Dynamic AdS/QCD

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• AdS/QCD summary
• Conformal window dynamics
• Walking Technicolor
• Techni-dilatons

Bjelasnica Mountain 2014
Holography — a remarkable new way to think about RG flow

— clearly a deep link between string theory and gravity

We treat RG scale as a direction of space-time....

Conformal theories eg

\[ \int d^4 x \ (\partial \phi)^2 \]

have a dilatation symmetry

\[ \phi \rightarrow e^\alpha \phi \]

\[ x_{3+1} \rightarrow e^{-\alpha} x_{3+1} \]

Reproduced in AdS metric when

\[ u \rightarrow e^\alpha u \]

So u can represent an energy scale...
Holography

Now solve the Klein Gordon equation in AdS....

C and C’ are objects in the gauge theory... they have dimension \( \Delta \) and \( 4-\Delta \)

They have the same symmetry properties (as \( \phi \))

We associate them with gauge invariant J and O

AdS/CFT has taught us that AMAZINGLY the bulk is weakly coupled when N=4 SYM is strongly coupled...
Traditional AdS/QCD

\[ S = \int_{r_0}^{\infty} d^5x \sqrt{-g} Tr \left\{ |D X|^2 + 3|X|^2 - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\} \]

Parameter count

- \( r_0 \)
- \( c \)
- \( m \)
- \( g_5 \)
Example Numerics - Meson Masses

The gauge field equation of motion

\[
\left[ \partial_z \left( \frac{1}{z} \partial_z V_{\mu}^a(q, z) \right) + \frac{q^2}{z} V_{\mu}^a(q, z) \right]_{\perp} = 0.
\]

\[
V(z) = V(z) e^{-i q \cdot x}, \quad q^2 = -M^2
\]

Numerically shoot from UV... seek solutions with \( V'(\text{wall})=0 \)...

Sub Vs back into the action and integrate over radial coordinate to determine decay constants....

Fix external currents normalization by matching to UV QCD \( \Pi_{\nu\nu}/\Pi_{\alpha\alpha} \).
The basic ideas are remarkably good... But... to systematically move to QCD we would need to

IR improve – include all operators that are non-zero in the vacuum and back react them on each other

UV perfect – match the running of operators to the true perturbative QCD values in the UV....

Of course as in any effective description of QCD this is overwhelming and you end up just re-parameterizing the data....

AdS/QCD should be good for understanding qualitative behaviour of RG flow..
The Conformal Window

SU(Nc) gauge theory with Nf fundamental quarks

\[ \frac{N_f}{N_c} \approx 4 \]

\[ Y_{\text{amawaki, Appelquist, Terning, Sannino, ...}}, \quad \chi_{\text{SB}} \]

m

\[ \gamma_m^{(1)} = \mu \frac{d \ln m_q}{d \mu} = \frac{3(N_c^2 - 1)}{4N_c \pi} \alpha. \]

Nf/Nc = 3.. 4.5

If critical \( \gamma = 1 \), then Nf/Nc \( \sim 4 \)

Yamawaki, Appelquist, Terning, Sannino, ...
Top-Down Models of Chiral Symmetry Breaking and Confinement

Chiral Symmetry Breaking and Pions in Non-Supersymmetric Gauge/Gravity Duals

J. Babington, J. Erdmenger, N. Evans, Z. Guralnik and I. Kirsch

Towards a holographic dual of large-$N_c$ QCD

Martín Kruczenski, David Mateos, Robert C. Myers and David J. Winters

Mesons in Gauge/Gravity Duals

A Review

Johanna Erdmenger, Nick Evans, Ingo Kirsch and Ed Threlfall

Flavoured Large $N$ Gauge Theory in an External Magnetic Field

Veselin G. Filev, Clifford V. Johnson, R. C. Rashkov and K. S. Viswanathan

Towards a Holographic Model of the QCD Phase Diagram

Nick Evans, Astrid Gebauer and Maria Magou

Low Energy Hadron Physics in Holographic QCD

Tadakatsu SAKAI and Shigeki SUGIMOTO

Dynamic AdS/QCD

\[ S = \int d^4x \, d\rho \, \text{Tr} \rho^3 \left[ \frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 + \frac{1}{2\kappa^2} (F_V^2 + F_A^2) \right] \]

\[ X = L(\rho) \, e^{2i\pi^a T^a}. \]

\[ ds^2 = \frac{d\rho^2}{(\rho^2 + |X|^2)} + (\rho^2 + |X|^2) dx^2, \]

X is now a dynamical field whose solution will determine the condensate as a function of m.

We use the top-down IR boundary condition on mass-shell: \( X'(\rho=X) = 0 \).

X enters into the AdS metric to cut off the radial scale at the value of m or the condensate.

