

AdS/CFT and The Axial Sector of large- N Yang-Mills theory

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**Work with U. Gursoy, I. Iatrakis, E. Kiritsis, L. Mazzanti, A. O'Bannon,
0707.1349, 0903.2859, 1212.3894**

Work in progress with some of the above plus Regensburg Lattice group

A case study in AdS/CFT phenomenology

- The AdS/CFT correspondence translates the strongly coupled regime of four dimensional Large- N gauge theories into the language of classical gravity.
- In this talk I will discuss a *bottom up* AdS/CFT description of the CP -odd sector of YM theory, controlled by the operator $Tr F\tilde{F}$.
- This is a simple and hopefully useful **case study** in how to use ideas from holography in association with other techniques to have **quantitative results** and (possibly) **predictions**.

Topological charge at large- N

$$\mathcal{L}_{YM} = \frac{1}{4g^2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{32\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

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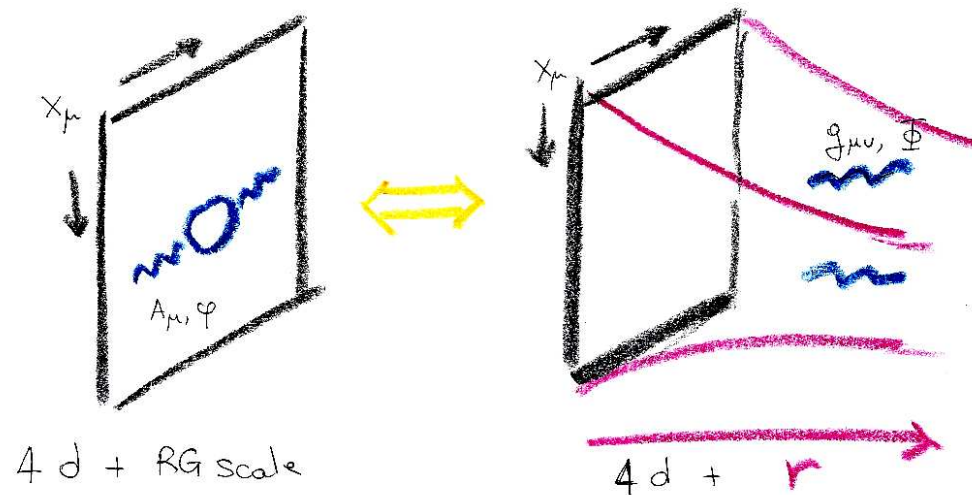
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- Quantities computed from $L[\lambda, \theta/N]$ have a well-defined large- N limit
- $\theta \in [0, 2\pi] \Rightarrow$ the contribution of the topological term to glue dynamics is suppressed at large N . For example (Witten):

$$\mathcal{E}(\lambda, \theta) \approx N^2 \mathcal{E}(\lambda, 0) + \frac{1}{2} \chi \theta^2, \quad \chi = \mathcal{E}''(\lambda, 0)$$

