RESEARCH WITH PLYMOUTH UNIVERSITY

Excited QCD 2014

Many-flavour QCD
and the conformal window

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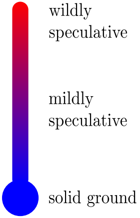
Conformal window

$$\mathcal{L} = -\frac{1}{2g^2}\,\mathrm{tr}\;F^2 + i\bar{\psi}\,\not{D}\,\psi$$



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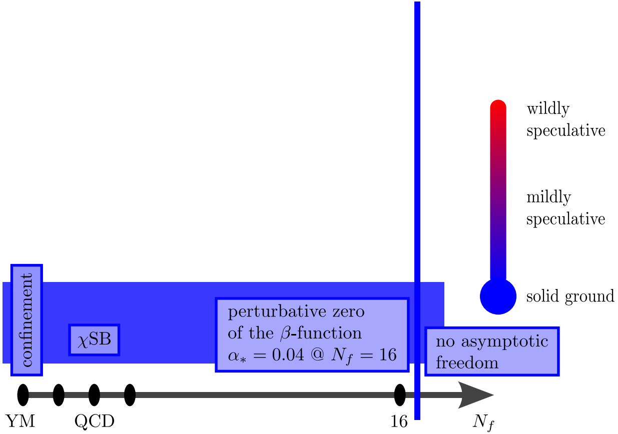

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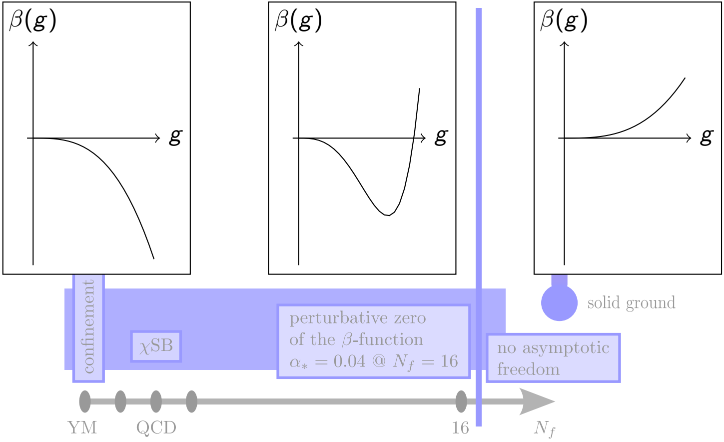

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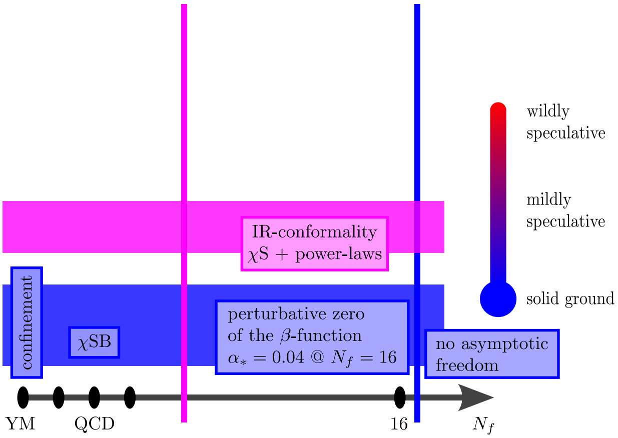

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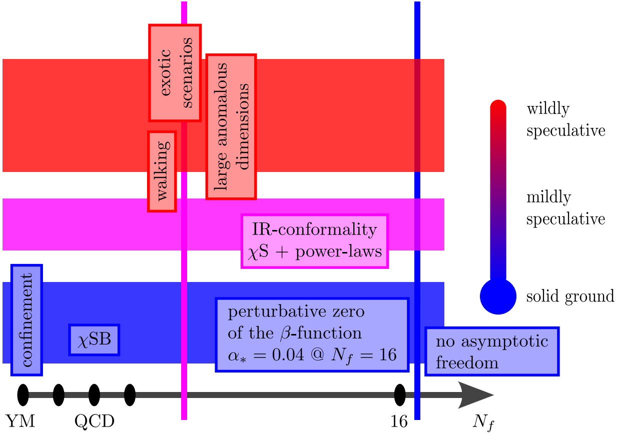
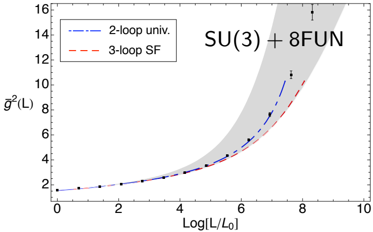
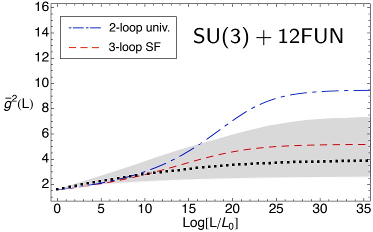
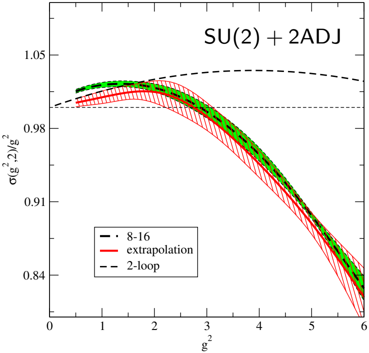


