Exotic Hadrons and Large Nc QCD

An Overview


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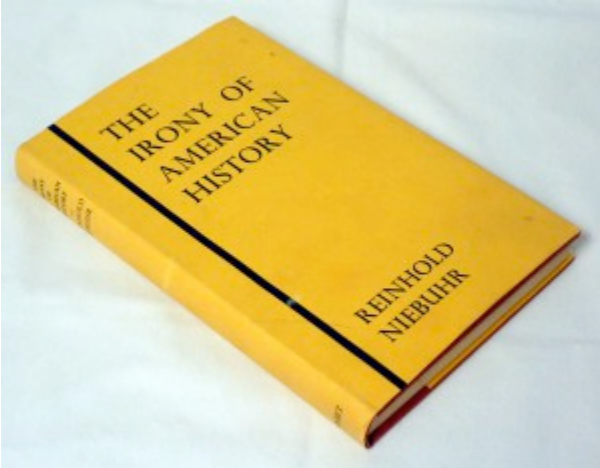
An overview

- Introduction
  - QCD vs the Quark Model
    - Exotics
  - Why Large Nc?
    - Glueballs, Hybrids
- The Conventional Wisdom
  - Tetraquarks can't exist at large Nc
- Weinberg's critique
  - "Proof" that narrow tetraquarks don't exist is flawed
- QCD(AS)
  - Narrow tetraquarks must exist at large Nc
- QCD(F) & Implications for the real world

Introduction

- History is full of Irony



- The history of QCD is no exception: the naïve quark model was an essential ingredient in the development of QCD, but given the existence of QCD it very hard to understand why the quark model works at all.

What does "quark" mean?

- It is a type of cheese which exists in the US but is really popular in Germany

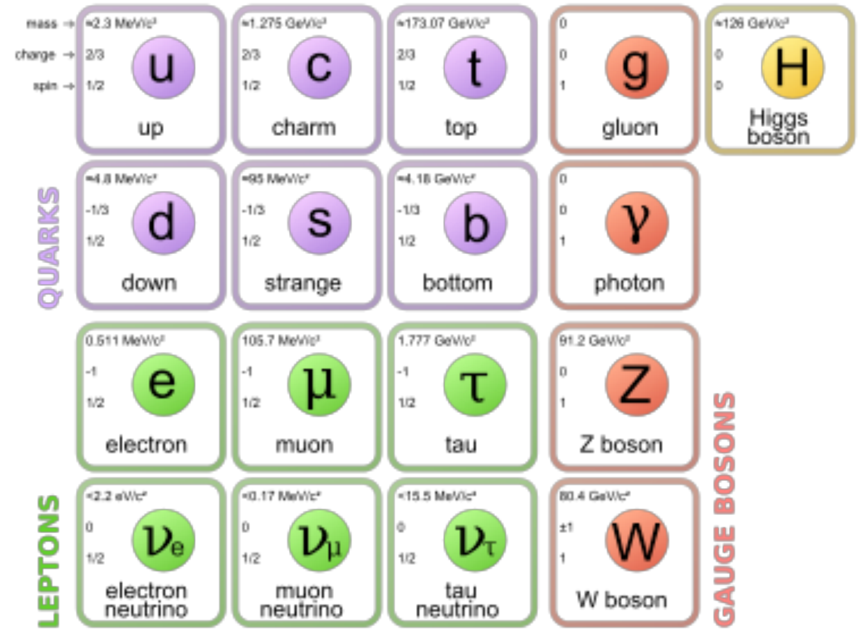
- It Is a nonsense word invented by James Joyce.

- It is an effective degree of freedom in the quark model.

- It is a fundamental degree of freedom in QCD.

All these meanings are fundamentally different

In what sense is the quark model a “model”?



- Note that from the modern perspective, the quark model is a more like a “model” in the sense of a model ship rather than the standard model. It captures *some* aspects of the real thing but misses others.

- It is not a complete theory like the standard model.

- It is definitely not beautiful like a supermodel.

- However, the simple quark model still strongly influences the language that we use to describe hadrons and remains a basic way most hadronic physicists think about states.
- Exotic hadrons are ones which do not fit into a quark model description and are important in that they help clarify what QCD has and the quark model does not.
  - There are two types
    - Quantum number exotics. States which by their quantum numbers **cannot** be made in the quark model. (eg. an isospin 2 meson)
    - Cryptoexotics. States which by their quantum numbers can be made in the simple quark model but which dynamically are dominated by components which are not of the quark model type.

- Why large Nc?
  - Many of the issues associated with the status of whether states are “really” quark model states contain intrinsic ambiguity. The hadronic states in nature are not typically bound states but resonances, while the naïve quark model describes bound states. Moreover, there can be mxing between different kinds of states (eg. a meson and a glueball).
  - In many cases these ambiguities vanish in the formal large Nc limit of QCD (‘t Hooft 1973).

    For example

    - glueballs must exist as narrow states only weakly coupled to meson in the large Nc limit (Witten 1979)
    - Quantum number extotic hybrids (quark+glue states) must exist as narrow resonance(Cohen 1998)

- In many cases the large Nc limit provides a crude cartoon of the real world of Nc=3 and this enables one to get insights into the real world.

