Dynamic AdS/QCD

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- AdS/QCD summary
- Conformal window dynamics
- Walking Technicolor
- Techni-dilatons

Bjelasnica Mountain 2014

Holography – a remarkable new way to think about RG flow

– clearly a deep link between string theory and gravity

We treat RG scale as a direction of space-time….

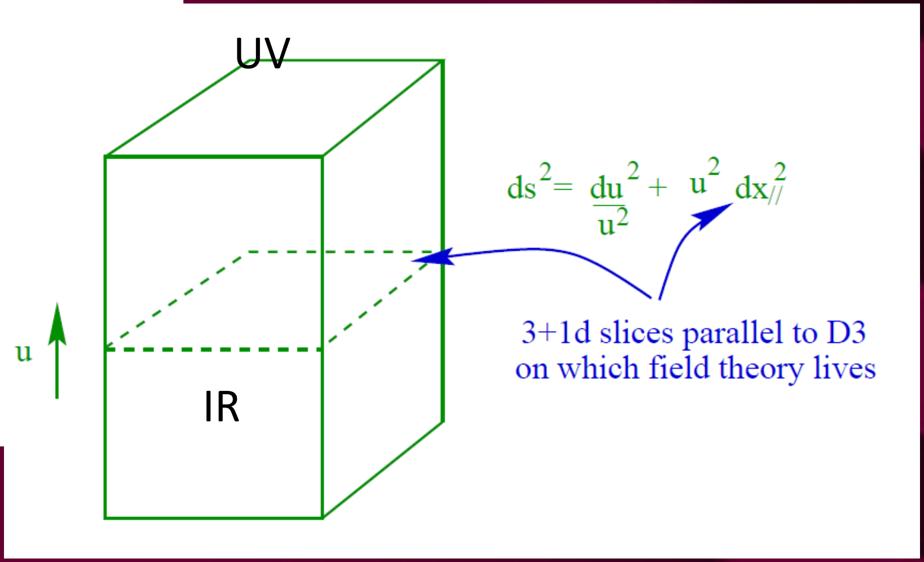
Conformal theories eg

$$\int d^4x \, (\partial\phi)^2$$

have a dilatation symmetry

$$\phi \rightarrow e^{\alpha} \phi$$

$$x_{3+1} \rightarrow e^{-\alpha} x_{3+1}$$



Reproduced in AdS metric when

$$u \rightarrow e^{\alpha} u$$

So u can represent an energy scale…

Holography

Now solve the Klein Gordon equation in AdS….

C and C' are objects in the gauge theory… they have dimension

Δ and $4-\Delta$

They have the same symmetry properties (as ϕ)

We associate them with gauge invariant J and O

$$\int d^4x \; JO$$

$$S \sim \int d^4x \; du \; \sqrt{-g}(g^{MN}\partial_M\phi\partial_N\phi + M^2\phi^2)$$

Consider spatially homogenous solutions where $\phi = \phi(u)$

$$S \sim \int d^4x \; du \; u^3(u^2\partial_u\phi\partial_u\phi + M^2\phi^2)$$

E-L equation:

$$\partial_u \left[u^5\partial_u\phi\right] - u^3M^2\phi = 0$$

has solution

$$\phi = \frac{C}{u^\Delta} + \frac{C'}{u^{(4-\Delta)}}, \qquad \Delta(\Delta-4) = M^2$$

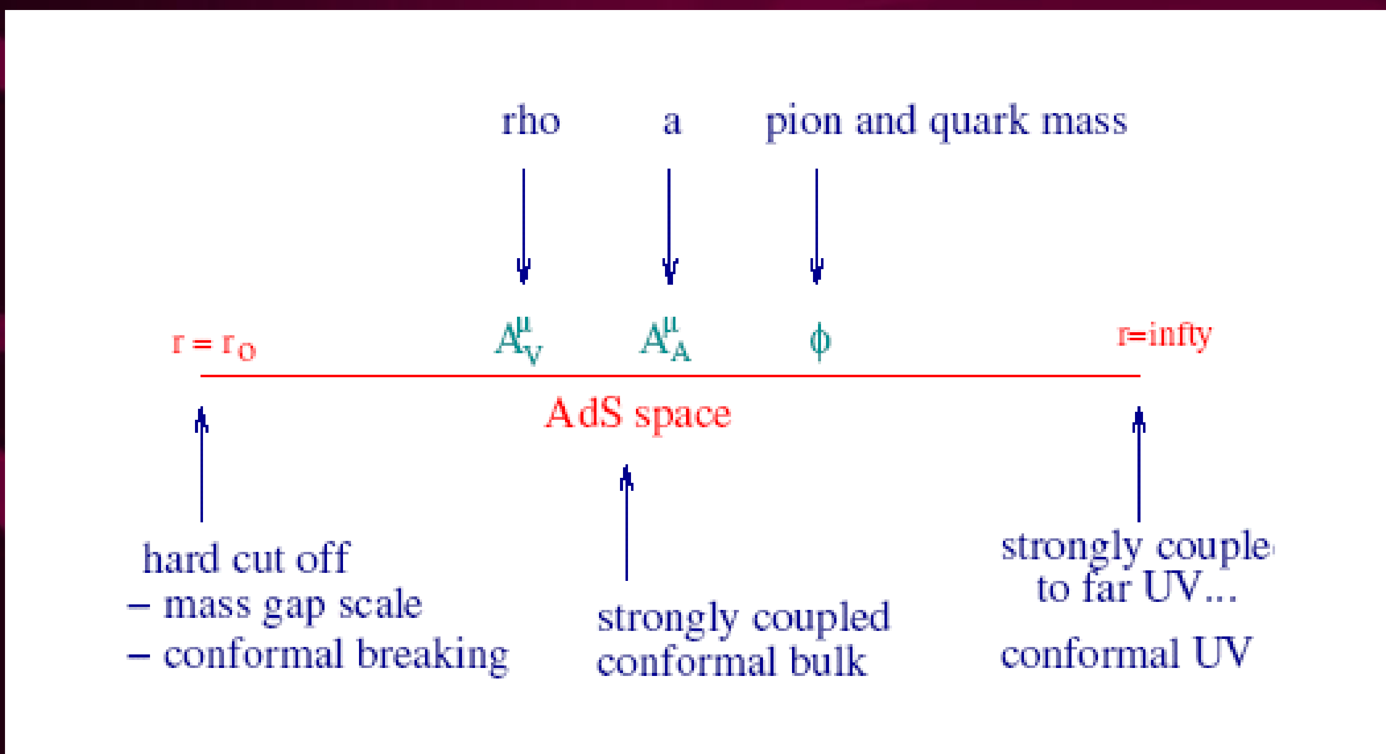
AdS/CFT has taught us that AMAZINGLY the bulk is weakly coupled when N=4 SYM is strongly coupled…

