Low x evolution equation for proton Green function

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Excited QCD 2014, Sarajevo

Outline

- Definitions
- Motivation
- Shock wave formalism
- Our results

Definitions

Introduce the light cone vectors n_1 and n_2

$$n_1 = (1,0,0,1)\,, \quad n_2 = \frac{1}{2}(1,0,0,-1)\,, \quad n_1^+ = n_2^- = n_1 n_2 = 1$$

For any p define $p^{\pm}$

$$p^+ = pn_2 = \frac{1}{2}\left(p^0 + p^3\right)\,, \qquad p^- = pn_1 = p^0 - p^3\,,$$

$$p^2 = 2p^+p^- - \vec{p}^{\,\mathbf{2}};$$

The scalar products:

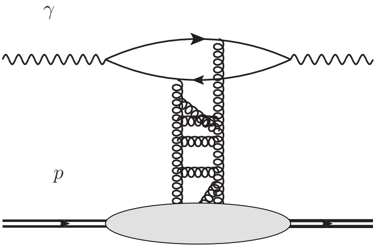
$$p = p^+ n_1 + p^- n_2 + p_\perp\,, \quad (p\,k) = p^\mu k_\mu = p^+ k^- + p^- k^+ - \vec{p}\vec{k}.$$

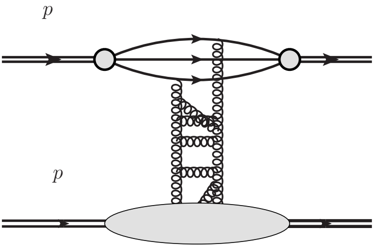
Definitions

Wilson line describing interaction with external field made of slow gluons with $p^+ < e^\eta$

$$U^\eta_{\vec{z}} = Pe^{ig\int_{-\infty}^{+\infty} dz^+ b^-_\eta\left(z^+,\vec{z}\right)}, \quad b^-_\eta = \int \frac{d^4p}{(2\pi)^4} e^{-ipz} b^-\left(p\right)\theta(e^\eta - p^+).$$

Motivation





Dipole picture

$s \gg Q^2 \gg \Lambda^2_{QCD}$

?

$$\sigma_{\gamma*}(s, Q^2) = \int d^2\mathbf{r} |\Psi_{\gamma*}(\mathbf{r}, Q^2)|^2 \sigma_{dip}(\mathbf{r}, s), \quad \sigma_{dip}(\mathbf{r}, s) = 2\int d\mathbf{b}(1 - \frac{1}{N_c} F(\mathbf{b}, \mathbf{r}, s))$$

$\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ — dipole size, $\mathbf{b} = \frac{1}{2}(\mathbf{r_1} + \mathbf{r_2})$ — impact parameter, $F = tr(U_1 U_2^\dagger)$, — dipole Green function, $U_i = U_i^\eta$ — Wilson lines, describing fast moving quarks interacting with the target.

η — rapidity divide, gluons with $p^+ > e^\eta$ belong to photon wavefunction, gluons with $p^+ < e^\eta$ belong to Wilson lines, describing the field of the target.

Motivation

$tr(U_1 U_2^\dagger)$ obeys the LO Balitsky-Kovchegov evolution equation

$$\frac{\partial tr(U_1 U_2^\dagger)}{\partial \eta} = \frac{\alpha_s}{2\pi^2} \int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2} \vec{z}_{42}^{\,2}} \left[tr(U_1 U_4^\dagger) tr(U_4 U_2^\dagger) - N_c tr(U_1 U_2^\dagger) \right].$$

LO equation was obtained in 1996-99, NLO — in 2007-2010 (Balitsky and Chirilli).

Shock wave

For a fast moving particle with the velocity $-\beta$ and the field strength tensor $\mathbb{F}(x^+, x^-, \vec{x})$ in its rest frame, in the observer's frame the field will look like

$$\mathfrak{F}^{-i}\left(y^+, y^-, \vec{y}\right) = \lambda \mathbb{F}^{-i}(\lambda y^+, \frac{1}{\lambda} y^-, \vec{y}) \to \delta\left(y^+\right) \mathfrak{F}^{i}\left(\vec{y}\right),$$

$$\mathfrak{F}^{-i} \gg \mathfrak{F}^{\cdots}$$

in the Regge limit $\lambda \to +\infty$, $\lambda = \sqrt{\frac{1+\beta}{1-\beta}}$.

