Non-perturbative features of the three-gluon vertex in Landau gauge

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The three-gluon vertex in Landau gauge, arXiv: 1402.1365

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Excited QCD, Bjelasnica Mountain, BIH, 02.-08. February 2014.

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Outline of talk

1. Introduction: Strong QCD.
2. Greens functions and Dyson-Schwinger equations (DSEs).
3. Greens functions of QCD.
4. Three-gluon vertex: calculation setup and results.
5. Summary and outlook.

Strong QCD and non-perturbative methods

- QCD coupling α_S large at low energies $\rightarrow$ Strong QCD.
- Non-perturbative phenomena: $D_\chi SB$, confinement, ...
- Studies in this regime require non-perturbative tools.
  1. Lattice QCD.
  2. Functional renormalisation group (FRG).
  3. Effective field theories (ChPT...)
  4. Dyson-Schwinger/Bethe-Salpeter/Faddeev equations.
- These tools are used for hadron phenomenology.
  1. Masses.
  2. Decay constants.
  3. Form factors.

Greens functions

- Basic objects in QFTs are Greens functions.
- n-point Greens function encodes interactions of n 'particles'.
- Continuum version of lattice QCD correlation functions.
- Example: two-point Greens function (propagator) given by

$$
\begin{aligned}
G(x_1, x_2) &= \frac{1}{N} \int \mathcal{D}_\phi e^{-i(S+\int_x J\cdot\phi)} \phi_1(x_1)\phi_2(x_2) \qquad (1) \\
&= \langle \phi_1(x_1)\phi_2(x_2) \rangle
\end{aligned}
$$

S = action, J = 'sources'

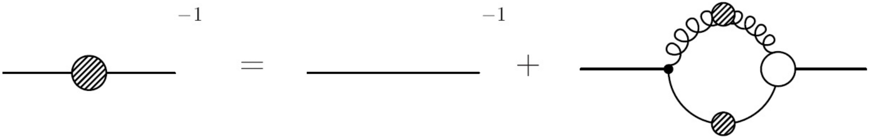
Dyson-Schwinger equations

- Theory invariant under field translations $\phi(x) \to \phi(x) + \epsilon(x)$
  $\Rightarrow$ Dyson-Schwinger equations (DSEs).

$$\left\langle \frac{\delta S[\phi]}{\delta \phi(x)} \right\rangle_J = J(x) \tag{2}$$

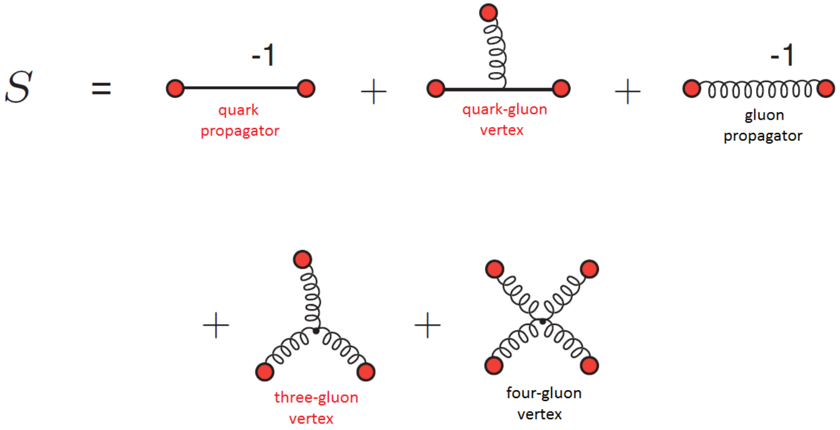
- DSEs connect various *n*-point Greens functions.
- Infinite tower of coupled integral equations.
- Exact, continuous formulation of a quantum field theory. ✓
- Perturbation theory can be extracted from DSEs. ✓
- Practical calculations neccessitate truncations. ✗

Quark propagator DSE



- Exact equation, with full gluon propagator and quark-gluon vertex.
- Satisfy own DSEs, with higher-point Greens functions $\rightarrow \infty$ tower.

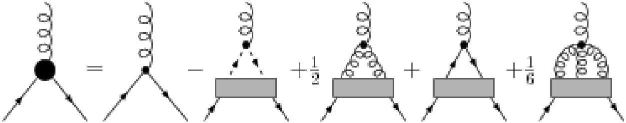
QCD action



QCD action (not gauge-fixed, no ghosts)

Quark propagator and quark-gluon vertex

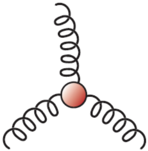
- Quark propagator
  $\to$ central object in bound state equations (Bethe-Salpeter, Fadeev).
- Quark-gluon vertex: connects matter and gluon fields.
  1. Essential ingredient in quark propagator DSE.
  2. Structure of bound state equations, Ward-Takahashi identity.



