$a_0(980)$ as a dynamically generated resonance in the extended linear sigma model

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this work was done in collaboration with Francesco Giacosa and Dirk Rischke

Outline

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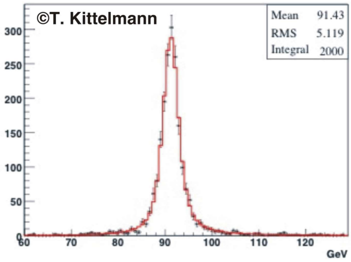
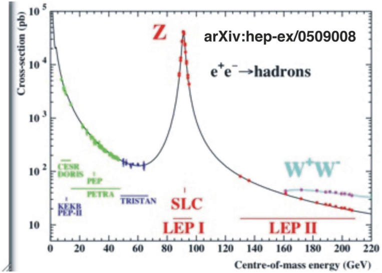
What is a resonance?

- in QFT: *particle* is an excitation of the fields (like a scalar field S) that are able to propagate over sufficiently large time scales
- extremely short-lived unstable particles with mean life times on the order of 10^{-22} s are called *resonances*
- cannot be directly observed, yet it is possible to establish their existence from a scattering process ($\rightarrow$ invariant mass distribution)





The problematic scalar sector

- enhancement of the cross section near $s \approx M^2$:

relativistic *Breit–Wigner* formula (if $\Gamma \ll M$)

$$d\sigma(a + b \to C + D) \sim \frac{M\Gamma}{(s - M^2)^2 + M^2\Gamma^2}$$

- naive quark model: works fine ($\to$ multiplet structures) for a wide range of unstable particles and resonances
- cannot be applied to the scalars: large widths, huge background and several decay channels (with short mass intervals, e.g. $K\bar{K} \sim 1$ GeV and $\eta\eta \sim 1.1$ GeV)
- one expects non-$q\bar{q}$ objects

The problematic scalar sector

- introduce complex mass poles of the form

T-Matrix pole $\sqrt{s}$ (in MeV)

$$\sqrt{s} = M - i\frac{\Gamma}{2}$$

- above parameterization is said to be stable against gauge and field-redefenition transformations (T. Bhattacharya and S. Willenbrock, PR **D47** (1993))
- BW-parameterization does not fulfill these properties

  (S. Willenbrock and G. Valencia, PL **B259** (1991); A. Sirlin, PL **B267** (1991))
- in particular, only for $\Gamma \ll M$ there is a reasonable connection between BW- and pole parameters

The problematic scalar sector

- Törnqvist: hadronic loop contributions dress bare states and dominate dynamics (N. A. Törnqvist, Z. Phys. **C68** (1995); G. Höhler, Zeits. f. Phys. **152** (1958))
- this means: dynamical effects distort correspondence between observed scalar mesons and underlying quark content

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Dynamical generation of scalar mesons

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(Received 18 March 2002; published 12 June 2002)

The problematic scalar sector

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- this means: dynamical effects distort correspondence between observed scalar mesons and underlying quark content

Institute for Particle I ... *m*

hundreds of MeV lighter than one would simply deduce from the constituent structure of the mesons.

In Ref. [1], Tornqvist presented a model in which the central focus is to consider the loop contributions given by the hadronic intermediate states that each meson can access: it is via these hadronic loops that the bare states become "dressed" and, in the case of scalar mesons, hadronic loop contributions totally dominate the dynamics of the process. He shows that the mass shift, which is a direct consequence of the presence of strongly coupled hadronic intermediate states, is so dramatic that it completely spoils the one-to-one correspondence between the resonances we observe and the underlying constituent structure. Though we follow Tornqvist's modelling quite closely, very similar models have

The problematic scalar sector

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easily infer its quark structure. A similar picture works for the tensors.

For scalar mesons the situation is different and the one-to-one correspondence between the observed scalar mesons and their underlying quark content is distorted by dynamical effects. This is because they couple strongly to more than one meson-meson channel, creating overlapping and interfering resonance structures. Furthermore, since the interactions are S waves, the opening of each threshold produces a more dramatic s dependence in the propagator. At each threshold,

Dynamical generation?

- mass and width of a resonance are then determined by the position of the **complex pole** of the full interacting propagator in the appropriate **unphysical Riemann sheet**

  (R. E. Peierls, Proceed. of the Glasgow Conf. on Nuclear and Meson Physics (1954))

- this requires the understanding of the **rich analytic properties** of this propagator
- maybe more than that:

> The present work focuses on the study of the $I=1$ and $I=1/2$ sector of the light scalar meson spectroscopy. Previous papers from Tornqvist and Roos [1,5] seemed to suggest that using a simple model based on the hadronic "dressing" of bare seeds, one could generate more than one, possibly a whole family of mesons, with the same quantum numbers, starting with one bare seed only. This is certainly a very interesting possibility, since we know that experiment has

(M. Boglione and M. R. Pennington, PR **D65** (2002))

Outline

Some words to the model...

