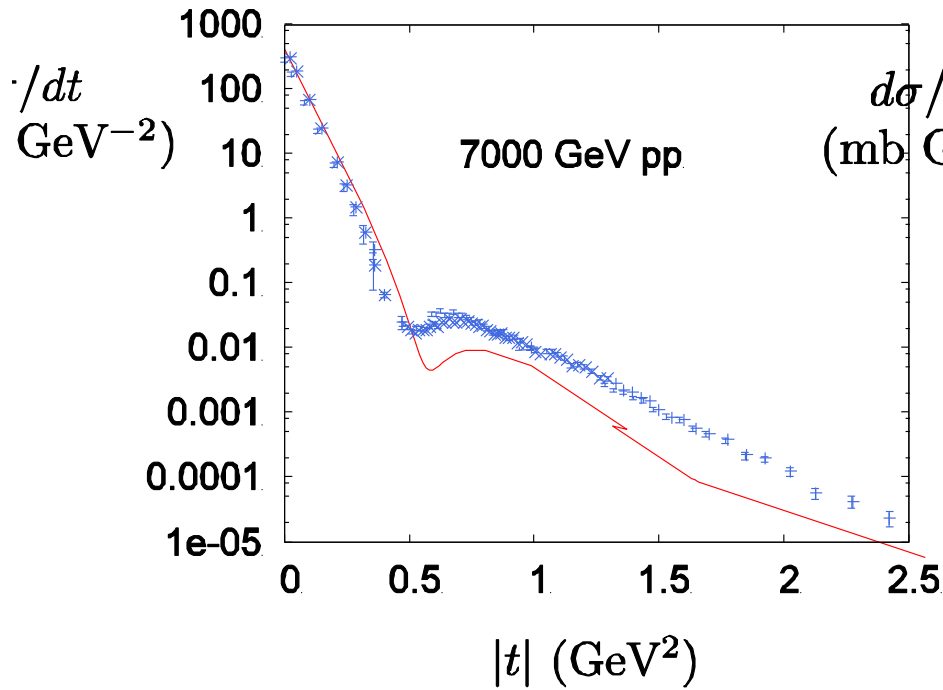


| CNI term |

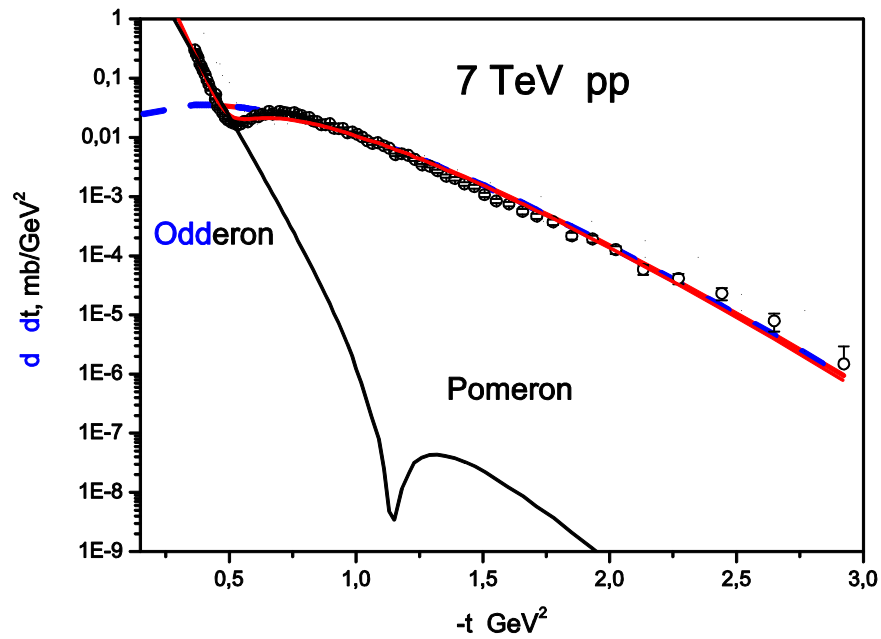


$$\frac{1}{\pi} \frac{d\sigma(s,t)}{dt} = (F_C^2(t) + (1 + \rho^2(s,t)) \text{Im} F_N^2(s,t) + 2(\rho(s,t) + \alpha\varphi(s,t)) F_C \text{Im} F_N)$$





