Infrared behaviour of propagators and running coupling in the conformal window of QCD

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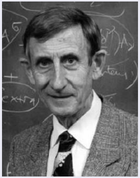
Der Wissenschaftsfonds.

Doktoratskolleg Graz "Hadrons in Vacuum, Nuclei and Stars"
FWF DK W1203-N16

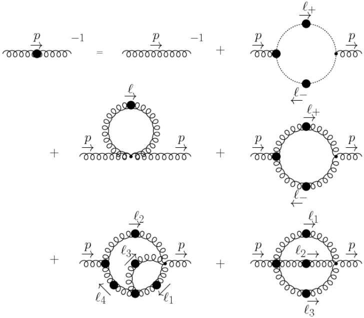
Outline

A Theoretical Description - The Formalism

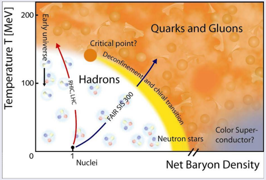
The  -  Approach

(F.J. Dyson) (J. Schwinger)

- we work in:
  - Landau gauge
  - Euclidean space-time



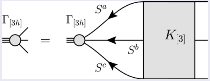
- **exploring the QCD phase diagram using DSEs**
  - study critical endpoint
    ⇝ cf. e.g. Luecker, Fischer, ...
  - investigate CFL/CSC phases
    ⇝ cf. e.g. Mueller, Nickel, Buballa, ...
  - scalar QCD @ $T \neq 0$ [in preparation]



- **exploring the conformal window**
  - turn up the number of flavours
    - $SU(3)$: $\gtrsim$ **8 fundamental flavours**
    - $SU(2)$: $\gtrsim$ **2 adjoint flavours**

technicolor

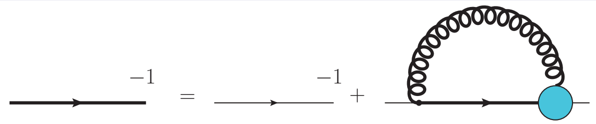
- **bound-state equations**
  - Bethe-Salpeter/Fadeev equations
    cf. e.g. Bhagwat, Eichmann, Maris, Nicmorus, Swanson, Tandy, Vujinovic, Williams, ...



Prerequisites for these Applications

- **details of the relevant quark-gluon vertex tensor structure**
  - **non-perturbative contributions**
  - **vertex dressing functions obtain non-zero values due to $D\chi SB$**
  - **how are these effects accounted for in models?**

Dyson-Schwinger Equation for the Quark Propagator



$$S^{-1}(p) = Z_2 S_0^{-1}(p) + g^2 Z_{1F} C_F \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \, S(q) \, \boldsymbol{\Gamma^\nu(p,q;k)} \, D_\gamma^{\mu\nu}(k)$$

Some Comments

- **Quark-Gluon Vertex**: rainbow truncation, BC/CP-type vertex constructions, ...
  - $D\chi SB \Leftrightarrow$ effective interaction strength
  - put effective interaction e.g. into gluon propagator
  - and/or use sophisticated vertex models
- **Goal**: **full self-consistent solution**
  - at vanishing temperature $\rightsquigarrow$ 12 tensors
  - use transversal projector $\rightsquigarrow$ 8 tensors

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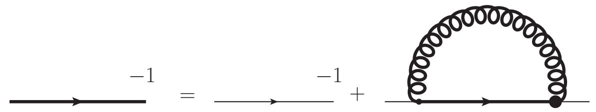

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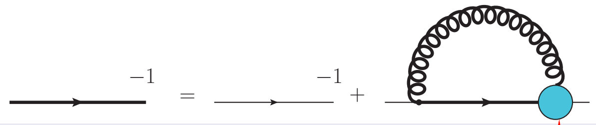
Dyson-Schwinger Equations - The Beginning

Dyson-Schwinger Equation for the Quark Propagator



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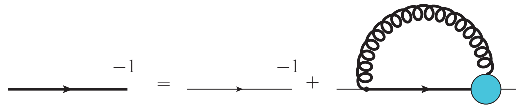
Basis (if $T = \mu = 0$)

$$\Gamma^\nu \rightsquigarrow \begin{Bmatrix} \mathbb{1} \\ \not{k} \\ \not{p} \\ \not{k}\not{p} \end{Bmatrix} \otimes \begin{Bmatrix} \gamma^\nu \\ k^\nu \\ p^\nu \end{Bmatrix}$$