The gauge DYNAMICS is input through \( \Delta m \).

\[ \Delta m^2 = -2\gamma = -\frac{3(N_c^2 - 1)}{2N_c \pi} \alpha \]

\[ m^2 = \Delta (\Delta - 4) \]

The only free parameters are m and \( \Delta \).
Stability Bound

We model the $\bar{q}q$ condensate by a scalar in AdS with a running mass

Breitenlohmer-Freedman Bound

A scalar in AdS is stable until

$$m^2 < -4$$

$$\Delta < 2$$

A hard prediction that matches gap equation-ology... using 2-loop $\alpha$ plus 1-loop $\gamma$ we predict

$$N_{f\_critical} = 4N_c$$

ie 12 in QCD (same algebra as gap equations)
At a fixed point $\gamma$ is constant and $0 < \gamma < 1$

\[ L = \frac{m}{\rho^\gamma} + \frac{\bar{q}q}{\rho^{2-\gamma}} \]

$|X| = L$

Imposing the IR boundary condition gives analytically

\[ \bar{q}q = \frac{\gamma}{2\gamma - 2} \left( \frac{2\gamma - 2}{\gamma - 2} \right)^{\frac{3-\gamma}{1+\gamma}} m^{\frac{3-\gamma}{1+\gamma}} \]

This is the hyperscaling relation of Del Debbio & Zwicky – ie dimensional analysis as the fixed point....
In The UV

In the UV we impose the perturbative log running of $\Delta m$

$$L = \frac{m}{(\ln \rho)^k} + \frac{\bar{q}q}{\rho^2}(\ln \rho)^k, \quad |X| = L$$

Imposing the IR boundary condition (for large m) gives analytically

$$\bar{q}q = -\frac{k}{2(\ln \frac{L_0}{\Lambda})^{4k+1}} m^3$$

This is dimensional analysis $qq \sim m^3$ upto log running....
The $\rho$ mass

A plot of the $\rho$ mass against the quark mass with the full two loop running included...

We can extract $\gamma$ as a function of RG scale from

$$M \sim m^{1/1+\gamma}$$

And compare to underlying running... small effects from $\Lambda$
We model the $q\bar{q}$ condensate by a scalar in AdS with a running mass.

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A transition due to a violation of the BF bound in the deep IR is of holographic BKT type...

The Schroedinger equation for the mesonic fluctuations at $m^2 = -4$ has an infinite number of unstable modes...

FIG. 4: The dots show numerical results for the quark condensate as a function of $N_f$. The dashed line is the BKT fit $a \exp(-3b/(N_f^c - N_f)^{1/2})$ with parameters $a = 63.090$ and $b = 5.111$. 

(Miransky scaling)
Walking Dynamics

Just above the CW regime theories have an enhanced UV quark condensate

\[ \langle \bar{q}q \rangle_{UV} \sim \Lambda_{UV} \langle \bar{q}q \rangle_{IR} \sim \Lambda_{UV} \Lambda_{IR}^2 \]

\[ f_\pi \sim \Lambda_{IR} \]

- Is the sigma particle light – a techni-dilaton?
- Do these make good inflation theories?
Enhancement reproduced....

Cf FCNC problem in TC

Suppression reproduced...

Cf a light higgs in TC...
The BKT transition is continuous... so we expect vector-axial symmetry to be continuously restored as we approach the transition... so we impose...

\[ \kappa^2 = 3.34(N_f - N_f^c). \]

Which imposes rho-a degeneracy at the transition...
Inflation

\[ \frac{\bar{q}q}{V_0^{3/4}} \rightarrow \infty \]

FIG. 3: The quark condensate \( \langle \bar{q}q \rangle \) against \( A \) in the model of (7) and \( q \) in (11) in each case for fixed vacuum energy \( \Delta V \).

\[ V \simeq \frac{\Delta V}{v^4} (\phi^2 - v^2)^2 \]
The roll from false vacuum to true...

The meson masses of the unstable vacuum. There is a single inflaton... whose mass falls to zero as one approaches the transition...

You can also time the roll in an explicit time dependent computation and show it increases in length as one approaches the CW transition...

If you’re willing to fine tune to the transition you can get an arbitrarily flat potential/long roll time...
Conclusions

- Dynamic AdS/QCD is a very simple model that reproduces all the known lore about the CW and walking dynamics. Useful toy for lattice practitioners?

- Holography predicts the conformal window becomes unstable when $\gamma=1$

- Reproduces hyper-scalings in UV/IR (and transition)

- Holography predicts the transition is of BKT nature

- If the gradient of $\gamma$ falls to zero at and around $\gamma=1$ then a techni-dilaton is observed

- Tuning to the exit point gives a very flat effective potential for $\bar{q}q$ suitable for naïve slow roll inflation
Jarvinen & Kiritsis Holographic Model

- 5d supergravity

\[ ds^2 = e^A dr^2 + dx_4^2 \]

- \(\lambda\) scalar to represent running coupling

- \(V(\lambda, A)\) – impose preferred IR and UV behaviour

- \(\phi\) scalar to represent \(\bar{q}q\) condensate

- \(V(\phi, \lambda, A)\) – to determine if \(\bar{q}q\) condenses

\[ N_f/N_c \sim 4 \]