deRold, Pomarol

Traditional AdS/QCD

Son, Stephanov,
Erlich, Katz

$$S = \int_{r_0}^{\infty} d^5x \sqrt{-g}\, Tr \left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$



Parameter count

r_0

c

m

g_5

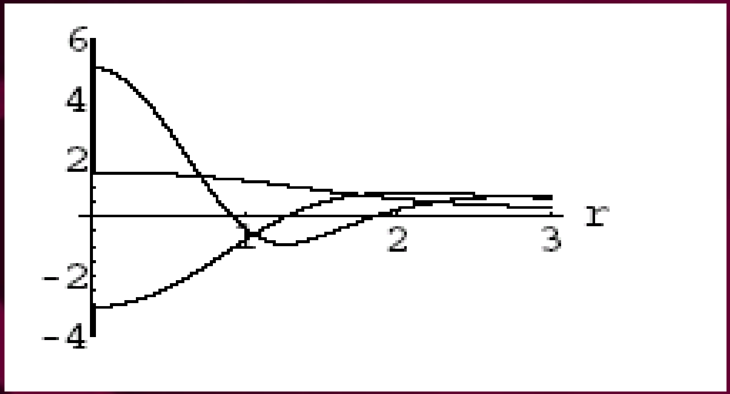
Example Numerics - Meson Masses

The gauge field equation of motion

$$\left[\partial_z\left(\frac{1}{z}\partial_z V^a_\mu(q,z)\right)+\frac{q^2}{z}V^a_\mu(q,z)\right]_\perp=0.$$

$$V\sim V(z)e^{-iq.x},\qquad q^2=-M^2$$

Numerically shoot from UV… seek solutions with V’(wall)=0…



Sub Vs back into the action and integrate over radial coordinate to determine decay constants….

Fix external currents normalization by matching to UV QCD Πvv/ Πaa

Observable	Measured (MeV)	AdS A (MeV)	AdS B (MeV)
m_π	139.6 ± 0.0004	139.6*	141
m_ρ	775.8 ± 0.5	775.8*	832
m_{a_1}	1230 ± 40	1363	1220
f_π	92.4 ± 0.35	92.4*	84.0
$F_\rho^{1/2}$	345 ± 8	329	353
$F_{a_1}^{1/2}$	433 ± 13	486	440

The basic ideas are remarkably good… But… to systematically move to QCD we would need to

IR improve – include all operators that are non-zero in the vacuum and back react them on each other

UV perfect – match the running of operators to the true perturbative QCD values in the UV….

Of course as in any effective description of QCD this is overwhelming and you end up just re-parameterizing the data….

AdS/QCD should be good for understanding qualitative behaviour of RG flow..

The Conformal Window

SU(Nc) gauge theory with Nf fundamental quarks

Nf=11/2 Nc

No AF

CFT

Nf = ? Nc

χ SB

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}N_c - \frac{2}{3}N_f\right\} - \frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}N_c^2 - \frac{N_f}{N_c}\left[\frac{13}{3}N_c^2 - 1\right]\right\} + \cdots$$

Using the 't Hooft coupling, and setting $\frac{N_f}{N_c} \to x$ we obtain

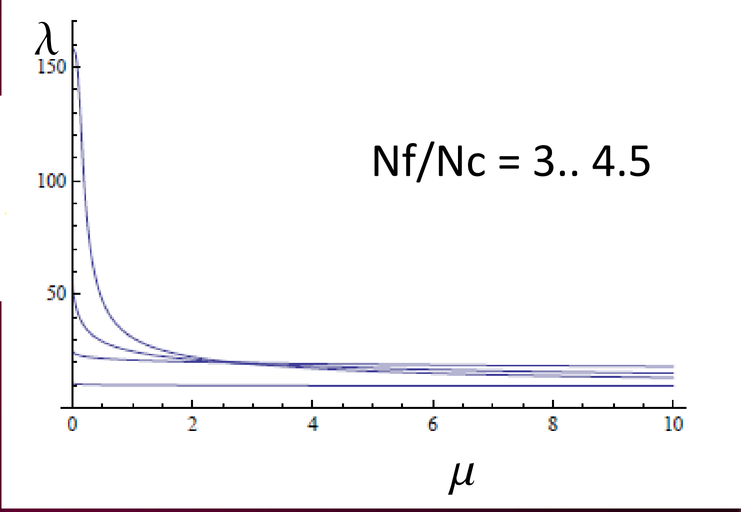
$$\lambda \equiv g^2 N_c \quad , \quad \dot{\lambda} = -b_0\lambda^2 + b_1\lambda^3 + \mathcal{O}(\lambda^4)$$