- starting point: reduce complexity of QCD interaction by effective hadron-hadron interactions with with hadronic dofs and symmetries known from the QCD Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm meson} + \mathcal{L}_{\rm baryon} + \mathcal{L}_{\rm dilaton} + \mathcal{L}_{\rm weak}$$

$$\begin{aligned}
\mathcal{L}_{\rm meson} = {} & \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
& + c_1(\det\Phi - \det\Phi^\dagger)^2 + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
& + \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2+R_\mu^2)\right] + \frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu,L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu,R^\nu]\}) \\
& + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\,\mathrm{Tr}(L_\mu^2+R_\mu^2) + h_2\,\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\,\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \\
& + \text{chirally invariant vector and axialvector four-point interaction vertices}
\end{aligned}$$

$\rightarrow$ extended Linear Sigma Model (eLSM)

(S. Janowski, D. Parganlija, F. Giacosa and D. H. Rischke, PR **D84** (2011);

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, PR **D87** (2013))

Some words to the model...

- mesons are assigned as $q\bar{q}$-states:

(Pseudo-)Scalars $\Phi_{ij} \sim \langle q_L \bar{q}_R \rangle_{ij} \sim \frac{1}{\sqrt{2}}(q_i \bar{q}_j - q_i \gamma_5 \bar{q}_j)$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0)}{\sqrt{2}} + \frac{i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0)}{\sqrt{2}} + \frac{i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

Lefthanded $L^\mu_{ij} \sim \langle q_L \bar{q}_L \rangle_{ij} \sim \frac{1}{\sqrt{2}}(q_i \gamma^\mu \bar{q}_j + q_i \gamma_5 \gamma^\mu \bar{q}_j)$

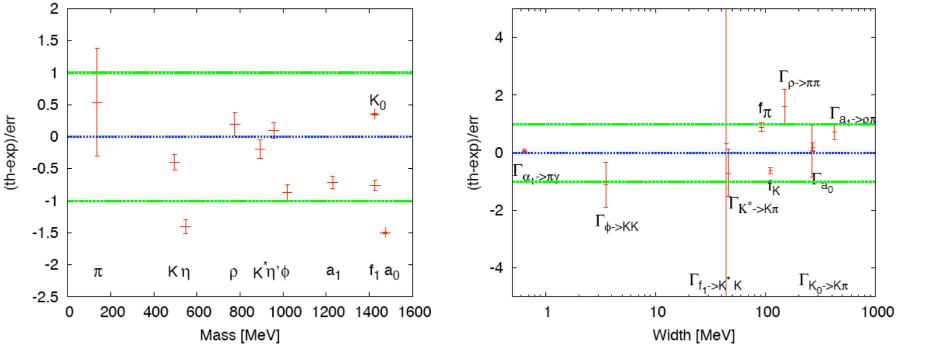
$$L^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{\star +} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} + K_1^0 \\ K^{\star -} + K_1^- & \bar{K}^{\star 0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu$$

Righthanded $R^\mu_{ij} \sim \langle q_R \bar{q}_R \rangle_{ij} \sim \frac{1}{\sqrt{2}}(q_i \gamma^\mu \bar{q}_j - q_i \gamma_5 \gamma^\mu \bar{q}_j)$

$$R^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{\star +} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} - K_1^0 \\ K^{\star -} - K_1^- & \bar{K}^{\star 0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu$$

Some words to the model...

- main results of the model:



D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, PR **D87** (2013))

- e.g., the a_0 is the $a_0(1450)$, a $q\bar{q}$-state with mass of about 1363 MeV

Some words to the model...

- π-η-Lagrangian:

$$\begin{aligned}\mathcal{L}_{a_0\eta\pi} &= (A_{a_0\eta_N\pi}\cos\varphi_\eta + A_{a_0\eta_S\pi}\sin\varphi_\eta)a_0^0\eta\pi^0\\ &+ B_{a_0\eta_N\pi}\cos\varphi_\eta a_0^0\partial_\mu\eta\partial^\mu\pi^0\\ &+ C_{a_0\eta_N\pi}\cos\varphi_\eta\partial_\mu a_0^0(\pi^0\partial_\mu\eta + \eta\partial^\mu\pi^0)\end{aligned}$$