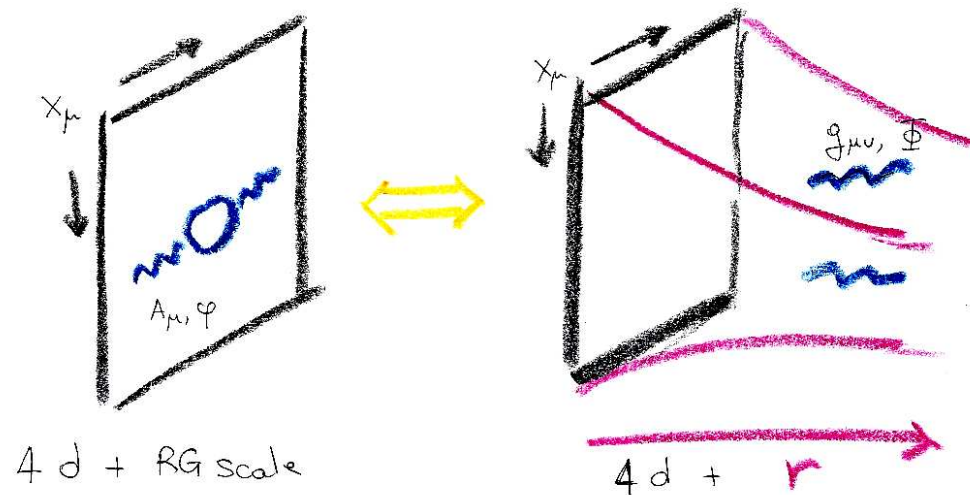
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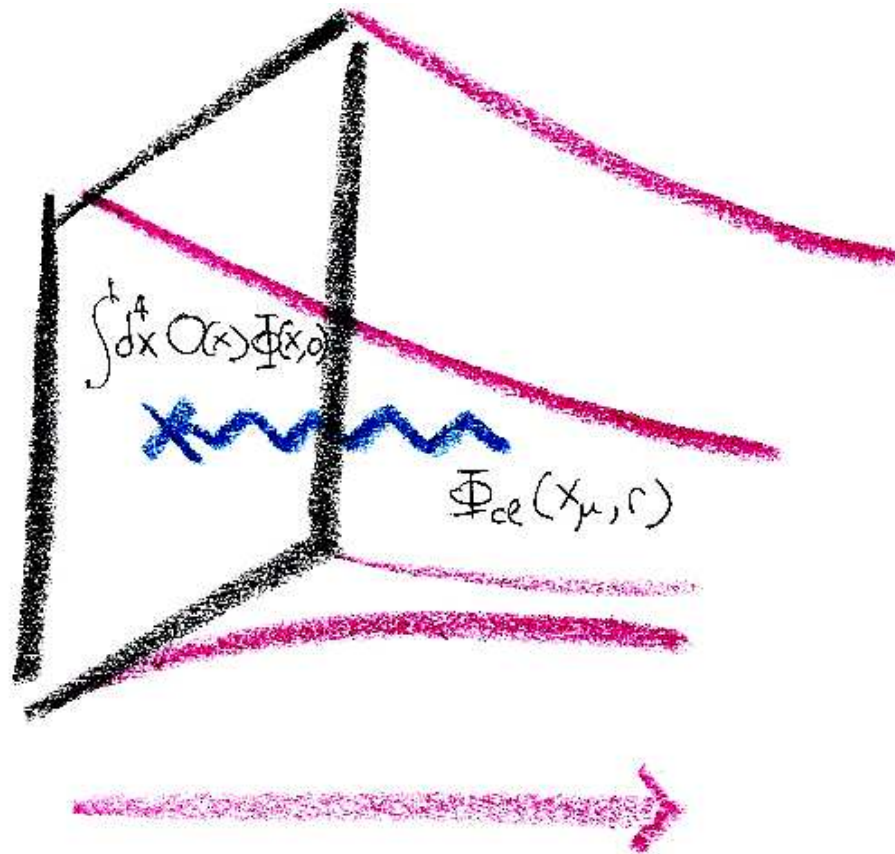
The AdS/CFT duality: conjecture that certain quantum field theories are equivalent to theories of gravity in higher dimensions.



- Conformally invariant theory \Leftrightarrow bulk AdS spacetime
 $ds^2 = r^{-2}(dr^2 + dx_\mu^2)$
- RG scale \Leftrightarrow radial coordinate of the extra dimension;
- Field theory $UV \Leftrightarrow$ large volume region ($r \rightarrow 0$);
- Deformation of AdS \Leftrightarrow breaking of conformal invariance;

Field/Operator correspondence

An operator $O(x)$ with dimension Δ corresponds to a bulk field $\Phi(x, r)$ with mass $m^2 = \Delta(4 - \Delta)$. Φ represents a **source** for O :



Setup

Consider a 5-dimensional model

$$S_{bulk} = N^2 S_{bkg}[g_{\mu\nu}, \Phi_I]$$



Background geometry

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probe pseudoscalar $a(x, r)$

- $a(x, r) \Leftrightarrow \tilde{O} = Tr F \tilde{F} \quad \Delta = 4 \Rightarrow m_a^2 = 0$

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- $a(x, r) \Leftrightarrow \tilde{O} = Tr F \tilde{F} \quad \Delta = 4 \Rightarrow m_a^2 = 0$
- Shift symmetry in the large- N limit \Rightarrow No potential for a .
- Neglect backreaction of a on the geometry.
- $Z(\Phi_I)$ to be fixed phenomenologically.