with

$$b_0 = \frac{2}{3}\frac{(11-2x)}{(4\pi)^2} \quad , \quad \frac{b_1}{b_0^2} = -\frac{3}{2}\frac{(34-13x)}{(11-2x)^2}$$

m $\bar{q}q$

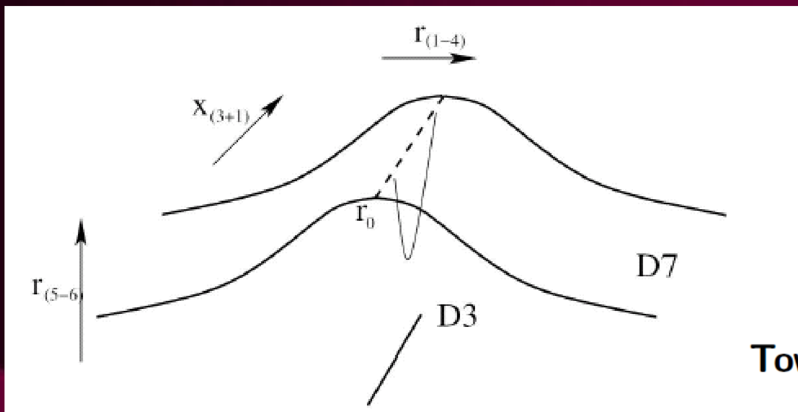
$$\gamma_m^{(1)} = \mu\frac{d\ln m_q}{d\mu} = \frac{3(N_c^2-1)}{4N_c\pi}\alpha$$



If critical γ = 1.... Nf/Nc ~ 4

Yamawaki, Appelquist, Terning, Sannino,...

Top-Down Models of Chiral Symmetry Breaking and Confinement



Chiral Symmetry Breaking and Pions in Non-Supersymmetric Gauge/Gravity Duals

J. Babington [a], J. Erdmenger [a], N. Evans [b], Z. Guralnik [a] and I. Kirsch [a*]

Towards a holographic dual of large-N_c QCD

Martín Kruczenski,[a] David Mateos,[b] Robert C. Myers [b,c] and David J. Winters [b,d]

Mesons in Gauge/Gravity Duals
A Review

Johanna Erdmenger [a], Nick Evans [bc], Ingo Kirsch [d] and Ed Threlfall [b *]

Flavoured Large N Gauge Theory in an External Magnetic Field

Veselin G. Filev[*], Clifford V. Johnson[*], R. C. Rashkov[†1] and K. S. Viswanathan[†]

Towards a Holographic Model of the QCD Phase Diagram

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School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, UK

Keun-Young Kim[§]

Low Energy Hadron Physics in Holographic QCD

Tadakatsu SAKAI[1,*)] and Shigeki SUGIMOTO[2,**)]

Dynamic AdS/QCD

arXiv:1307.4896; arXiv:1302.4553

$$S = \int d^4x \, d\rho \, \mathrm{Tr} \, \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 + \frac{1}{2\kappa^2} (F_V^2 + F_A^2) \right]$$

$$X = L(\rho) \, e^{2i\pi^a T^a}.$$

$$ds^2 = \frac{d\rho^2}{(\rho^2 + |X|^2)} + (\rho^2 + |X|^2) dx^2,$$

X is now a dynamical field whose solution will determine the condensate as a function of m

We use the top-down IR boundary condition on mass-shell: $X'(\rho=X) = 0$

X enters into the AdS metric to cut off the radial scale at the value of m or the condensate

The gauge DYNAMICS is input through Δm

$$m^2 = \Delta(\Delta - 4)$$

$$\Delta m^2 = -2\gamma = -\frac{3(N_c^2 - 1)}{2N_c \pi} \alpha$$

The only free parameters are m and Λ

Stability Bound

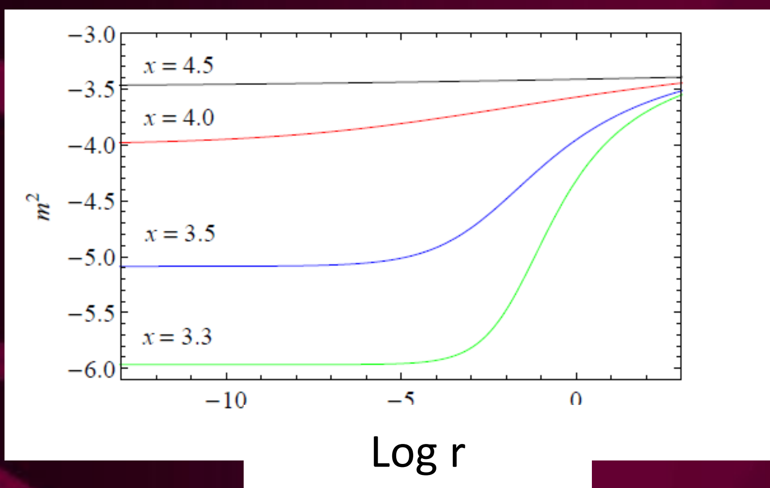
No AF

Nf=11/2 Nc

CFT

Nf = ? Nc

χ SB



We model the $\bar{q}q$ condensate by a scalar in AdS with a running mass

Breitenlohmer-Freedman Bound

A scalar in AdS is stable until

$m^2 < -4$

ie $\Delta < 2$

A hard prediction that matches gap equation-ology… using 2-loop α plus 1-loop γ we predict

