ECT* European Centre for Theoretical Studies in Nuclear Physics and Related Areas

FONDAZIONE BRUNO KESSLER

Effects of divergent ghost loops on the QCD Green's functions

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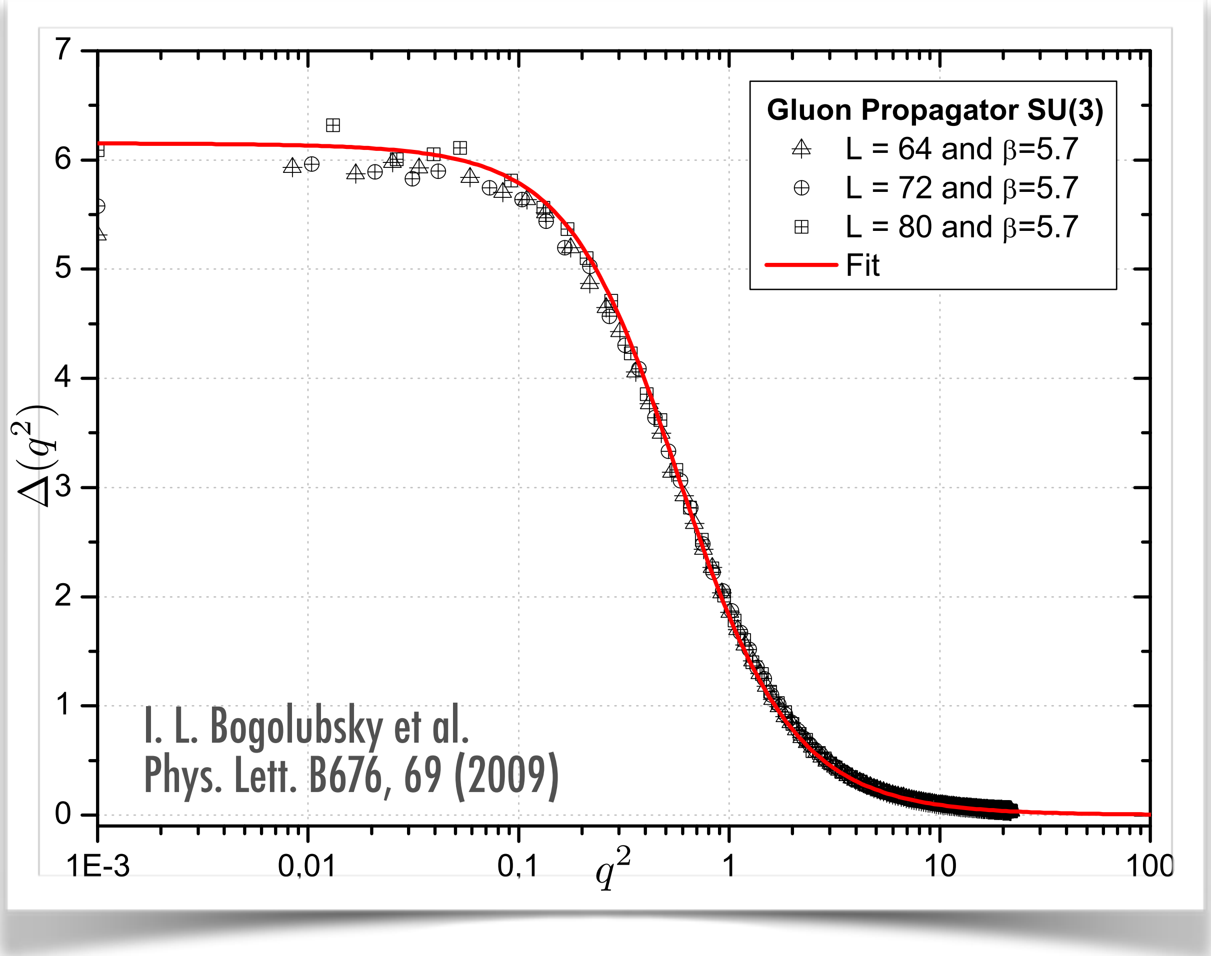
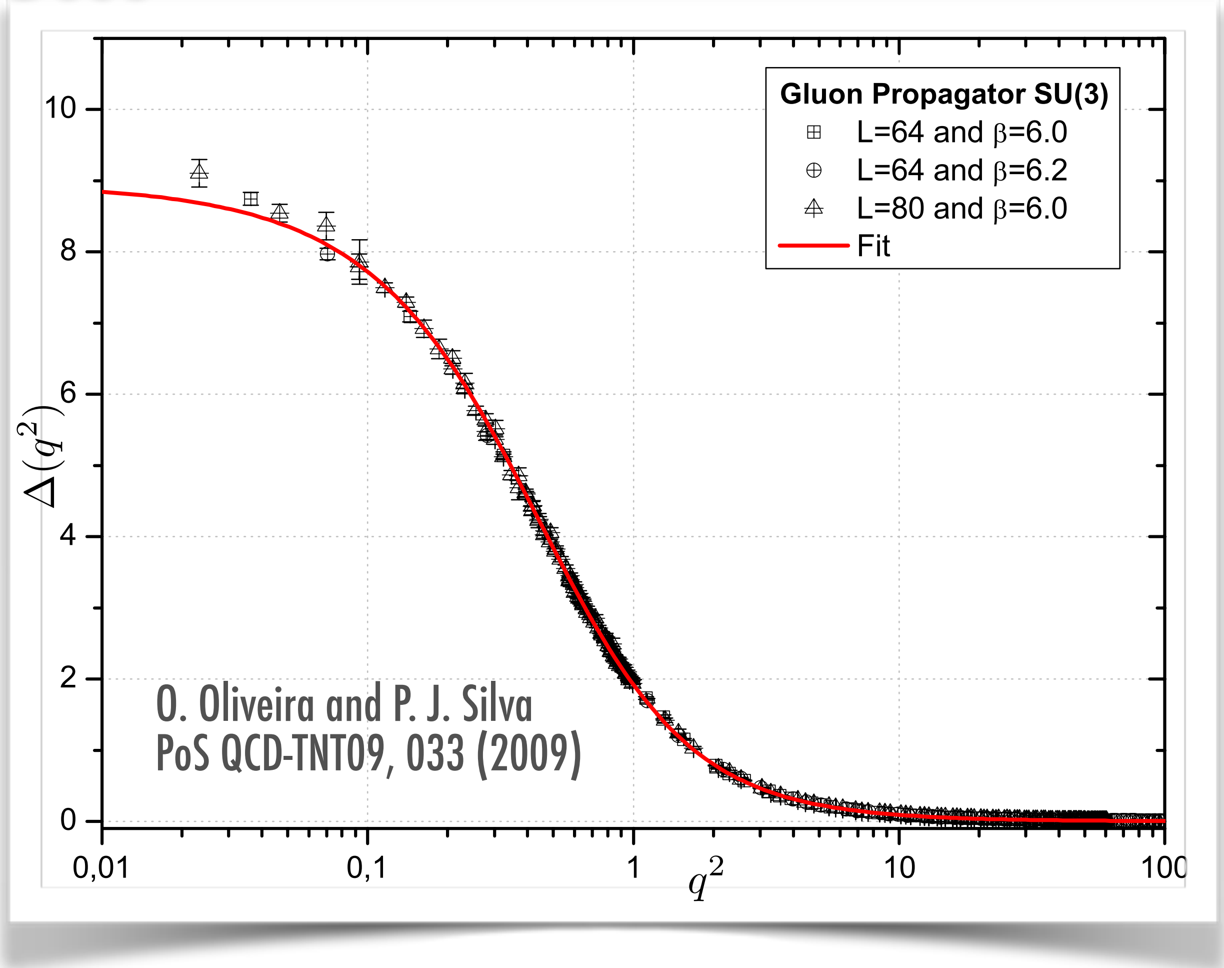
eQCD - Bjelasnica mountain
02|02|2014-08|02|2014