Action and Field Equation

A vacuum background is specified by a solution of Einstein field equations for $(g_{\mu\nu}, \Phi_I)$, in the form:

$$g_{\mu\nu} = b^2(r) [dr^2 + \eta_{\mu\nu} dx^\mu dx^\nu], \quad \Phi_I = \Phi_I(r)$$

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Once this is specified, the axion action takes the universal form:

$$S_a = \int d^4x dr \frac{A(r)}{2} [(\partial_r a)^2 + \eta^{\mu\nu} \partial_\mu a \partial_\nu a], \quad A(r) = b^3(r) Z(\Phi_I(r))$$

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The field equations are linear:

$$\partial_r [A(r) \partial_r a(x, r)] + A(r) \partial_\mu \partial^\mu a(x, r) = 0$$

Vacuum Topological Susceptibility

Homogeneous solutions ($\partial_\mu a = 0$)

$$(A(r)a')' = 0 \quad \Rightarrow \quad a(r) = a_0 + a_1 \int_0^r \frac{dr'}{A(r')}$$

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- Imposing $a \rightarrow 0$ in the IR fixes a_1 . The action on shell (= vacuum energy) is:

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- One can compute χ on the lattice \Rightarrow **fix normalization by matching lattice result.**

Axial Glueball Spectrum

- In AdS/CFT, normalizable modes in the bulk correspond to gauge-invariant states in the Hilbert space of the field theory.
- the spectrum of normalizable modes of the axion field is identified with the spectrum of composite states associated to $Tr F \tilde{F}$, i.e. 0^{-+} glueballs.
- This can be recast in searching the spectrum of bound states in a 1-d Schrödinger equation.

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$$\psi'' - \left[\frac{1}{2} \frac{A''}{A} - \frac{1}{4} \left(\frac{A'}{A} \right)^2 \right] \psi - k^2 \psi = 0$$

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- 0^{-+} spectrum \Leftrightarrow spectrum of normalizable solutions. If the background $A(r)$ comes from a confining geometry \Rightarrow infinite tower of discrete eigenvalues m_n

Field theory correlators from the gravity side

The boundary value of $a(x, r)$ represents a **source** for $Tr F \tilde{F}(x)$:

$$a(x, r) \simeq_{r \rightarrow 0} \alpha(x) + \dots \quad \Leftrightarrow \quad S_{QFT} = S_0 + \int d^4x \alpha(x) Tr F \tilde{F}(x)$$

Quantum generating functional $Z_{QFT}[\alpha(x)]$ (in the large- N limit):

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- Compute correlators as in QFT:

$$\langle Tr F \tilde{F}(x_1) \dots Tr F \tilde{F}(x_n) \rangle = \frac{\delta}{\delta \alpha(x_1)} \dots \frac{\delta}{\delta \alpha(x_n)} Z_{QFT}[\alpha]$$

Two-point function and decay constants

Beyond the spectrum: full two-point function from AdS/CFT . one can show that:

$$\langle \tilde{O}(k)\tilde{O}(-k) \rangle = \lim_{r \rightarrow 0} [A(r)a_{-k}(r)a'_k(r)] / 2$$

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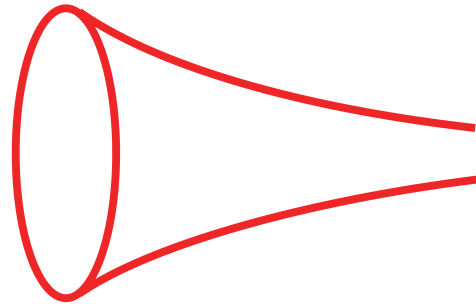
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- The residues (glueball decay constants) f_n are given by:

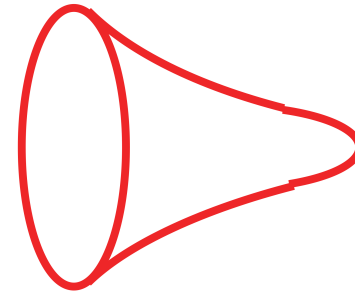
$$f_n = \sqrt{A(0)} \left| \psi'_n(0) - \frac{1}{2} \frac{A'(0)}{A(0)} \psi_n(0) \right|$$

Deconfined phase

- Above the deconfinement transition, the relevant gravity solution is a 5d black hole.