Therefore the natural choice for the gauge is $b^{i,+} = 0$,

b^- is the solution of the equations

$$\frac{\partial b^-}{\partial y^i} = \delta(y^+) \mathfrak{F}^{i}\left(\vec{y}\right), i.e.$$

$$b^{\mu}\left(y\right) = \delta(y^+) B\left(\vec{y}\right) n_2^{\mu}$$

It is the shock-wave field.

Propagator in the shock wave background

Choose the gluon field $\mathcal{A}$ in the gauge $\mathcal{A}n_2 = 0$ as a sum of external classical b and quantum A.

$$\mathcal{A} = A + b, \quad b^{\mu}(x) = \delta(x^+) B(\vec{x})\, n_2^{\mu}.$$

The A-b interaction lagrangian has only one vertex

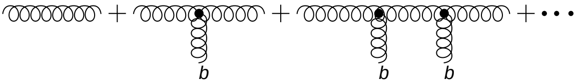
$$\mathcal{L}_i = \frac{g}{2} f^{acb} (b^-)^c g_{\perp}^{\alpha\beta} \left[A_{\alpha}^{a} \frac{\overleftrightarrow{\partial}}{\partial x^-} A_{\beta}^{b} \right].$$

The free propagator $G_0^{\mu\nu}(x^+, p^+, \vec{p}) =$

$$= \frac{-d_0^{\mu\nu}(p^+, p_{\perp})}{2p^+} e^{-i\frac{\vec{p}^{\,2} x^+}{2p^+}} \left(\theta(x^+)\theta(p^+) - \theta(-x^+)\theta(-p^+)\right) + n_2^{\mu} n_2^{\nu} \ldots,$$

$$d_0^{\mu\nu}(p) = g_{\perp}^{\mu\nu} - \frac{p_{\perp}^{\mu} n_2^{\nu} + p_{\perp}^{\nu} n_2^{\mu}}{p^+} - \frac{n_2^{\mu} n_2^{\nu} \vec{p}^{\,2}}{(p^+)^2}.$$

Propagator in the shock-wave background



Sum the diagrams

- b does not depend on x^-, hence the conservation of p^+,
- $b \sim \delta(x^+)$, hence $e^{-i\frac{\vec{p}^{\,2}(x_1^+ - x_2^+)}{2p^+}} \to 1$ in every internal vertex,
- $g_\perp^{\mu\nu} d_{0\nu\rho} g_\perp^{\rho\sigma} = g_\perp^{\mu\sigma}$, hence no dependence on $\vec{p} \Longrightarrow$ conservation of $\vec{x}$ in every internal vertex

Propagator in the shock-wave background:

$$G_{\mu\nu}(x,y)|_{x^+>0>y^+} = 2i A^\mu(x) \int d^4z\, \delta(z^+) F^{+i}(z) \overset{\longleftrightarrow}{\frac{U_{\vec{z}}}{\frac{\partial}{\partial z^-}}} F^{+i}(z) A^\nu(y) \,.$$

where the interaction with b is through Wilson line

$$U_{\vec{z}} = P e^{ig\int_{y^+}^{x^+} dz^+ b^-(z^+, \vec{z})}.$$

Baryon Wilson loop

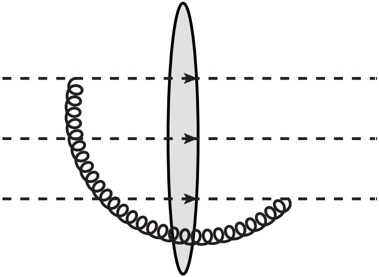
$$B_{123} = \varepsilon^{i'j'h'} \varepsilon_{ijh} U(\vec{z}_1)^i_{i'} U(\vec{z}_2)^j_{j'} U(\vec{z}_3)^h_{h'} = U_1 \cdot U_2 \cdot U_3.$$

- B_{123} is gauge invariant since under a gauge rotation the Wilson lines change

$$U(\vec{z}_1)^i_{i'} \to V(x)^i_k U(\vec{z}_1)^k_{k'} V(y)^{k'}_{i'}, \quad V \in SU(3).$$

- $\varepsilon^{i'j'h'} U^i_{i'} U^j_{j'} U^h_{h'} = \varepsilon^{ijh}$,
- $\varepsilon_{ijh}\varepsilon^{i'j'h'} U^i_{i'} U^j_{j'} = 2(U^\dagger)^{h'}_h, \quad \varepsilon_{ijh}\varepsilon^{i'j'h'} (U^\dagger)^i_{i'} (U^\dagger)^j_{j'} = 2U^{h'}_h$,
- $U_i \cdot U_j \cdot U_k = (U_i U_l^\dagger) \cdot (U_j U_l^\dagger) \cdot (U_k U_l^\dagger)$.
- $B^\eta_{iij} = U_i \cdot U_i \cdot U_j = 2tr(U_j U_i^\dagger)$, i.e. quark-diquark and quark-antiquark systems are described by the same operator.