- π-η'-Lagrangian:

$$\begin{aligned}\mathcal{L}_{a_0\eta'\pi} &= (-A_{a_0\eta_N\pi}\sin\varphi_\eta + A_{a_0\eta_S\pi}\cos\varphi_\eta)a_0^0\eta'\pi^0\\ &+ (-B_{a_0\eta_N\pi}\sin\varphi_\eta)a_0^0\partial_\mu\eta'\partial^\mu\pi^0\\ &+ (-C_{a_0\eta_N\pi}\sin\varphi_\eta)\partial_\mu a_0^0(\pi^0\partial^\mu\eta' + \eta'\partial^\mu\pi^0)\end{aligned}$$

- K-K-Lagrangian:

$$\begin{aligned}\mathcal{L}_{a_0KK} &= A_{a_0KK}a_0^0(K^0\bar{K}^0 - K^-K^+)\\ &+ B_{a_0KK}a_0^0(\partial_\mu K^0\partial^\mu\bar{K}^0 - \partial_\mu K^-\partial^\mu K^+)\\ &+ C_{a_0KK}\partial_\mu a_0^0(K^0\partial^\mu\bar{K}^0 + \bar{K}^0\partial\mu K^0 - K^-\partial^\mu K^+ - K^+\partial^\mu K^-)\end{aligned}$$

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How we calculate the loops

- calculate imaginary part of self-energy loop $\Pi_{ij}(s)$ by the optical theorem (regularized by Gaussian 3d-cutoff function):

$$\int d\Gamma \, |-i\mathcal{M}_{ij}|^2 = \sqrt{s} \, \Gamma_{ij}^{\text{tree}}(s) = -2 \, \text{Im} \, \Pi_{ij}(s)$$

- calculate the corresponding real part through a dispersion relation:

$$\text{Re} \, \Pi_{ij}(s) = \frac{1}{\pi} \fint ds' \, \frac{\text{Im} \, \Pi_{ij}(s)}{s - s'}$$

- perform the analytic continuation, $s \to z$, and the continuation into the appropriate Riemann sheet(s) by:

Analytic continuation

$$\Pi_{ij}^{c}(z) = \Pi_{ij}(z) + \text{Disc} \, \Pi_{ij}(z) \; , \quad \text{Disc} \, \Pi_{ij}(s) = 2i \lim_{\epsilon \to 0^+} \text{Im} \, \Pi_{ij}(s + i\epsilon)$$

Outline

Canonical quantization[1]

- theory with two scalar fields: $\mathcal{L}_{\text{int}} = gS\partial_\mu\phi\partial^\mu\phi$
- to quantize, we write down the Hamiltonian by using conjugate momenta:

$$\pi_S = \partial^0 S \quad , \quad \pi_\phi = \partial^0\phi + 2gS\partial^0\phi$$

- the Hamiltonian then reads

$$\begin{aligned}\mathcal{H} &= \pi_S\partial^0 S + \pi_\phi\partial^0\phi - \mathcal{L} \\ &= \mathcal{H}_S + \frac{1}{2}\pi_\phi\pi_\phi(1+2gS)^{-1} + \frac{1}{2}\vec{\nabla}\phi\cdot\vec{\nabla}\phi + \frac{1}{2}m^2\phi^2 + gS\vec{\nabla}\phi\cdot\vec{\nabla}\phi\end{aligned}$$

- in contrast to an 'ordinary' interaction:

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_\phi - gS\phi\phi$$

[1] hats and indices are written explicitly on this and the next slide

Canonical quantization

- expanding the denominator gives

$$\mathcal{H}_{\text{int}} = -gS\pi_\phi\pi_\phi + gS\vec{\nabla}\phi\cdot\vec{\nabla}\phi + 2g^2S^2\pi_\phi\pi_\phi + \mathcal{O}(g^3)$$

- finally $S \to \hat{S}$, $\phi \to \hat{\phi}$, $\pi_S \to \hat{\pi}_S$, $\pi_\phi \to \hat{\pi}_\phi$ and commutation relations
- <u>but</u>: for pertubation theory we need the formulation in the *interaction picture*:

$$\hat{S}^I = \hat{U}\hat{S}\hat{U} \quad , \quad \hat{\phi}^I = \hat{U}\hat{\phi}\hat{U} \quad , \quad \hat{\pi}^I_S = \partial^0\hat{S}^I \quad , \quad \hat{\pi}^I_\phi = \partial^0\hat{\phi}^I$$

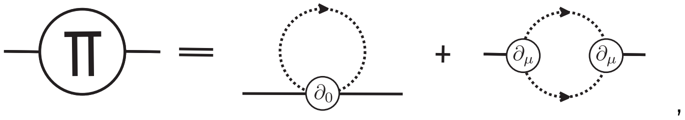
- which results in

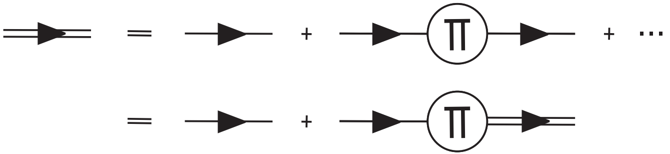
$$\hat{\mathcal{H}}^I_{\text{int}} = -\hat{\mathcal{L}}^I_{\text{int}} + 2g^2\hat{S}^I\hat{S}^I\partial_0\hat{\phi}^I\partial^0\hat{\phi}^I + \mathcal{O}(g^3) \ ,$$

so an infinite number of vertices in our Feynman rules

Contractions with derivatives[2]