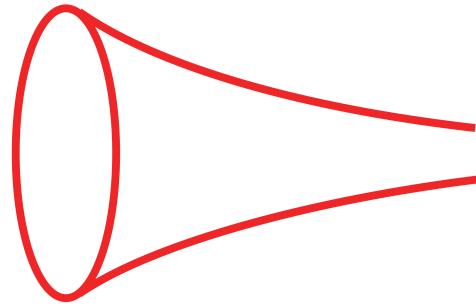
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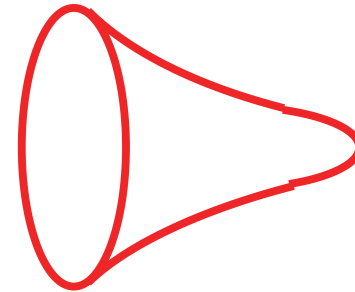
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- The only solution $a(r)$ regular at the BH horizon is $a(r) = \text{const.}$

$$\Rightarrow \chi_{top} = 0 \quad T > T_c$$

This agrees with what is found numerically
(Vicari, Panagopoulos, '08) and with large N arguments.

Chern-Simons diffusion

The low frequency limit of the correlator gives a *diffusion constant*

$$\Gamma_{CS} = \lim_{\omega \rightarrow 0} \frac{2T}{\omega} \text{Im} G_{ret}(\omega, \vec{k} = 0) \quad G_{ret}(t) = i\theta(t) \left\langle \left[\tilde{O}(t), \tilde{O}(0) \right] \right\rangle$$

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- This quantity plays an important role in the chiral magnetic effect [Kharzayev, Pisarsky, Tytgat '00](#)
- *AdS/CFT* can easily compute real-time correlators: they are obtained by the mode solutions $a_\omega(r)$ which are infalling at the black hole horizon.

$$\Gamma_{CS} = \frac{sT}{N^2} \frac{Z(r_h)}{2\pi}$$

Towards the real world

Take a background generated by a single scalar λ , dual to $Tr F^2$, and representing the running t'Hooft coupling [Gursoy, Kiritsis, Mazzanti, FN 08-09](#)

$$S_{bkg} = N^2 \int d^5x \sqrt{-g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right]$$

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- UV asymptotic freedom, confinement, 0^{++} and 2^{++} glueball spectrum and thermodynamics in agreement with lattice, can be achieved by an appropriate choice of $V(\lambda)$.
- The solution has asymptotics:

$$g_{ab} \simeq \eta_{ab} \begin{cases} \ell^2 \\ \overline{\ell^2} \\ e^{-2\Lambda^2 r^2} \end{cases} \quad \lambda(r) \simeq \begin{cases} \frac{1}{\beta_0 \ln r} & r \rightarrow 0 \\ r e^{3\Lambda^2 r^2/2} & r \rightarrow \infty \end{cases}$$

Parametrizing the axion Lagrangian

$$S_a = \frac{1}{2} \int \sqrt{-g} Z(\lambda) (\partial a)^2$$

$$Z(\lambda) = Z_0 (1 + c_1 \lambda + c_4 \lambda^4)$$

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Finite χ_{top} Universal Regge slopes

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Free parameters to fix by matching lattice/experiment

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Discrete 0^{-+} spectrum with asymptotics (from WKB method)

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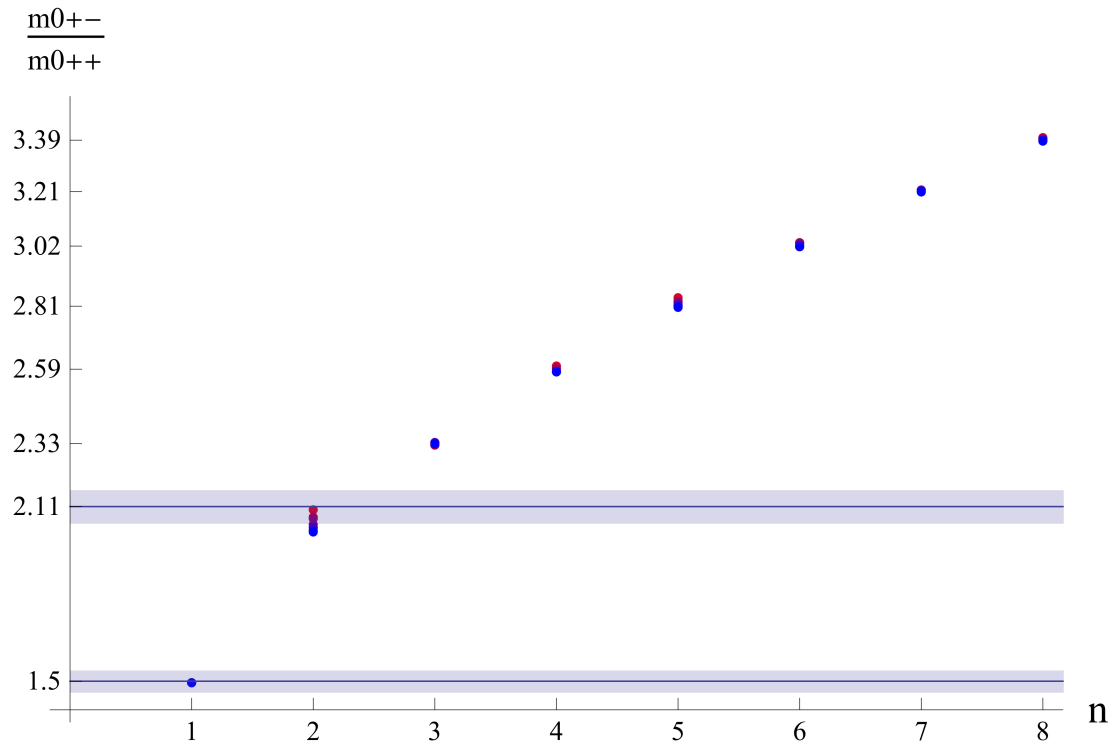
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For $c_1 = 0$, $c_4 = 0.26$ one finds a good match with Lattice result for the lowest lying 0^{-+} states.