  For example:

  - The OZI rule becomes exact at large Nc
  - Explains why baryons are heavier than mesons
  - Explains why decays with the smallest number of mesons typically dominate decays

- In some cases the large Nc limit provides a semi-quantitative understanding in a particular for baryons

  - An contracted SU(2Nf) spin-flavor symmetry emerges at large Nc and makes quantative predictions with corrections of order 1/Nc or $1/Nc^2$ Gervais and Sakita 1983; Dashen and Manhar 1993

- Given the usefulness of the large Nc limit and 1/Nc expansion one might hope that insights from large Nc to the real world for exotics.
- What is the real world situation?
  - The existence of glueballs remains controversial
  - There does seem to be strong evidence for least one hybrid state, the π_1(1400)
  - While there are believed to be heavy tetraquark states (but these probably are special to heavy quark physics; the heavy quark limit and large Nc limit do not, in general, commute) no known exotic tetraquarks composed of light quarks exist. However, there is a long history of identifying scalars (eg. fo(980)) as crypto-tetraquarks (Jaffe 1977)

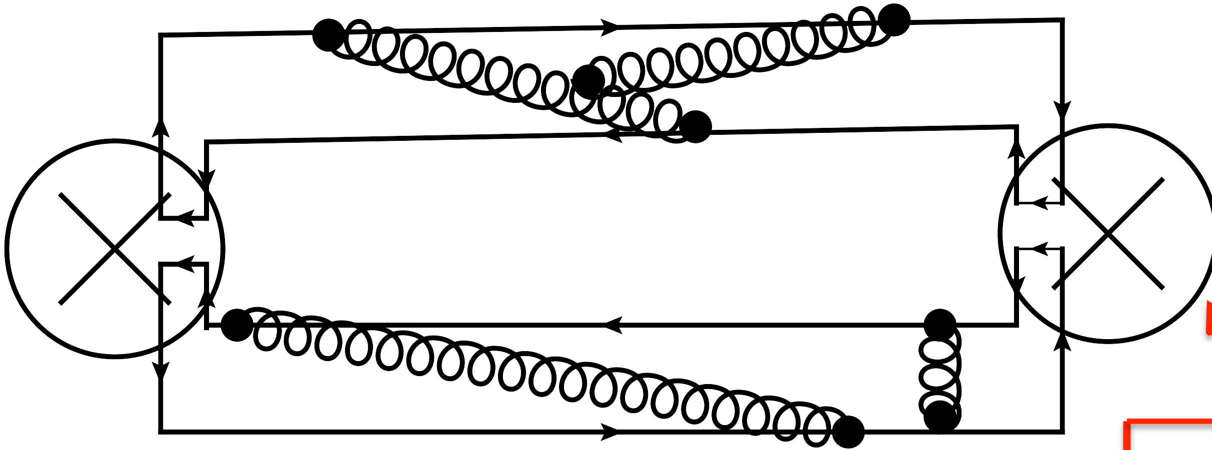
The Conventional Wisdom

- Tetraquarks do not exist at large Nc. (Witten 1979; Coleman 1985)

Basic argument:

The standard method to study hadrons at large Nc is via a study of the correlation functions for sources with the appropriate quantum numbers. It is easy to show that with a minimal tetraquark source of two bilinears at the same point, the leading order diagram ($\mathcal{O}(N_c^2)$) is just a disconnected diagram which behaves like two non-interacting mesons. It does not act like a tetraquark.

Disconnected graphs $\mathcal{O}(N_c^2)$



a typical diagram at quark/glue level: dominated by loops with planar gluons inside

Source

$$J = \overline{q}^a(x)q_a(x)\overline{q}^a(x)q_b(x)$$

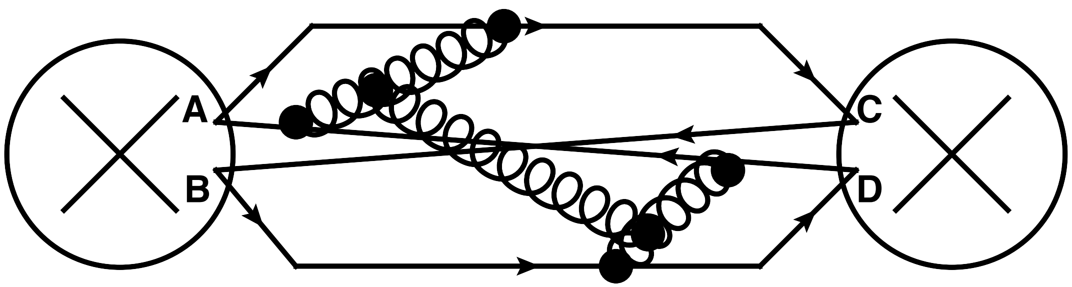
a,b are color indices

hadronic level two mesons

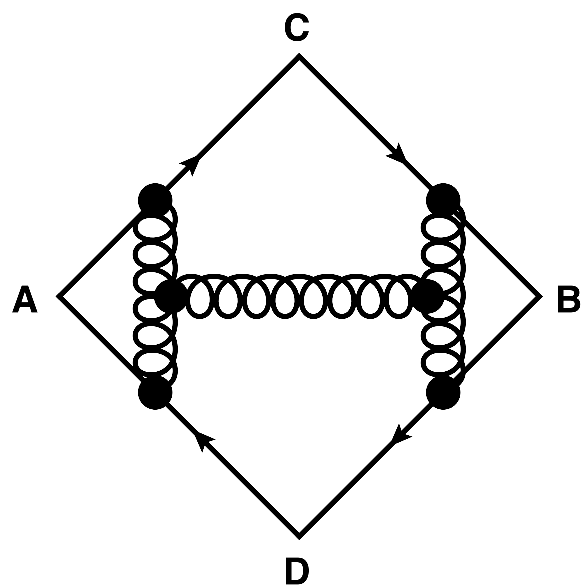
Weinberg's Critique

- Recently this has been called into question. Weinberg pointed out in a PRL last year that the standard argument is not valid.
- The argument is wrong for a very simple reason: the fact that leading order correlator implies that that the tetraquark operator "makes two meson and nothing else" is irrelevant. One needs to look at the leading diagrams in which the four quarks all interact---i.e. the leading connected diagram---to see states which look like to interacting mesons or tetraquarks. *Whether or not these resonate into tetraquarks is separate question from whether the leading diagrams only make noninteracting mesons.*

Connected graphs $\mathcal{O}(N_c)$



A typical diagram at quark/glue level : dominated by a single loop with planar gluons inside. Written as a sensible looking space-time type diagram, it does not seem to be by a single loop with planar gluons inside.