Lengyel-Tarics (2012) –arXiv:1206.5817
(odderon contribution)



General Parton Distributions -GPDs

Electromagnetic
form factors
(charge
distribution)

Gravitation
form factors
(matter distribution)

Elastic nucleon scattering

$$T_B(s, t) = h_1 G_E^2(t) \left(\frac{s}{s_0}\right)^{\Delta_1} e^{\alpha_1 t \ln(s/s_0)} + h_2 G_B^2(t) \left(\frac{s}{s_0}\right)^{\Delta_1} e^{\alpha_1/4 t \ln(s/s_0)}$$



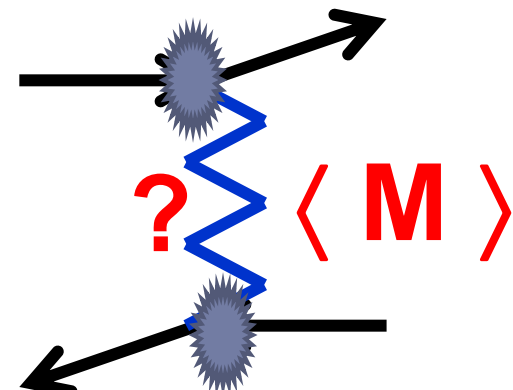
Scattering process described in terms of **Helicity Amplitudes** ϕ_i

All dynamics contained in the **Scattering Matrix** M

(Spin) Cross Sections expressed in terms of

observables:
3 x-sections
5 spin asymmetries

{	spin non-flip	$\phi_1(s,t) = \langle ++ M ++ \rangle$
	double spin flip	$\phi_2(s,t) = \langle ++ M -- \rangle$
	spin non-flip	$\phi_3(s,t) = \langle +- M +- \rangle$
	double spin flip	$\phi_4(s,t) = \langle +- M -+ \rangle$
	single spin flip	$\phi_5(s,t) = \langle ++ M +- \rangle = -\langle ++ M -+ \rangle$



identical spin 1/2 particles

- GPDs → electromagnetic FF

- GPDs → gravimagnetic FF



Summary

1. Proposed the new simple t - dependence of the GPDs
2. The well description of the proton and neutron electromagnetic form factors are obtained.
3. It is shown that in the framework of the same model assumptions the “Rosenbluth” and “Polarization” data of FF can be obtained using the difference slopes of the F_2 .
4. The compare the calculations and full row of the data shows the preference the “Polarization” case.
5. The corresponding gravitation form factors of the proton are calculated.

The forward magnetic densities - \mathcal{E} is proportional to the \mathcal{H} with additional factor *as in* Cuidal et al. hep-ph:041025 (v.1-2005 – v.2-2009)

$$\mathcal{E}_1^u(x) = \frac{k_u}{N_u} (1-x)^{\eta_u} u_\nu(x); \quad \mathcal{E}_1^d(x) = \frac{k_d}{N_d} (1-x)^{\eta_d} d_\nu(x);$$

Where the normalization factors :

$$N_u = \int_0^1 dx (1-x)^{\eta_u} u_\nu(x); \quad N_d = \int_0^1 dx (1-x)^{\eta_d} d_\nu(x);$$

$$F_2^u(t) = \int_0^1 dx (1-x)^{\eta_u} u_\nu(x) G_u(x,t); \quad F_2^d(t) = \int_0^1 dx (1-x)^{\eta_d} d_\nu(x) G_d(x,t);$$



Elastic scattering amplitude

$$pp \rightarrow pp \qquad p\bar{p} \rightarrow p\bar{p}$$

$$\frac{d\sigma}{dt} = 2\pi \left[|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2 \right]$$

$$\Phi_i(s,t) = \Phi_i^h(s,t) + \Phi_i^e(t) e^{i\alpha\varphi}$$

$$\varphi(s,t) = \mp \left[\gamma + \ln(B(s,t) |t| / 2) + \nu_1 + \nu_2 \right]$$

$\gamma = 0,577\dots$ (the Euler constant)

ν_1 and ν_2 are small correction terms

