Ghosts in Keldysh-Schwinger formalism

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based on: A. Czajka & St. Mrówczyński, arXiv: 1401.5773

Excited QCD, 2 – 8 Feb 2014, Sarajevo, Bosnia & Herzegovina

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Motivation

- Computations in covariant gauges are usually much simpler than those in physical ones like the Coulomb gauge.

- Covariant gauges require ghosts to compenasate unphysical degrees of freedom.

How to introduce ghosts in the Keldysh-Schwinger formalism?

What is the Green's function of free ghosts?

Keldysh–Schwinger formalism

Description of non-equilibrium many-body systems

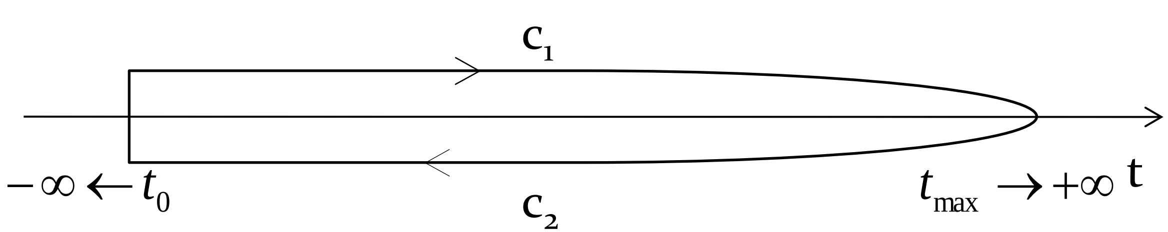
Contour Green function of gauge field

$$i\mathcal{D}_{ab}^{\mu\nu}(x,y) \overset{\text{def}}{=} \left\langle \tilde{T} A_a^{\mu}(x) A_b^{\nu}(y) \right\rangle$$

$$\langle \ldots \rangle = \mathrm{Tr}[\hat{\rho}(t_0)\ldots]$$

$\tilde{T}$ - ordering along the contour

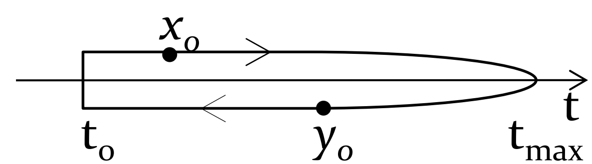
$$\tilde{T}A(x)B(y) = \Theta(x_0, y_0)A(x)B(y) \pm \Theta(y_0, x_0)B(y)A(x)$$



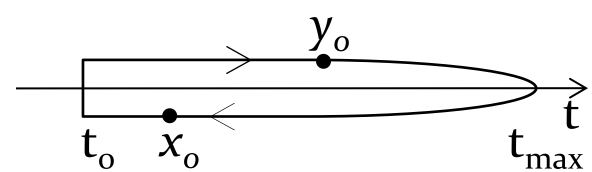
Keldysh–Schwinger formalism

Contour Green's function includes 4 Green's functions
with real time arguments:

$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{>}(x,y) = \left\langle A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$

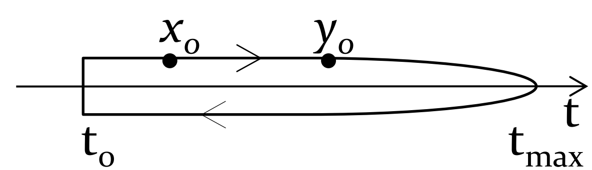


$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{<}(x,y) = \left\langle A^{\nu}_{b}(y) A^{\mu}_{a}(x) \right\rangle$$



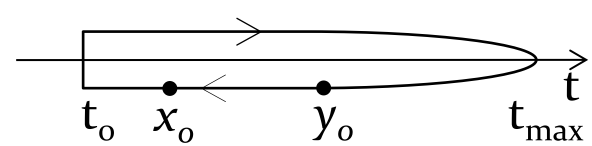
$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{c}(x,y) = \left\langle T^{c} A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$

Chronological time ordering



$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{a}(x,y) = \left\langle T^{a} A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$