- at one-loop level (in particular upon resummation) only terms of $\mathcal{O}(g^2)$ contribute:





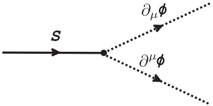
- full (inverse) propagator takes the form $\Delta_S^{-1}(s) = s - M_0^2 - \Pi(s)$

[2]hats and indices are omitted; field operators are in the interaction picture!

Contractions with derivatives

- in momentum space one usually writes $\partial_\mu \to \pm i k_\mu$, e.g. the decay amplitude for $S \to \phi\phi$ reads

$$-2ig \underbrace{\left(-\frac{s-2m^2}{2}\right)}_{=-k\cdot k} =$$



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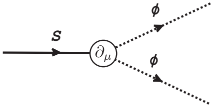
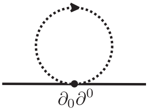


- this is OK since no additional vertex from $\mathcal{H}_{\text{int}}$ enters here
- there is also no problem for the tadpole diagram in the self-energy,

$$\langle 0|\mathcal{T}\left\{\partial_0^x \phi(x)\partial^{0,x}\phi(x)\right\}|0\rangle \;\sim$$



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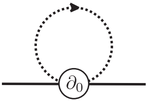


,

because time-ordering is obsolet

Contractions with derivatives

- this is different for the one-loop diagram; usually the contractions equal Feynman propagators:

$$\underbrace{\phi(x_1)\phi}(x_2) = \langle 0|\mathcal{T}\{\phi(x_1)\phi(x_2)\}|0\rangle = i\Delta_F^{\phi}(x_1 - x_2) = i\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{e^{-ik\cdot(x_1-x_2)}}{k^2 - m^2 + i\epsilon}$$

- the contractions in our loop diagram are found by using the time-ordered product

$$\langle 0|\mathcal{T}\{\phi(x_1)\phi(x_2)\}|0\rangle = \langle 0|\phi(x_1)\phi(x_2)|0\rangle\Theta(x_1^0 - x_2^0) + \langle 0|\phi(x_2)\phi(x_1)|0\rangle\Theta(x_2^0 - x_1^0)$$

Contractions with derivatives

- the action of one derivative on the Feynman propagator gives

$$\begin{aligned} i\partial_\nu^{x_2}\Delta_F^\phi(x_1-x_2) &= \partial_\nu^{x_2}\langle 0|\mathcal{T}\left\{\phi(x_1)\phi(x_2)\right\}|0\rangle \\ &= \dots \\ &= \langle 0|\mathcal{T}\left\{\phi(x_1)\partial_\nu^{x_2}\phi(x_2)\right\}|0\rangle \\ &\quad - \eta_{\nu 0}\delta(x_1^0-x_2^0)\langle 0|\underbrace{\left[\phi(x_1),\phi(x_2)\right]}_{=0}|0\rangle \end{aligned}$$

- while another derivative leads to

$$\begin{aligned} i\partial_\mu^{x_1}\partial_\nu^{x_2}\Delta_F^\phi(x_1-x_2) &= \partial_\mu^{x_1}\langle 0|\mathcal{T}\left\{\phi(x_1)\partial_\nu^{x_2}\phi(x_2)\right\}|0\rangle \\ &= \dots \\ &= \langle 0|\mathcal{T}\left\{\partial_\mu^{x_1}\phi(x_1)\partial_\nu^{x_2}\phi(x_2)\right\}|0\rangle \\ &\quad + \eta_{\mu 0}\delta(x_1^0-x_2^0)\langle 0|\underbrace{\left[\phi(x_1),\partial_\nu^{x_2}\phi(x_2)\right]}_{\neq 0}|0\rangle \end{aligned}$$

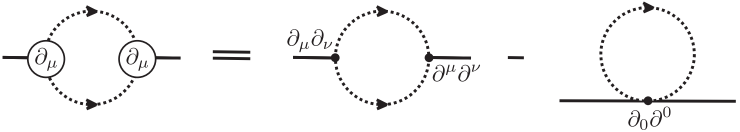
Contractions with derivatives

- for the extra term we find

$$\eta_{\mu 0}\delta(x_1^0 - x_2^0)\langle 0|\left[\phi(x_1), \partial_\nu^{x_2}\phi(x_2)\right]|0\rangle = i\eta_{\mu 0}\eta_{\nu 0}\delta^{(4)}(x_1 - x_2) \; ,$$

which breaks Lorentz invariance explicitly

- this extra term makes the loop diagram to split into



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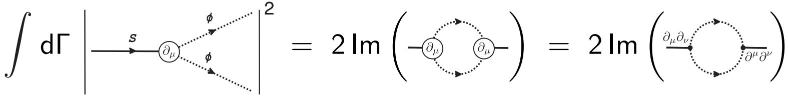
where the latter term cancels the first tadpole diagram