But topologically it is, and the Nc counting only depends on the topology

- If tetraquarks do exist they can be found in the dynamics of these connected diagrams.
- Note that the logic by which tertraquarks must be absent since they do not appear in the $\mathcal{O}(N_c^2)$leading order contribution to the correlator must be wrong
  - The same argument could be applied to a 4-quark source with the nonexotic quantum number of two-pions combined to a vector-isovector. The $\mathcal{O}(N_c^2)$ leading order contribution indeed just makes two non-interacting pions. However one cannot deduce from that a ρ meson does not exist. They do, and can be seen in the leading order connected contribitons $\mathcal{O}(N_c)$

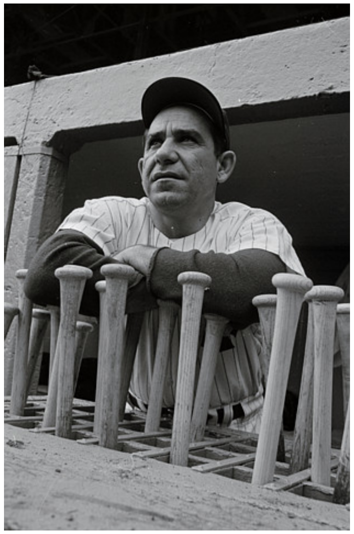
- It is important to note, however, that Weinberg has **NOT** shown that tetraquarks do exist as narrow resonances at large Nc. Merely that the argument to disprove the existence of tetraquark is wrong.
- At this stage there is no known way to show that tetraquarks exist with the standard version of large Nc QCD. Indeed, I will argue at the end that they do not!!
- However there is a variant of large Nc QCD in which it is possible to show that quantum number exotics must exist and become narrow as $N_c \to \infty$.

QCD (AS)

- **The large Nc limit of QCD is not unique**
  - **For gluons there is a unique prescription SU(3)→SU(Nc)**
  - **However for quarks, we can choose different representations of the gauge group**
  - **Asymptotic freedom restricts the possibilities to the fundamental (F), adjoint (Adj), two index symmtetric (S), two index anti-symmtetric (S).**
    - **Adj transforms like gluons (traceless fundamental color-anticolor); dimension Nc^2-1; 8 for Nc=3 (unlike our world).**
    - **S transforms like two colors (eg fundamental quarks) with indices symmetrized; dimension ½Nc(Nc+1); 6 for Nc=3 (unlike our world).**
    - **AS transforms like two colors (eg fundamental quarks) with indices antisymmetrized; dimension ½Nc(Nc-1); 3 for Nc=3 (just like our world).**

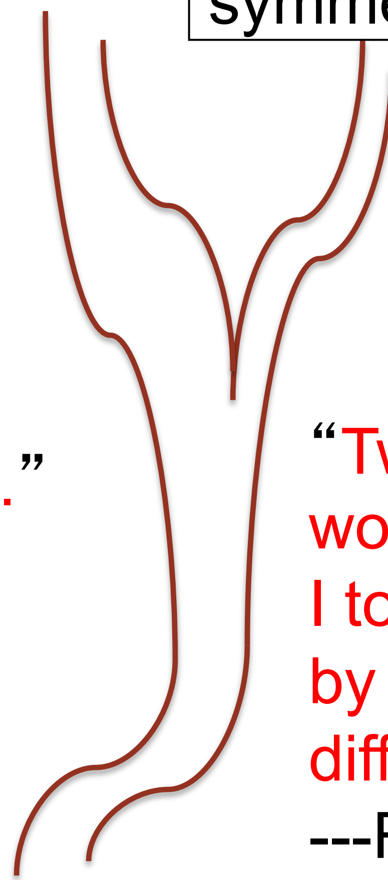
- Note that Nc=3 quarks in the AS representation are indistinguishable from the (anti-)fundamental. (In essence antisymmetric r b is the same as $\bar{g}$.)
- However quarks in the AS and F extrapolate to large Nc in **different ways.**
  - The large Nc limits are physically different
  - The 1/Nc expansions are different.
  - A priori it is not obvious which expansion is better
  - It may well depend on the observable in question
- The idea of using QCD (AS) at large Nc is old
  - Corrigan &Ramond (1979)
  - Idea was revived in early 2000's by Armoni, Shifman and Veneziano who discovered a remarkable duality that emerges at large Nc.