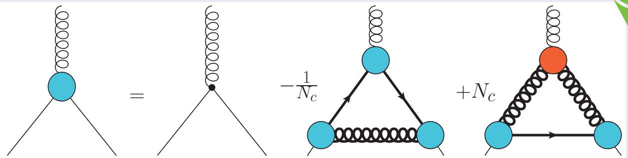
i.e. $\Gamma^\nu \propto \sum_{i=1}^{12} \lambda_i \Gamma_i$

The Coupled System

Dyson-Schwinger Equation for the Quark Propagator



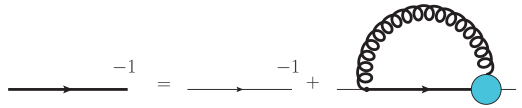
“Dyson-Schwinger Equation” for the Quark-Gluon Vertex



Remarks/Ingredients

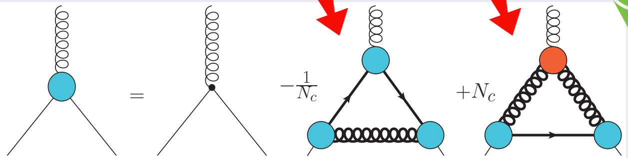
- **dress all vertices**, [Alkofer, Fischer, Llanes-Estrada, Schwenzer, Annals Phys. 324 (2009)]
  - correspondence to DSE-like equation in 3PI formalism, [Berges, PRD70 (2004)]
- **3-gluon vertex** $\rightarrow$ lattice/DSE/FRG results (cf. talk by M. Vujinovic)
- **gluon propagator** from lattice/<u>**DSE**</u> calculations $\rightsquigarrow$ **brute-force on GPUs**

Dyson-Schwinger Equation for the Quark Propagator



Abelian Diagram **non-Abelian Diagram**

"Dyson-Schwinger Equation" for the Quark-Gluon Vertex



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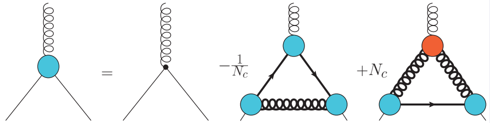
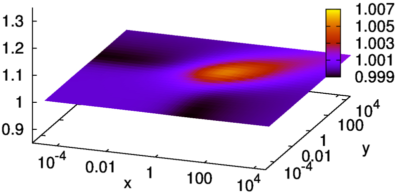
Detour: A Scalar Theory

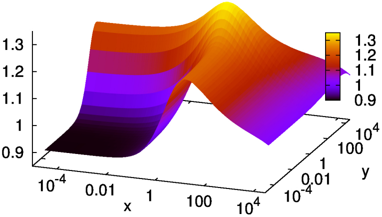
The Scalar-Gluon Vertex - Fundamental Representation - SU(3)

$$A\Big(\overbrace{p_1^2}^{x}, \overbrace{p_2^2}^{y}; \overbrace{\frac{p_1\cdot p_2}{|p_1||p_2|}}^{\zeta}\Big) \leftrightsquigarrow$$





Abelian contribution



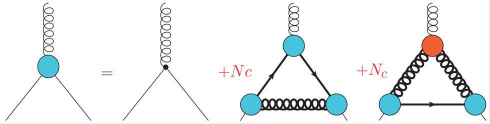
non-Abelian contribution

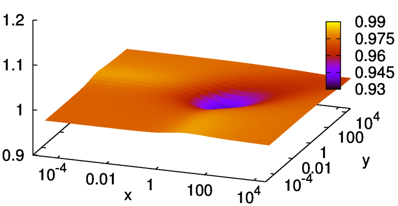
[MH, R. Alkofer, arXiv:1304.4360, ExcitedQCD 2013]

Detour: A Scalar Theory

The Scalar-Gluon Vertex - Adjoint Representation - SU(2)

$$A\Big(\overbrace{p_1^2}^{x},\ \overbrace{p_2^2}^{y};\ \overbrace{\frac{p_1\cdot p_2}{|p_1||p_2|}}^{\zeta}\Big) \leftrightsquigarrow$$





