Why $f_0(500)$ must be narrower?

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White!!! Bjelašnica BiH

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Schedule

- **for unfamiliar with this subject** ($\sim$ 8 min),
- **for all**

what is the meson $f_0(500)$?

- formally $f_0(500)$ (informally σ) with mass and width $\sim$ 500 MeV,
- the lightest scalar-isoscalar meson with $I^G J^{PC} = 0^+ 0^{++}$, decays into $\pi\pi$,
- had a rich but difficult life,

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rich but difficult life of the σ meson

- until 1976 called ϵ or σ,
- excluded from Particle Data Tables from 1978 to 1992 and replaced by *correlated two pions*,
- since 1994: $f_0(400-1200)$,
- in years 2002-2010: $f_0(600)$,
- now (since 2012): $f_0(500)$

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- very important for e.g.
  - calculation of quark condensate mass,
  - determination of $q\bar{q} - gg$ couplings,
  - parameterization of $\pi\pi$ S wave amplitudes in e.g. many heavy meson decays (FSI)
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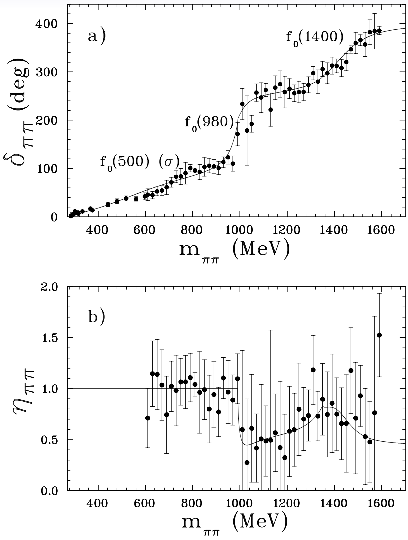
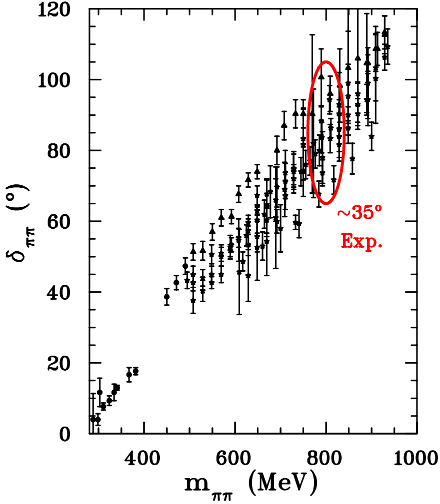
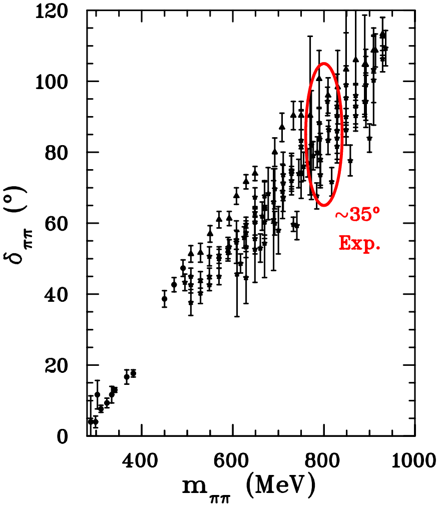


GKPY dispersion equations with imposed



GKPY dispersion equations with imposed crossing symmetry condition

Madrid-Kraków group 2005-2011



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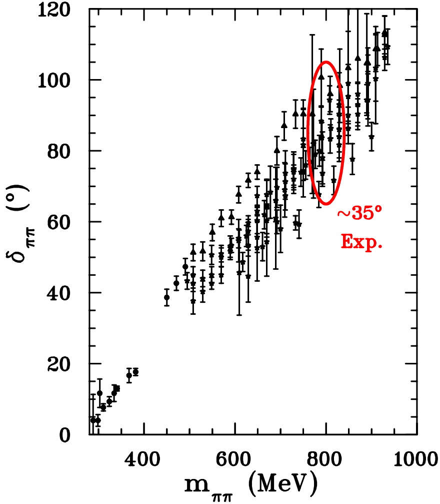