⇒ at one-loop level all extra terms (coming from the additional vertex and contractions with derivatives) cancel each other

Contractions with derivatives

- but: when using dispersion relations with cutoff-functions, one needs to take into account the first tadpole diagram since



  will give an imaginary part that used in the dispersion relation yields the *wrong* diagram, i.e, the loop from the middle
- we need to correct:

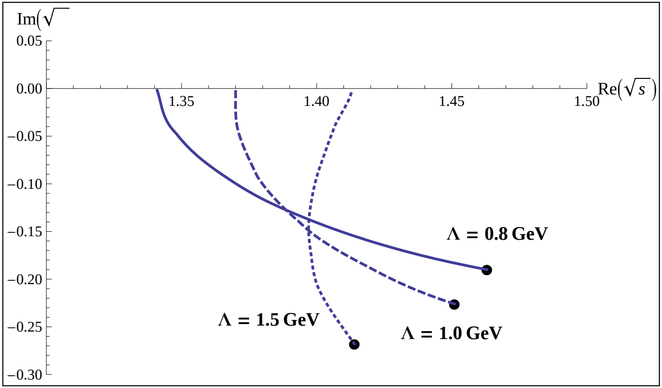
Correction for derivative interactions

In case of a loop with **two connected** derivative vertices, subtract a tadpole diagram in the inverse propagator.

Outline

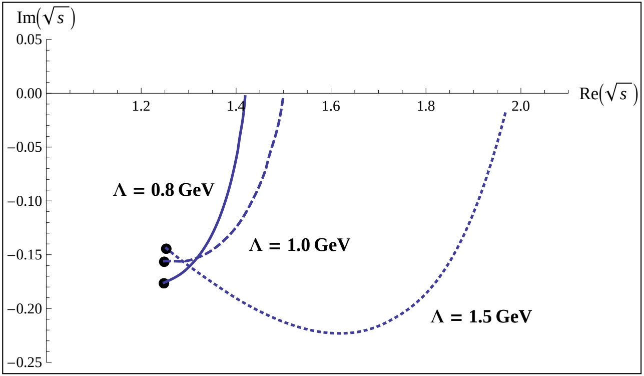
No derivative interactions

- $\mathcal{L}_{\text{int}} = g\left(Aa_0^0\eta\pi^0 + Ba_0^0\eta'\pi^0 + Ca_0^0(K^0\bar{K}^0 - K^-K^+)\right)\ , \quad g = 0\ldots1$



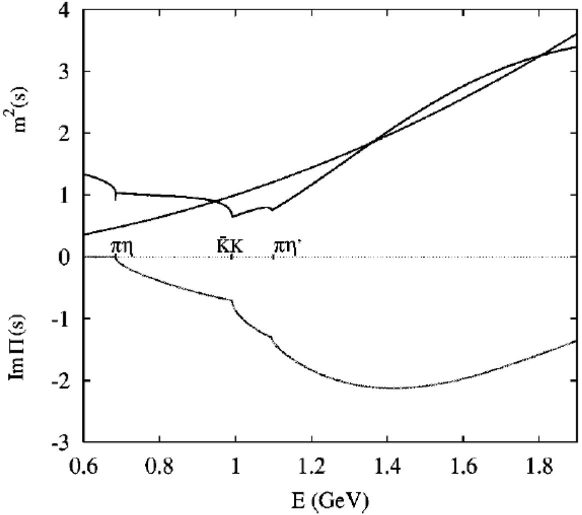
Only derivative interactions

- $\mathcal{L}_{\text{int}} = g\big(Aa_0^0\partial_\mu\eta\partial^\mu\pi^0 + Ba_0^0\partial_\mu\eta'\partial^\mu\pi^0 + Ca_0^0(\partial_\mu K^0\partial^\mu\bar{K}^0 - \partial_\mu K^-\partial^\mu K^+)\big)\ , \quad g = 0\ldots1$