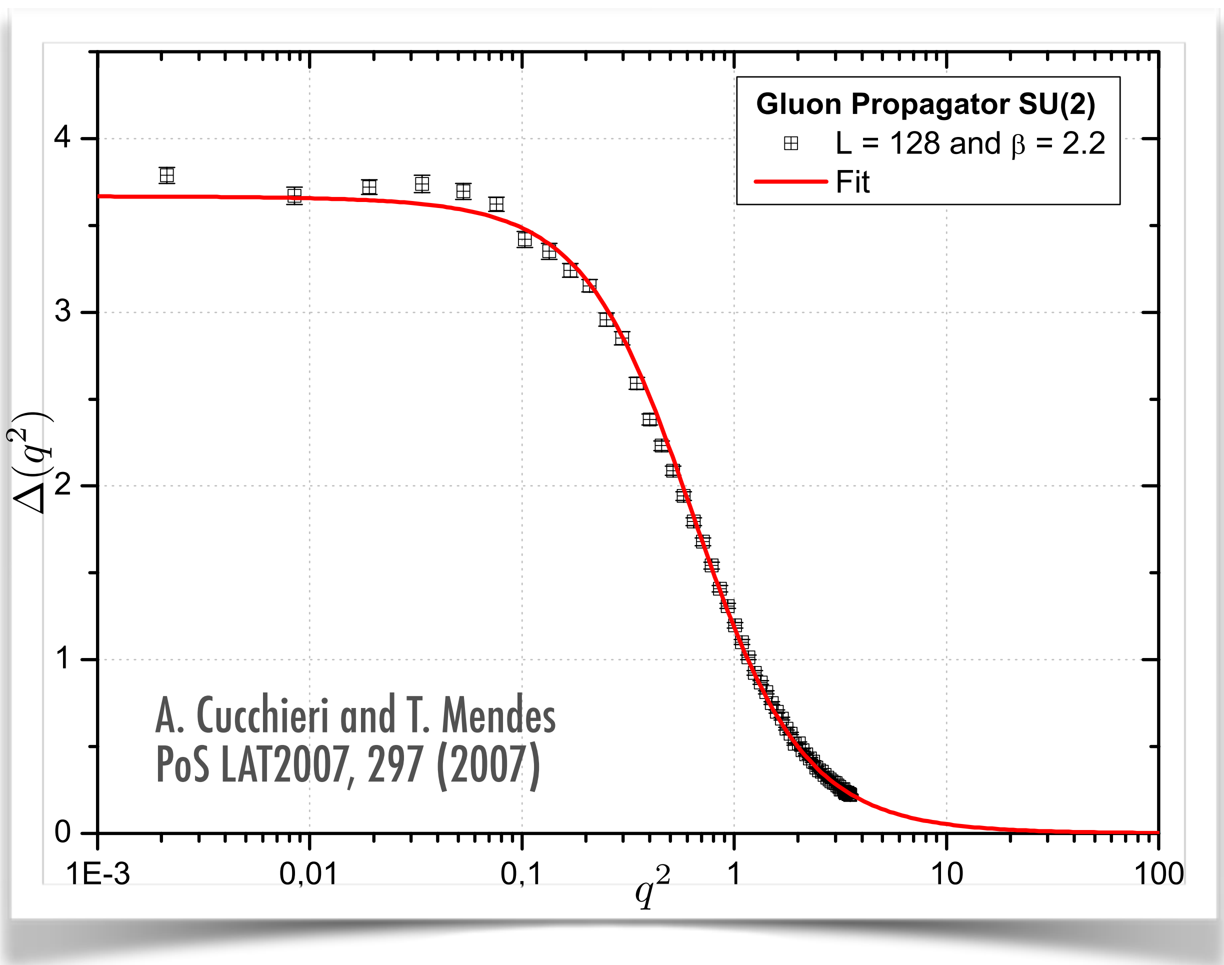
Prolegomena

gluon and ghost propagators from lattice QCD

- **Massive gluon** propagator







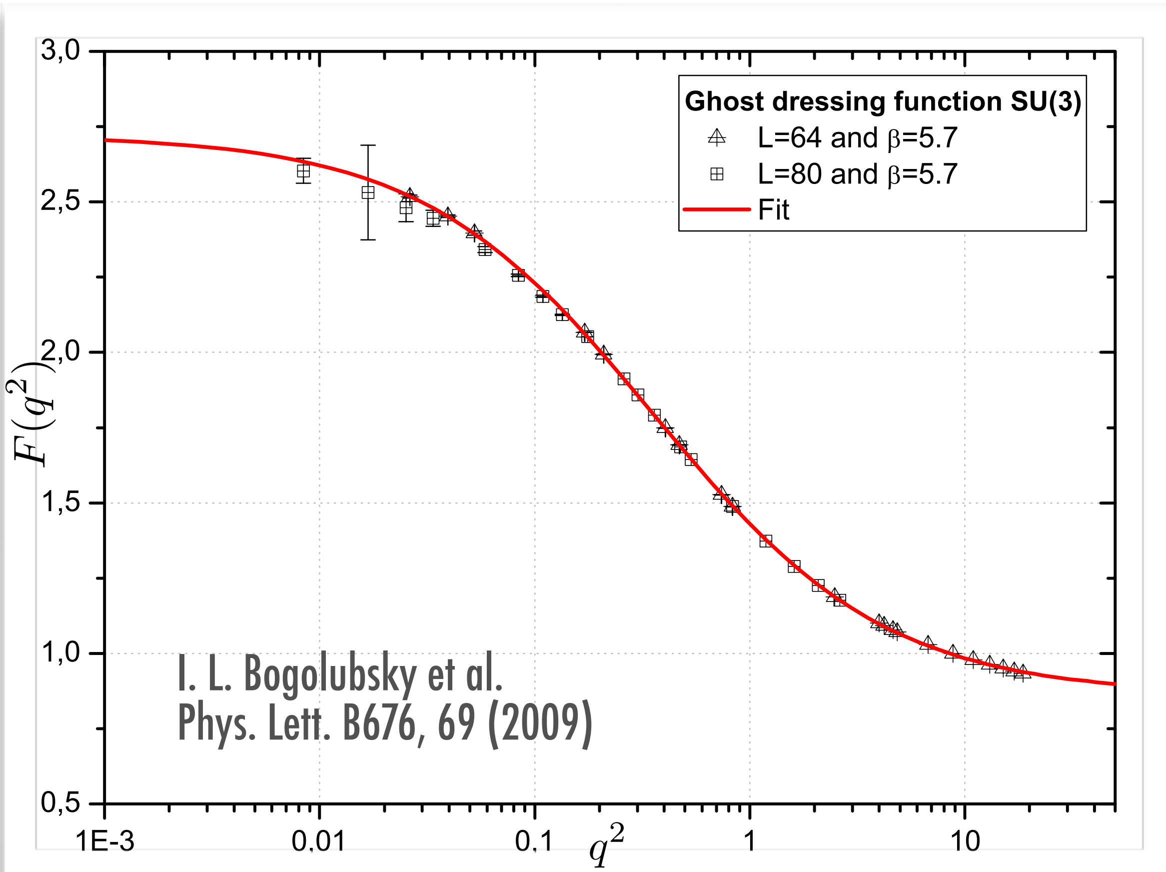
- Fitted by the following function

$$\Delta^{-1}(q^2) = m^2 + q^2\left[1 + \frac{13C_A g_f^2}{96\pi^2}\ln\left(\frac{q^2+\rho m^2}{\mu^2}\right)\right]$$

gluon and **ghost propagators from lattice QCD**

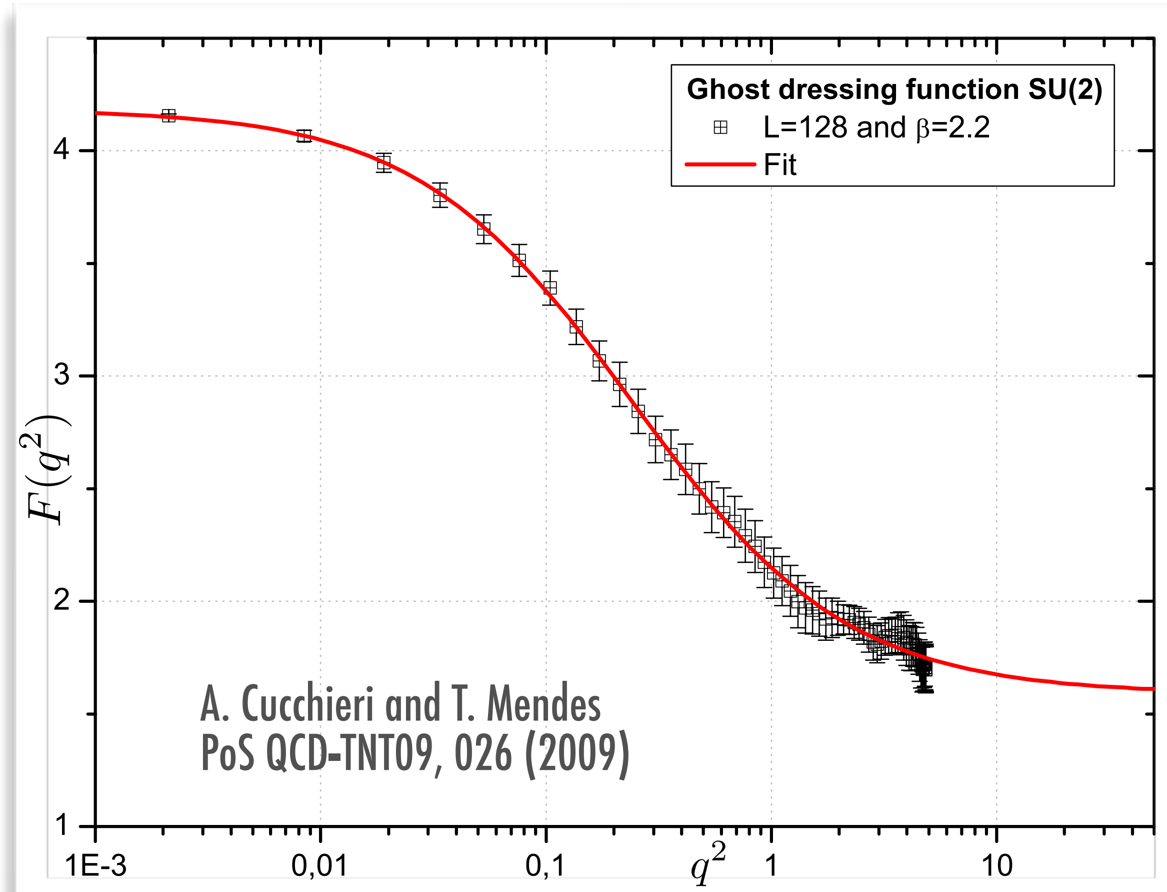
Ph. Boucaud et al., Phys. Rev. D74, 034505 (2006)

Ghost dressing function $F(q^2)$ saturates ($D=F/q^2$)



- **Ghost** propagator is still **IR divergent** (i.e., there is no ghost mass)
- **Ghosts** seem to **play** a **marginal role** in the game (as opposed to ghost-dominance)

R. Alkofer, L. von Smekal, C. Fisher, A. Maas,. . .

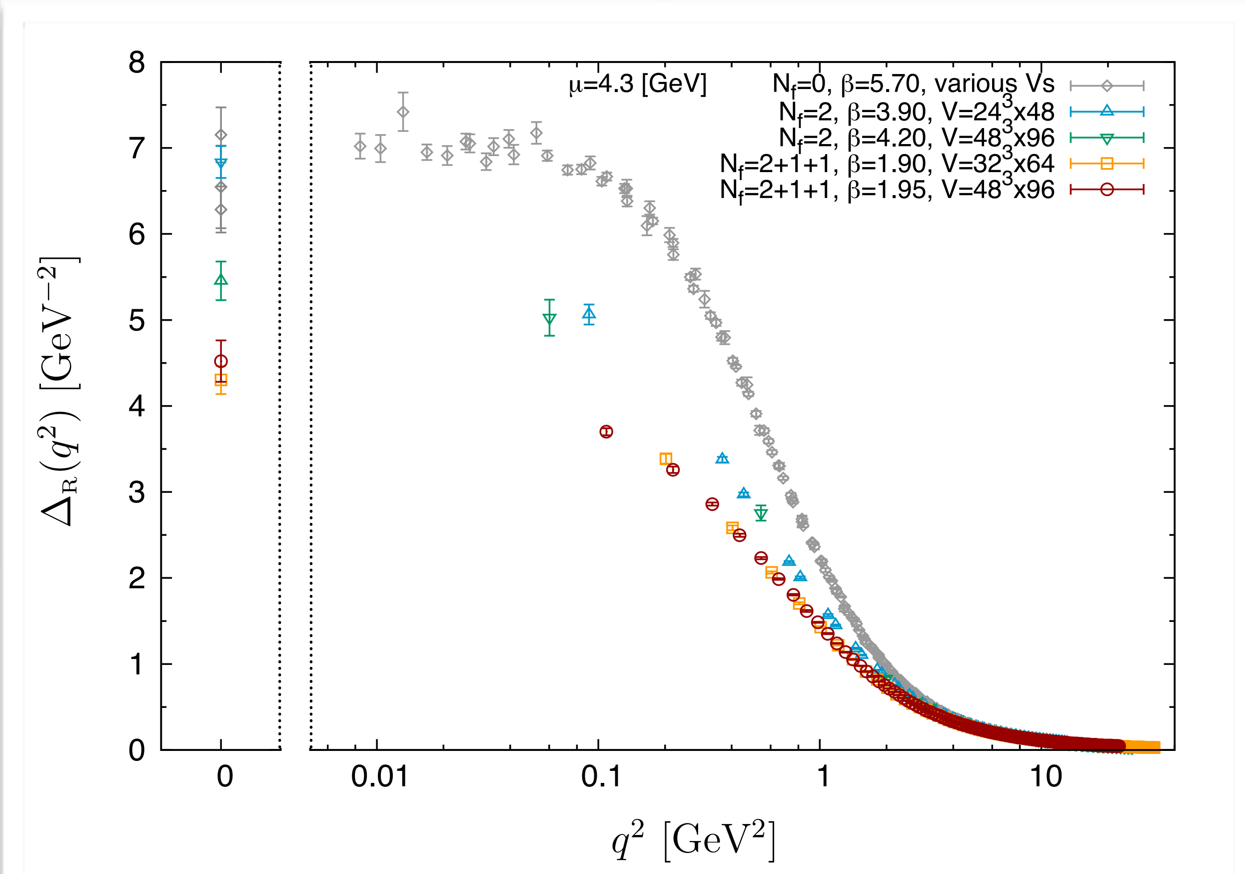


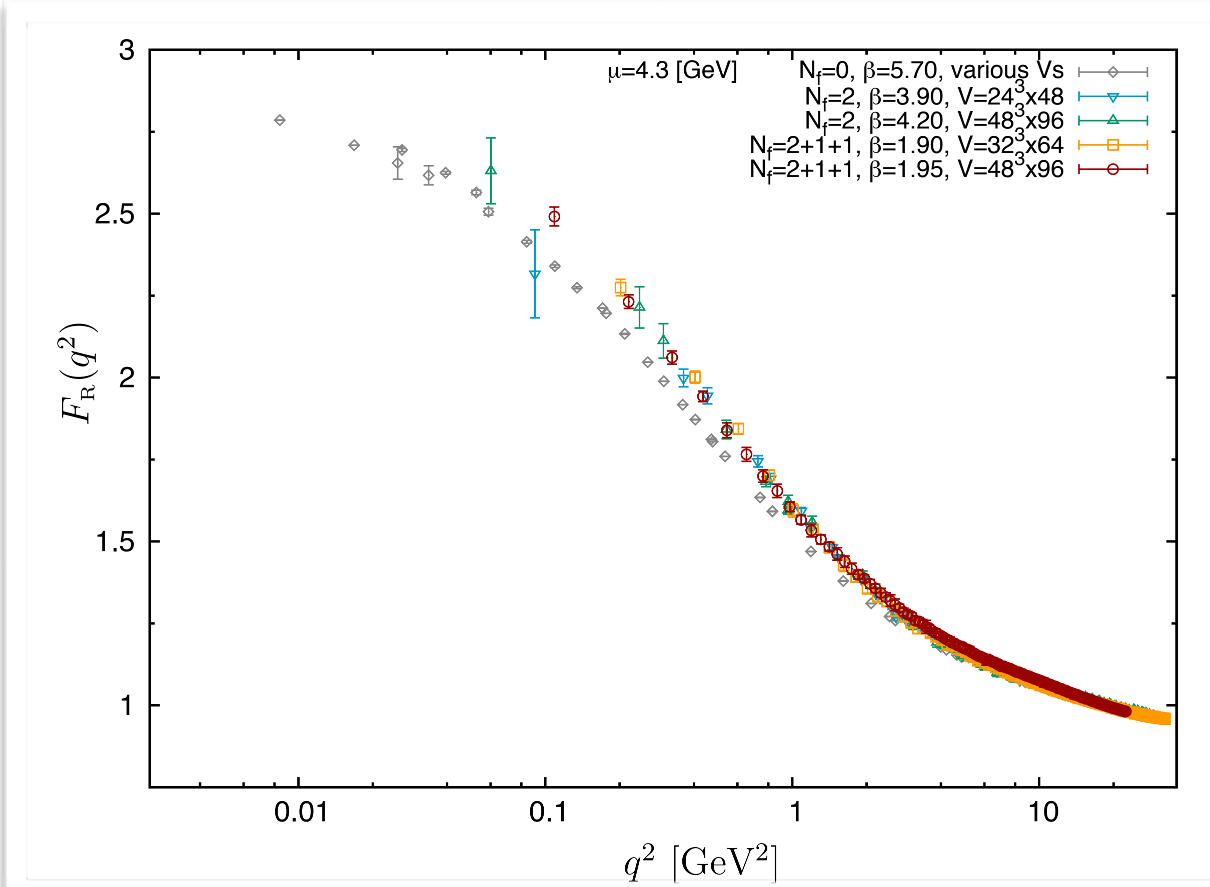
Fitted by the following function

$$F(p^2) = \frac{a_1 - a_2}{1 + (p^2/p_1^2)^\gamma} + a_2$$

gluon and ghost propagators from lattice QCD

- **Results persists in full QCD**





A. Ayala, A. Bashir, D. B., M. Cristoforetti and J. Rodriguez-Quintero, Phys. Rev. D86, 074512 (2012)

dynamical gluon
mass generation

J. S. Schwinger, Phys. Rev. 125, 397 (1962)
J. S. Schwinger, Phys. Rev. 128, 2425 (1962)

- **Dyson resum**

$$\Delta(q^2) = \frac{1}{q^2\left[1+\Pi(q^2)\right]}$$

Idea

If $\Pi(q^2)$ has a pole at $q^2=0$ the gauge boson is massive even though it is massless in the absence of interactions