LO Evolution equation for a baryon Wilson loop



$$\Delta B_r = \frac{\Delta\eta\alpha_s}{\pi^2}\int d\vec{z}_4 (U_4^{\eta_2})^{ab}$$

$$\times\left[\frac{1}{\vec{z}_{41}^{\,2}}\left(t^a U_1^{\eta_2} t^b\right)\cdot U_2^{\eta_2}\cdot U_3^{\eta_2} + (1\leftrightarrow 2) + (1\leftrightarrow 3)\right.$$

$$+\frac{\vec{z}_{41}\vec{z}_{42}}{\vec{z}_{41}^{\,2}\vec{z}_{42}^{\,2}}\left((t^a U_1^{\eta_2})\cdot(U_2^{\eta_2} t^b) + (U_1^{\eta_2} t^b)\cdot(t^a U_2^{\eta_2})\right)\cdot U_3^{\eta_2}$$

$$\left.+(1\leftrightarrow 3) + (2\leftrightarrow 3)\right].$$

Evolution equation for Baryon Green function

Using SU(3) identities

$$U_4^{ba} = 2tr(t^b U_4 t^a U_4^\dagger), \quad t_{ij}^a t_{kl}^a = \frac{1}{2}\delta_{il}\delta_{kj} - \frac{1}{2N_c}\delta_{ij}\delta_{kl}$$

the evolution equation reads

$$\frac{\partial B_{123}^\eta}{\partial \eta} = \frac{\alpha_s}{2\pi^2}\int d\vec{z}_4 \left[\left\{\frac{C_1}{\vec{z}_{41}^{\,2}} + (1\leftrightarrow 2) + (1 \leftrightarrow 3)\right\}\right.$$

$$\left. + \left\{\frac{\vec{z}_{41}\vec{z}_{42}}{\vec{z}_{41}^{\,2}\vec{z}_{42}^{\,2}} C_{12} + (1\leftrightarrow 3) + (2\leftrightarrow 3)\right\}\right].$$

where

$$C_1 = tr\left(U_1^\eta U_4^{\eta\dagger}\right) B_{423}^\eta - 3B_{123}^\eta,$$

$$C_{12} = 2B_{123}^\eta - \left(U_2^\eta U_4^{\eta\dagger} U_1^\eta + U_1^\eta U_4^{\eta\dagger} U_2^\eta\right) \cdot U_4^\eta \cdot U_3^\eta.$$

Evolution equation for baryon Green function

Then we can use the SU(3) identity

$$\left(U_2 U_4^\dagger U_1 + U_1 U_4^\dagger U_2\right) \cdot U_4 \cdot U_3 = -B_{123}^\eta +$$

$$+\frac{1}{2}(B_{144}^\eta B_{324}^\eta + B_{244}^\eta B_{314}^\eta - B_{344}^\eta B_{214}^\eta)$$

to rearrange the equation in the closed way

$$\frac{\partial B_{123}^\eta}{\partial \eta} = \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \left[\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{41}^{\,2} \vec{z}_{42}^{\,2}} (-B_{123}^\eta + \right.$$

$$\left. + \frac{1}{6}(B_{144}^\eta B_{324}^\eta + B_{244}^\eta B_{314}^\eta - B_{344}^\eta B_{214}^\eta)) \quad + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right].$$

In the large N_c limit $\langle B_{144}^\eta B_{324}^\eta \rangle \to \langle B_{144}^\eta \rangle \langle B_{324}^\eta \rangle$.