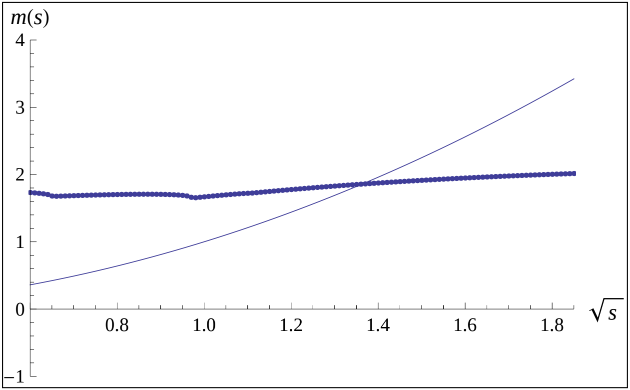
Running mass plot

- $m^2(s) = M_0^2 + \mathrm{Re}\,\Pi(s)$ (M. Boglione and M. R. Pennington, PR **D65** (2002))



No derivative interactions

- decreasing the cutoff: $\Lambda = 1.5$ GeV, 1.0 GeV, 0.8 GeV



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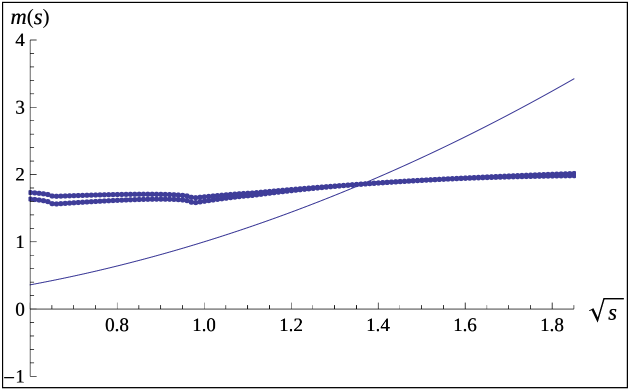

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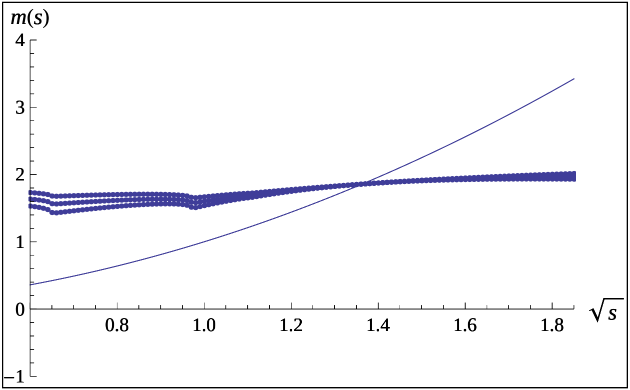

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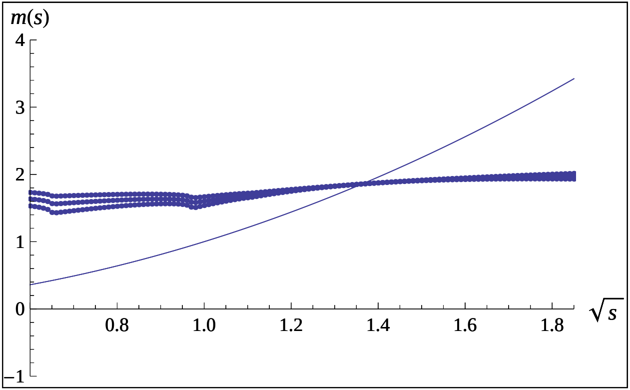

- there are no additional poles

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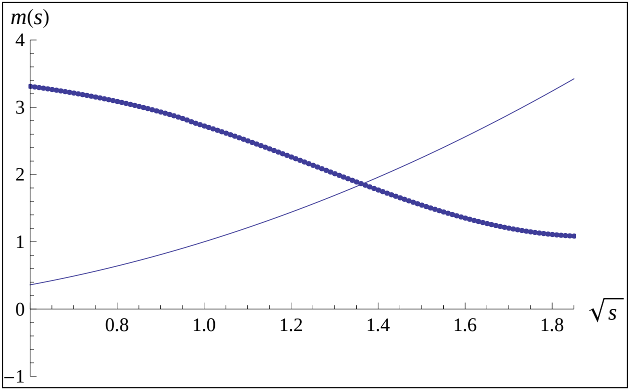

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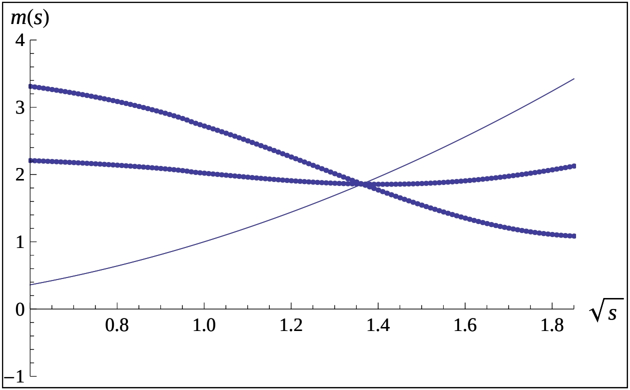