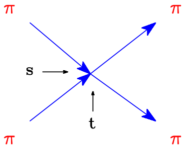

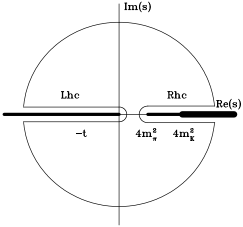

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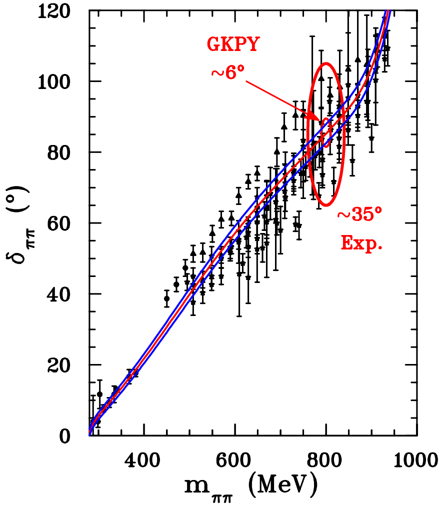

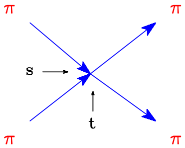

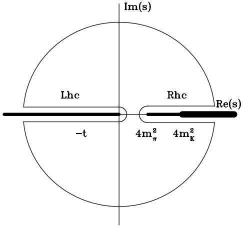

GKPY equations and poles of the $\pi\pi$ amplitudes

partial waves: *JI*

experiment

F1 D2

S0 D0

S2 P1

GKPY equations and poles of the $\pi\pi$ amplitudes

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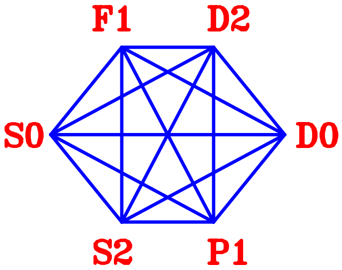
experiment + theory (GKPY)



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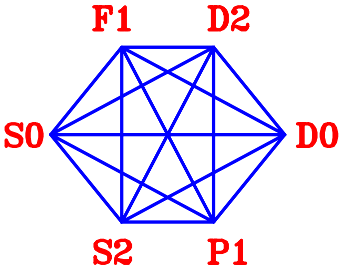
$$\int_{4m_\pi^2}^{\infty}$$



GKPY equations:

$$\text{Re}\, t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C^{II'}\, t_0^{I'(IN)}(4m_\pi^2) + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \int_{4m_\pi^2}^{\infty} ds'\, K_{\ell\ell'}^{II'}(s,s')\,\text{Im}\, t_{\ell'}^{I'(IN)}(s')$$

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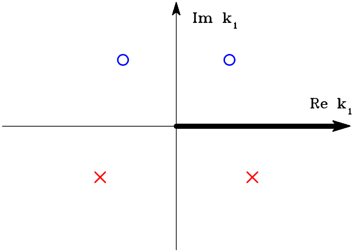
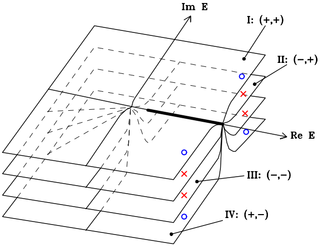
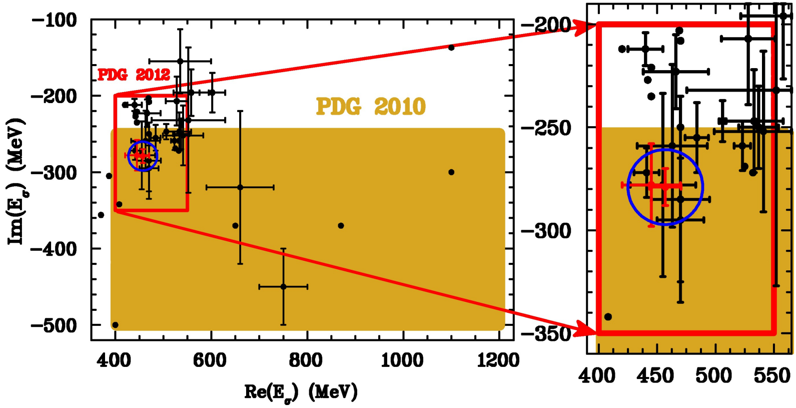
and poles of the $\pi\pi$ amplitudes:





$$M = Re(E_{pole}), \quad \Gamma = -2 \times Im(E_{pole})$$



Before 2012

Citation: C. Amsler *et al.* (Particle Data Group), PL **B667**, 1 (2008) and 2009 partial update for the 2010 edition (URL: http://pdg.lbl.gov)

$f_0(600)$ or σ

$I^G(J^{PC}) = 0^+($

A REVIEW GOES HERE – Check our WWW

$f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*		*TECN*
(400–1200)−*i*(250–500) OUR ESTIMATE			
• • • We do not use the following data for averages, fits, limits, et			
$(455 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$	[1] CAPRINI	08	RVUE
$(463 \pm 6^{+31}_{-17}) - i(259 \pm 6^{+33}_{-34})$	[2] CAPRINI	08	RVUE
$(552^{+84}_{-106}) - i(232^{+81}_{-72})$	[3] ABLIKIM	07A	BES2
$(466 \pm 18) - i(223 \pm 28)$	[4] BONVICINI	07	CLEO
$(484 \pm 17) - i(255 \pm 10)$	GARCIA-MAR..	07	RVUE
$(441^{+16}_{-8}) - i(272^{+9}_{-12.5})$	[5] CAPRINI	06	RVUE
$(470 \pm 50) - i(285 \pm 25)$	[6] ZHOU	05	RVUE
$(541 \pm 39) - i(252 \pm 42)$	[7] ABLIKIM	04A	BES2
$(528 \pm 32) - i(207 \pm 23)$	[8] GALLEGOS	04	RVUE
$(440 \pm 8) - i(212 \pm 15)$	[9] PELAEZ	04A	RVUE
$(533 \pm 25) - i(247 \pm 25)$	[10] BUGG	03	RVUE
$532 - i272$	BLACK	01	RVUE
$(470 \pm 30) - i(295 \pm 20)$	[5] COLANGELO	01	RVUE

Since year 2012

Citation: J. Beringer *et al.* (Particle Data Group), PR **D86**, 010001 (2012) and 2013 p

$f_0(500)$ or σ was $f_0(600)$

$I^G(J^{PC}$

A REVIEW GOES HERE – Check our

$f_0(500)$ T-MATRIX POL

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(400–550)−*i*(200–350) OUR ESTIMATE		
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$(440 \pm 10) - i(238 \pm 10)$	[1] ALBALADEJO	12
$(445 \pm 25) - i(278^{+22}_{-18})$	[2,3] GARCIA-MAR..	11
$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	[2,4] GARCIA-MAR..	11
$(442^{+5}_{-8}) - i(274^{+6}_{-5})$	[5] MOUSSALLAM	11
$(452 \pm 13) - i(259 \pm 16)$	[6] MENNESSIER	10
$(448 \pm 43) - i(266 \pm 43)$	[7] MENNESSIER	10
$(455 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$	[8] CAPRINI	08
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$(466 \pm 18) - i(223 \pm 28)$	[11] BONVICINI	07
$(472 \pm 30) - i(271 \pm 30)$	[12] BUGG	07
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BUT!

some physicists still complain.