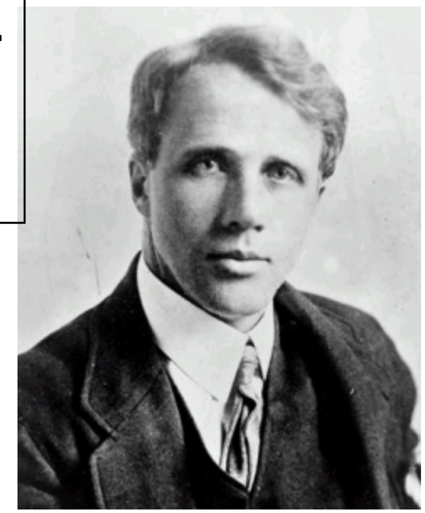
Two Roads to Large Nc QCD



Quarks in Fundamental

Quarks in 2-index anti-symmetric





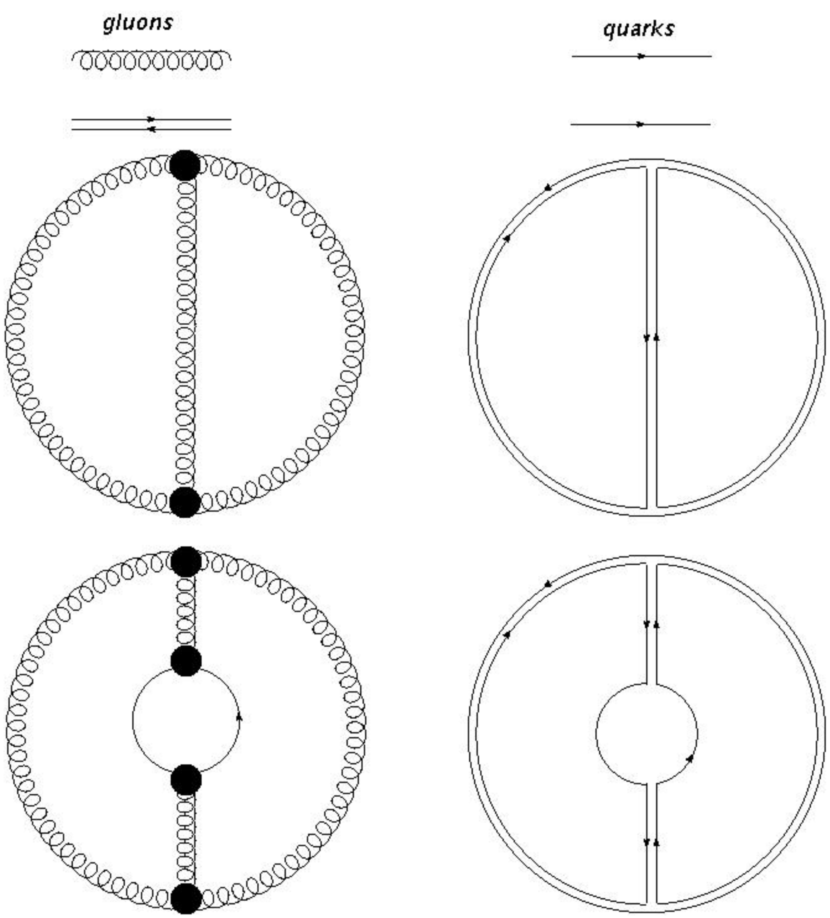
"When you come to a fork in the road, take it."
---Yogi Berra,
American baseball player, coach and part-time philosopher

"Two roads diverged in a wood, and I—
I took the one less traveled by And that has made all the difference."
---Robert Frost,
American poet

Large Nc QCD

Principal difference between QCD(AS) and QCD(F) at large Nc is in the role of quarks loops

Easy to see this using `t Hooft color flow diagrams



$$\left.\begin{array}{r} g^2 \sim 1/N_c \\ \text{3 color loops } N_c^3 \end{array}\right\} N_c^2$$

$$\left.\begin{array}{r} g^4 \sim 1/N_c^2 \\ \text{3 color loops } N_c^3 \end{array}\right\} N_c$$

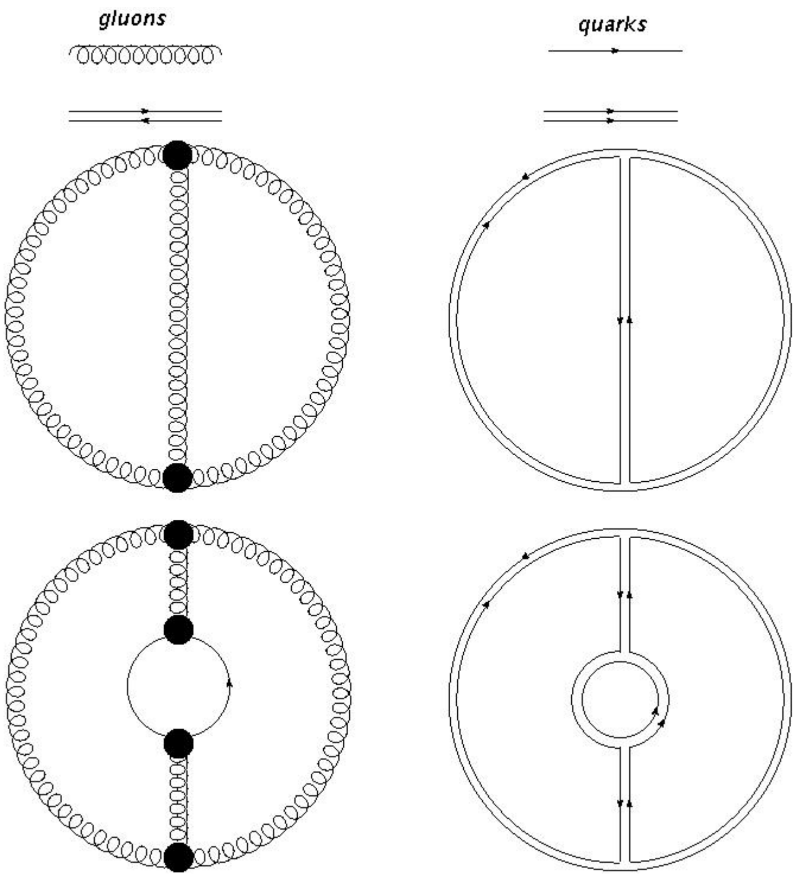
Recall the 't Hooft large Nc limit keeps g^2N_c fixed So $g^2 \sim 1/N_c$

QCD(F)

Insertion of a planar quark loops yields a 1/Nc suppression.

Leading order graphs are made of planar gluons

QCD(AS)



$$\left.\begin{matrix} g^2 \sim 1/N_c \\ \text{3 color loops } N_c^3 \end{matrix}\right\} N_c^2$$

Insertion of a planar quark loops does not lead to a 1/Nc suppression.

$$\left.\begin{matrix} g^4 \sim 1/N_c^2 \\ \text{4 color loops } N_c^4 \end{matrix}\right\} N_c^2$$

Leading order graphs are made of planar gluons and quarks

Principal phenomenological difference between the two is the inclusion of quark loop effects at leading order in QCD(AS). Whether this is a bug or a feature depends upon the observable. In baryon spectrscopy based on emergent symmetry, both QCD(AS) and QCD(F) appear to have predictive power (Cherman, Cohen & Lebed 2009, 2012).