Nf_critical = 4 N_c

ie 12 in QCD (same algebra as gap equations)

In The CW

In progress with Marc Scott

At a fixed point γ is constant and $0 < \gamma < 1$

$$L = \frac{m}{\rho^\gamma} + \frac{\bar{q}q}{\rho^{2-\gamma}},$$

|X|=L

Imposing the IR boundary condition gives analytically

$$\bar{q}q = \frac{\gamma}{2\gamma - 2}\left(\frac{2\gamma - 2}{\gamma - 2}\right)^{\frac{3-\gamma}{1+\gamma}} m^{\frac{3-\gamma}{1+\gamma}}$$

This is the hyperscaling relation of Del Debbio & Zwicky – ie dimensional analysis as the fixed point....

In The UV

In the UV we impose the perturbative log running of Δm

$$L = \frac{m}{(\ln\rho)^k} + \frac{\bar{q}q}{\rho^2}(\ln\rho)^k,$$

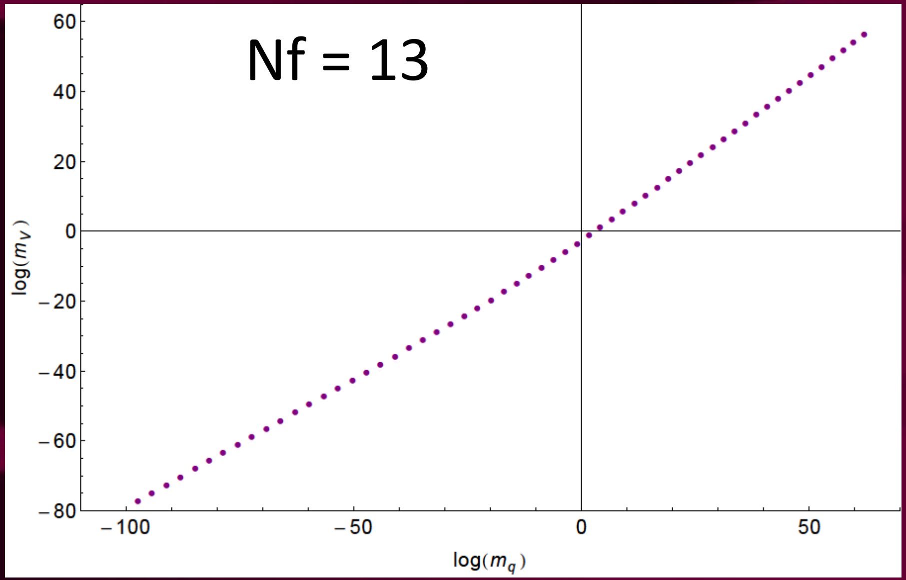
|X|=L

Imposing the IR boundary condition (for large m) gives analytically

$$\bar{q}q = -\frac{k}{2(\ln\frac{L_0}{\Lambda})^{4k+1}} m^3$$

This is dimensional analysis qq ~ m ^{3} upto log running....

The ρ mass

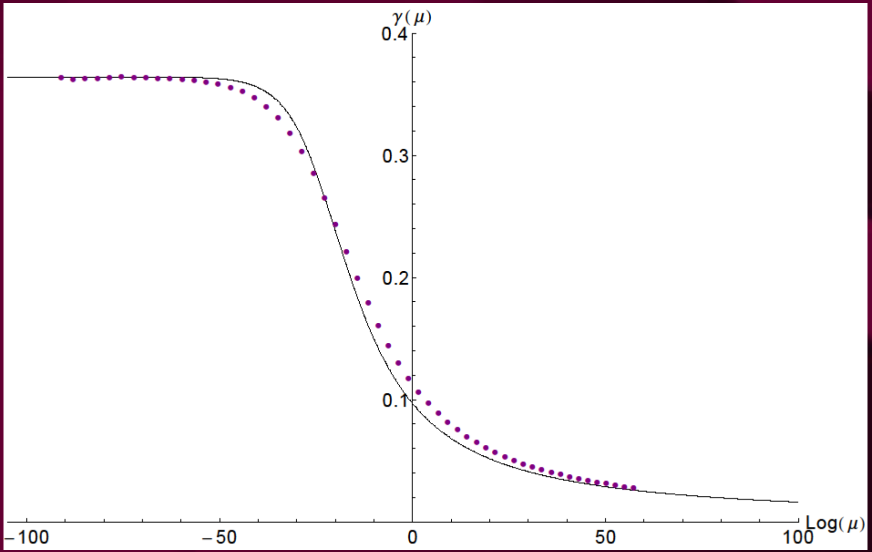


A plot of the ρ mass against the quark mass with the full two loop running included...