Anti-chronological time ordering



Retarded, advanced & symmetric Green's functions

$$\boxed{\mathcal{D}^{+}(x,y)} = \Theta(x_0 - y_0)\left(\mathcal{D}^{>}(x,y) - \mathcal{D}^{<}(x,y)\right)$$

$$\boxed{\mathcal{D}^{-}(x,y)} = \Theta(y_0 - x_0)\left(\mathcal{D}^{<}(x,y) - \mathcal{D}^{>}(x,y)\right)$$

$$\boxed{\mathcal{D}^{sym}(x,y)} = \mathcal{D}^{>}(x,y) + \mathcal{D}^{<}(x,y)$$

Meaning of the functions

$$\mathcal{D}^{<,>}(x,y)$$

- phase-space density
- mass-shell constraint
- real particles

$$\mathcal{D}^{\pm}(x,y)$$

- retarded & advanced propagator
- no mass-shell constraint
- virtual particles

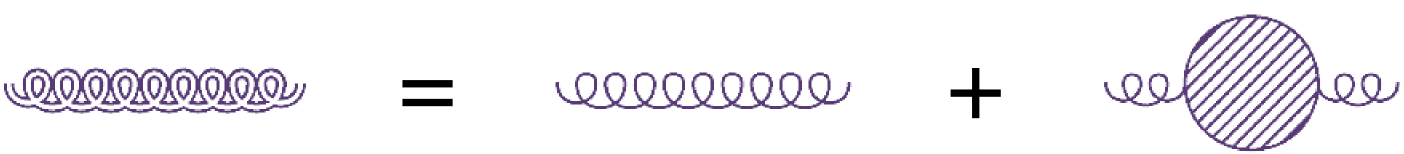
Full Green's function of gluon field

Contour–ordered Green's function has perturbative expansion similar to that of time-ordered Green's function.

Dyson – Schwinger equation

$$\mathcal{D} = D - D\Pi\mathcal{D}$$

1-st order



Full contour Green's function

Free contour Green's function

Contour polarization tensor

From general covarriant to Feynman gauge

Equation of motion of the free contour function in general covariant gauge

$$\left[\partial_x^2 g^{\mu\nu} - \left(1 - \frac{1}{\alpha}\right)\partial_x^\mu \partial_x^\nu\right] D_{\nu\rho}^{ab}(x,y) = g^\mu_{\ \rho}\delta^{ab}\delta_C^{(4)}(x,y)$$

$$\mathcal{L}_{gf} = -\frac{1}{2\alpha}(\partial^\mu A_\mu^a)^2$$

$$\delta_C^{(4)}(x,y) = \begin{cases} \delta^{(4)}(x-y) & \text{for } x_0, y_0 \text{ on the upper branch} \\ 0 & \text{for } x_0, y_0 \text{ on different branches} \\ -\delta^{(4)}(x-y) & \text{for } x_0, y_0 \text{ on the lower branch} \end{cases}$$

$$D_{ab}^{\mu\nu}(x,y) = D_{ab}^{\mu\nu}(x-y)$$ **homogeneity, translational invariance**

$$D^{>}(p) \propto \delta(p^2)\left(g^{\mu\nu} - (1-\alpha)\frac{p^\mu p^\nu}{p^2}\right)$$

mass-shell condition

→ Feynman gauge $\alpha = 1$

Free Green's functions of gluon field

Equations of motion of the real-time argument Green's functions

$$p^2 D^{>,<}(p) = 0 \qquad p^2 D^{c,a}(p) = \mp \delta^{ab} g_{\mu\nu}$$

$$D^{>}(p) = \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab}\left[\delta(E_p - p_0)[n_g(\mathbf{p}) - 1] + \delta(E_p + p_0) n_g(-\mathbf{p})\right]$$