- QCD(AS) naturally includes quark loops. Thus one might expect that in non-quantum number exotic channels tetraquarks will mix with ordinary mesons at leading order.
  - This can be shown to be correct.

- More interestingly, in quantum number exotic channels, QCD(AS) **MUST** have narrow tetraquarks at large Nc (i.e. narrow states which have at least 2 qyarks and 2 antiquaks)Cohen&Lebed 2014.

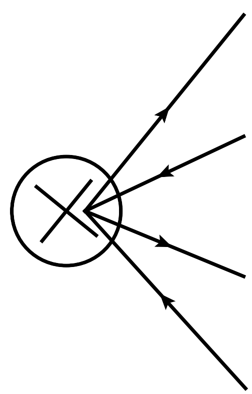
Key ingredient: there are single color trace tetraquark sources in QCD(AS). That is the source cannot be broken up into two separate color singlets (except for N_c^{-2} contbitutions). This cannot be done in QCD(F)

$$J(x)=\sum_{\substack{A,B\\a,b,c,d}} C_{AB}\,\overline{q}^{ab}(x)\Gamma_A q_{bc}(x)\overline{q}^{cd}(x)\Gamma_B q_{da}(x)$$

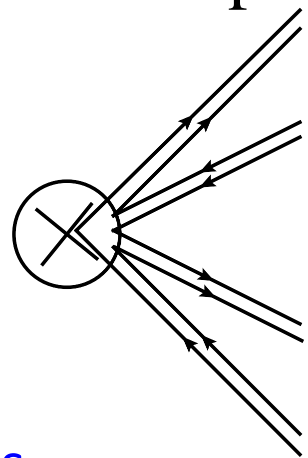
Γ_A,Γ_B are matrices in Dirac-flavor space.

a,b,c,d are fundamental color indices

choice of C_{AB} fixes quantum #s; for simplicty chose an exotic

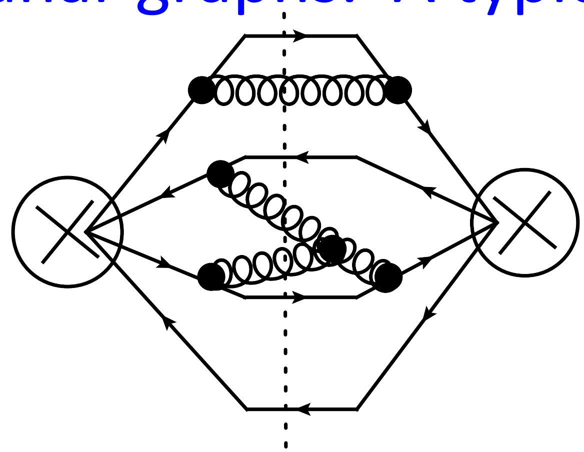


Source as a Feynman diagram

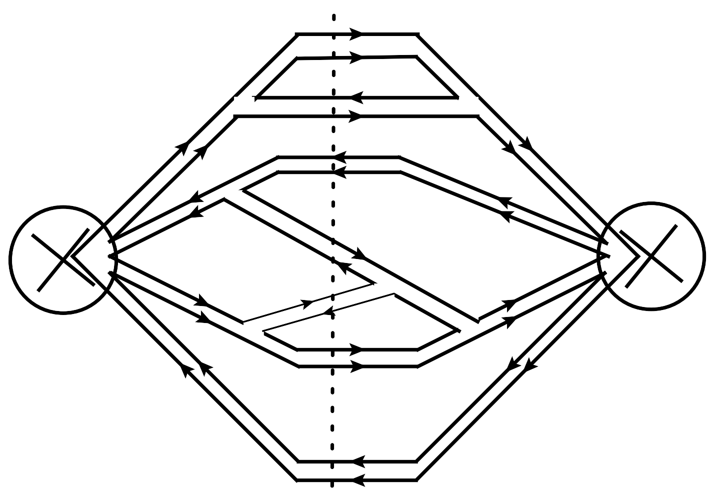


Source as a color-flow diagram

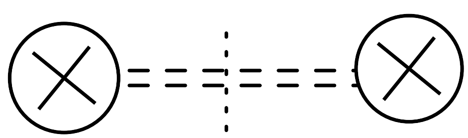
Look at the JJ correlation function. It is dominated by planar graphs. A typical diagram scales as N_c^4