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- *doubting semi specialists:* ... for sure your solution is not unique,
- *frightened:* ... DR groups (Madrid & Bern) got their results only due to specific choice parameterization of amplitudes,
- *really frightened:* ... not crossing symmetry in GKPY eqs but the limitation of this eqs to a single $\pi\pi$ channel leads to narrower and lighter σ,
- *ignorants:* ... one can put poles by hand and look at single - closest to physical region ones,
- *beginners:*... GKPY eqs are not enough, they neglect information from other channels,
- *nervous:* ... left cut is enough, we do not need GKPY,
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Another group - "Bern" group:
H. Leytwyller, J. Gasser, G. Colangelo, I. Caprini ...

The Role of the input in Roy's equations for pi pi scattering" G. Wanders, Eur. Phys. J. C17 (2000) 323-336

In the abstract:

An updated survey of known results on the dimension of the manifold of solutions is presented. The solution is unique for a low energy interval with upper end at 800 MeV. We determine its response to small variations of the input: S-wave scattering lengths and absorptive parts above 800 MeV.

I.e.:
Fixed two boundary conditions for the $\pi\pi$ amplitude:

- at the threshold ($S0$ wave scattering length) and
- at 800 MeV

specific choice parameterization?

Madrid: $cot\delta_0^0 = \frac{\sqrt{s}}{2k}\frac{M_\pi^2}{s-\frac{1}{2}z_0^2}\left[B_0 + B_1 w(s) + B_2 w(s)^2 + B_3 w(s)^3\right], \quad w = \frac{\sqrt{s}-\sqrt{s_0-s}}{\sqrt{s}+\sqrt{s_0-s}}$

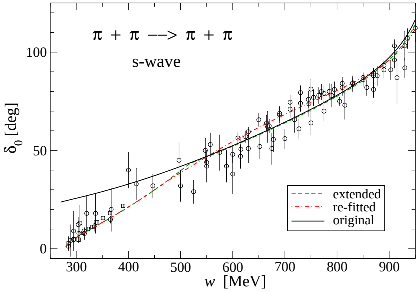
Test amplitude: $T(s) \sim \prod_{i=1}^N [w(s) - w_i], \quad w = \frac{\sqrt{s-s_2}+\sqrt{s-s3}}{\sqrt{s_3-s_2}}$

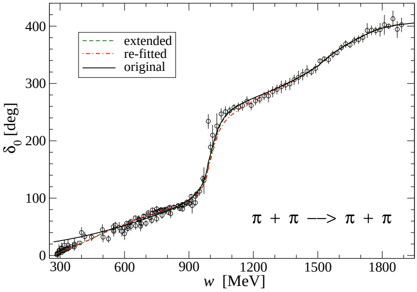
New low energy amplitude (up to $\sim 400 - 500$ MeV):

$$Ref_\ell^I(s) = \frac{\sqrt{s}}{4k} sin2\delta_\ell^I = m_\pi k^{2l}[a_\ell^I + b_\ell^I k^2 + c_\ell^I k^4 + d_\ell^I k^6 + O(k^8)]$$

above $\sim 400 - 500$ MeV - structure of amplitude not changed

repeated fit to the data (not changed) + GKPY equations





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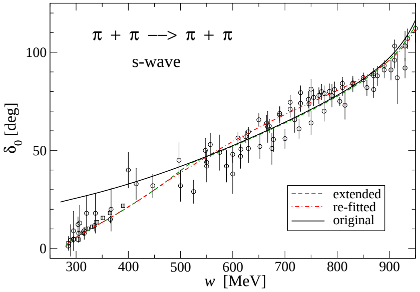