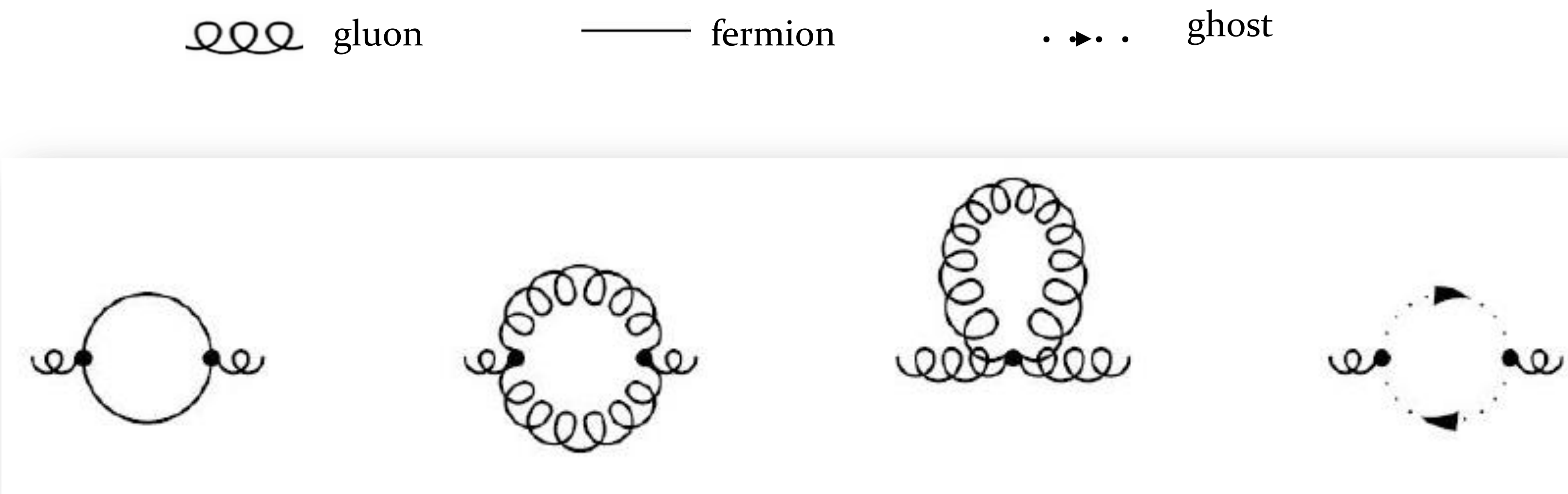
$$D^{<}(p) = \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab}\left[\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0)[n_g(-\mathbf{p}) - 1]\right]$$

$$D^{c}(p) = -g_{\mu\nu}\delta^{ab}\left[\frac{1}{p^2 + i0^+} - \frac{i\pi}{E_p}\left(\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) n_g(-\mathbf{p})\right)\right]$$

$$D^{a}(p) = g_{\mu\nu}\delta^{ab}\left[\frac{1}{p^2 - i0^+} + \frac{i\pi}{E_p}\left(\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) n_g(-\mathbf{p})\right)\right]$$

$n_g(\mathbf{p})$ - gluon distribution function

Polarization tensor in QCD



How to find the ghosts Green's functions?

One can solve the equation of motion of the scalar field.
But what is the distribution function of ghosts?

How to get Green's function of free ghosts?

Ghost sector should be determined by the gauge symmetry of the theory!

$$A_\mu^a \to \left(A_\mu^a\right)^U = A_\mu^a + f^{abc}\omega^b A_\mu^c - \frac{1}{g}\partial_\mu \omega^a$$

↓ gauge symmetry of the theory

Slavnov-Taylor identities

Generating functional

$$W_0[J,\chi,\chi^*] = N_0 \int_{BC} \mathcal{D}A\,\mathcal{D}c\,\mathcal{D}c^*\, e^{i\int_C d^4x\,\mathcal{L}_{\text{eff}}(x)}$$

all fields are on the contour

boundary conditions:
the fields are fixed in $t = -\infty \pm i0^+$

Lagrangian:

$$\mathcal{L}_{\text{eff}}(x) = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \bar{\psi}(i\gamma_\mu D^\mu - m)\psi - \frac{1}{2\alpha}(\partial^\mu A_\mu^a)^2$$
$$- c_a^*(\partial^\mu\partial_\mu\delta_{ab} - g\partial^\mu f^{abc}A_\mu^c)c_b + J_\mu^a A_a^\mu + \chi_a^* c_a + \chi_a c_a^*$$

$$W[J,\chi,\chi^*] = N\int DA'\,Dc'\,Dc^{*\prime}\,DA''\,Dc''\,Dc^{*\prime\prime}$$
$$\times \rho[A',c',c^{*\prime}|A'',c'',c^{*\prime\prime}]\,W_0[J,\chi,\chi^*]$$

density matrix

Generating functional

$$W[J,\chi,\chi^*] = N\int DA' Dc' Dc^{*\prime} DA'' Dc'' Dc^{*\prime\prime} \times \rho[A',c',c^{*\prime} | A'',c'',c^{*\prime\prime}]\, W_0[J,\chi,\chi^*]$$