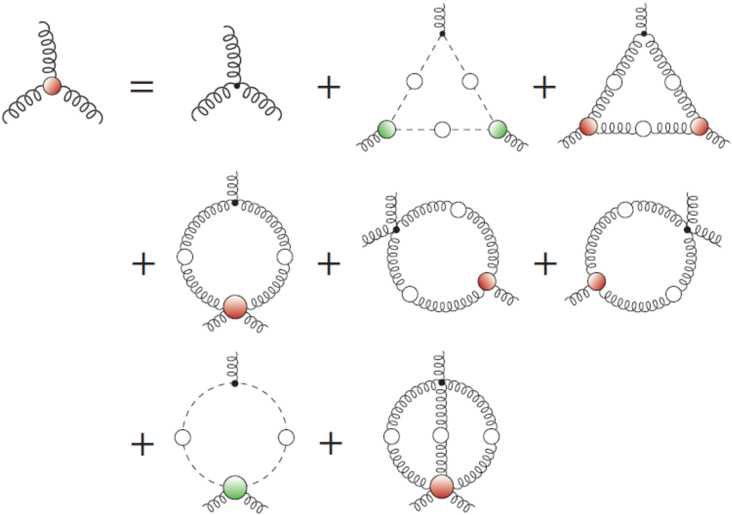
Full DSE for quark-gluon vertex.

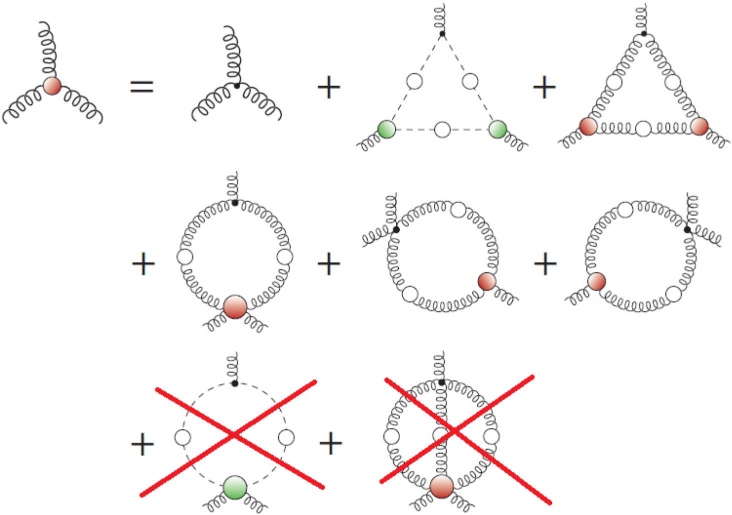
Three-gluon vertex Γ_{3g}

- Γ_{3g} and Γ_{4g} epitomise self-interaction of gluon field.
- Importance of three-gluon vertex:
  1. Driving term in quark-gluon vertex DSE.
  2. Enters bound state equations through quark-gluon vertex.
  3. Irreducible three-body force in Faddeev equation.
  4. Closing truncated system of DSEs in ghost/gluon sector.
- Γ_{3g} is the current focus of our study.



Three-gluon vertex DSE

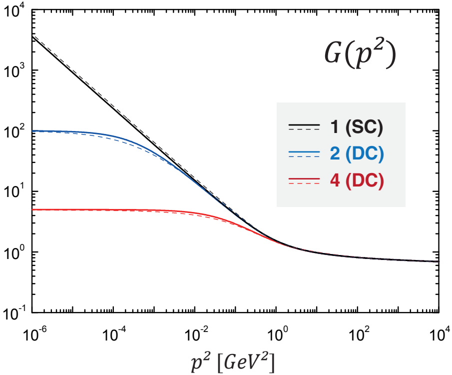


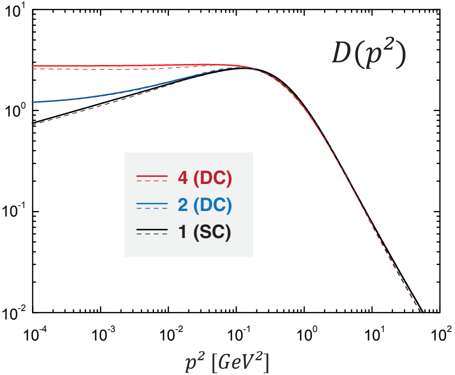


Inputs for calculation

1. Ghost/gluon propagators $\rightarrow$ use fit functions.
2. Ghost-gluon vertex $\rightarrow$ taken to be bare.
   A. Cucchieri, A. Maas, T. Mendes, PRD **77**, 2008
3. Three-gluon vertex $\rightarrow$ self-consistently back-coupled.