Feynman diagram

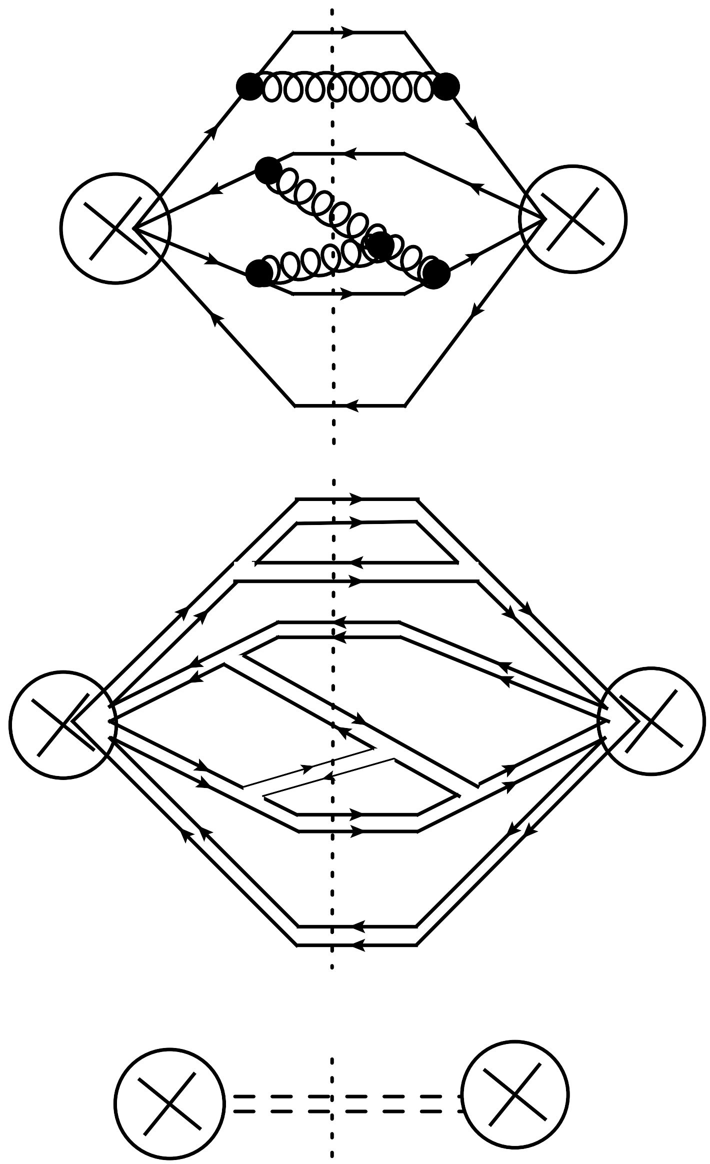


Color-flow diagram; 7 color loops $\sim N_c^7$; 6 factors of g $\sim N_c^{-3}$; overall scaling $\sim N_c^4$



Hadronic level diagram: propagation of a single tetraquark

The reason this corresponds to a single tetra hadrons can be understood in terms of a cut of the diagram.



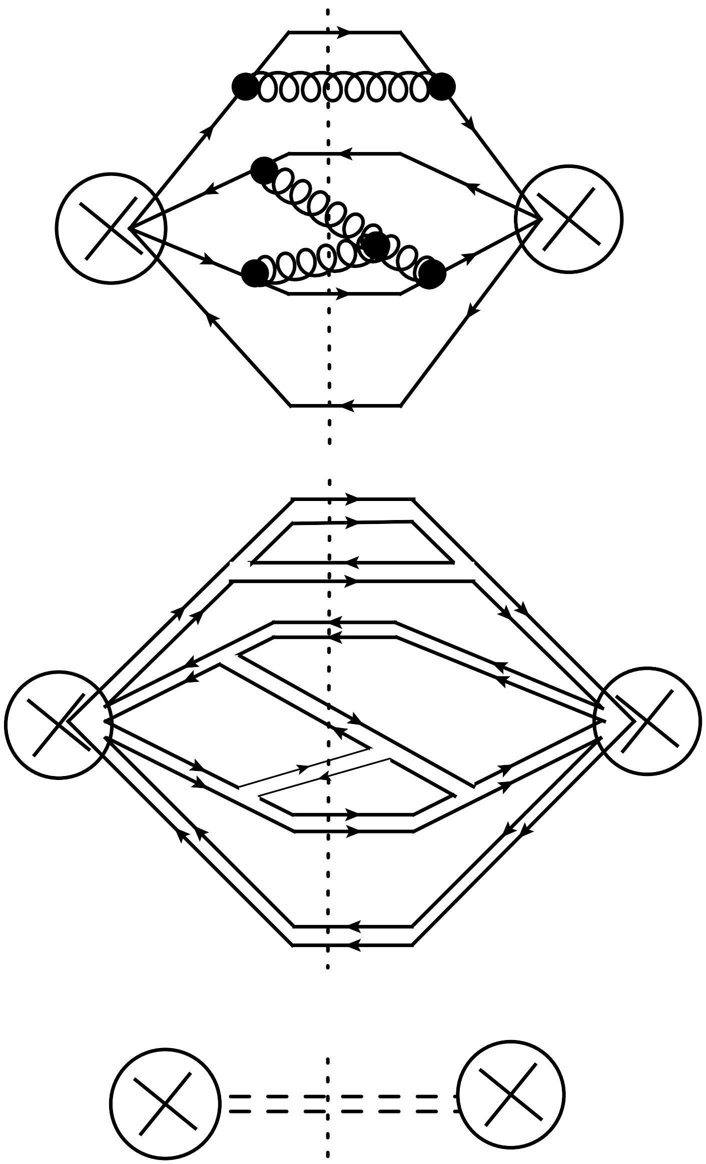
Short dashed line indicates a cut which reveals the intermediate state structure of the diagram.

The cut shown here corresponds to a state of the form

$$\bar{q}^{ab} q_{bc} A^{c}_{\ d} A^{d}_{\ e} \bar{q}^{ef} A^{g}_{f} q_{ga}$$

This is a single color-trace object. It can not be divided into two separate color singlets (except by a $1/N_c^2$ contribution)

This is generic: all cuts yield single-color trace objects



If one includes confinement, this implies that the state must be a single hadron at leading order. It cannot break up into two color singlet hadrons since all intermediate states consist of a single indivisible color singlet.

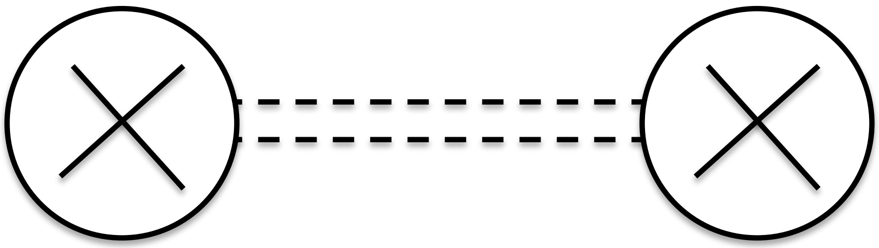
It must be narrow as components with more than one hadron are suppressed in the 1/Nc expansion.

- This can be seen to be true self-consistently
  - One can use standard kind of large Nc analysis for correlators with appropriate changes to account for QCD(AS) to deduce that a generic multi-hadron vertex scales as $N_c^{\,2-n}$ where n is the number of hadrons (mesons, glueballs, hybrids & tetraquarks).