SF running coupling





Appelquist, Fleming, Nail, PRL 100:171607 (2008)



Rantaharju, Rummukainen, Tuominen, arXiv:1301.2373

Fake fixed points

- Consider Yang-Mills. The static potential at short distance goes like:

$$V(r) \simeq -k \frac{g^2_{MS}(r^{-1})}{r}$$

- A running coupling can be defined like:

$$g^2(r^{-1}) = \frac{r^2 V'(r)}{k}$$

- At short distance (or high energy)

$$r^{-1} \to \infty \quad : \qquad g^2(r^{-1}) \simeq g^2_{MS}(r^{-1})$$

- At large distance (or low energy)

$$r^{-1} \to 0 \quad : \qquad g^2(r^{-1}) \simeq \frac{\sigma}{k} r^2$$

the coupling constant diverges.

Fake fixed points

- Consider Yang-Mills. The static potential at short distance goes like:

$$V(r) \simeq -k \frac{g^2_{MS}(r^{-1})}{r}$$

- A running coupling can be defined like:

$$g^2(r^{-1}) = \frac{r^2 V'(r)}{k + r^2 V'(r)}$$

- At short distance (or high energy)

$$r^{-1} \to \infty \quad : \qquad g^2(r^{-1}) \simeq g^2_{MS}(r^{-1})$$

- At large distance (or low energy)

$$r^{-1} \to 0 \quad : \qquad g^2(r^{-1}) \simeq 1$$

the coupling constant goes to constant!

Chiral behaviour

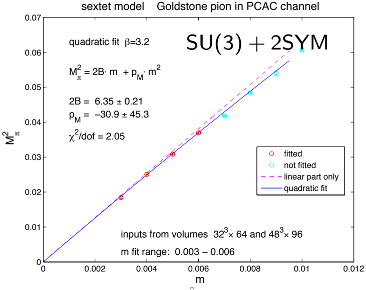
- Scaling of IR physical quantities in the chiral limit.

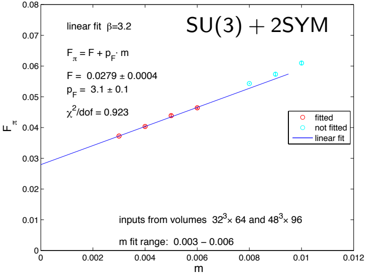
χSB	IR-conformality
$m_\pi \propto m^{1/2}$	$m_\pi \propto m^{\frac{1}{1+\gamma}}$
$m_\rho \simeq const$	$m_\rho \propto m^{\frac{1}{1+\gamma}}$
$F_\pi \simeq const$	$F_\pi \propto m^{\frac{1}{1+\gamma}}$
$\frac{m_\rho}{m_\pi} \simeq \infty$	$\frac{m_\rho}{m_\pi} \simeq const$

- In actual simulation there are at least two parameters that break conformality, the mass and the volume. One should do finite-size-scaling analysis.

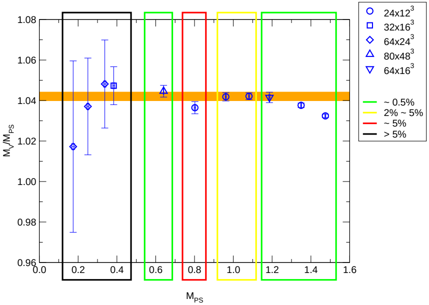
$$m_\pi = \frac{1}{L} f(m^{\frac{1}{1+\gamma}} L)$$

Chiral behaviour





SU(2) + 2ADJ



Fodor, Holland, Kuti, Nogradi, Schroeder, Wong, arXiv:1211.6164

Del Debbio, Lucini, Pica, AP, Rago, Roman, arXiv:1311.5597

Dirac operator

- Spectral density of the massless Dirac operator

  $$\rho(\lambda) = \langle \sum_i \delta(\lambda - \lambda_i) \rangle$$

  On the lattice it is convenient to calculate the mode number

  $$\nu(\Lambda) = 2 \int_0^{\sqrt{\Lambda^2 - m^2}} d\lambda \; \rho(\lambda) = \text{number of ev's of } \not{D}_m \text{ in } [-\Lambda, \Lambda]$$