Ghost dressings and gluon 'propagators'





Solid lines: DSE solutions, data by C. Fischer

Dashed lines: Fit functions

Four-gluon vertex

- DSE study of Γ_{4g}, without full tensor/color basis.
  C. Kellermann, C. Fischer, PRD **78**, 2008
- We model Γ_{4g}, dressed tree-level structure.

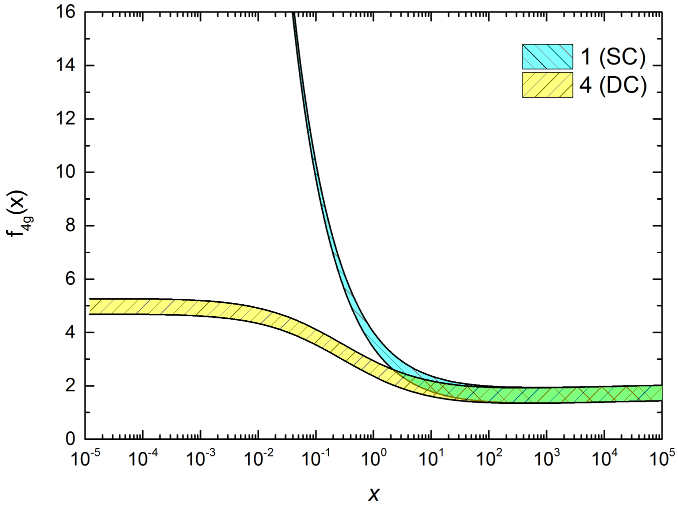
$$\Gamma^{\mu\nu\rho\sigma}(p_1, p_2, p_3, p_4) = f_{4g}(x) \cdot \Gamma^{\mu\nu\rho\sigma}_{(0)} \tag{3}$$

$$x \propto (p_1^2 + p_2^2 + p_3^2 + p_4^2)$$

- $f_{4g}(x)$ is 'correct' in IR and UV.
- Study dependence of results on Γ_{4g} model:

$$f_{4g}(x) \rightarrow f_{4g}(x) + (0 \ldots 0.6) \tag{4}$$

Γ_{4g} dressing



Stability of solution

- For f_{4g} too weak, solution is unstable (gluon triangle diverges).
- Different iteration methods do not help.
- Obtained stable solutions by strenghtening f_{4g}.
- Through f_{4g} we account for:
  1. Neglected 2-loop terms in vertex DSE.
  2. Insufficient knowledge of 'true' four-gluon vertex.

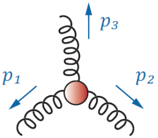
Basis for three-gluon vertex

- 3 Lorentz indices, 2 momenta $\Rightarrow$ 14 tensor structures.
- In Landau gauge 4 tensors describe Γ_{3g}.
- Use orthonormal basis internally.

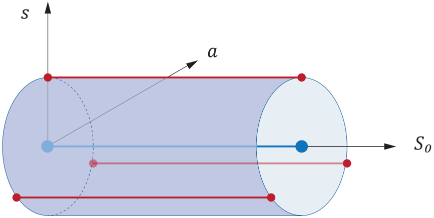
$$\tau^{i}_{\mu\nu\rho} \cdot \tau^{j}_{\mu\nu\rho} = \delta^{ij}, \quad i,j = 1 \dots 4 \tag{5}$$

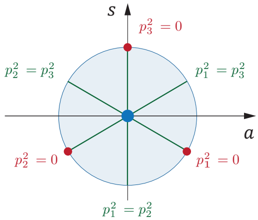
- Rotate onto Bose-symmetric BC-like basis for results.
  J. S. Ball, T. W. Chiu, PRD **22**, 2550 (1980)
- DSE calculation of Γ_{3g} with 1 tensor structure:
  A. Blum, M. Huber, M. Mitter, L. von Smekal, arXiv:1401.0713