- Requires **massless, longitudinally coupled** Goldstone like **poles** (~ $1/q^2$)
- **Occur dynamically** (even in the **absence** of canonical **scalar fields**) as **composite excitations** in a **strongly coupled** gauge theory

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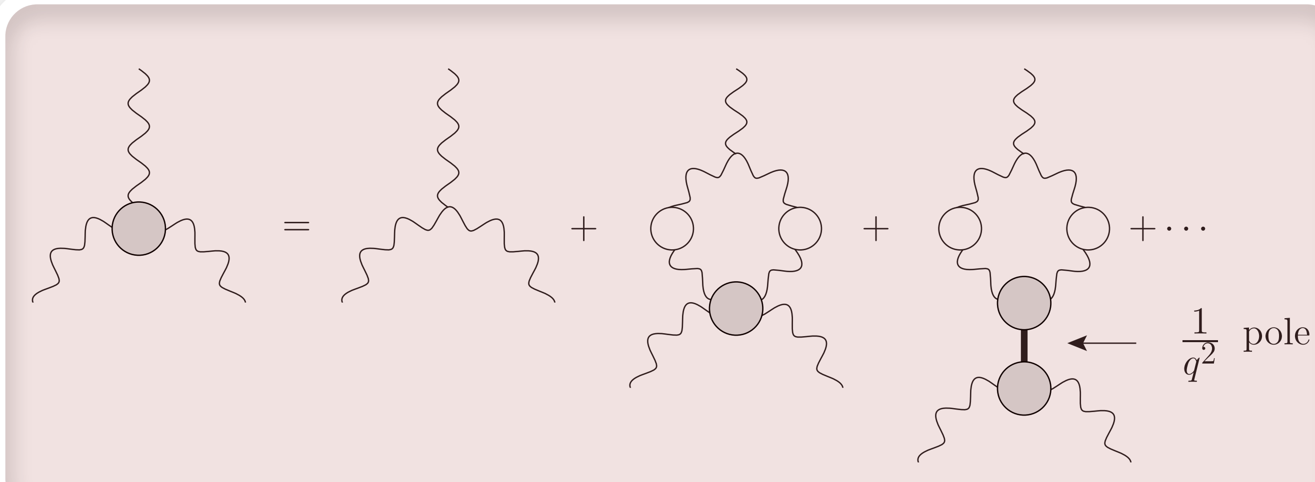
R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973)
J. M. Cornwall and R. E. Norton, Phys. Rev. D8, 3338 (1973)
E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974)

- Dynamics enters through the **three-gluon vertex**
  - **Longitudinally coupled** massless **poles**
    - **Not** a **kinematic singularity**, rather **bound states poles** non-perturbatively produced
    - **Do not appear** in the S matrix of the theory ("eaten-up" by the gluons to become massive)
  - Instrumental for ensuring that



- $\Delta^{-1}(0) > 0$

dynamical gluon mass generation and confinement

With massive gluons where does the **long range force** associated with confinement come from **?**

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

- Massive gauge invariant QCD
- Theory predicts a number of **quantum solitons**

  → **long-range** (massless) pure gauge term

  topological charge corresponding to the *SU(N)* center *Z(N)*

- **Center vortices**
  - Center vortices are **not singular, thick** ($\sim 1/m$) and possess **finite action**
  - **Entropy** (per unit size) larger than **action**

  → **Vortex condensation** $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle \neq 0$ → **Confinement**: **area law for Wilson loops in the fundamental rep.**

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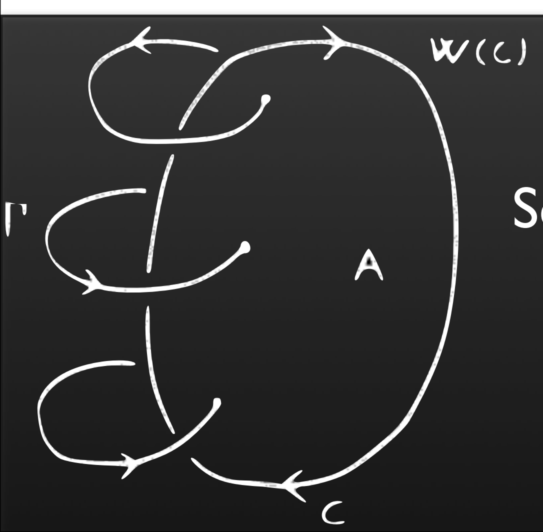


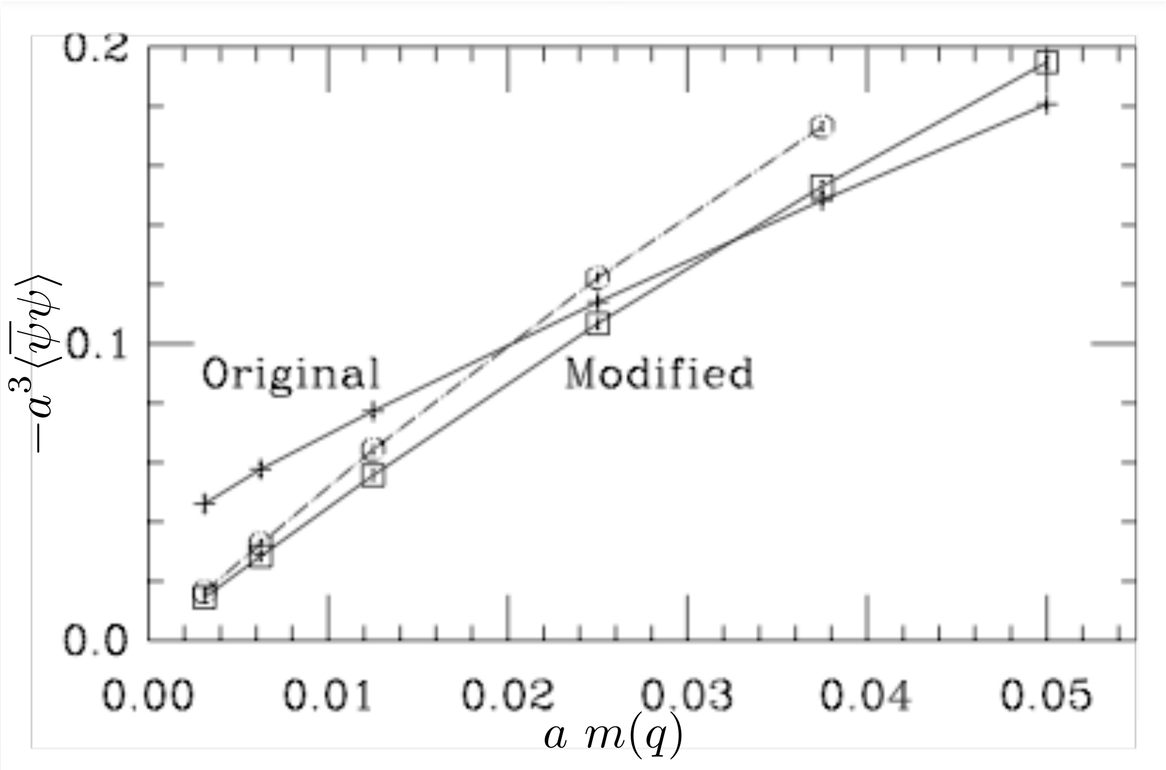
Confinement and **dynamical gluon mass** are intertwined

Schwinger's mech. (IR slavery) → **gluon mass gen** → massive QCD effective theory → center vortex solutions → **vortex condensation** ⇄ **gluon mass gen**

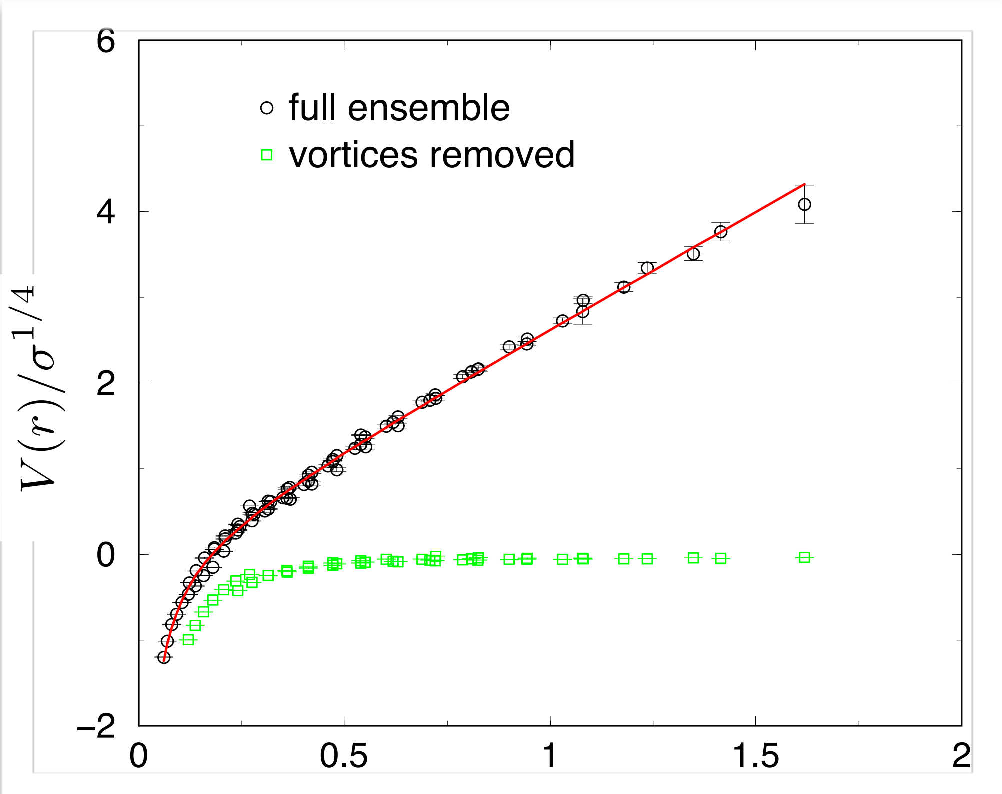
vortex condensation → **confinement**

center vortices and lattice calculations

- **χSB disappears** without vortices
  - Ph. de Forcrand and M. D'Elia, Phys. Rev. Lett. 82, 4582 (1999)