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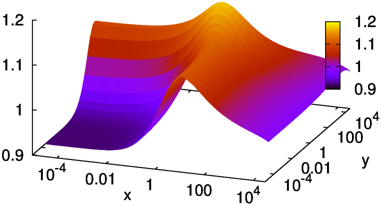
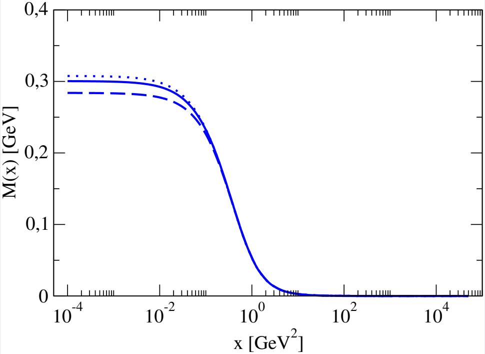


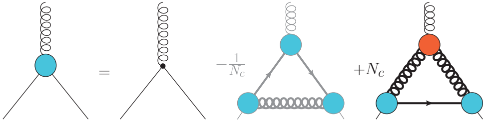
non-Abelian contribution

[MH, R. Alkofer, arXiv:1304.4360, ExcitedQCD 2013]

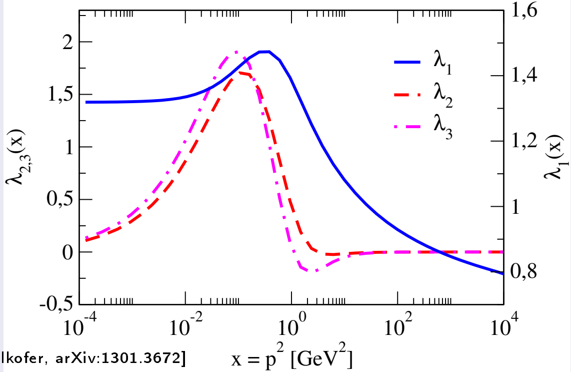
FERMIONIC CASE - Full Calculation with non-Abelian Diagram



- full calculation taking non-Abelian diagram into account

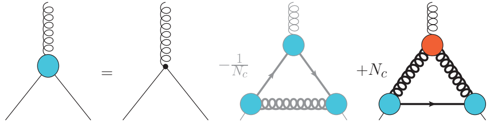


Some Selected Quark-Gluon Vertex Dressing Functions



[MH, Windisch, Alkofer, arXiv:1301.3672]

- full calculation taking non-Abelian diagram into account



Tensor Decomposition of the Quark-Gluon Vertex

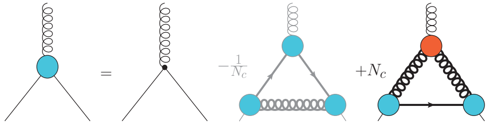
$$\Gamma^\mu(p,q;k) = g_0\,\Gamma_0^\mu + g_1\,\Gamma_1^\mu + g_2\,\Gamma_2^\mu + g_3\,\Gamma_3^\mu + \ldots + g_6\,\Gamma_6^\mu + g_7\,\Gamma_7^\mu$$

- use **transversal/orthonormal basis**
- **basis not unique** $\rightsquigarrow$ **rotate to Ball/Chiu basis in the end**

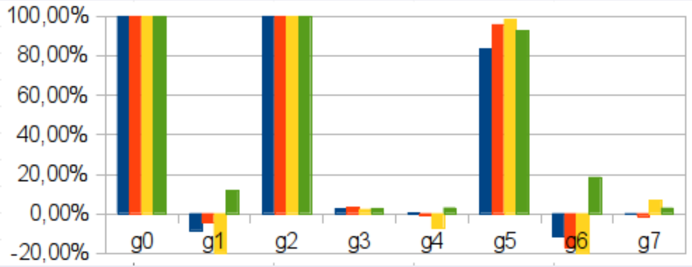
Relevance of Different Tensor Structures - *the Procedure*