SU(3) identity

Using $U_i \cdot U_j \cdot U_k = (U_i U_l^\dagger) \cdot (U_j U_l^\dagger) \cdot (U_k U_l^\dagger)$, one can rewrite the identity as

$$-\left(U_2 U_4^\dagger U_1 U_4^\dagger + U_1 U_4^\dagger U_2 U_4^\dagger\right) \cdot E \cdot (U_3 U_4^\dagger)$$

$$= (U_1 U_4^\dagger) \cdot (U_2 U_4^\dagger) \cdot (U_3 U_4^\dagger)$$

$$-tr(U_1 U_4^\dagger)(U_3 U_4^\dagger) \cdot (U_2 U_4^\dagger) \cdot E - tr(U_2 U_4^\dagger)(U_3 U_4^\dagger) \cdot (U_1 U_4^\dagger) \cdot E$$

$$+tr(U_3 U_4^\dagger)(U_2 U_4^\dagger) \cdot (U_1 U_4^\dagger) \cdot E.$$

and prove it expanding the Levi-Civita symbols as

$$\varepsilon_{ijh}\varepsilon^{i'j'h'} = \begin{vmatrix} \delta_i^{i'} & \delta_i^{j'} & \delta_i^{h'} \\ \delta_j^{i'} & \delta_j^{j'} & \delta_j^{h'} \\ \delta_h^{i'} & \delta_h^{j'} & \delta_h^{h'} \end{vmatrix}.$$

Quark-diquark limit

$B^{\eta}_{122} = U_1 \cdot U_2 \cdot U_2 = 2tr(U_1 U_2^{\dagger})$, i.e. quark-diquark and quark-antiquark systems are described by the same operator. The evolution equation should go into the dipole Balitsky-Kovchegov evolution equation as $\vec{z}_{23} \to 0$

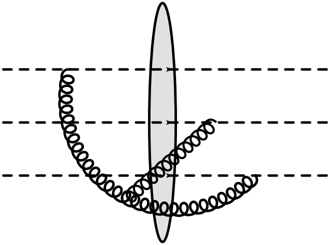
$$\frac{\partial tr(U_1 U_2^{\dagger})}{\partial \eta} = \frac{\alpha_s}{2\pi^2} \int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2} \vec{z}_{42}^{\,2}} \left[tr(U_1 U_4^{\dagger}) tr(U_4 U_2^{\dagger}) - N_c tr(U_1 U_2^{\dagger}) \right].$$

Indeed this is the case

$$\frac{\partial B^{\eta}_{122}}{\partial \eta} = \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \left[\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{41}^{\,2} \vec{z}_{42}^{\,2}} (-B^{\eta}_{122} + \right.$$

$$\left. + \frac{1}{6}(B^{\eta}_{144} B^{\eta}_{224} + B^{\eta}_{244} B^{\eta}_{214} - B^{\eta}_{244} B^{\eta}_{214})) \quad + (1 \to 2) + (2 \leftrightarrow 2) \right].$$

NLO corrections: under check now



$$\frac{\partial B_{123}}{\partial \eta} = \frac{\alpha_s(\mu^2)}{8\pi^2} \int d\vec{r}_0 \left[(B_{100}B_{320} + B_{200}B_{310} - B_{300}B_{210} - 6B_{123}) \right.$$

$$\times \left\{ \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} - \frac{3\alpha_s}{4\pi}\frac{11}{3}\left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}}\right) - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} \ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right)\right]\right\}$$

$$- \frac{\alpha_s}{\pi} \ln\frac{\vec{r}_{20}^{\,2}}{\vec{r}_{21}^{\,2}} \ln\frac{\vec{r}_{10}^{\,2}}{\vec{r}_{21}^{\,2}} \left\{ \frac{1}{2}\left[\frac{\vec{r}_{13}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{30}^{\,2}} - \frac{\vec{r}_{32}^{\,2}}{\vec{r}_{30}^{\,2}\vec{r}_{20}^{\,2}} \right] (B_{100}B_{320} - B_{200}B_{310}) \right.$$

$$\left.\left. - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{20}^{\,2}} \left(9B_{123} - \frac{1}{2}\left[2\left(B_{100}B_{320} + B_{200}B_{130}\right) - B_{300}B_{120}\right]\right)\right\} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right]$$

NLO corrections: under check now

$$
-\frac{\alpha_s^2}{8\pi^4}\int d\vec{r}_0 d\vec{r}_4\ \Big[\{\tilde{L}_{12}\left(U_0 U_4{}^{\dagger} U_2\right)\cdot\left(U_1 U_0{}^{\dagger} U_4\right)\cdot U_3
$$

$$
+L_{12}\Big[\left(U_0 U_4{}^{\dagger} U_2\right)\cdot\left(U_1 U_0{}^{\dagger} U_4\right)\cdot U_3 + tr\left(U_0 U_4{}^{\dagger}\right)\left(U_1 U_0{}^{\dagger} U_2\right)\cdot U_3\cdot U_4
$$