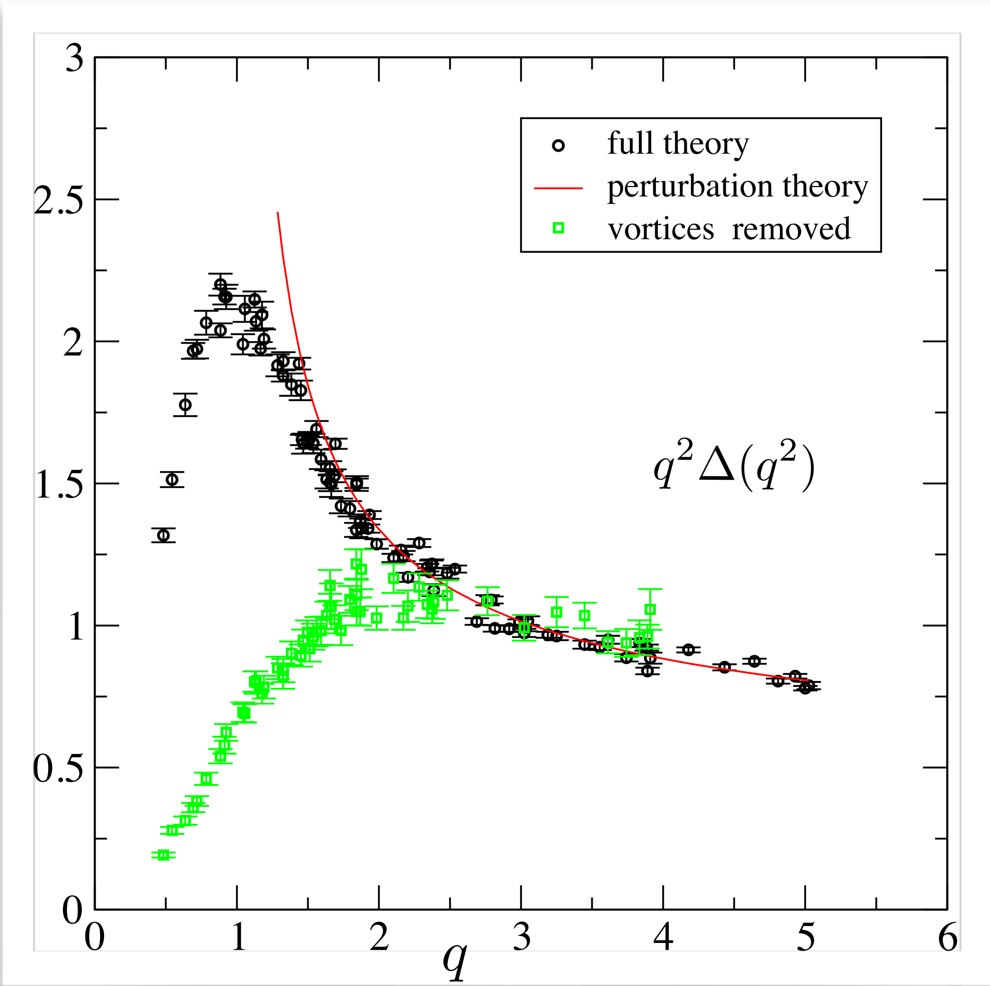


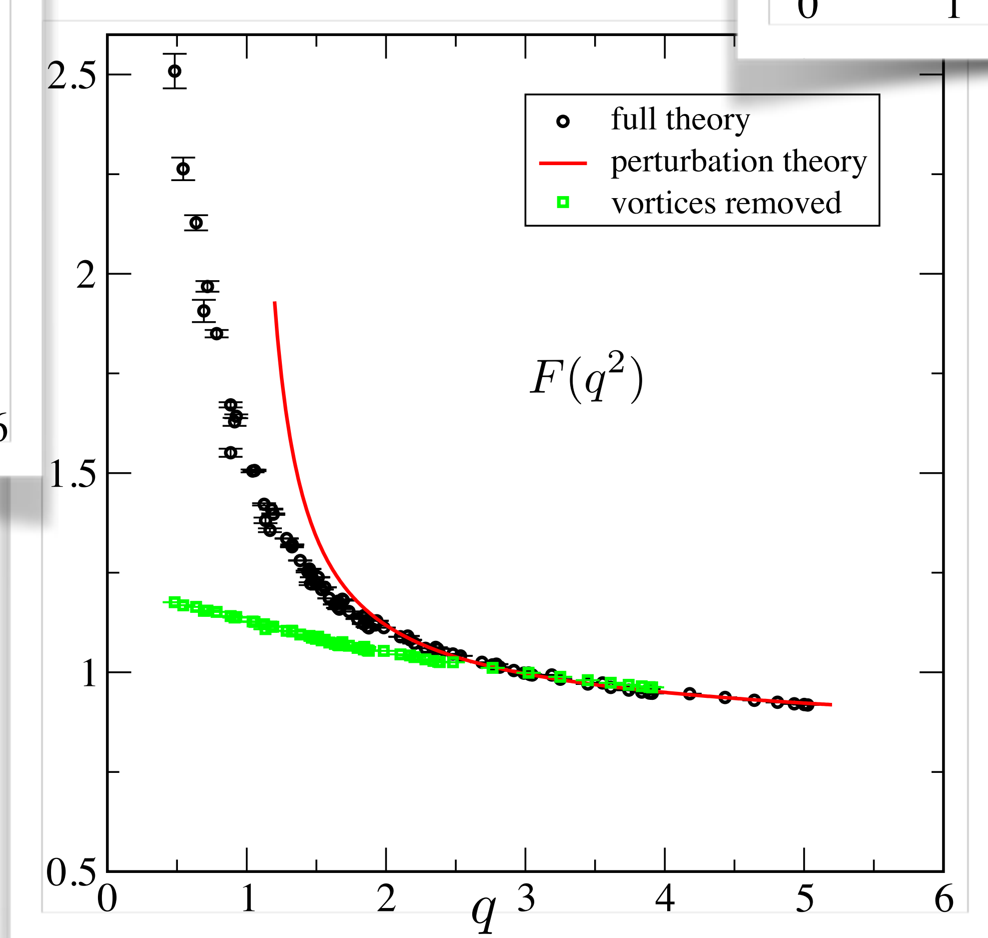
- **No linear rising** quark potential
  - J. Gattnar, K. Langfeld and H. Reinhardt, Nucl. Phys. B621,131 (2002)
    J. Greensite and S. Olejnik, Phys. Rev. D67, 094503 (2003)

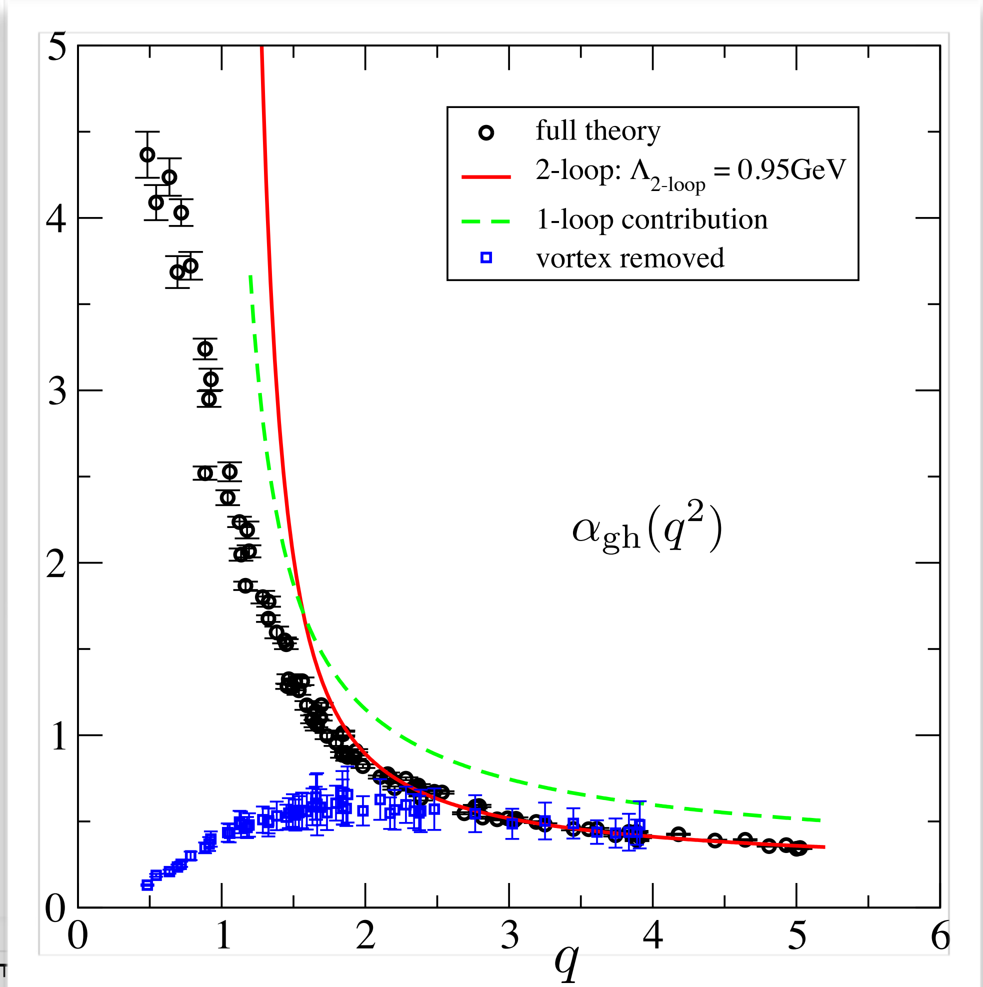


center vortices and lattice calculations

- Vortices impact the shape of Green's functions
  - J. Gattnar, K. Langfeld and H. Reinhardt, Phys. Rev. Lett. 93, 061601 (2004)







(problems with) conventional formalism

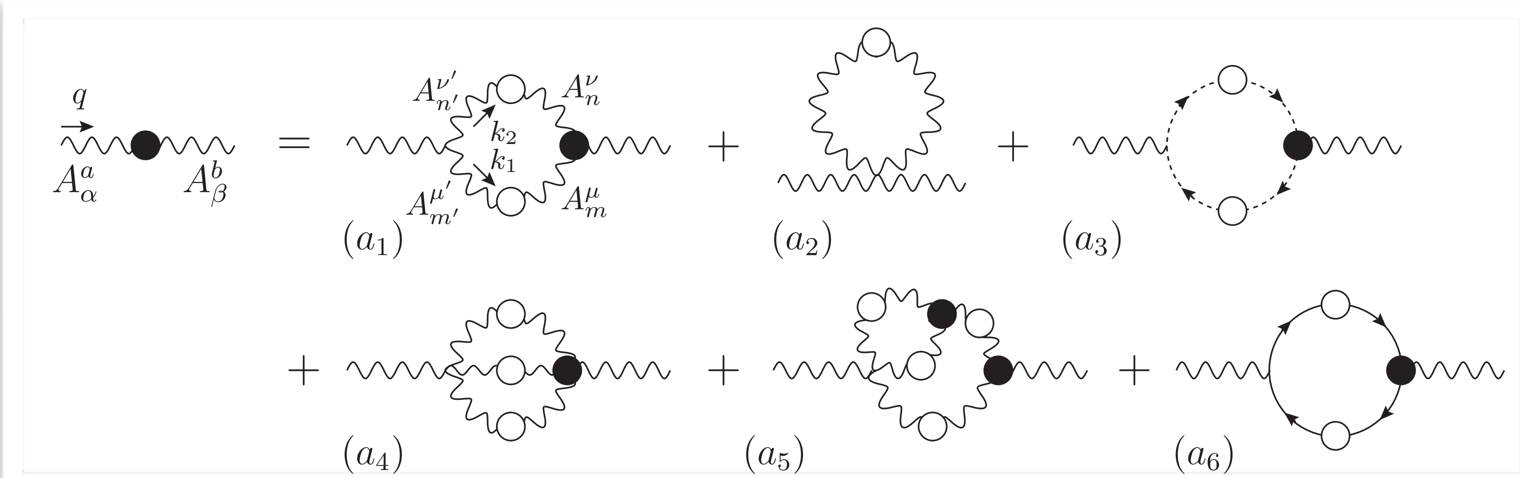
- Schwinger-Dyson eqs: way of **treating purely non-perturbative phenomena** (e.g., mass gap generation)

  - Infinite system of **coupled non-linear integral equations**
    - captures the **full quantum e.o.m.**
    - expansion about the free-field vev, but finally no reference to it

  - Require a **truncation scheme**
    - gauge and renormalization group invariance should be respected

- **Gluon propagator**

  - BRST demands $q^{\alpha} \sum_{i=1}^{5} (a_i)_{\alpha\beta} = 0$
    - very difficult diagrammatic verification
    - **cannot truncate in any obvious way**



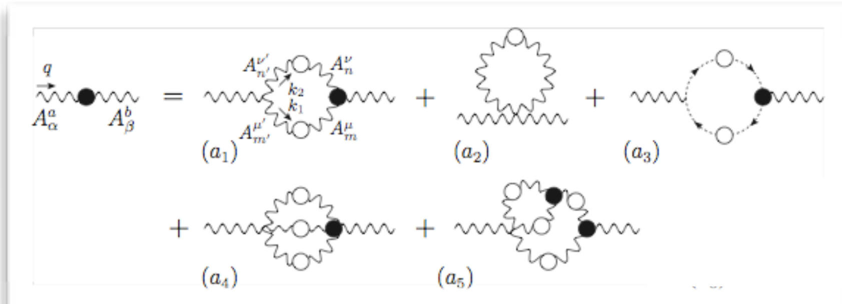
- Retaining (a_1) and (a_2) only is not correct even at one loop
  - $q^{\alpha}\Pi_{\alpha\beta}(q)|_{(a_1)+(a_2)} \neq 0$

- Adding (a_3) is not sufficient for a full analysis; beyond one loop
  - $q^{\alpha}\Pi_{\alpha\beta}(q)|_{(a_1)+(a_2)+(a_3)} \neq 0$