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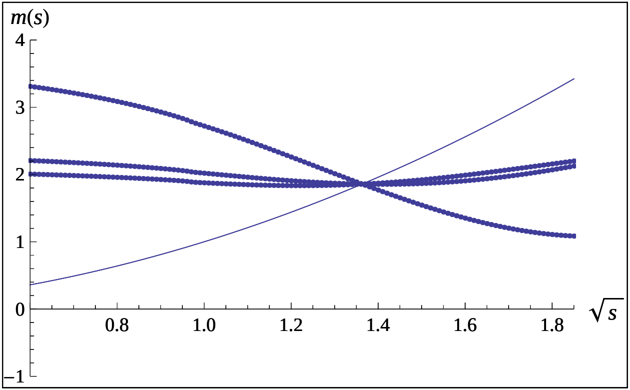

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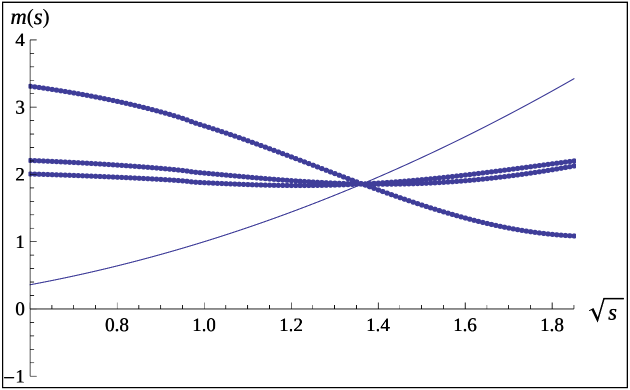


- there are no additional poles

Outline

Summary and outlook

- we have studied the popagator pole of the isovector state $a_0(1450)$ as it is determined by the eLSM
- single kind of loop corrections (vertices with/only derivatives) do not change the overall result of our model
- we find no *companion pole* that could be assigned as the $a_0(980)$

- we need to extend to the mixed case (vertices with and without derivatives) $\rightarrow$ ongoing
- one should include the contact terms that are present in the model

Thank you!

Breit–Wigner parameterization

- the Breit–Wigner mass M_{BW} and decay width Γ_{BW} are defined as

Breit–Wigner parameterization

$$M_{\text{BW}}^2 = M_0^2 + \operatorname{Re}\Pi(M_{\text{BW}}^2)\ , \qquad \Gamma_{\text{BW}} = -\frac{Z}{M_{\text{BW}}}\operatorname{Im}\Pi(M_{\text{BW}}^2)$$

- if $\operatorname{Im}\Pi(M_{\text{BW}}^2)$ small, neglect the full energy dependence of $\Pi(p^2)$:

$$\begin{aligned}
\Delta_S(p^2) &\simeq \frac{Z}{p^2 - M_{\text{BW}}^2 - iZ\operatorname{Im}\Pi(M_{\text{BW}}^2)} \\
&\simeq \frac{Z}{p^2 - M_{\text{BW}}^2 + iM_{\text{BW}}\Gamma_{\text{BW}} + \frac{\Gamma_{\text{BW}}^2}{4}} \\
&= \frac{Z}{p^2 - \left(M_{\text{BW}} - i\frac{\Gamma_{\text{BW}}}{2}\right)^2}
\end{aligned}$$

General problem

- complex root function:

$$f : \mathbb{C} \to \mathbb{C},\ z \mapsto +\sqrt{z} = \sqrt{z} = w\ ,$$
$$f(z) = \sqrt{z} = \sqrt{\rho}e^{i\frac{\varphi}{2}}\ ,\ \text{for } \varphi \in (-\pi, \pi]$$

- behaviour of f by approaching the negative real axis:

$$\begin{aligned}
\lim_{\epsilon \to 0^+} f(-\rho + i\epsilon) &= \sqrt{\rho}e^{i\frac{\pi}{2}} \\
&= i\sqrt{\rho}\ , \\
\lim_{\epsilon \to 0^+} f(-\rho - i\epsilon) &= \sqrt{\rho}e^{-i\frac{\pi}{2}} \\
&= -i\sqrt{\rho}
\end{aligned}$$

$\Rightarrow$ f is **not** well-defined

General problem

- discontinuity across the cut:

$$\begin{aligned} \mathrm{Disc}\, f(-\rho) &= \lim_{\epsilon\to 0^+} \Big[f(-\rho+i\epsilon) - f(-\rho-i\epsilon) \Big] \\ &= i\sqrt{\rho} - (-i\sqrt{\rho}) \\ &= 2i\sqrt{\rho} \end{aligned}$$