We can extract γ as a function of RG scale from

$$M \sim m^{1/1+\gamma}$$

And compare to underlying running... small effects from Λ



Stability Bound

Nf=11/2 Nc — No AF

CFT

Nf = ? Nc

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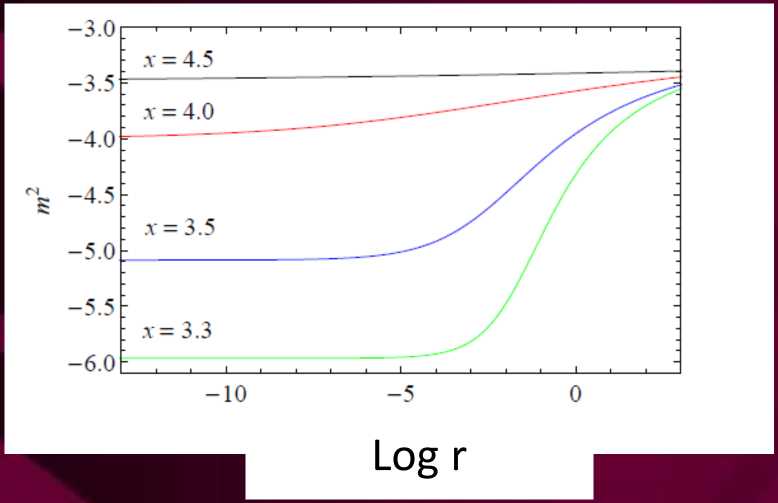
We model the $q\bar{q}$ condensate by a scalar in AdS with a running mass



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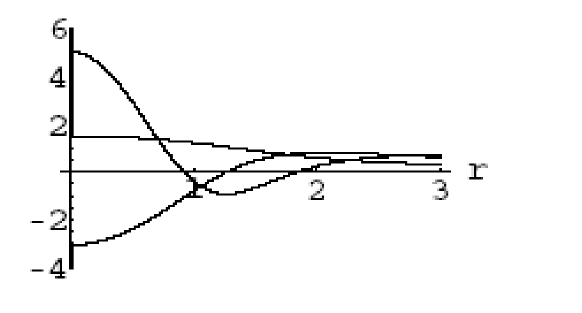
BKT transition

arXiv:0905.4752[hep-th]; arXiv:1002.3159 [hep-th]

A transition due to a violation of the BF bound in the deep IR is of holographic BKT type...

$$L = L_0 + \delta(\rho) e^{ikx}$$

$$k^2 = -M^2$$



The Schroedinger equation for the mesonic fluctuations at $m^2 = -4$ has an infinite number of unstable modes...

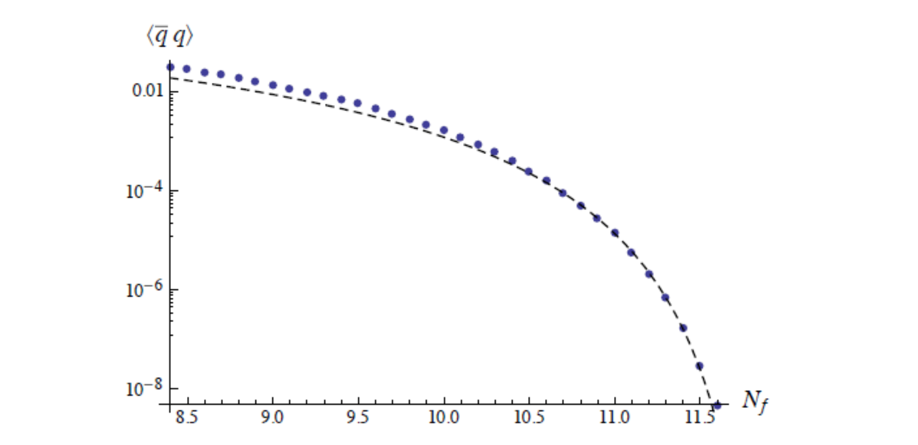
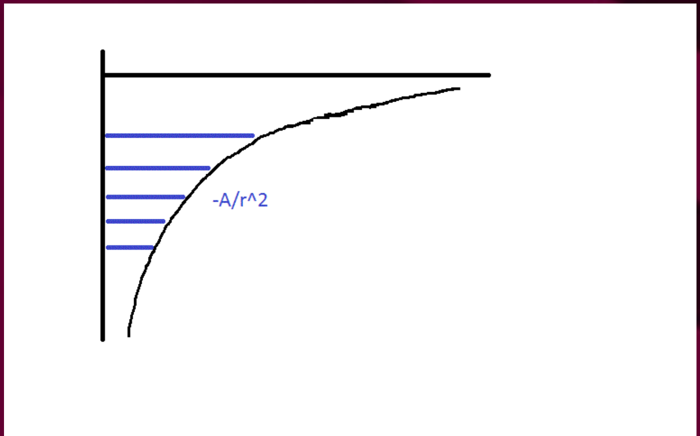


FIG. 4: The dots show numerical results for the quark condensate as a function of N_f. The dashed line is the BKT fit $a \exp(-3b/(N_f^c - N_f)^{1/2})$ with parameters $a = 63.090$ and $b = 5.111$.