- If chiral symmetry is broken, Banks-Casher relation:

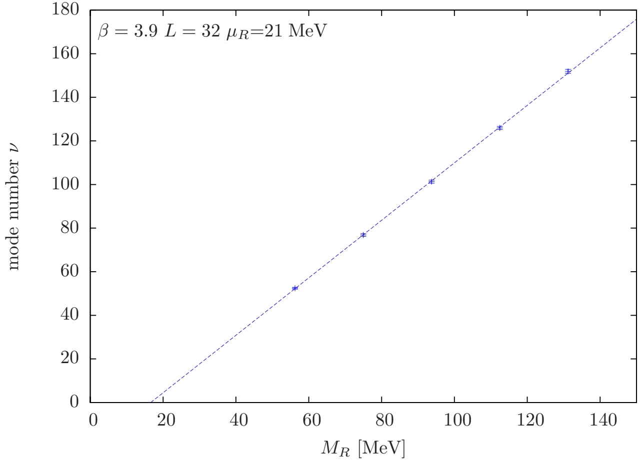
  $$\lim_{\lambda \to 0} \lim_{m \to 0} \rho(\lambda) = -\frac{1}{\pi} \langle \bar{\psi}\psi \rangle$$

  $\lim_{m \to 0} \nu(\Lambda)$ is linear at small Λ

- In the conformal window, power law for small Λ:

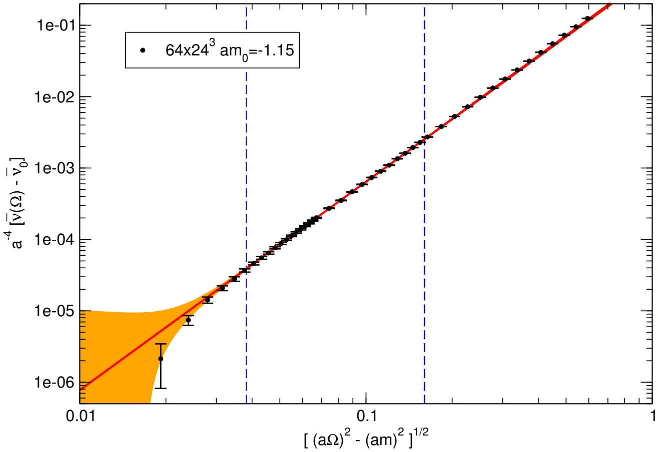
  $$\lim_{m \to 0} \nu(\Lambda) \simeq a\Lambda^{\frac{4}{1+\gamma_*}}$$

Dirac operator



QCD $N_f = 2$
Cichy, Garcia-Ramos, Jansen, Shindler, arXiv:1312.3535 [hep-lat].

Dirac operator



SU(2) + 2ADJ $\gamma_* = 0.371(20)$

Trace anomaly

- Energy-momentum tensor $\theta_{\mu\nu}$. One can use the arbitrariness in defining the vacuum energy by requiring

$$\langle \theta_{\mu\nu} \rangle = 0$$

- Scale invariance in QCD is broken by the trace anomaly (evan in the chiral limit)

$$\theta^{\mu}_{\mu} \neq 0$$

- In IR-conformal theories, at small temperatures T:

$$\langle \theta^{\mu}_{\mu} \rangle_T \simeq a T^4$$

- We need to define the EM tensor on the lattice...

Giusti, Meyer, Phys.Rev.Lett. 106 (2011) 131601, arXiv:1011.2727

Giusti, Meyer, JHEP 1301 (2013) 140, arXiv:1211.6669

Robaina, Meyer, arXiv:1310.6075

Giusti, Meyer, arXiv:1310.7818

Suzuki, PTEP 8, 083B03 (2013), arXiv:1304.0533

Asakawa, Hatsuda, Itou, Kitazawa, Suzuki, arXiv:1312.7492

Del Debbio, AP, Rago, JHEP 1311, 212 (2013), arXiv:1306.1173

Conclusions

- What I have not talked about:
  - Running coupling in other schemes (e.g. Polyakov loop, Wilson flow)
  - Wilsonian rensormalization flow (MCRG)
  - Radial quantization
  - Spectrum hierarchy
  - Physics at zero mass and finite volume
  - ...

- Gauge theories in the conformal window are very different from QCD.

- So far we have mostly adapted analysis techniques that were devoloped for QCD, but this is not a very efficient way to proceed. We are starting to develop *ad hoc* techniques.