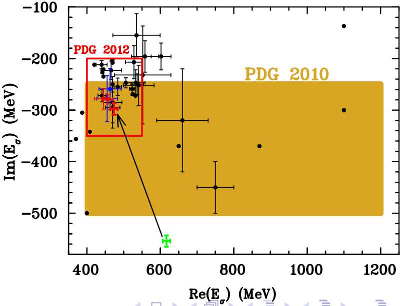

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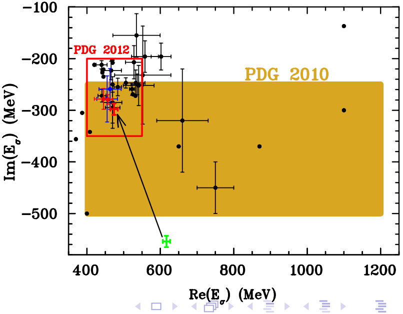
	χ_{tot}	χ_{GKPY}	m.p.S	m.p.P
start	5912	1330	500	500
end	626	59	390.2	648.2



one channel analysis leads to narrower and lighter σ

Let's make exercise:
fit of one pole at $k_r = \sqrt{s_r/4 - m_\pi^2}$ (one channel) to the phase shifts δ^{in} produced by 2 poles (2 channels) at $\sqrt{s} = 620 - i550$ MeV and at $650 - i550$ MeV.

$$S = \frac{-k - k_r}{k - k_r} \times \frac{-k + k_r^*}{k + k_r^*}$$

Analytically calculated $Re(k_r)$ and $Im(k_r) \longrightarrow \sqrt{s_r} = 615.5 - i544.6$ MeV
Fit to δ^{in} for $280 < \sqrt{s} < 1000$ MeV $\longrightarrow \quad \sqrt{s_r} = 618.2 - i539.3$ MeV

Almost no difference between two wide poles in the 2-channel case and one pole!

Reason is the that all poles lie on different Riemann sheets and play very different role (even opposite signs!) in the full amplitude. They can compensate each other in large extend.

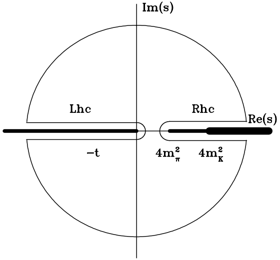
GKPY eqs are not enough, they neglect information from other, higher channels $K\bar{K}, \eta\eta$...
$\equiv$
one has to analyze poles of the σ on various Riemann sheets

$$\text{Re}\ t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C^{II'}\, t_0^{I'(IN)}(4m_\pi^2) + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \int_{4m_\pi^2}^{\infty} ds'\, K_{\ell\ell'}^{II'}(s,s')\ \text{Im}\ t_{\ell'}^{I'(IN)}(s')$$

where $t_{\ell'}^{I'}(s') \sim \eta(s')e^{2i\delta(s')}$

- above the $K\bar{K}$ threshold ($\sqrt{s'} \approx 990$ MeV) $t_\ell^{I(IN)}(s)$ must be multichannel therefore must have singularities on many Riemann sheets. Moreover $\text{Re}\ t_\ell^{I(OUT)}(s) \approx \text{Re}\ t_\ell^{I(IN)}(s)$,
- below 1100 MeV: $S_{\pi\pi} = \eta e^{2i\delta_{\pi\pi}}$ and $S_{K\bar{K}} = \eta e^{2i\delta_{K\bar{K}}}$ have the same η which is directly fitted to the GKPY eqs,

... left cut is enough, we do not need GKPY ...



Left hand cut in parameterizations of amplitudes:

- additional factor $e^{i\alpha}$ in the full $S = e^{2i\delta}$ matrix element,
- It has, however, nothing to do with crossing symmetry!
    - It does not provide any type of relationship $A(s,t) = C_{st}A(t,s)$,
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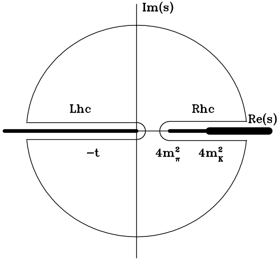

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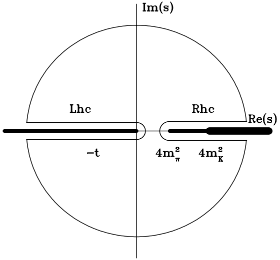

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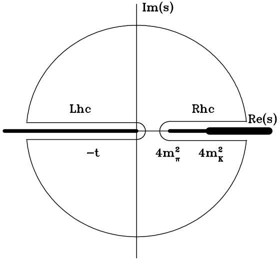

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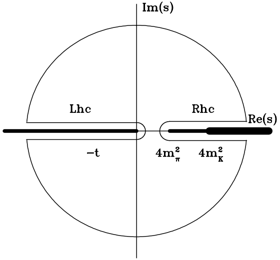

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what forces GKPY eqs to pull up-left the sigma pole?