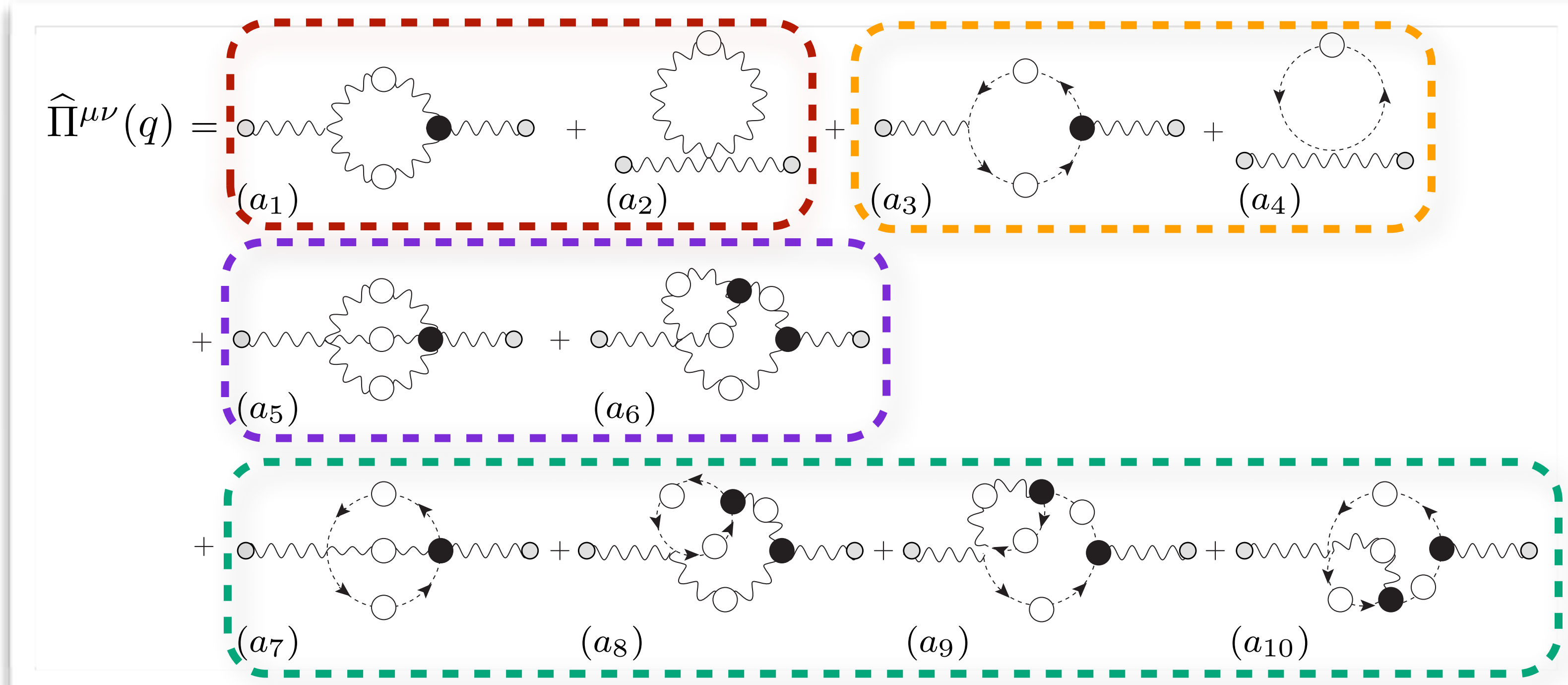
D. B. & J. Papavassiliou, Phys. Rev. D77, 061702(R) (2008)
D. B. & J. Papavassiliou, JHEP 11, 063 (2008)

PT-BFM resummed
Schwinger-Dyson series

- Apply the pinch technique to the Schwinger-Dyson equation of the gluon propagator



- graphs made out of **new vertices**, (inside **conventional props)**
- new vertices corresponds to **BFM vertices**
  - **external gluons** dynamically converted into **background gluons**



- New Schwinger-Dyson equation has a **special structure**
  - **Subgroups** (one-/two-loop dressed gluon/ghost) are **individually transverse**

Problem

Not a genuine Schwinger-Dyson equation (**mixes pinch technique** and **conventional** propagators)

- Express the **Schwinger-Dyson eq** in terms of a **BQI**
  - $$\Delta^{-1}(q^2)\left[1+G(q^2)\right]^2 P_{\alpha\beta}(q) = q^2 P_{\alpha\beta}(q) + \sum_{i=0}^{8}(d_i)_{\alpha\beta}$$
    $$\widehat{\Delta}(q^2) = \left[1+G(q^2)\right]^{-2}\Delta(q^2)$$
  - Since in $4d$ L is subleading the function G is directly related to the inverse of the ghost dressing function

- $F^{-1}(q^2) \approx 1 + G(q^2)$

D. B. & J. Papavassiliou, Phys. Rev. D77, 061702(R) (2008)
D. B. & J. Papavassiliou, JHEP 11, 063 (2008)

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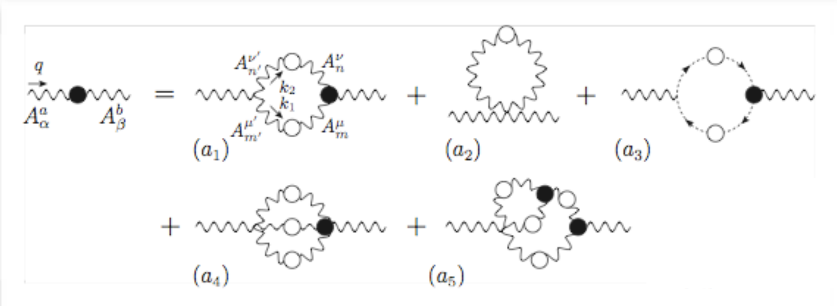

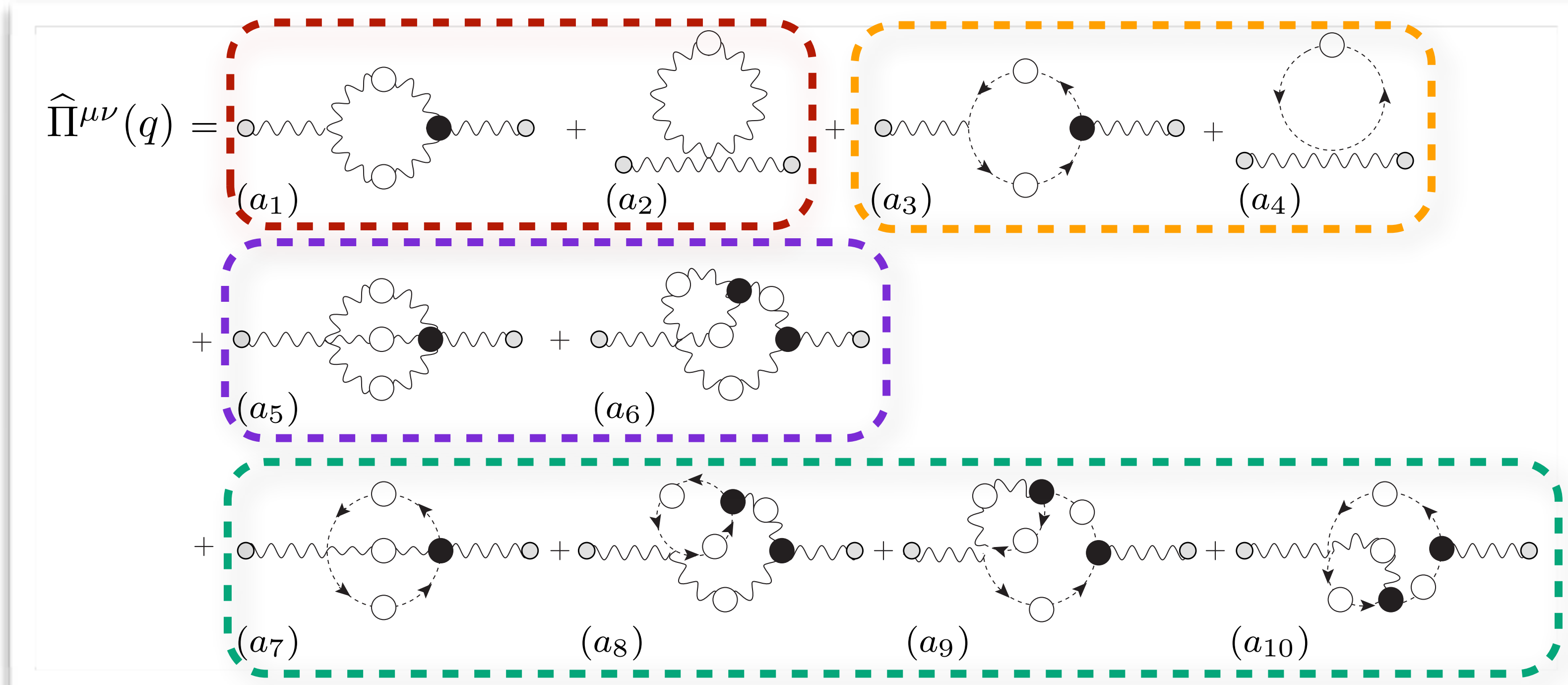

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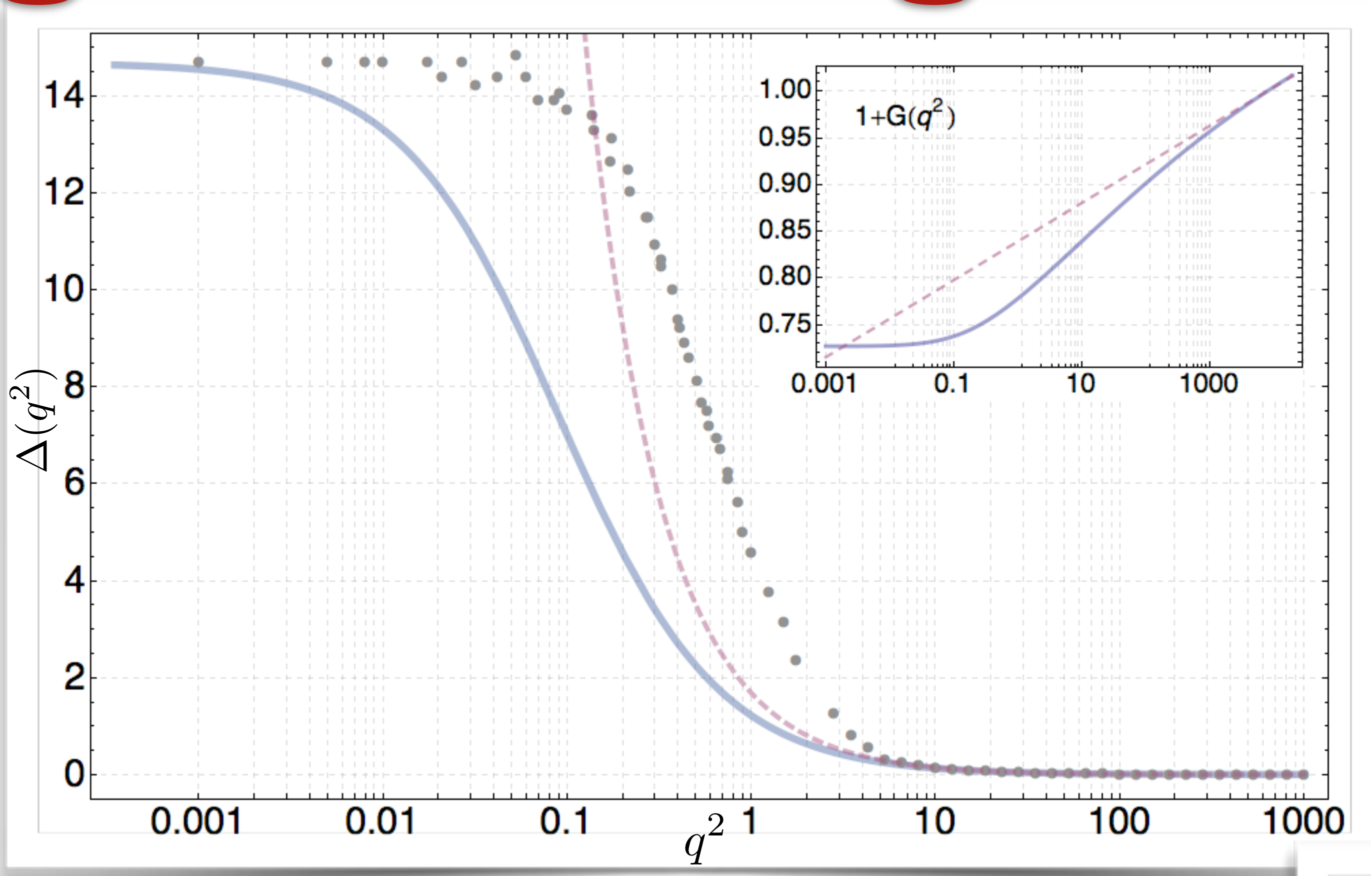
Very flexible framework

- Can **add/remove group** of diagrams gauge **invariantly**
- **Gauge invariance preserved at each step** (transverse projectors can be traced out)
- **Different options for truncation** (always retaining exact gauge invariance)
- Modest price to pay: **additional equation** for $G(q^2)$ (but this function is very worth the effort)
- **PT-BFM** and **conventional vertices** can be **treated independently**
- **Allow the gauge invariant study of ghost contributions**

results I (d=4)