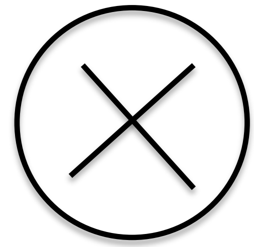
  - Thus, tetraquark width ~N_c^{-2}

  Thus as advertised the tetraquark is narrow

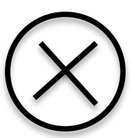
An example: the tetraquark-2 meson vertex. Proceed by studying the scaling of the appropriate correlator.



Recall at quark-gluon level we deduced that the tetraquark two point function scaled as N_c^4

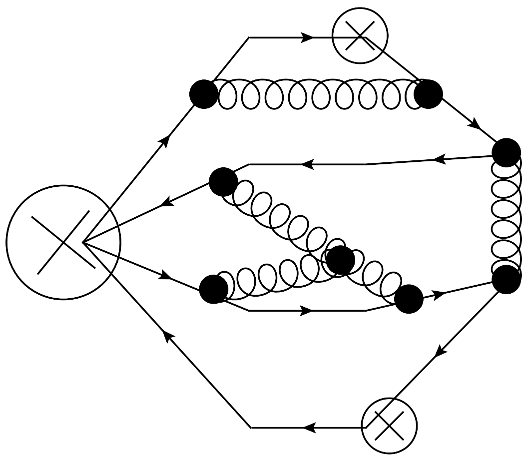


Thus the source scales as N_c^2



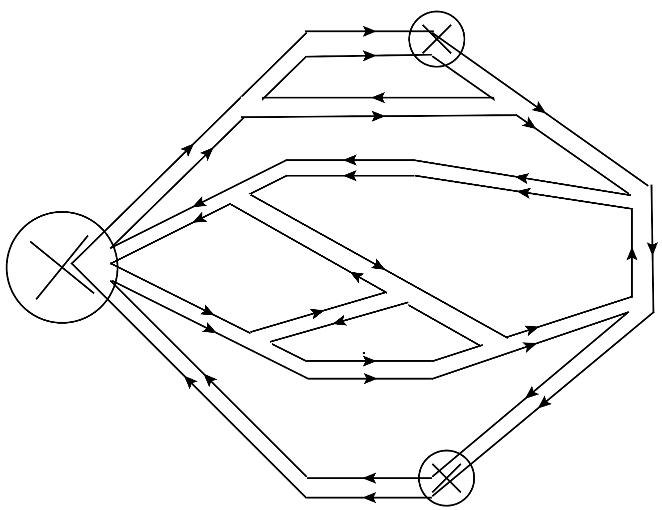
The meson source scales as N_c^1 by analogous reasoning

A typical graph

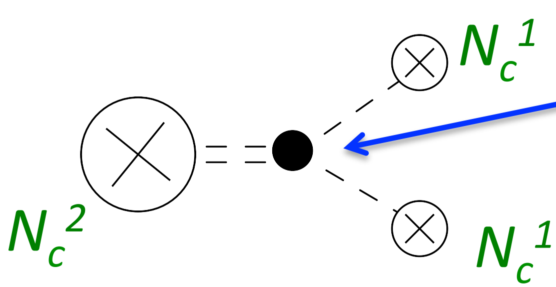


Feynman diagram: 8 factors of g

Overall N_c scaling $N_c^{\,3}$ = $(N_c^{\,-4})(N_c^{\,7})$



Color flow diagram: 7 color loops

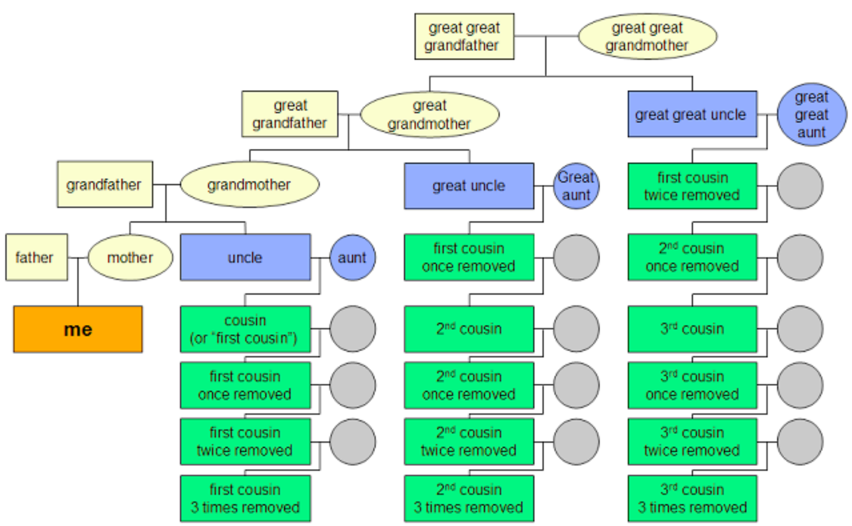


Hadronic level diagam

To match scaling at QCD level the tetraquark-2 meson vertex must scale as $N_c^{\,-1}$ yielding a width of $N_c^{\,-2}$.

Implications For the Real World

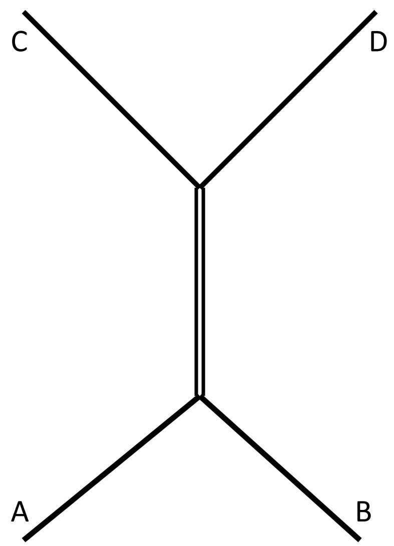
- Minimally, this result shows that nothing in the structure of gauge theories such as QCD excludes exotic tetraquark states made from light quarks as narrow resonances---a cousin of QCD has been shown to have them.
- The question is "how close a cousin"?