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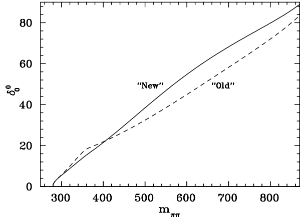

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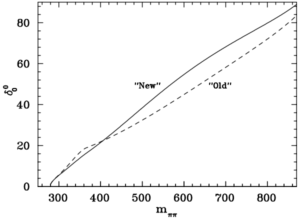

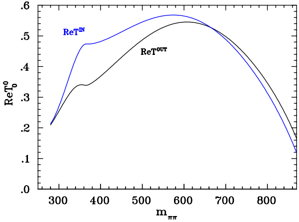

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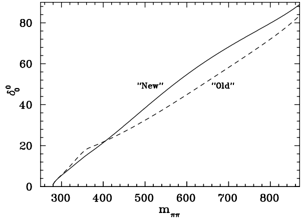

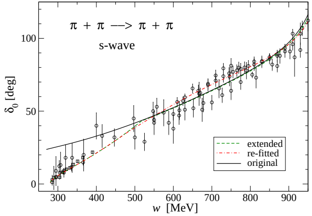

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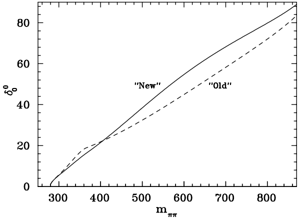

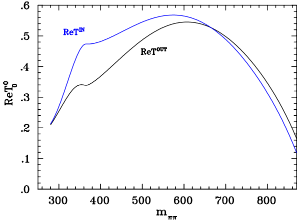

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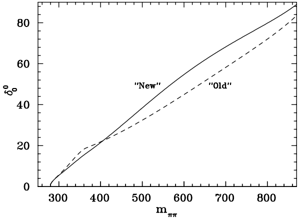

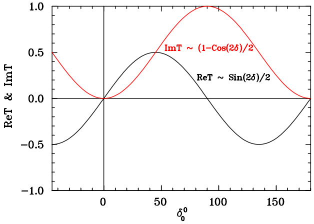

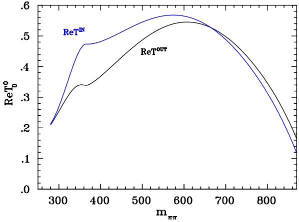

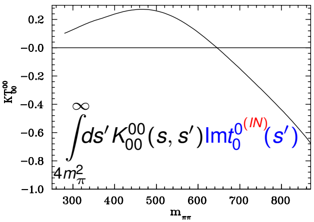

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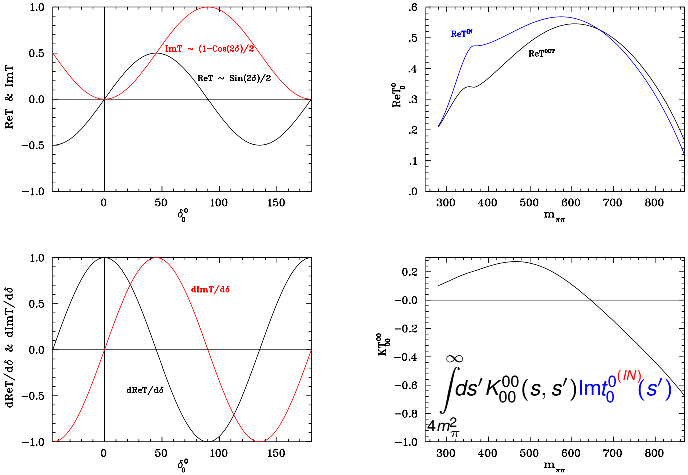