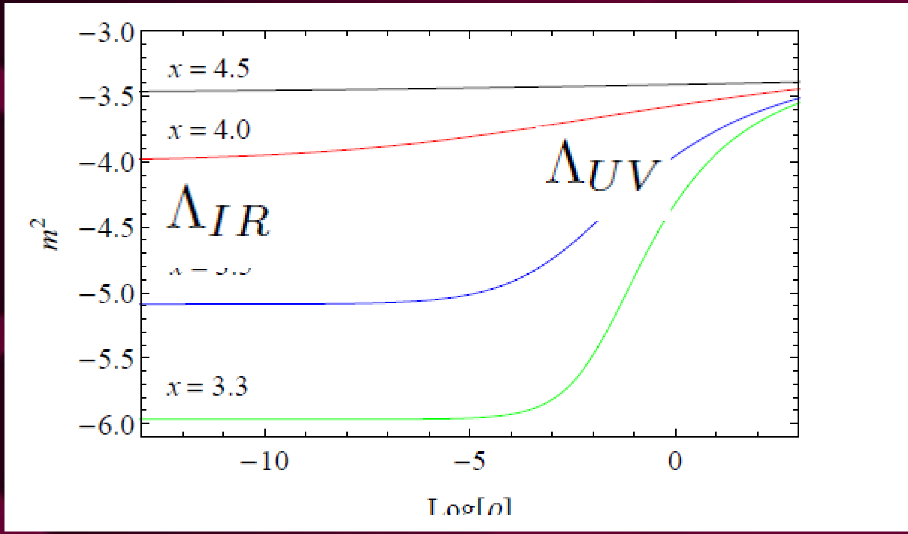
(Miransky scaling)

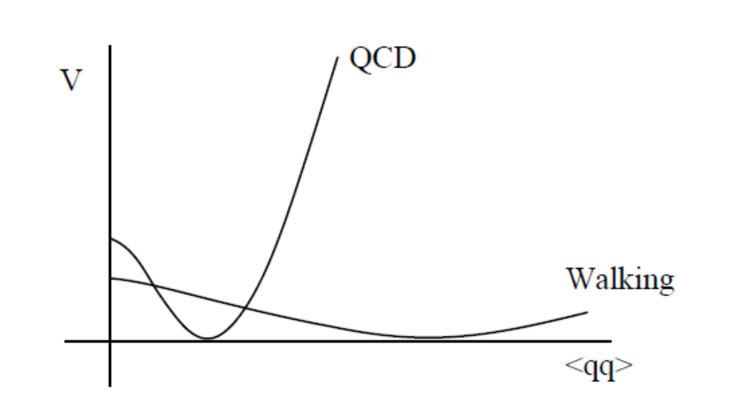
Walking Dynamics

Just above the CW regime theories have an enhanced UV quark condensate

$$\langle \bar{q}q \rangle_{UV} \sim \Lambda_{UV} \langle \bar{q}q \rangle_{IR} \sim \Lambda_{UV} \Lambda_{IR}^2$$

$$f_\pi \sim \Lambda_{IR}$$





- Is the sigma particle light – a techni-dilaton?
- Do these make good inflation theories?

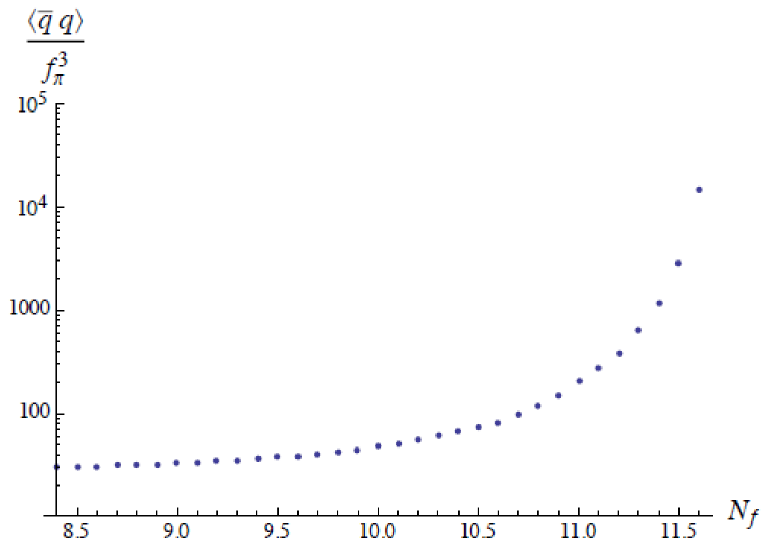


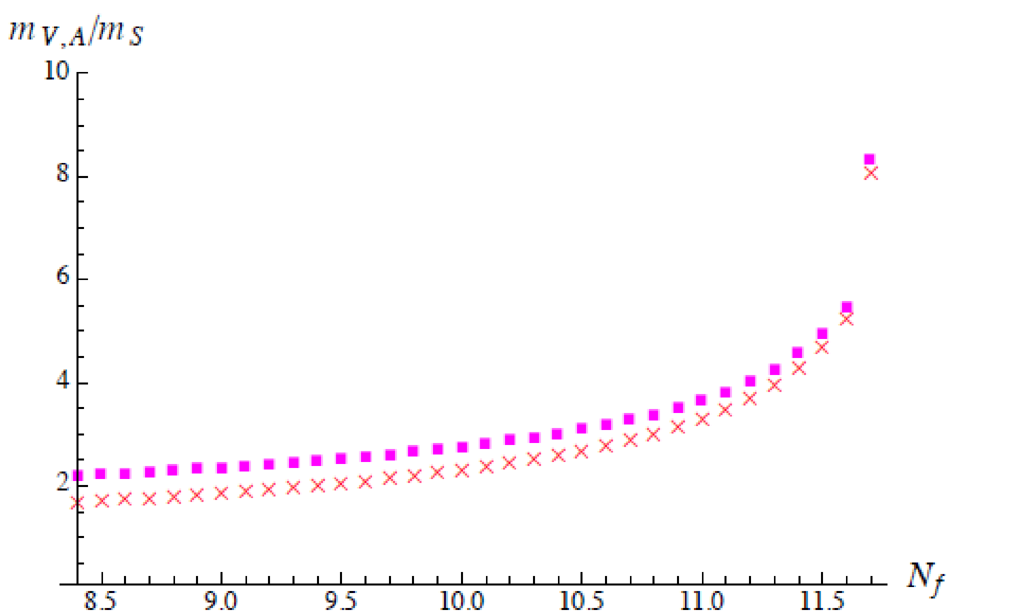
FIG. 5: The quark condensate normalized by f_π^3 vs N_f.