	5d model	lattice hep-lat/9901004
$m_{0^{-+}}/m_{0^{++}}$	1.50	1.50(4)
$m_{0^{*-+}}/m_{0^{++}}$	2.10	2.11(6)

Matching the lattice 0^{-+} spectrum

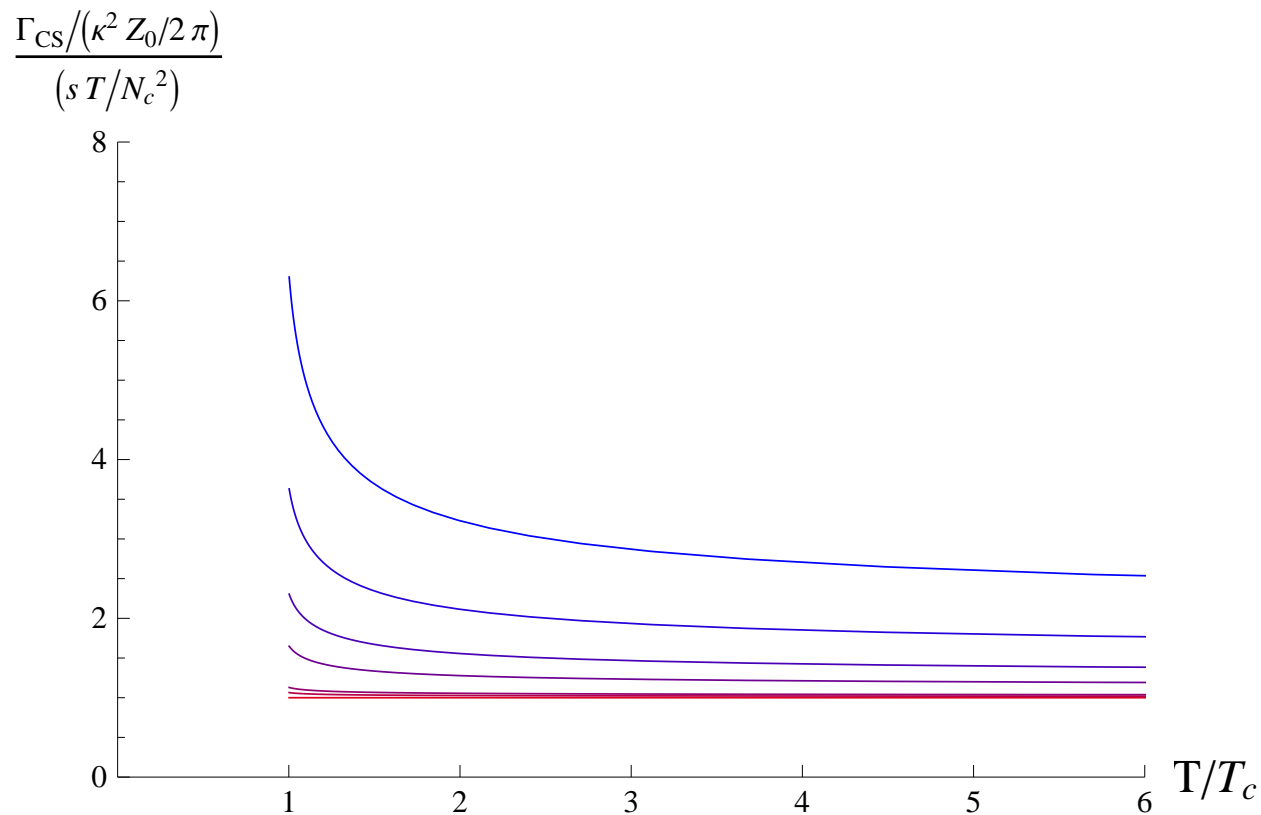
The spectrum is rather insensitive to the details of $Z(\lambda)$.