$$
-\frac{3}{4}[B_{144}B_{234}+B_{244}B_{134}-B_{344}B_{124}]+\frac{1}{2}B_{123}\Big]
$$

$$
+(M_{13}-M_{12}-M_{23}+M_2)\Big[\left(U_0 U_4{}^{\dagger} U_3\right)\cdot\left(U_2 U_0{}^{\dagger} U_1\right)\cdot U_4
$$

$$
+\left(U_1 U_0{}^{\dagger} U_2\right)\cdot\left(U_3 U_4{}^{\dagger} U_0\right)\cdot U_4\Big]+(\text{all 5 permutations } 1\leftrightarrow 2\leftrightarrow 3)\}+\ (0\leftrightarrow 4)\Big]\,.
$$

NLO corrections: under check now

$$L_{12}=\left[\frac{1}{\vec r_{01}{}^2\vec r_{24}{}^2-\vec r_{02}{}^2\vec r_{14}{}^2}\left(-\frac{\vec r_{12}{}^4}{8}\left(\frac{1}{\vec r_{01}{}^2\vec r_{24}{}^2}+\frac{1}{\vec r_{02}{}^2\vec r_{14}{}^2}\right)+\frac{\vec r_{12}{}^2}{\vec r_{04}{}^2}-\frac{\vec r_{02}{}^2\vec r_{14}{}^2+\vec r_{01}{}^2\vec r_{24}{}^2}{4\vec r_{04}{}^4}\right)\right.$$

$$\left.+\frac{\vec r_{12}{}^2}{8\vec r_{04}{}^2}\left(\frac{1}{\vec r_{02}{}^2\vec r_{14}{}^2}-\frac{1}{\vec r_{01}{}^2\vec r_{24}{}^2}\right)\right]\ln\left(\frac{\vec r_{01}{}^2\vec r_{24}{}^2}{\vec r_{14}{}^2\vec r_{02}{}^2}\right)+\frac{1}{2\vec r_{04}{}^4}.$$

$$\tilde L_{12}=\frac{\vec r_{12}{}^2}{8}\left[\frac{\vec r_{12}{}^2}{\vec r_{01}{}^2\vec r_{02}{}^2\vec r_{14}{}^2\vec r_{24}{}^2}-\frac{1}{\vec r_{01}{}^2\vec r_{04}{}^2\vec r_{24}{}^2}-\frac{1}{\vec r_{02}{}^2\vec r_{04}{}^2\vec r_{14}{}^2}\right]\ln\left(\frac{\vec r_{01}{}^2\vec r_{24}{}^2}{\vec r_{14}{}^2\vec r_{02}{}^2}\right).$$

$$M_{12}=\frac{\vec r_{12}{}^2}{16}\left[\frac{\vec r_{12}{}^2}{\vec r_{01}{}^2\vec r_{02}{}^2\vec r_{14}{}^2\vec r_{24}{}^2}-\frac{1}{\vec r_{01}{}^2\vec r_{04}{}^2\vec r_{24}{}^2}-\frac{1}{\vec r_{02}{}^2\vec r_{04}{}^2\vec r_{14}{}^2}\right]\ln\left(\frac{\vec r_{01}{}^2\vec r_{02}{}^2}{\vec r_{14}{}^2\vec r_{24}{}^2}\right).$$

$$M_2=\left(\frac{\vec r_{12}{}^2\vec r_{23}{}^2}{\vec r_{01}{}^2\vec r_{02}{}^2\vec r_{24}{}^2\vec r_{34}{}^2}-\frac{\vec r_{14}{}^2\vec r_{23}{}^2}{\vec r_{01}{}^2\vec r_{04}{}^2\vec r_{24}{}^2\vec r_{34}{}^2}-\frac{\vec r_{03}{}^2\vec r_{12}{}^2}{\vec r_{01}{}^2\vec r_{02}{}^2\vec r_{04}{}^2\vec r_{34}{}^2}+\frac{\vec r_{13}{}^2}{\vec r_{01}{}^2\vec r_{04}{}^2\vec r_{34}{}^2}\right)$$

$$\times\frac{1}{4}\ln\left(\frac{\vec r_{02}{}^2}{\vec r_{24}{}^2}\right).$$

Summary

- The nonlinear LO low-x evolution equation (closed in color space) for a Baryon Green function.
- NLO evolution equation under check now.
- Transformation of the NLO equation to the quasi-conformal form.

Work in progress and plans

- Checking.
- Solution of the equation.

Thank you for your attention