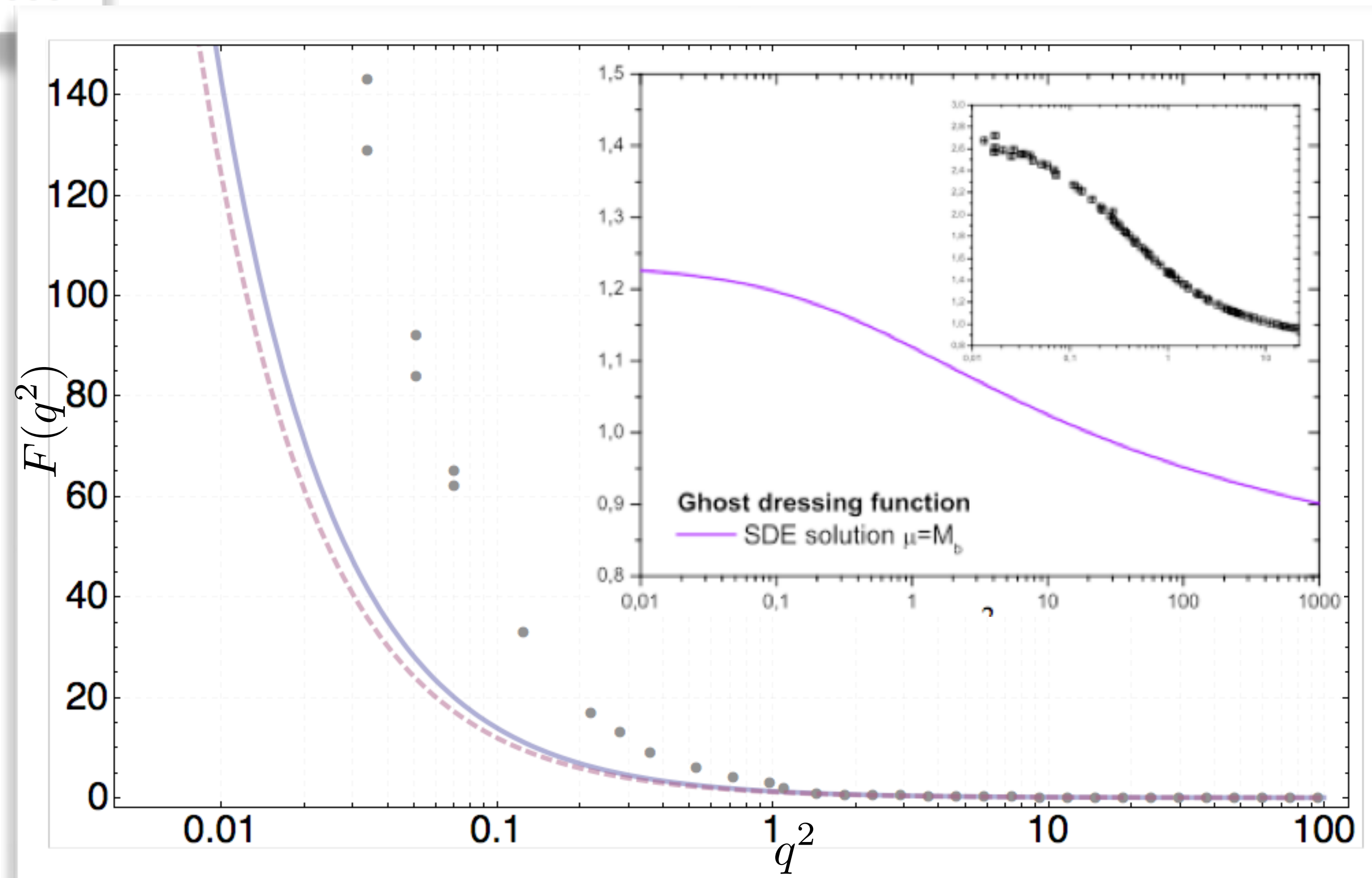
gluon and ghost propagators



A. C. Aguilar, D. B. and J. Papavassiliou, Phys. Rev. D78, 02510 (2008)
SU(3) lattice data: I. L. Bogolubsky et al. Phys. Lett. B676, 69 (2009)

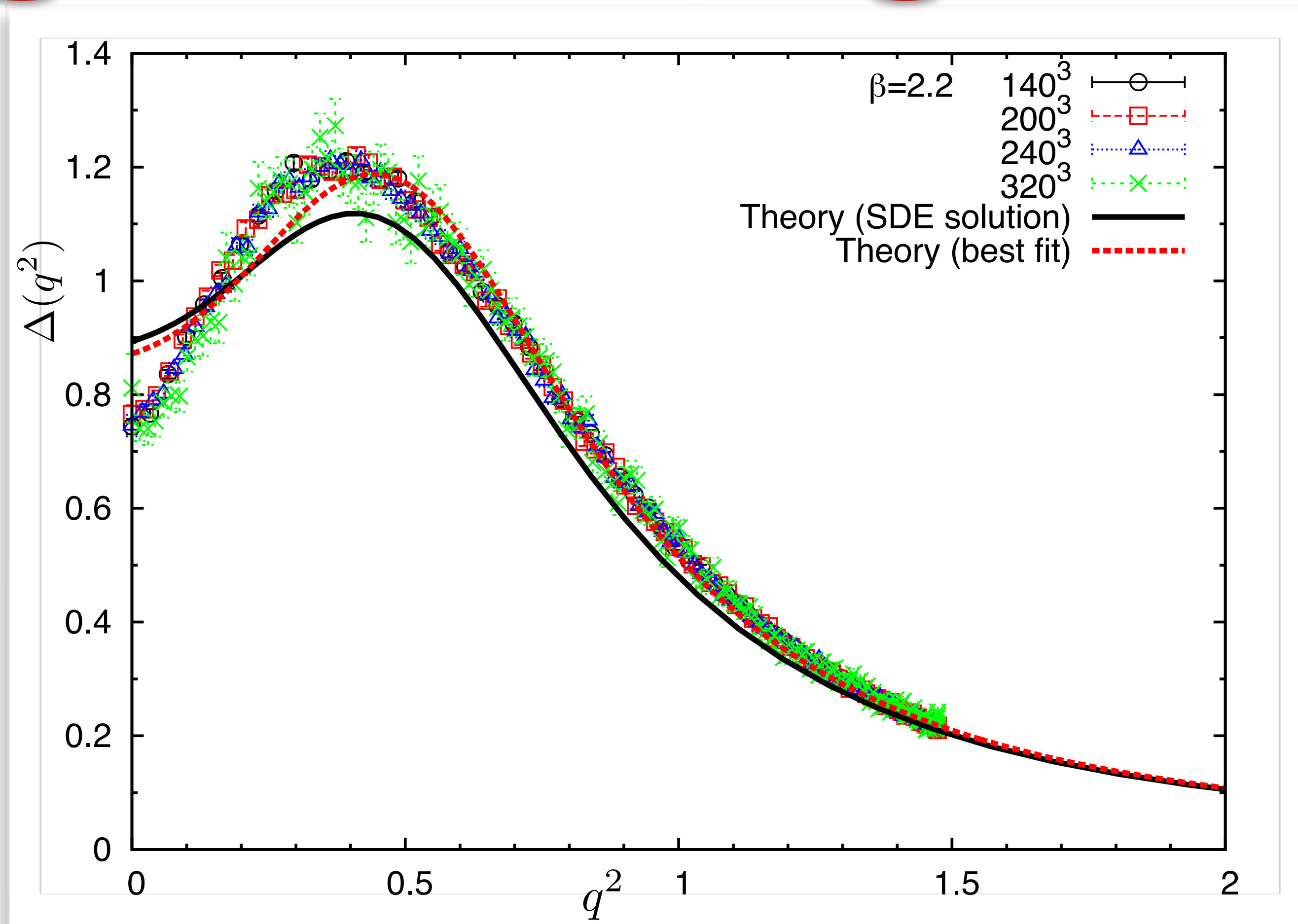
- **Solution of the 4d system**
  - Three unknowns: G, Δ and D (or F)
  - IR condition

$$\lim_{q^2\to 0} \Delta^{-1}(q^2) = \Delta^{-1}_{\text{lat}}(0)$$



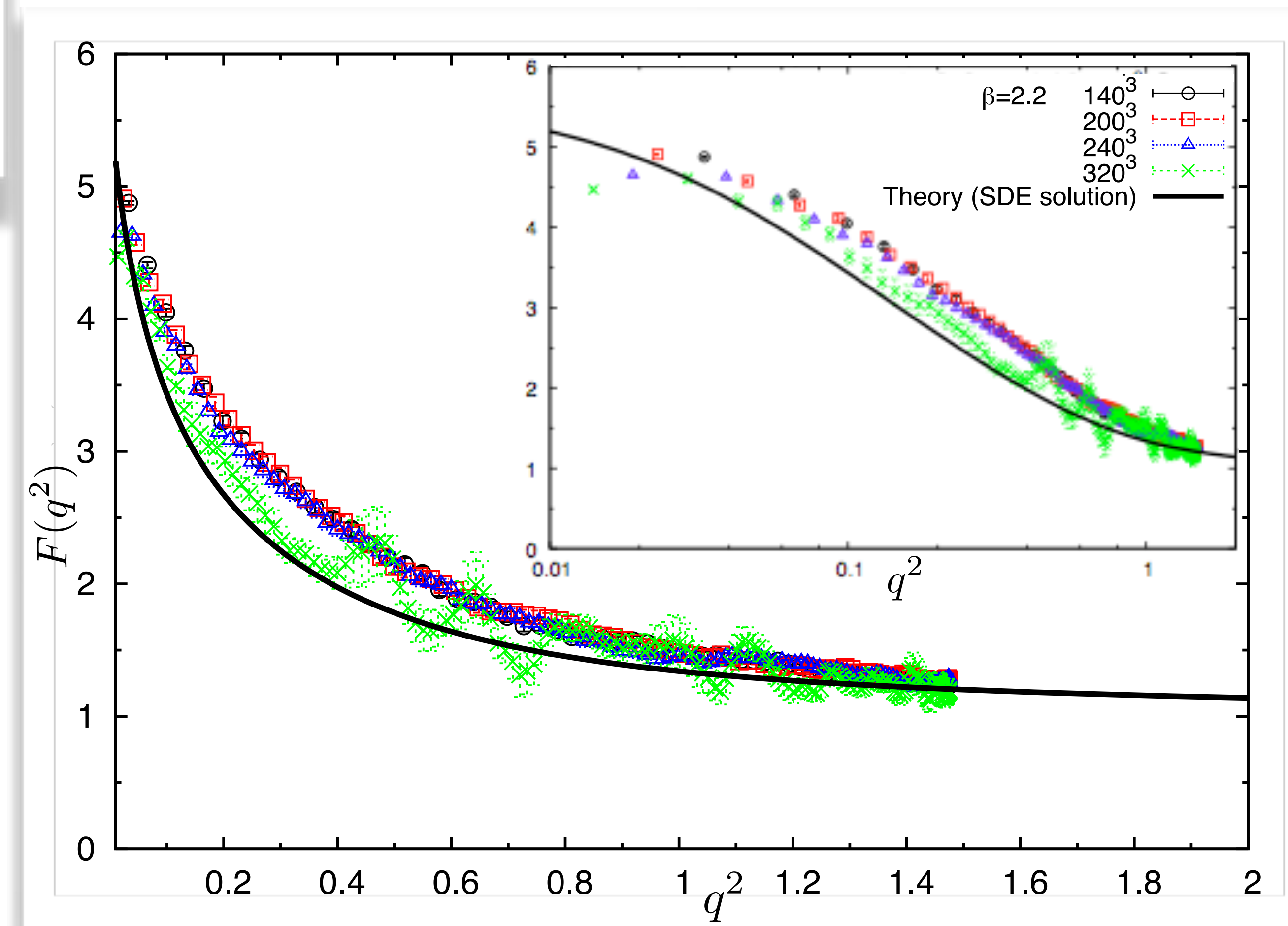
results II (d=3)

gluon and ghost propagators



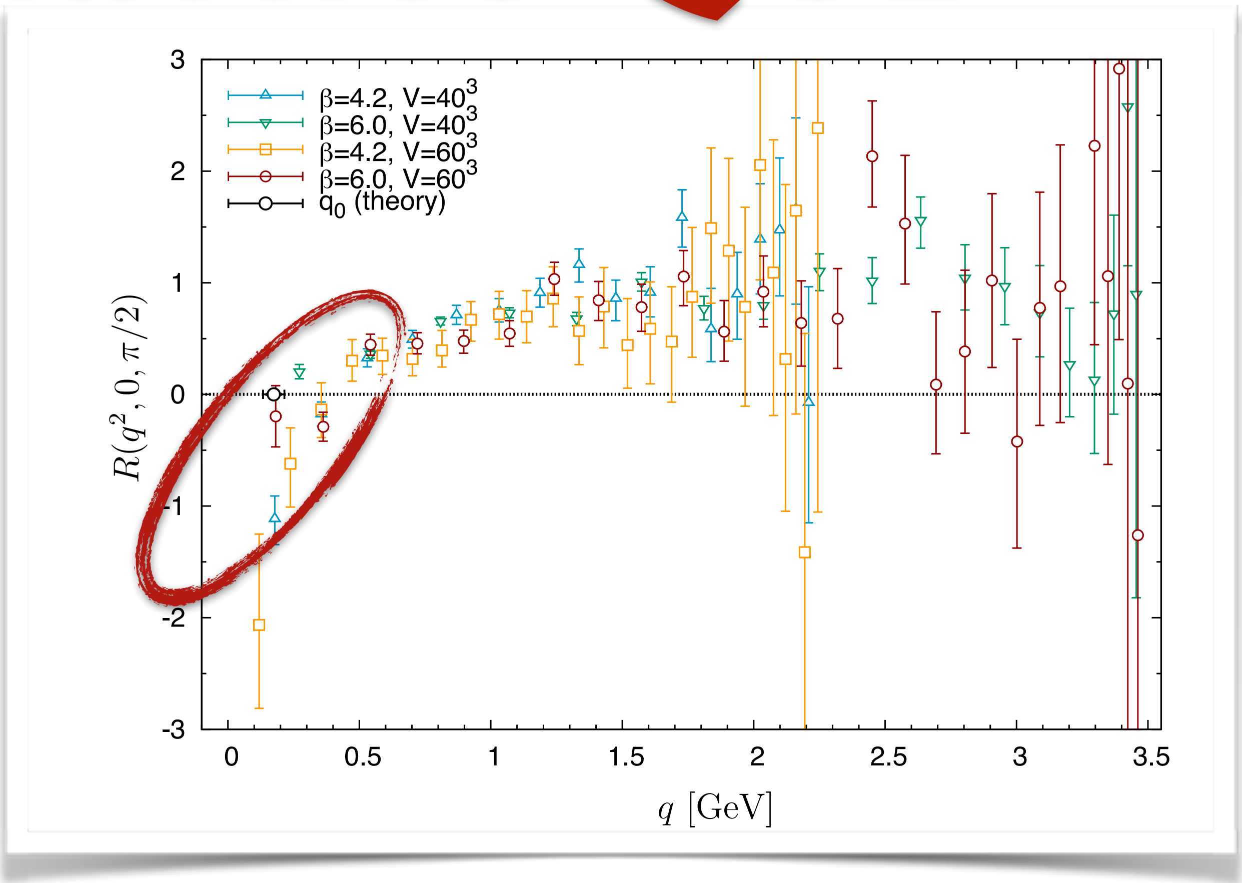
A. C. Aguilar, D. B. and J. Papavassiliou, Phys. Rev. D81 125025 (2010)
SU(2) lattice data: A. Cucchieri and T. Mendes, PoS QCD-TNT09, 026 (2009)