The full Green's function can be generated through

$$i\mathcal{D}^{ab}_{\mu\nu}(x,y) = (-i)^2 \frac{\delta^2}{\delta J^a_\mu(x)\delta J^b_\nu(y)} W[J,\chi,\chi^*]\Big|_{J=\chi=\chi^*=0}$$

density matrix $\rho[A',c',c^{*\prime} | A'',c'',c^{*\prime\prime}]$ is not specified

⇓

the explicit form of the functional and the Green's function cannot be found

The functional provides various relations among Green's functions.

General Slavnov-Taylor identity

$$W[J,\chi,\chi^*] = N\int_{BC} \mathcal{D}A\,\Delta[A] e^{i\int_C d^4x\,\mathcal{L}(x)}$$

analog of the Fadeev-Popov determinant

$$\Delta[A] \equiv \int_{BC} \mathcal{D}c\,\mathcal{D}c^*\, e^{-i\int_C d^4x\left(-c_a^*(\partial^\mu\partial_\mu\delta_{ab} - g\partial^\mu f^{abc}A_\mu^c)c_b + \chi_a^* c_a + \chi_a c_a^*\right)}$$

The invariance of $W[J,\chi,\chi^*]$ under the transformations

$$A_\mu^a \to \left(A_\mu^a\right)^U = A_\mu^a + f^{abc}\omega^b A_\mu^c - \frac{1}{g}\partial_\mu\omega^a$$

leads to

$$\left\{ i\partial^\mu_{(y)} \frac{\delta}{\delta J_d^\mu(y)} - \int_C d^4x\, J_a^\mu(x)\left(\partial_\mu^{(x)}\delta^{ab} + igf^{abc}\frac{\delta}{\delta J_c^\mu(x)}\right) M_{bd}^{-1}\left[\frac{1}{i}\frac{\delta}{\delta J}\middle| x,y\right]\right\} W[J,\chi,\chi^*] = 0$$

Slavnov-Taylor identity for gluon Green's function

$$\frac{\delta}{\delta J_e^\nu(z)} \left\{ i\partial^\mu_{(y)} \frac{\delta}{\delta J_d^\mu(y)} - \int_C d^4x\, J_a^\mu(x) \left(\partial_\mu^{(x)} \delta^{ab} + igf^{abc} \frac{\delta}{\delta J_c^\mu(x)} \right) M_{bd}^{-1} \left[\frac{1}{i} \frac{\delta}{\delta J} \middle| x, y \right] \right\} W[J, \chi, \chi^*] = 0$$

$$J = \chi = \chi^* = 0$$

$$-p^\mu \mathcal{D}_{\mu\nu}^{ab}(p) = p_\nu \Delta_{ab}(p)$$

free ghosts Green's function

The longitudinal component of the gluon Green's function is free.

Ghost functions

$$-p^{\mu} D_{\mu\nu}^{ab}(p) = p_{\nu}\Delta_{ab}(p)$$

$$\Delta^{>}(p) = -\frac{i\pi}{E_p}\delta^{ab}\left[\delta(E_p - p_0)[n_g(\mathbf{p})+1] + \delta(E_p + p_0)n_g(-\mathbf{p})\right]$$