Enhancement reproduced….

Cf FCNC problem in TC

Suppression reproduced…

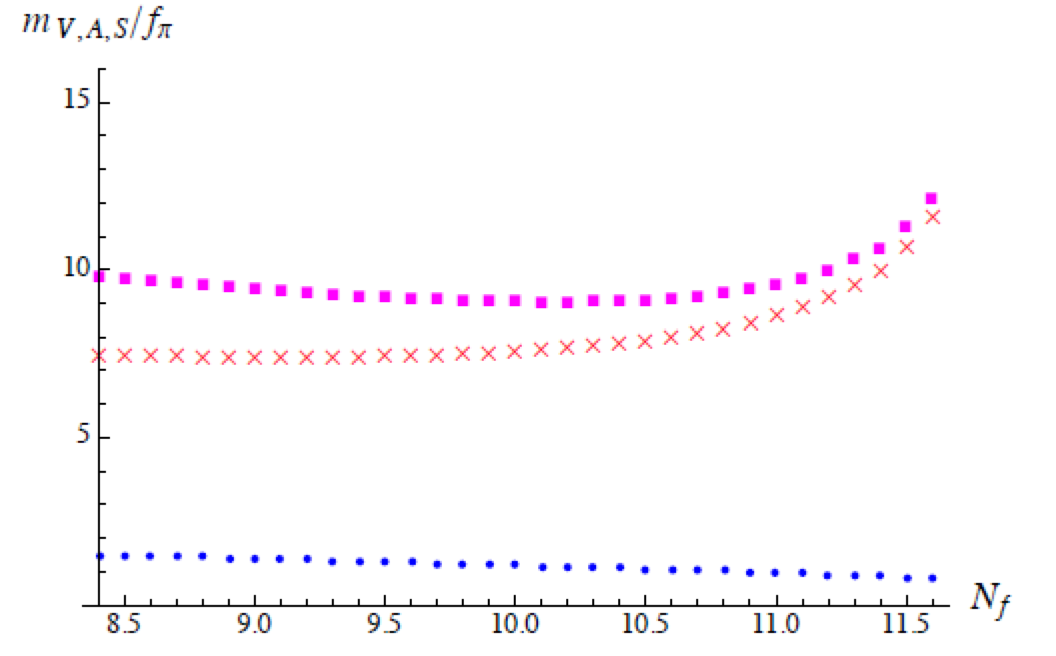
Cf a light higgs in TC…

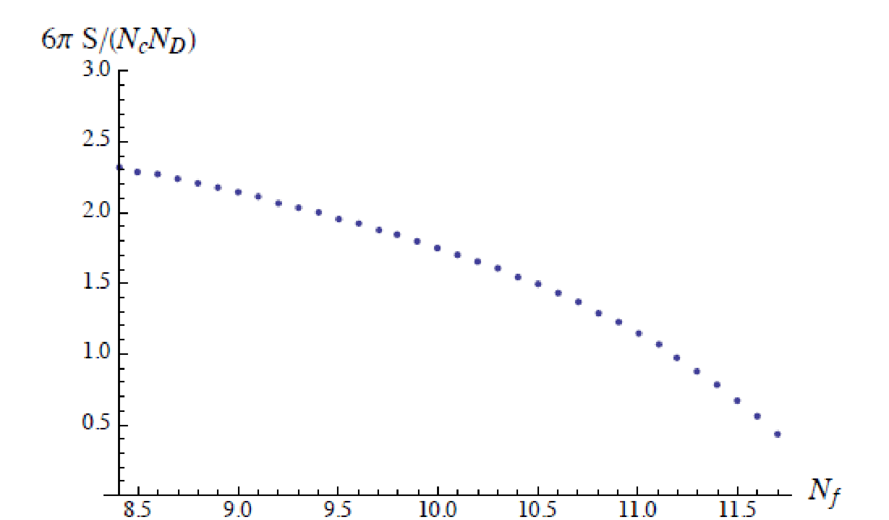


The BKT transition is continuous… so we expect vector-axial symmetry to be continuously restored as we approach the transition… so we impose…

$$\kappa^2 = 3.34(N_f - N_f^c).$$

Which imposes rho-a degeneracy at the transition…

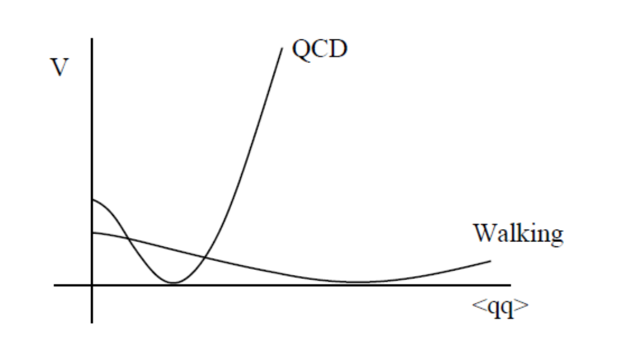




$$S = 4\pi\left(\Pi'_{AA}(0) - \Pi'_{VV}(0)\right),$$

Inflation

arXiv:1009.5678 [hep-th]; arXiv:1208.3060 [hep-ph]