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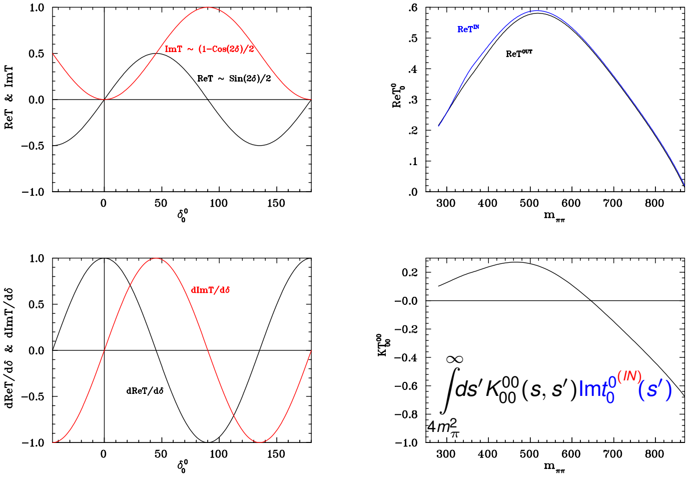

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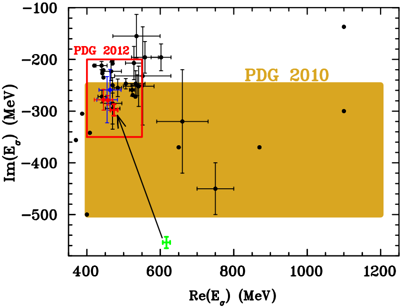

Two things: trigonometry and crossing symmetry algebra lead to narrower and lighter σ.

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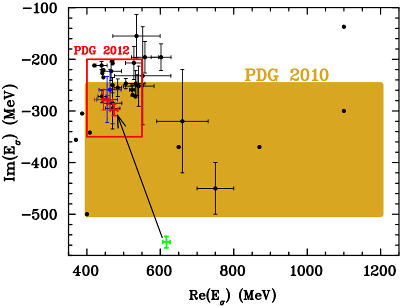
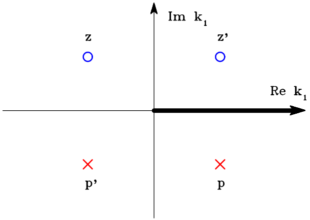


Two things: trigonometry and crossing symmetry algebra lead to narrower and lighter σ.

Nothing more and nothing instead of it is needed.

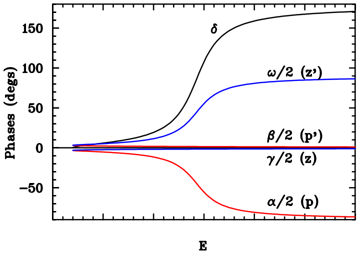
single pole - closest to physical region one



Unitarity: $S(k) = S^*(-k^*)$

$$S = \frac{-k-p}{k-p} \times \frac{-k-p'}{k-p'}$$

Pole p':
needed for unitarity but gives also a small contribution the the phase shifts.



how important is pole p'?

exercise with $\rho(770)$ (P-wave)

PDG Tables'2012:
$M = 771.1 \pm 0.9$ MeV,
$\Gamma = 149.2 \pm 0.7$ MeV
It corresponds to $\delta = 90^o$ and includes influence of the symmetric pole p'.

Analytical continuation of the amplitude to the complex s plane gives
$M = 766.1$ MeV, $\Gamma = 149.2$ MeV
for the single p pole.

Some analyses find difference ~ 10 MeV for mass (Res_{pole}) of the ρ.

$\rho(770)$