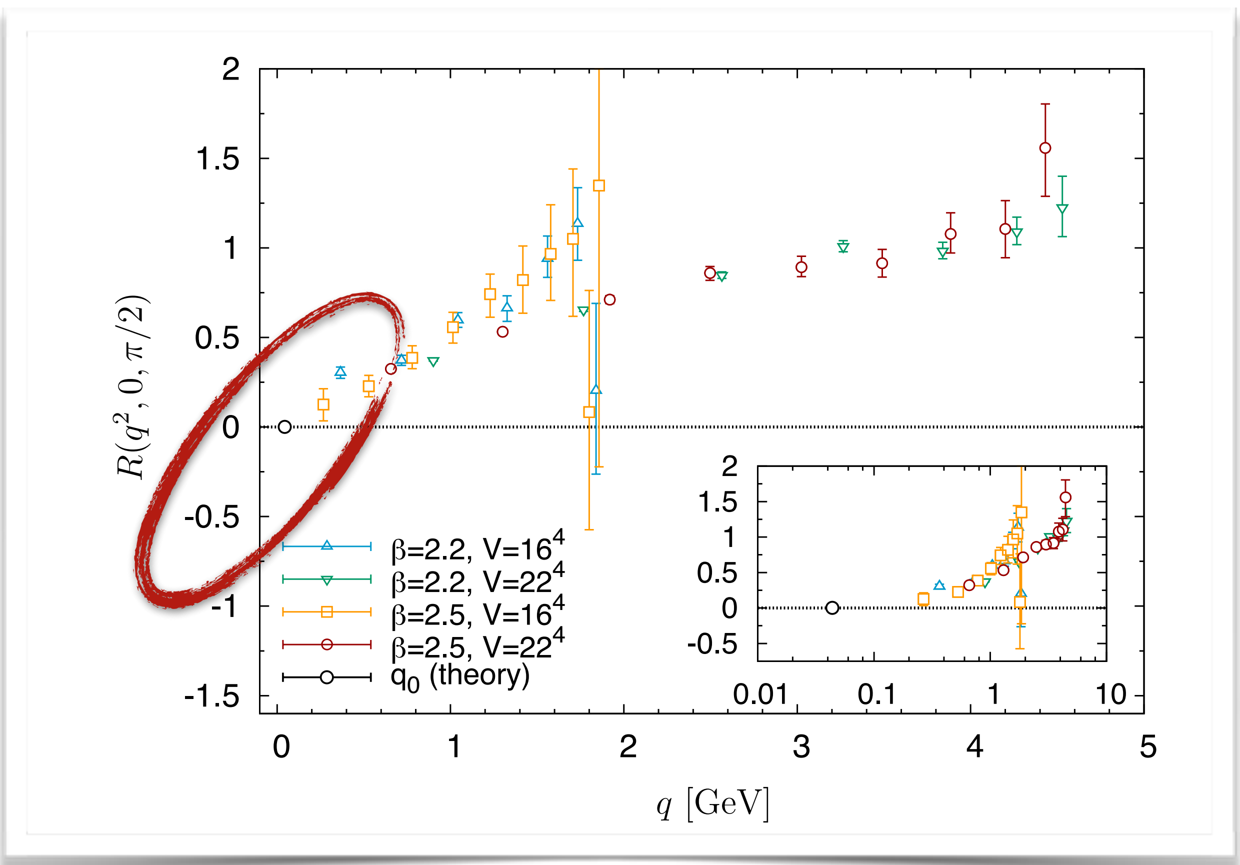
- **Solution of the 3d system**
  - One-loop approximations for Δ with a tree-level (hard mass)
  - Full ghost equation for D (or F)



three gluon vertex in lattice QCD

SU(2) lattice data: A. Cucchieri, A. Maas and T. Mendes, Phys.Rev. D77 (2008) 094510





- Lattice results for the R projector

$$R(q,r,p) = \frac{\Gamma^{(0)}_{\alpha\mu\nu}(q,r,p)P^{\alpha\rho}(q)P^{\mu\sigma}(r)P^{\nu\tau}(p)\Gamma_{\rho\sigma\tau}(q,r,p)}{\Gamma^{(0)}_{\alpha\mu\nu}(q,r,p)P^{\alpha\rho}(q)P^{\mu\sigma}(r)P^{\nu\tau}(p)\Gamma^{(0)}_{\rho\sigma\tau}(q,r,p)}$$

- Somewhat surprising results
  - Negative IR divergence in the deep IR (evidence in d=3, indication in d=4)

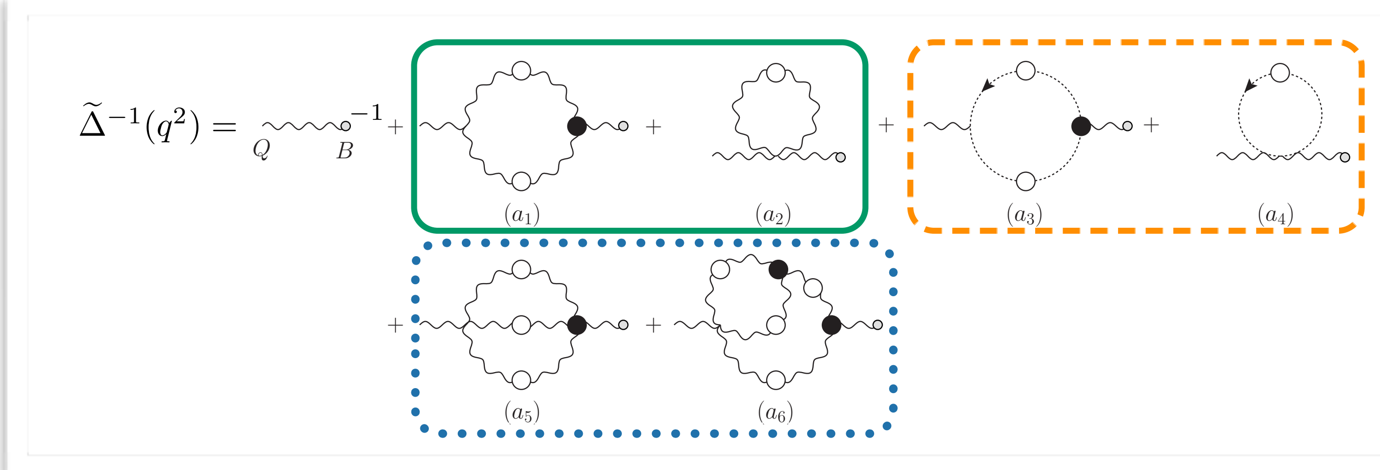
- How can we understand this in terms of IR finite propagators/dressing functions?
  - $\Delta(q^2) = q^2 J(q^2) + m^2(q^2)$
  - $R(q^2) \sim F(0) J(q^2)$

Not arbitrary: $m^2(q^2)$ completely determined by an exact integral equation

- The divergence must be contained in J

A toy model

a one-loop
toy model



- Perturbative one-loop setting with a **massive gluon** and a **massless ghost**

  - Gluon contribution to the inverse dressing

$$J_{a_1}(q^2) \sim \begin{cases} \ln\left[(q^2+m^2)/\mu^2\right], & d=4; \\ (1/q)\arctan(q/2m), & d=3, \end{cases}$$

  - Ghost contribution to the inverse dressing

$$J_{a_3}(q^2) \sim \begin{cases} \ln\left(q^2/\mu^2\right), & d=4; \\ 1/q, & d=3. \end{cases}$$

- Then, the gluon propagator becomes

$$\begin{aligned} \Delta^{-1}(q^2) &= q^2 J(q^2) + m^2 \\ &= q^2[1 + c_1 J_{a_1}(q^2) + c_3 J_{a_3}(q^2)] + m^2 \end{aligned}$$

The coefficients are determined at the one-loop level

  - **Four** dimensional case

$$c_1 = 2\left(\frac{\alpha C_A}{4\pi}\right); \quad c_3 = \frac{1}{6}\left(\frac{\alpha C_A}{4\pi}\right)$$

$c_i > 0$ and $c_1 \gg c_3$

  - **Three** dimensional case

$$c_1 = -\left(\frac{25 g^2 C_A}{32\pi}\right); \quad c_3 = -\left(\frac{g^2 C_A}{32}\right)$$

$c_i < 0$ and $c_1 \gg c_3$

toy model
features

- The gluon propagator displays a maximum for any d

The massless log is a sufficient condition for a maximum

$$[\Delta^{-1}(q^2)]' = c_3 \ln\left(q^2/\mu^2\right) + \left\{1 + c_1 \ln\left[(q^2+m^2)/\mu^2\right] + \frac{c_1 q^2}{q^2+m^2} + c_3\right\}$$

$$[\Delta^{-1}(q^2)]'' = \frac{c_1}{q^2+m^2} + \frac{c_1 m^2}{(q^2+m^2)^2} + \frac{c_3}{q^2} > 0$$

$$[\Delta^{-1}(q^2)]' = 1 + \frac{c_1}{2q}\arctan(q/2m) + \frac{c_3}{2q} + \frac{c_1 m}{q^2+4m^2}$$

$$q_\Delta/m = -\frac{c_3/m}{2+c_1/m} \qquad \frac{m}{2g^2} \gtrsim 0.14$$

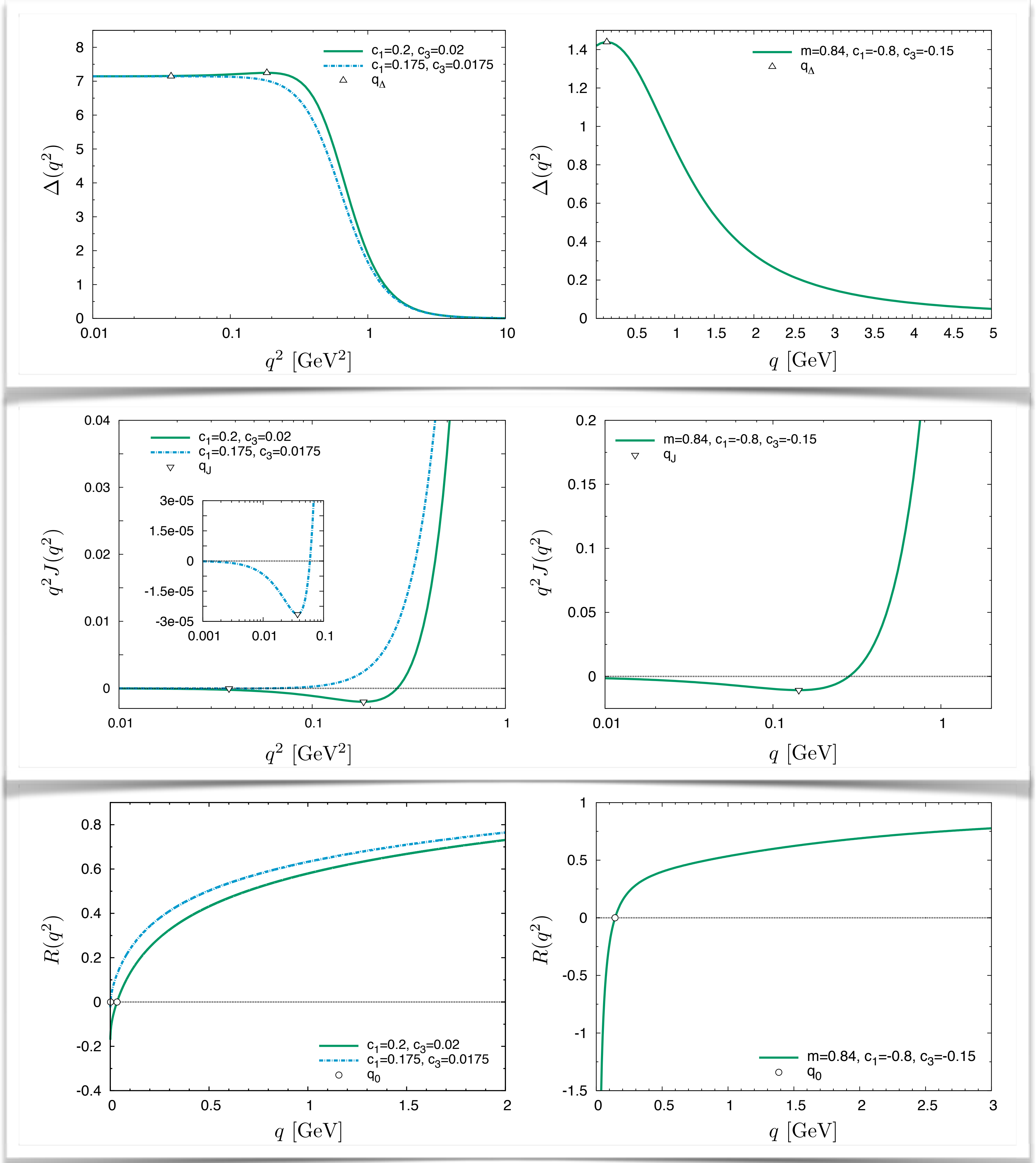
- The combination $q^2 J(q^2)$ displays a minimum (located at the same position)
- The R projector displays a (negative) divergence