	$\langle\bar\psi\psi\rangle$	Γ_π	$M(0)$		error	[%]	$\overline{error}$ [%]
$\{g_i\}$	**292.75**	**86.77**	**282.53**	BLUE	RED	YELLOW	GREEN
$\{g_i\}\backslash g_0$	181.15	68.05	231.82	38.12	21.57	17.95	**25.88**
$\{g_i\}\backslash g_1$	268.08	81.72	300.08	8.43	5.82	-6.21	**6.82**
$\{g_i\}\backslash g_2$	131.10	47.46	172.10	55.22	45.30	39.09	**46.54**
$\{g_i\}\backslash g_3$	285.10	83.95	282.09	2.61	3.25	0.16	**2.01**
$\{g_i\}\backslash g_4$	291.20	86.76	286.80	0.53	0.01	-1.51	**0.68**
$\{g_i\}\backslash g_5$	239.15	67.07	206.05	18.31	22.70	27.07	**22.69**
$\{g_i\}\backslash g_6$	309.35	94.82	338.13	-5.67	-9.28	-19.68	**11.54**
$\{g_i\}\backslash g_7$	286.97	82.53	242.22	1.97	4.89	14.27	**7.04**

Relevance of Different Tensor Structures - *Self-consistent Treatment*



mass generation stability mass generation

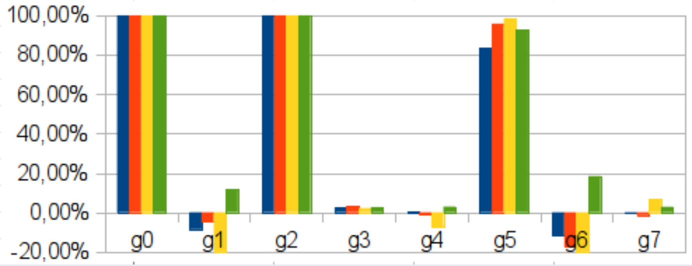


	error [%]			$\overline{error}$ [%]
$\{g_i\}\backslash g_{3,4,7}$	2.05	1.26	2.55	**1.95**
$\{g_i\}\backslash g_{1,3,4,6,7}$	-3.66	-27.49	-80.58	**37.24**

Relevance of Different Tensor Structures - *Self-consistent Treatment*

- find optimized basis
  - probably **only three relevant tensor structures**

mass generation stability mass generation



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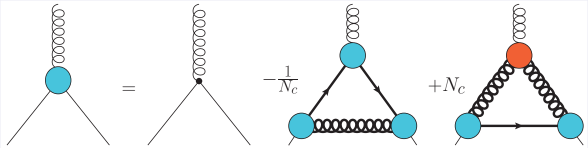
Including the Abelian Diva



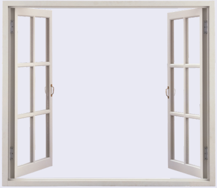
Relevance of Different Tensor Structures

	error [%]			$\overline{error}$ [%]
$\{g_i\}\backslash g_{3,4,7}$	2.05	1.26	2.55	**1.95**
$\{g_i\}\backslash g_{3,4,7}$ +***Abelian***	1.32	0.22	5.56	$\lesssim$ **5**

Abelian Diagram

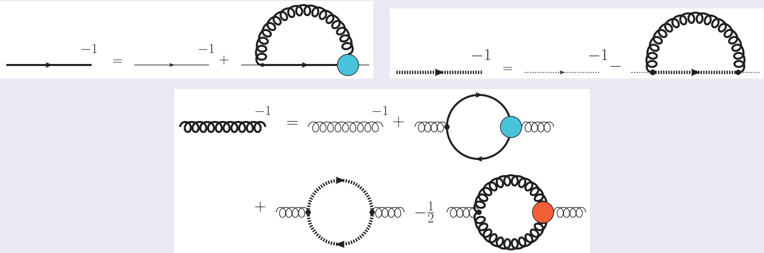
- **brute force approach**: $\approx$ **100.000 terms**
- **improved basis set**:
  - much less terms: **100.000** $\rightarrow$ **8.000 terms**
  - feasible with CUDA (on C2070 GPUs at least)
- **contributes marginally - even in CHIRAL LIMIT**

QCD with a Large Number of Flavours - Probing the Conformal Window



- **technicolor (TC) models** $\rightarrow$ phenomenological discrepancies/inconsistencies
  - *e.g.* **large FCNC's** - not observed in experiment
- improvements - **walking/conformal technicolor**
  - **minimal walking TC** based on $SU(2)$ with 2 adjoint Dirac fermions
  - **large N_f QCD**
  - **large distance behaviour different from QCD** $\rightsquigarrow$ (approx.) IRFP