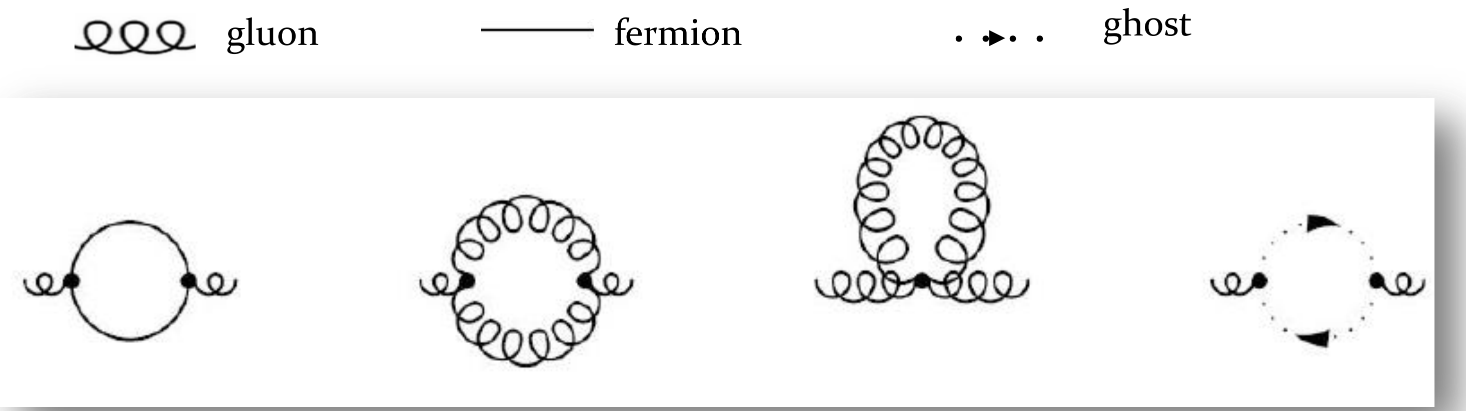
$$\Delta^{<}(p) = -\frac{i\pi}{E_p}\delta^{ab}\left[\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)[n_g(-\mathbf{p})+1]\right]$$

$$\Delta^{c}(p) = \delta^{ab}\left[\frac{1}{p^2 + i0^+} - \frac{i\pi}{E_p}\left(\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)n_g(-\mathbf{p})\right)\right]$$

$$\Delta^{a}(p) = -\delta^{ab}\left[\frac{1}{p^2 - i0^+} + \frac{i\pi}{E_p}\left(\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)n_g(-\mathbf{p})\right)\right]$$

$n_g(\mathbf{p})$ - gluon distribution function

Contributions to the contour polarization tensor



$$\Pi^{\mu\nu}_{ab}(x,y) = -ig^2 N_c \delta_{ab}\, \mathrm{Tr}[\gamma^\mu S(x,y)\gamma^\nu S(y,x)]$$

quark-loop contribution to <u>contour</u> polarization tensor

From contour to retarded polarization tensor

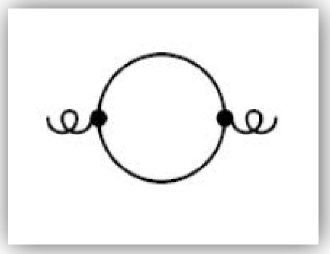
$$\Pi^{contour}(x,y) \to \Pi^{<,>}(x,y) \to \Pi^{\pm}(x,y)$$

position of x_o and y_o on the contour

identity

$$\Pi^{\pm}_{\mu\nu}(x,y) = \pm\Theta(\pm x_0 \mp y_0)(\Pi^{>}_{\mu\nu}(x,y) - \Pi^{<}_{\mu\nu}(x,y))$$

Fermion-loop contribution to retarded polarization tensor



$$\left(\Pi^{+}(k)\right)^{\mu\nu}_{ab} = -\frac{ig^{2}}{2} N_c \delta_{ab} \int \frac{d^4 p}{(2\pi)^4} \times$$
$$\times \mathrm{Tr}[\gamma^{\mu} S^{+}(p+k)\gamma^{\nu} S^{\mathrm{sym}}(p) + \gamma^{\mu} S^{\mathrm{sym}}(p)\gamma^{\nu} S^{-}(p-k)]$$

Free quark Green's functions:

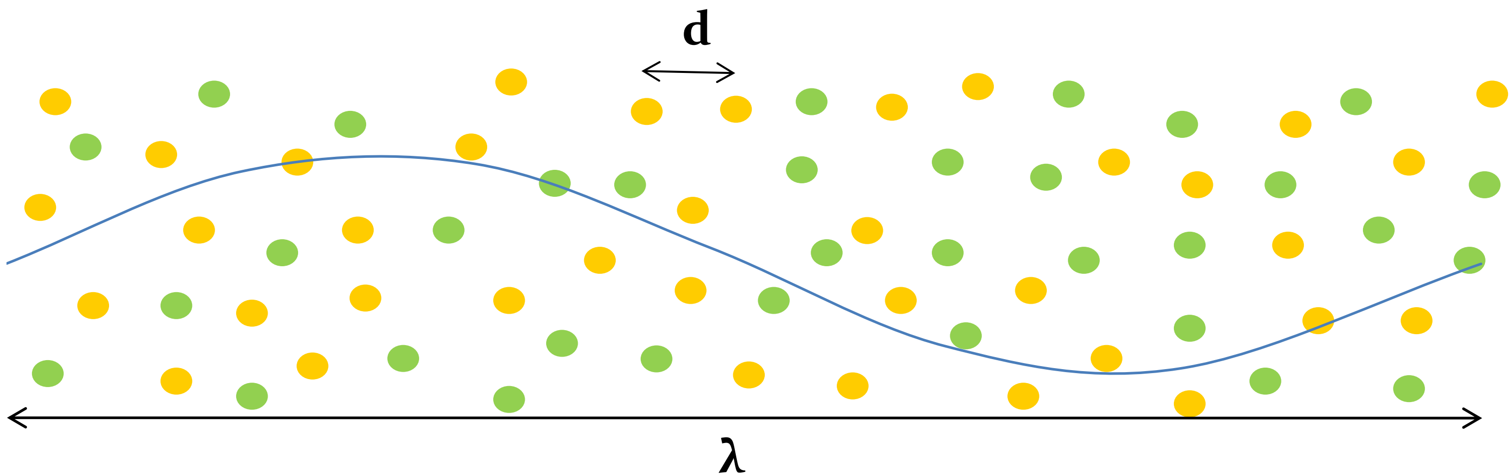
$$S^{\pm}(p) = \frac{p^{\mu}\gamma_{\mu}}{p^2 \pm i p_0 0^{+}} \qquad S^{\mathrm{sym}}(p) = S^{>}(p) + S^{<}(p)$$

$$S^{>}(p) = \frac{i\pi}{E_p} \hat{p}\left[\delta(E_p - p_0)[n_q(\mathbf{p}) - 1] + \delta(E_p + p_0)\bar{n}_q(-\mathbf{p})\right]$$

$$S^{<}(p) = \frac{i\pi}{E_p} \hat{p}\left[\delta(E_p - p_0) n_q(\mathbf{p}) + \delta(E_p + p_0)[\bar{n}_q(-\mathbf{p}) - 1]\right]$$

For gluon and ghosts loop and tadpole there are analogical formulas

Hard Loop Approximation



Wavelength of a quasi-particle is much bigger than inter-particle distance in the plasma:

$$\lambda >> d$$

$$k^{\mu} << p^{\mu}$$

Polarization tensor

$$\Pi^{\mu\nu}_{ab}(k) = g^2\delta_{ab}\int\frac{d^3p}{(2\pi)^3}\frac{f(\mathbf{p})}{E_p}\frac{k^2p^\mu p^\nu - [p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu}(k\cdot p)](k\cdot p)}{(k\cdot p + i0^+)^2}$$

distribution function

$$f(\mathbf{p}) \equiv 2N_c n_g(\mathbf{p}) + n_q(\mathbf{p}) + \bar{n}_q(\mathbf{p})$$

(vacuum effect is subtracted)

- **symmetric** $\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)$
- **transversal** $k_\mu\Pi^{\mu\nu}(k) = 0$

Gauge independence!

Ghosts work properly!

Conclusions

- The generating functional of QCD in the Keldysh-Schwinger formalism was constructed.
- The general Slavnov-Taylor identity was derived.
- The ghost Green's function was expressed through the gluon one.
- The computed polarization tensor in the hard loop approximation is automatically transverse.
- **QCD calculations in Keldysh-Schwinger formalism are possible in the Feynman gauge.**