Vertex kinematics

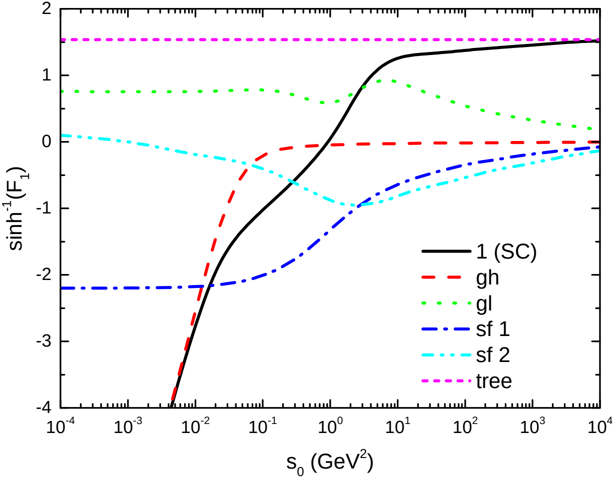


$$S_0 = \frac{1}{6}(p_1^2 + p_2^2 + p_3^2), \;\; s = \sqrt{3}\frac{p_2^2 - p_1^2}{p_1^2 + p_2^2 + p_3^2}, \;\; a = \frac{p_1^2 + p_2^2 - 2p_3^2}{p_1^2 + p_2^2 + p_3^2} \qquad (6)$$

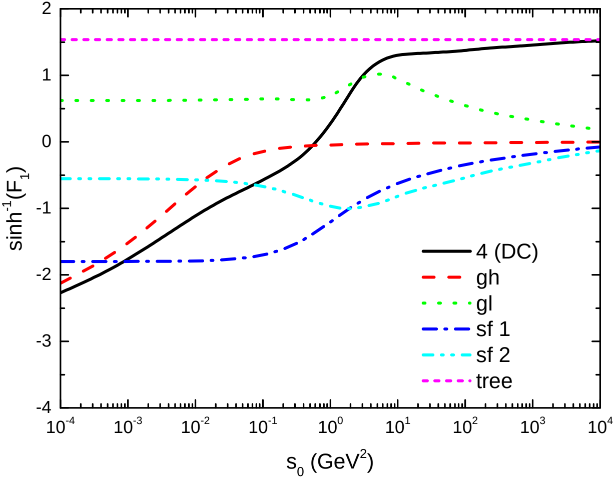




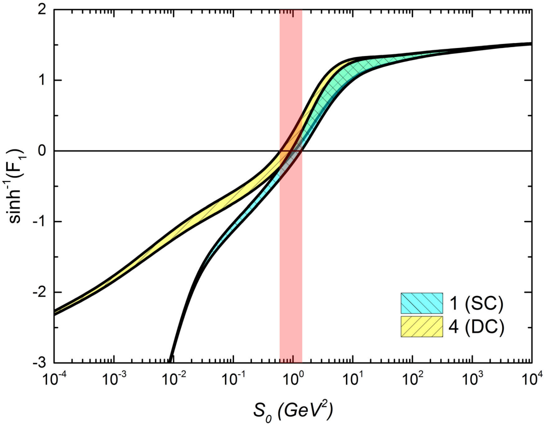
Contributions to $\Gamma^{\mu\nu\rho}_{(0)}$ dressing, scaling



Contributions to $\Gamma^{\mu\nu\rho}_{(0)}$ dressing, decoupling



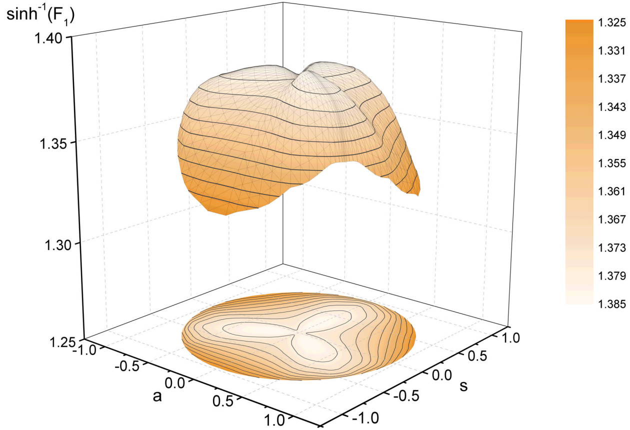
Zero crossing of $\Gamma^{\mu\nu\rho}_{(0)}$ dressing