The Coupled Quark System



- **model the quark-gluon vertex**: CP, BC, **1BC**, ...
- **model the three-gluon vertex**: **Fischer 03'**, Huber 12'
- **increase the number of flavours** N_f → investigate **influence of quark-loop**
- in the following: **large** N_f **QCD**, i.e. $SU(3)$
  - but also adjoint/$SU(2)$ possible

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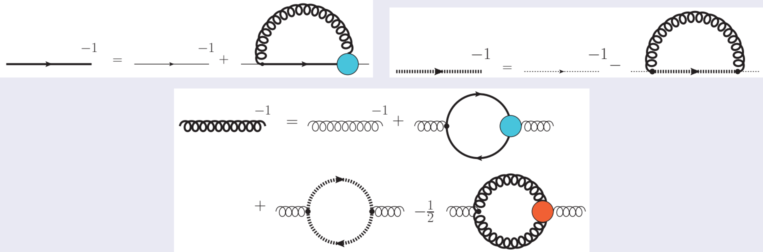
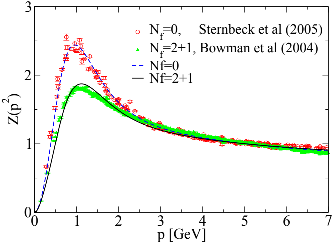
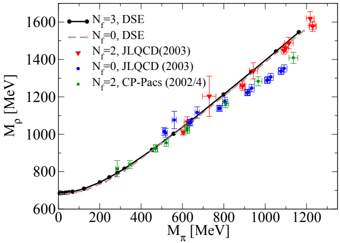


- some results obtained from these models





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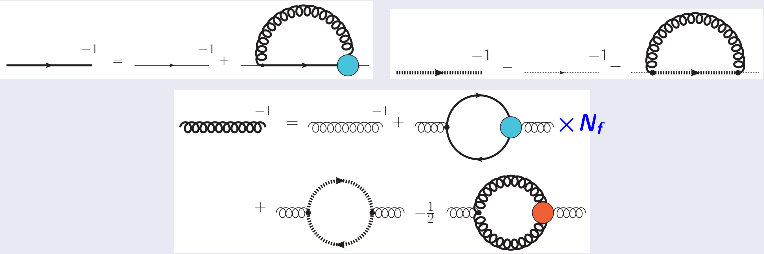

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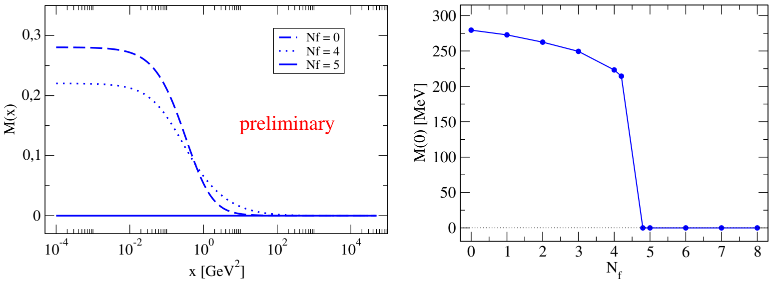
Including the quark-loop with a $1BCxG^2$ vertex ansatz

- look at **quark mass function** $M(p^2 = x)$ for $N_f \in \{0, 4, 5\}$
- system enters **chirally symmetric phase** for $N_f \geq N_f^{crit} \Leftrightarrow M(x) = 0$



- $N_f^{crit} \approx 4.5 \rightsquigarrow$ **too low! - but depends ONLY on quark-qluon vertex!**
- problem: scale fixing for gluon propagator!

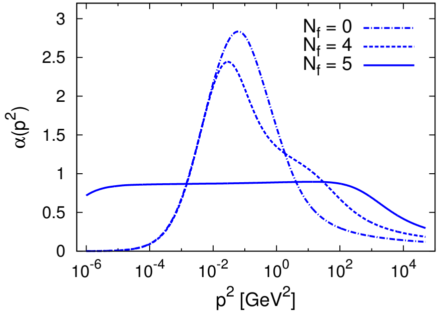
[MH, Fischer, Alkofer, in preparation]

Including the quark-loop with a $1BCxG^2$ vertex ansatz

- look at non-perturbative **running coupling**: $\alpha(x) \propto Z(x)G^2(x)$
- **scaling relation** between ghost and gluon dressing function for $N_f \geq N_f^{crit}$
- coupling drops significantly $\Rightarrow \boldsymbol{M(p^2 = x) = 0} \rightarrow$ **plateau** is formed