$$\frac{\bar{q}q}{V_0^{3/4}} \to \infty$$

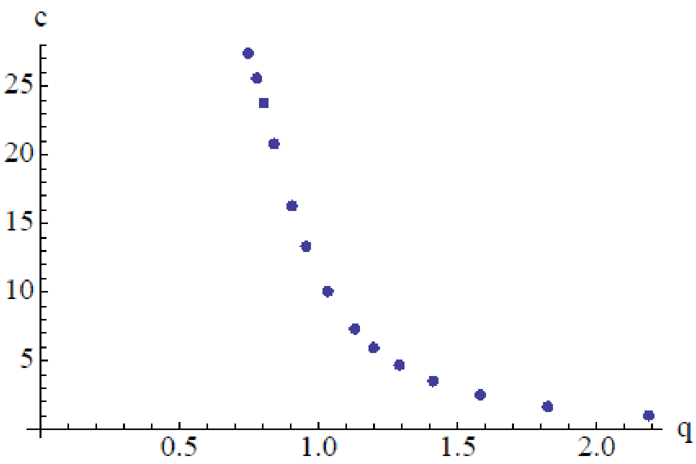
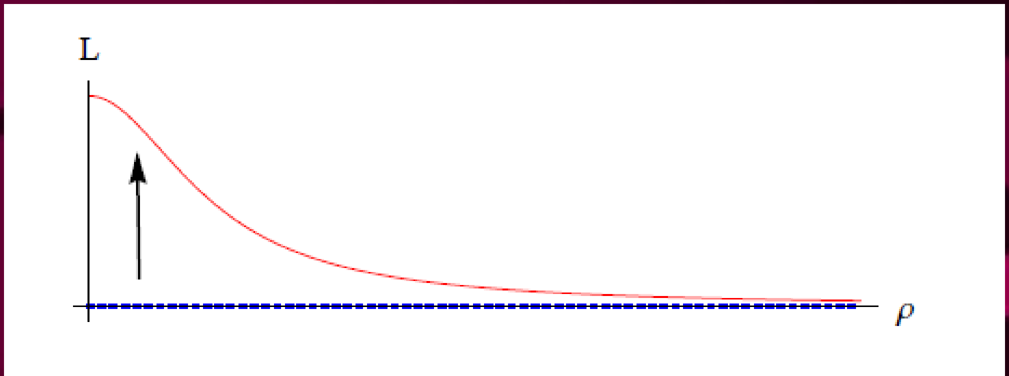
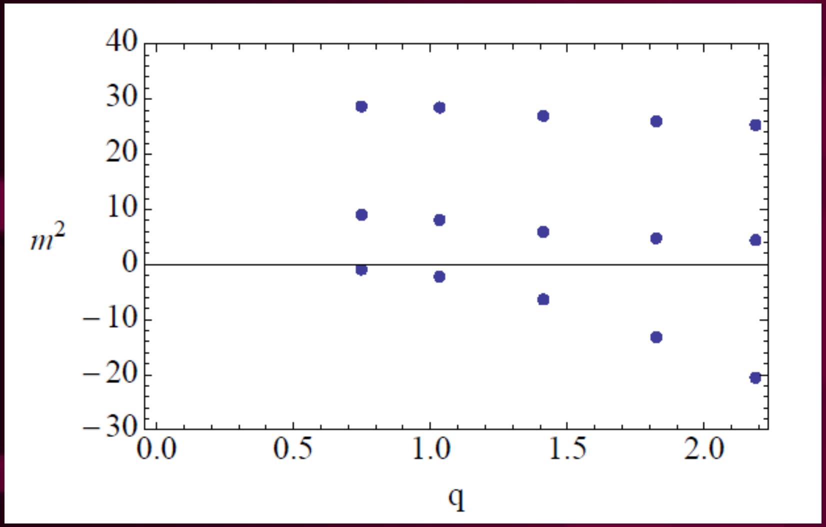


FIG. 3: The quark condensate $\langle \bar{q}q \rangle$ against A in the model of (7) and q in (11) in each case for fixed vacuum energy ΔV.

$$V \simeq \frac{\Delta V}{v^4}\left(\phi^2 - v^2\right)^2$$

The roll from false vacuum to true…





The meson masses of the unstable vacuum.. There is a single inflaton… whose mass falls to zero as one approaches the transition…

You can also time the roll in an explicit time dependent computation and show it increases in length as one approaches the CW transition…

If you’re willing to fine tune to the transition you can get an arbitrarily flat potential/ long roll time…

Conclusions

- Dynamic AdS/QCD is a very simple model that reproduces all the known lore about the CW and walking dynamics. Useful toy for lattice practitioners?
- Holography predicts the conformal window becomes unstable when γ=1
- Reproduces hyper-scalings in UV/IR (and transition)
- Holography predicts the transition is of BKT nature
- If the gradient of γ falls to zero at and around γ=1 then a techni-dilaton is observed
- Tuning to the exit point gives a very flat effective potential for $\bar{q}q$ suitable for naïve slow roll inflation

Jarvinen & Kiritsis Holographic Model

arXiv:1112.1261 [hep-ph]

- 5d supergravity $ds^2 = e^A dr^2 + dx_4^2$
- λ scalar to represent running coupling
- V(λ, A) – impose preferred IR and UV behaviour

Nf/Nc ~ 4

- ϕ scalar to represent $\bar{q}q$ condensate
- V(ϕ, λ, A) – to determine if $\bar{q}q$ condenses