Changing c_1 between zero and 100 (with c_4 conditioned to keep $m_{0^{-+}}$ fixed) only affects the first excited state in the tower.

Diffusion constant

On the other hand, the diffusion constant changes significantly if we move in (c_1, c_4) space

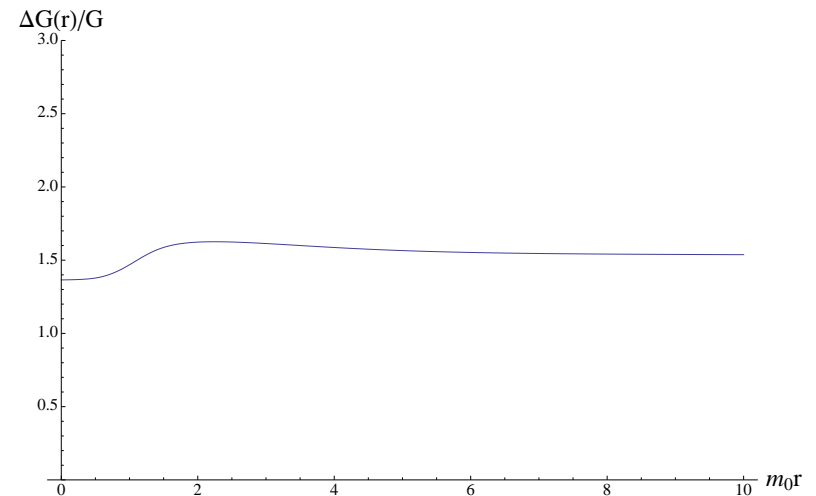
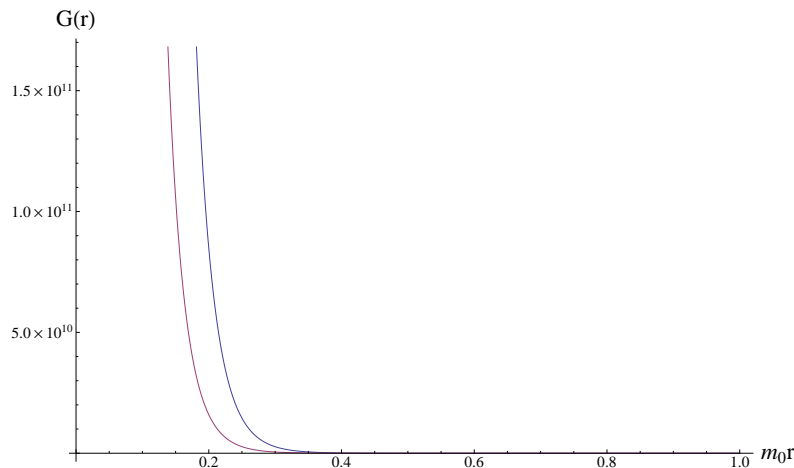


Increasing c_1 enhances the effect close to T_c

Full Correlator

The best shot at fixing $Z(\lambda)$ (and possibly predict $\Gamma_{CS}(T)$) is to compute the full correlator. We can do it in position space, and compare directly with a lattice computation (in progress)

$$\langle \tilde{O}(x)\tilde{O}(0) \rangle = \square^3 \left(\frac{1}{|x|} \sum_{n=0}^{\infty} \frac{f_n^2}{m_n^5} K_1(m_n|x|) \right)$$



the plots correspond to the two point function with $c_1 = 0$ and $c_1 = 5$.

Conclusion

- *AdS/CFT* is a great tool to get qualitative picture of strongly coupled dynamics.
- To get results at a quantitative level, it is inevitable to resort to a phenomenological approach.
- The most efficient approach is to use *AdS/CFT* in combination with other methods, with information flowing both ways.