IR analysis

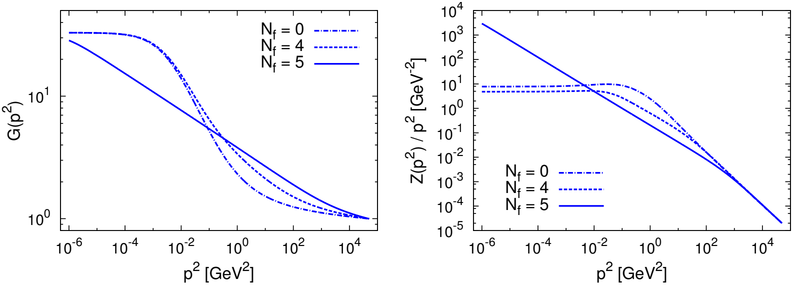
- assume power law
  - $Z(x \ll 1) \rightarrow A\ x^{\varrho_1}$
  - $G(x \ll 1) \rightarrow B\ x^{\varrho_2}$
- possible: $\rho_1 \overset{relat.}{\leftrightsquigarrow} \rho_2$
- scaling relation
  - $Z(x \ll 1) \rightarrow A\ x^{2\varrho}$
  - $G(x \ll 1) \rightarrow B\ x^{-\varrho}$
  - $\varrho \ldots$constant
- otherwise: decoupling



[MH, Fischer, Alkofer, in preparation]

Including the quark-loop with a $1BCxG^2$ vertex ansatz

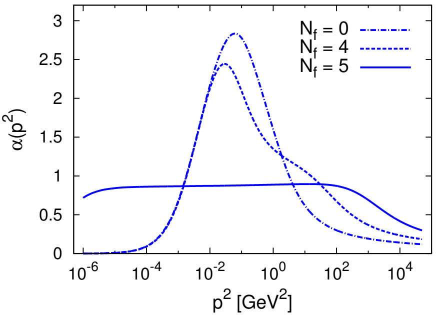
- **scaling relation** between ghost and gluon dressing function for $N_f \geq N_f^{crit}$
- power law behaviour over wide range of momenta $\rightarrow$ **sudden change!**
  - $Z(x) \rightarrow A\, x^{2\varrho}$
  - $G(x) \rightarrow B\, x^{-\varrho}\,, \quad \varrho(N_f) \approx 0.149 \,@\, N_f^{crit}$



[MH, Fischer, Alkofer, in preparation]

Including the quark-loop with a $1BCxG^2$ vertex ansatz

- obstacles: **quadratic divergengies** $\rightarrow$ analytical/numerical subtraction
- **NO INFLUENCE** on N_f^{crit} (≈ 4.5) - only minor qualitative changes



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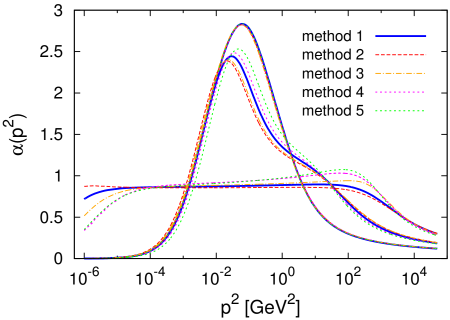

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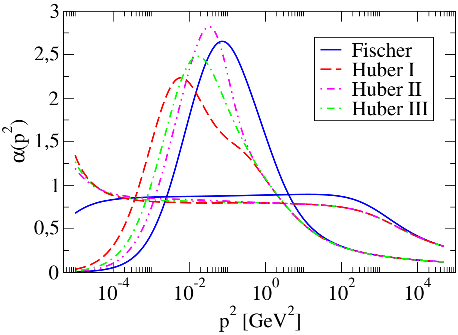
Including the quark-loop with a $1BCxG^2$ vertex ansatz

- dependence on **three-gluon vertex ansatz**
  - **NO INFLUENCE** on N_f^{crit} (≈ 4.5)
    - **qualitative changes within confinement phase**, i.e. $N_f < N_f^{crit}$
    - almost **no influence within conformal phase**, i.e. $N_f \geq N_f^{crit}$
  - do the calculation with a proper vertex → (near) future work