- One hint on this would be the behavior of QCD(F) (the more standard type of large Nc QCD)
- There is a somewhat subtle argument that despite Weinberg's critique of Coleman and Witten tetraquaeks in fact, do not exist at large Nc in QCD(F) cohen & Lebed in preparation
  - Reacall that Weinberg merely showed that the Witten/Coleman argument against tetraquarks is flawed, not that tetraquarks themselves exist.
  - The argument is too involved to be given in detail here.

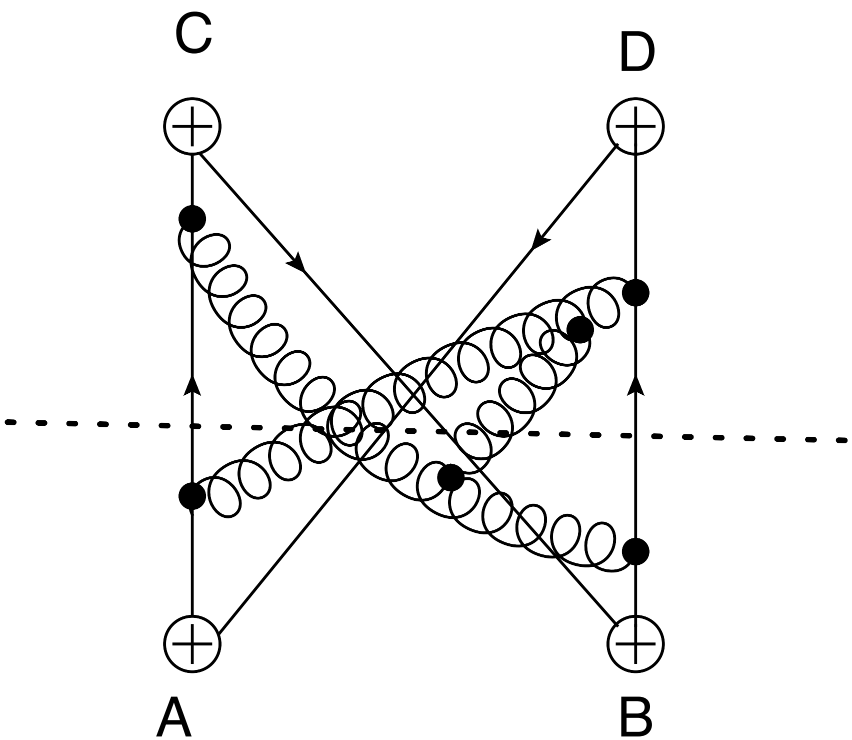
A brief sketch of the argument

If exotic tetraquarks exist they will couple to ordinary meson with a coupling strength ~$N_c^{-1/2}$. Thus it must appear as a singularity in the s-channel of scattering for incident mesons.

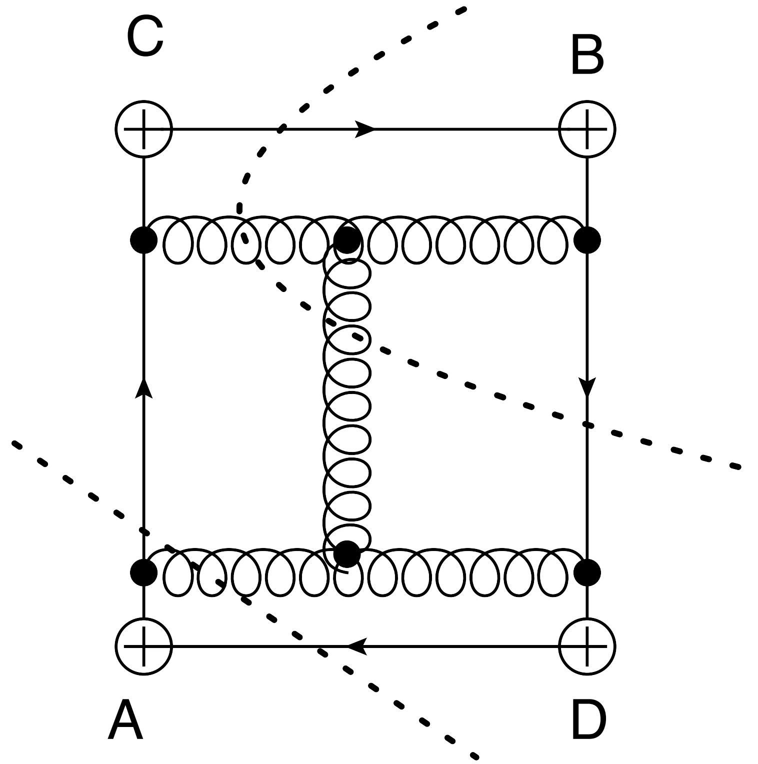


There is a topological argument that the connected 4-point function at leading order **every diagram** in an exotic channel only has singularities in the s-channel associated with the asymptotic mesons (either initial or final) in the sense that the cut has two color singlets carry the initial four momenta of each; thus there are no singularities associated with intermediate object.

A typical contribution



Space-time diagram



Topologically equivalent planar graph. Note that the cut has broken in two and each part carries the 4-momenta inserted at A or B

This behavior is generic and occurs in all diagrams associated with exotic channels

- Given this argument, whether tetraquarks exist in the real world depends on whether the real world is closer (in this aspect) to QCD(F) at large Nc or to QCD(AS) at large Nc. This is a dynamical question.

The bottom line

Whether narrow tetraquarks exist in the real world is a matter of dynamical detail which generic large Nc arguments cannot answer. Large Nc arguments show they cannot be excluded as incompatible with QCDlike theories