Zero crossing seen in 3D lattice and 4D continuum studies.
A. Blum, M. Huber, M. Mitter, L. von Smekal, arXiv:1401.0713
A. Aguilar, D. Binosi, D. Ibanez, J. Papavassiliou, arXiv:1312.1212v1

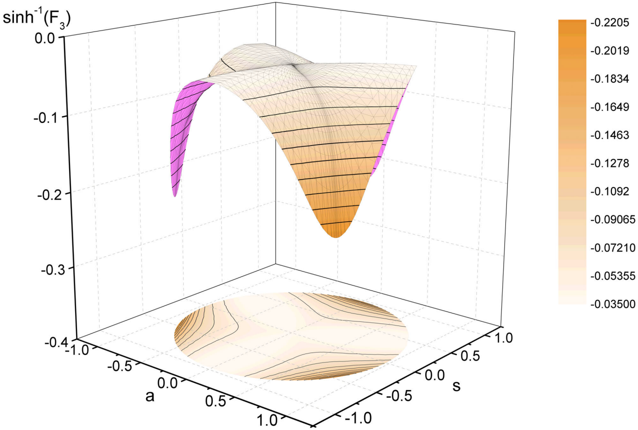
Model and truncation dependent!

Angular dependence of F_1 dressing



Angular dependence of F_1 dressing function at $S_0 = 102\ \text{GeV}^2$.

Angular dependence of F_3 dressing



Angular dependence of F_3 dressing function at $S_0 = 102\ \text{GeV}^2$.

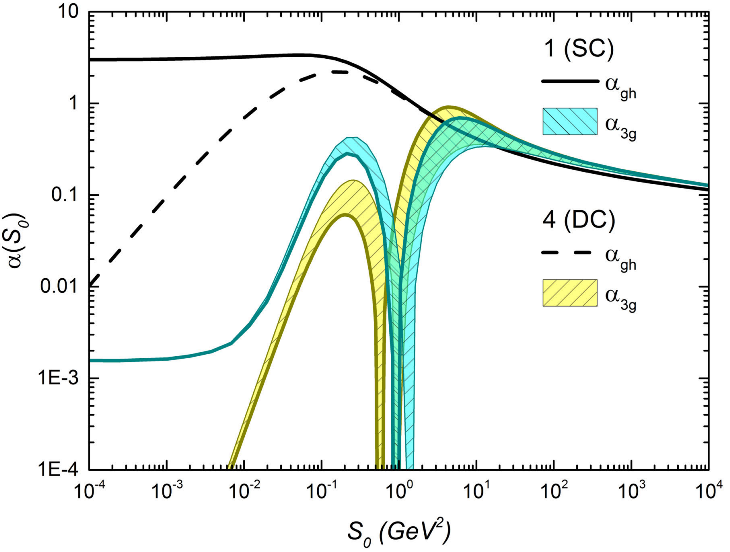
Running couplings

- RG invariant running couplings for different vertices:

$$\alpha_{gh} = \frac{g^2}{4\pi}\, Z \cdot G^2$$
$$\alpha_{3g}^{(n)} = \frac{g^2}{4\pi}\, Z \cdot G^2 \left[\frac{Z \cdot F_1}{G}\right]^n \qquad (7)$$

- We calculate $\alpha_{3g}^{(n)}$ for $n = 2$.
- In scaling scenario, IR fixed points of running couplings.
  R. Alkofer, C. Fischer, F. Llanes-Estrada, PLB **611**, (2005)
- Our result for fixed point: $\alpha_{3g}^{(2)}(0) = 0.0016$.

Results for α_{gh} and α_{3g}



Summary

1. First DSE study of Γ_{3g} with full tensor structure.
2. Expected all-scales divergences in the IR.
3. Soft collinear divergences in subleading components.
4. Zero crossing in F_1 at hadronic scales $\Lambda \approx 1$ GeV:
    - Mild 4g vertex model dependence.
    - Possible significant dependance on truncation.

Outlook: Applications for Γ_{3g}

1. Impact of zero crossing on hadronic observables.
2. Meson spectroscopy beyond rainbow ladder:
    - Excited states.
    - Exotic mesons, hybrids and glueballs.
3. Irreducible three-body force in baryons:
    - Two- vs. three-quark dominance in excited states.
    - Baryonic hybrids.
4. Investigations of (near) conformal window of QCD-like theories.
   see also talk by Markus Hopfer

THANK YOU FOR YOUR ATTENTION!