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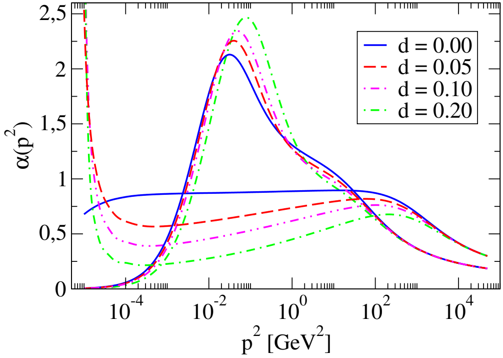
Including the quark-loop with a $1BCxG^2$ vertex ansatz

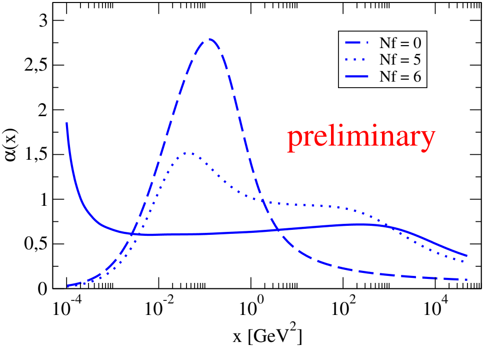
- dependence on **quark-gluon vertex ansatz**
  - varying the $1BC \times G^2$ model, $d > 0$ enhances IR interaction strength
  - corresponds to: **tree-level** $\times$ **function**
  - **NO INFLUENCE** on N_f^{crit} (≈ 4.5)



[MH, Fischer, Alkofer, in preparation]

Including the quark-loop with a $1BCxG^2$ vertex ansatz

- dependence on **quark-gluon vertex ansatz**
  - try e.g. $CPxG^2$ ansatz: **different N_f^{crit} ⇒ tensor structure important**
  - corresponds to: **(tree-level + additional tensor structure)** × **function**
  - do the calculation with a proper (full) vertex → (near) future work

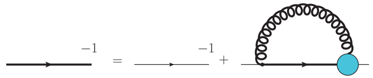


[MH, Fischer, Alkofer, in preparation]

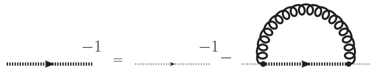
Outlook: Solving a proper System

The Coupled System

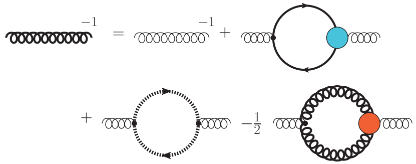
Quark Propagator



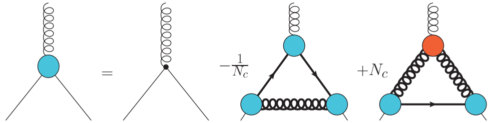
Ghost Propagator



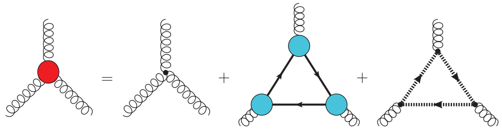
Gluon Propagator



Quark-Gluon Vertex



Three-Gluon Vertex



Summary

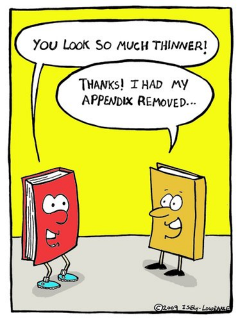
- **quark-gluon vertex**
  - **self-consistent solution** $\rightarrow$ also **scalar theory**
  - **dynamical mass** in quark propagator
  - $D_\chi SB$ also in vertex dressing functions
  - **isolate important tensor structures**
  - **useful for phenomenological model building**
- **large** N_f **QCD**
  - **transition to a chirally symmetric phase** above N_f^{crit}
  - **scaling relation** for YM propagators restored
  - **conformal running coupling**
  - **dependence on vertex ansatz** $\Rightarrow$ needs improvement

Outlook

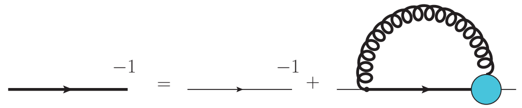
- **still work to do for quark-gluon vertex !**
- **redo the flavour study using a full quark-gluon vertex !**

Thank You For Your Attention!