- analytic continuation down into second Riemann sheet:

$$\begin{aligned} \lim_{\epsilon\to 0^+} f_{\mathrm{II}}(-\rho-i\epsilon) &= \lim_{\epsilon\to 0^+} f(-\rho+i\epsilon) \\ &= \lim_{\epsilon\to 0^+} f(-\rho-i\epsilon) + 2i\sqrt{\rho} \\ &= i\sqrt{\rho}\,, \\ \Rightarrow f_{\mathrm{II}}(z) &= -f(z) \\ &= -\sqrt{z} \end{aligned}$$

General problem

- we find in general for a function f with property $f(z) = f^*(z^*)$:

Discontinuity on real axis

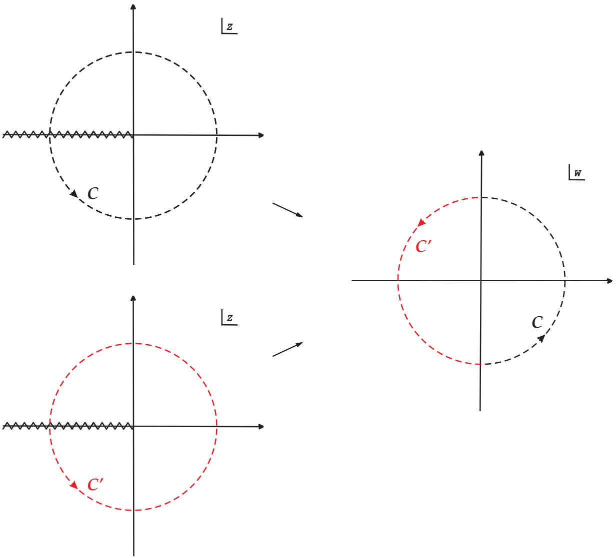
$$\text{Disc}\, f(x) = 2i \lim_{\epsilon \to 0^+} \text{Im}\, f(x + i\epsilon)$$

- the function f is either purely real on the real axis or has a branch cut with the discontinuity $\text{Disc}\, f(x)$
- analytic continuation into second Riemann sheet:

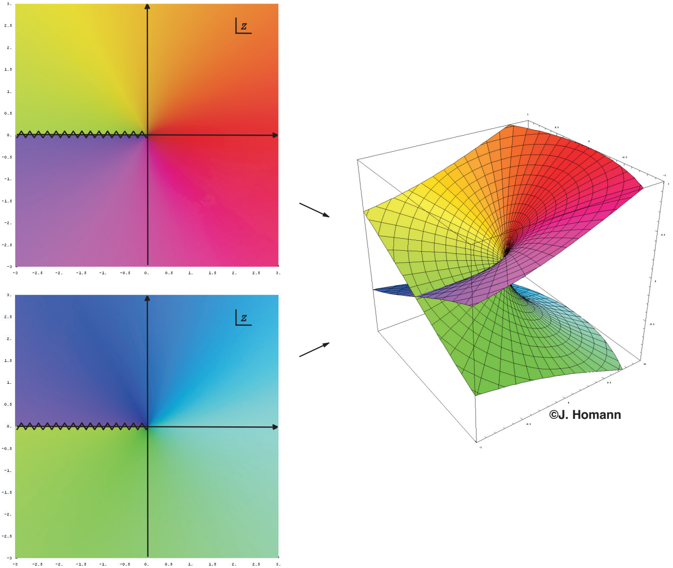
Analytic continuation

$$f_{II}(z) = f(z) + \text{Disc}\, f(z)$$

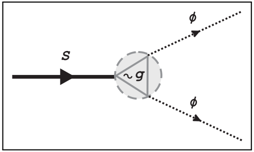
General problem



Introduce a *Riemann surface*



Regularization function



- the cutoff parameter Λ does **not** exist at the Lagrangian level
- it can be implemented by using a non-local interaction term (if $f_\Lambda(q) = f_\Lambda(|\mathbf{q}|)$), e.g.

$$\mathcal{L}_{\text{int}} = gS(x)\phi^2(x) \;\rightarrow\; \mathcal{L}_{\text{int}} = gS(x)\int \mathrm{d}^4y\,\phi(x+y/2)\phi(x-y/2)\Phi(y)$$

- changes also the tree-level result for the decay width:

$$\Gamma^{\text{tree}}(s) \;\rightarrow\; \Gamma^{\text{tree}}(s)\cdot f_\Lambda^2(p_{S\phi\phi})$$

- our choice:

Regularization function in our case

$$f_\Lambda(q) = \exp\left(-|\mathbf{q}|^2/\Lambda^2\right)$$

Results of the eLSM

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80