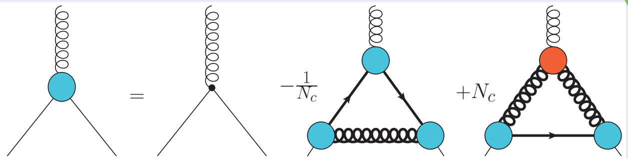
Appendix



Dyson-Schwinger Equation for the Quark Propagator



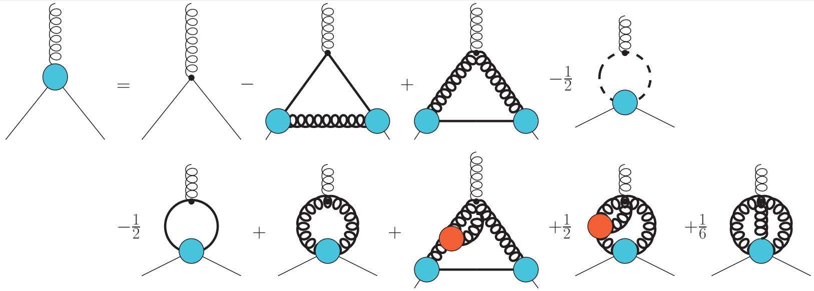
"Dyson-Schwinger Equation" for the Quark-Gluon Vertex



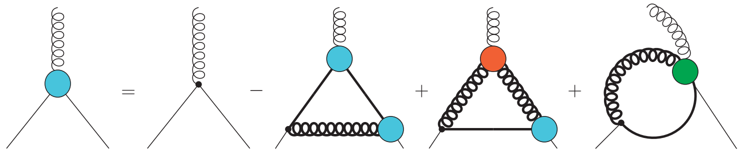
Remarks/Ingredients

- **dress all vertices**, [Alkofer, Fischer, Llanes-Estrada, Schwenzer, Annals Phys. 324 (2009)]
    - correspondence to DSE-like equation in 3PI formalism, [Berges, PRD70 (2004)]
- **gluon propagator** from lattice/DSE calculations
- **3-gluon vertex** → only models available

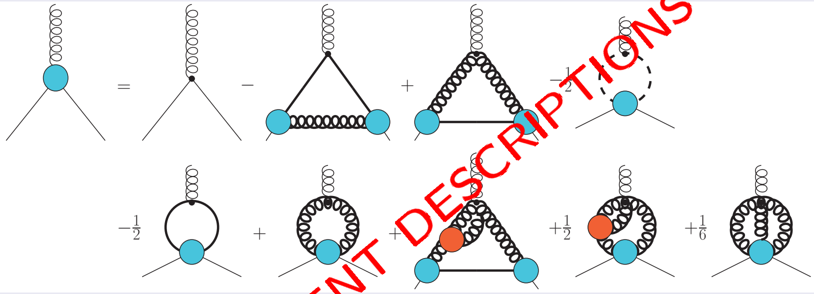
First 1PI Version



Second 1PI Version

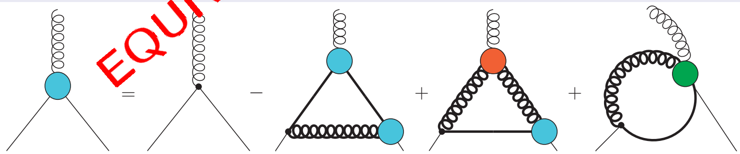


First 1PI Version

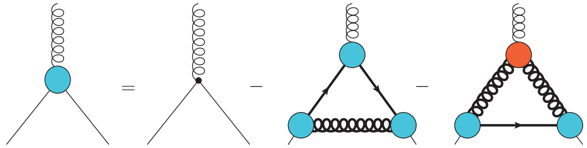


EQUIVALENT DESCRIPTIONS

Second 1PI Version



Analogy to 3PI Formalism [Berges, PRD70 (2004)]



- DSE-like equation derived in 3PI formalism

- (some) higher order corrections included
- **same IR behaviour** for all 3 equations,

  [Alkofer, Fischer, Llanes-Estrada, Schwenzer, Annals Phys. 324 (2009)]
- **makes the system treatable** $\Rightarrow$ **perfect, I'll buy it**
  - compare with upcoming **lattice data** in the end and see whether truncation is justified