  - $R(q^2) \sim [q^2 J(q^2)]'$

$$R(q^2) \underset{q^2\to 0}{\sim} c_3 J_{a_3}(q^2) \sim \begin{cases} \ln\left(q^2/\mu^2\right), & d=4; \\ -1/q, & d=3. \end{cases}$$

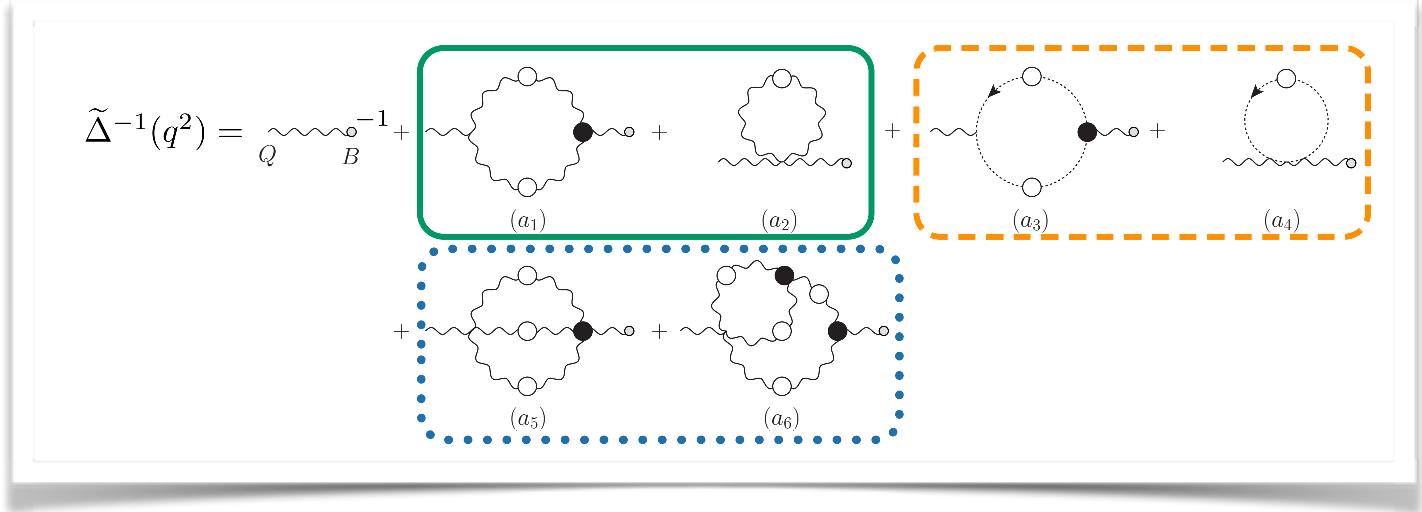
  - Zero crossing happens at the same position of the minimum of $q^2 J(q^2)$

toy model
numerics



Full np analysis

non perturbative calculations-I



- Need to evaluate the full ghost contributions to $q^2 J(q^2)$

  - $q^2 J_c(q^2) = C_d\, F(q^2)\left[4T(q^2) + q^2 S(q^2)\right]$

$$T(q^2) = \int_k \frac{F(k+q) - F(k)}{(k+q)^2 - k^2} + \left(\frac{d}{2} - 1\right)\int_k \frac{F(k)}{k^2}$$

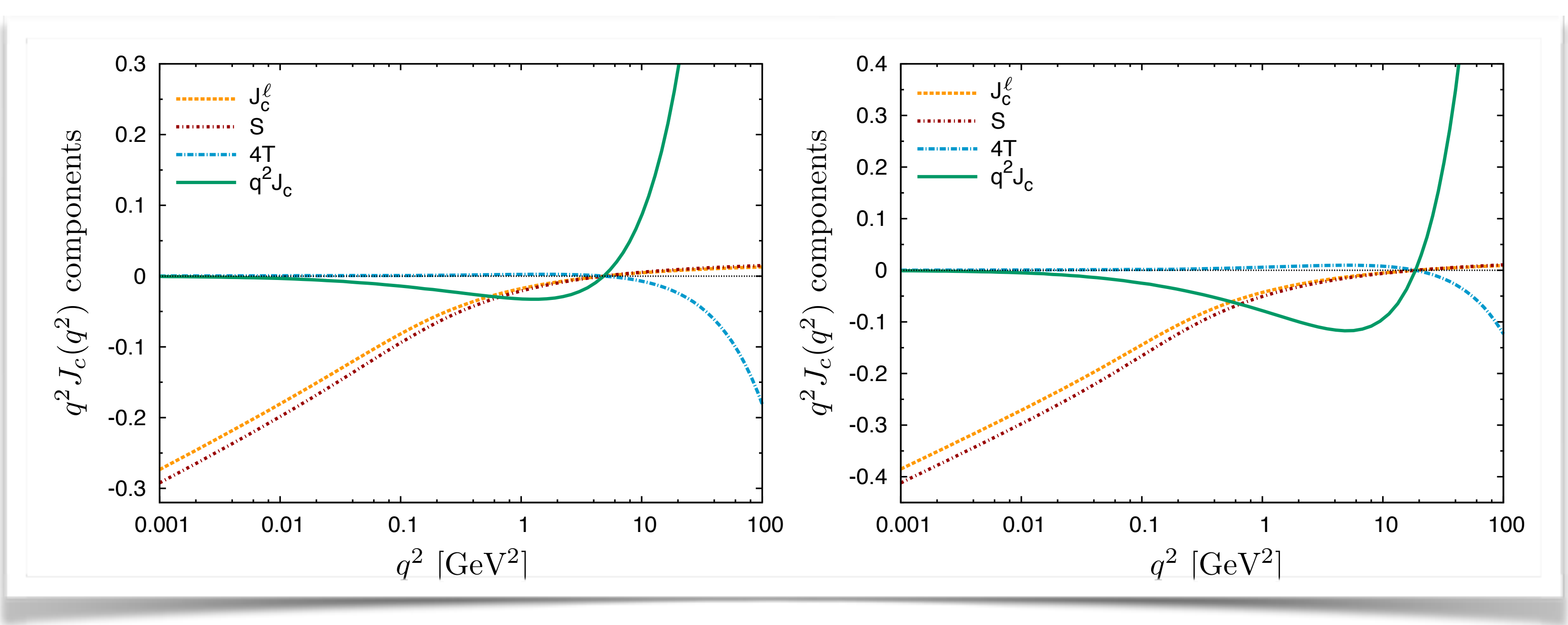
$$S(q^2) = \int_k \frac{F(k)}{k^2(k+q)^2} - \int_k \frac{F(k+q) - F(k)}{k^2[(k+q)^2 - k^2]}$$

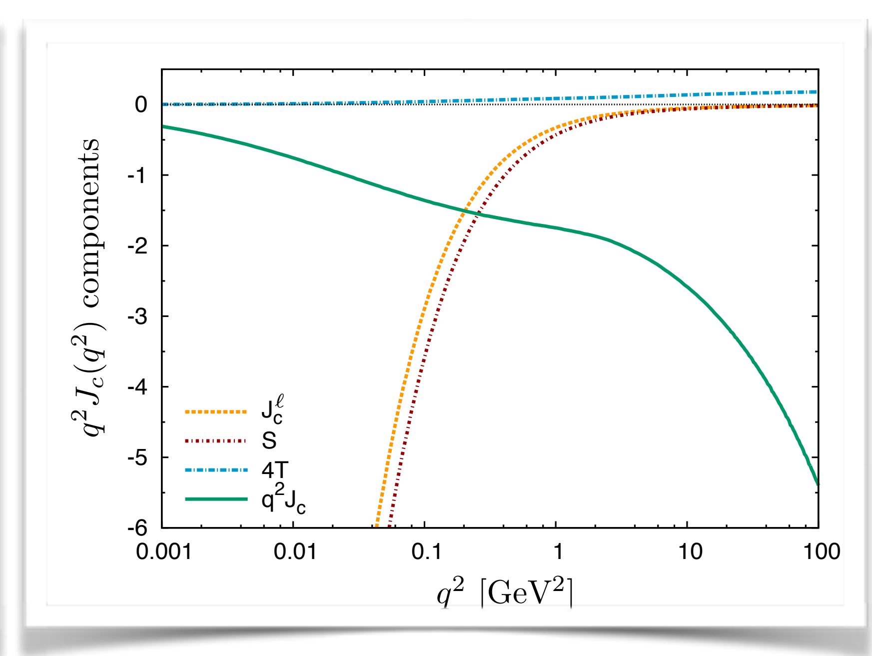
The ghost-gluon vertex is obtained by solving the WI neglecting the transverse part

- The IR behavior is similar to the one of the toy model

$$J_c(q^2) = J_c^{\ell}(q^2) + J_c^{s\ell}(q^2) \qquad J_c^{\ell}(q^2) \sim F(q^2)\int_k \frac{F(k)}{k^2(k+q)^2}$$

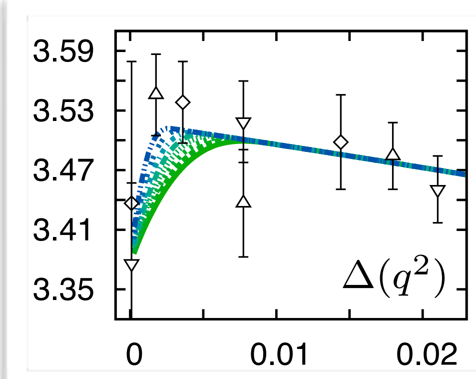
The IR leading contribution diverges in the IR

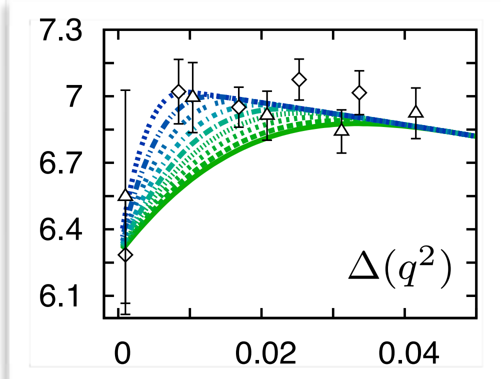


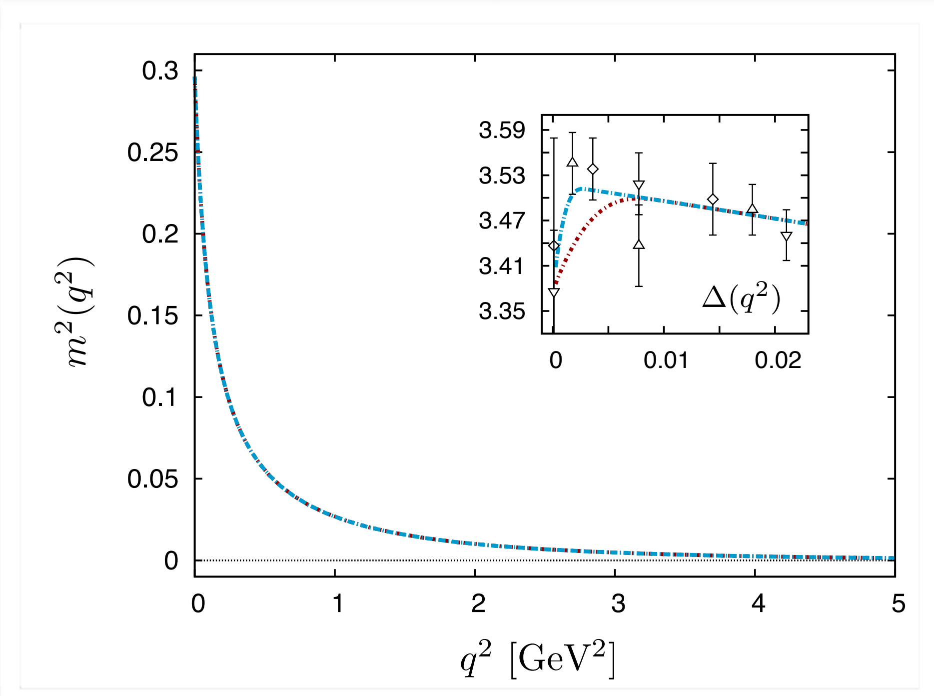


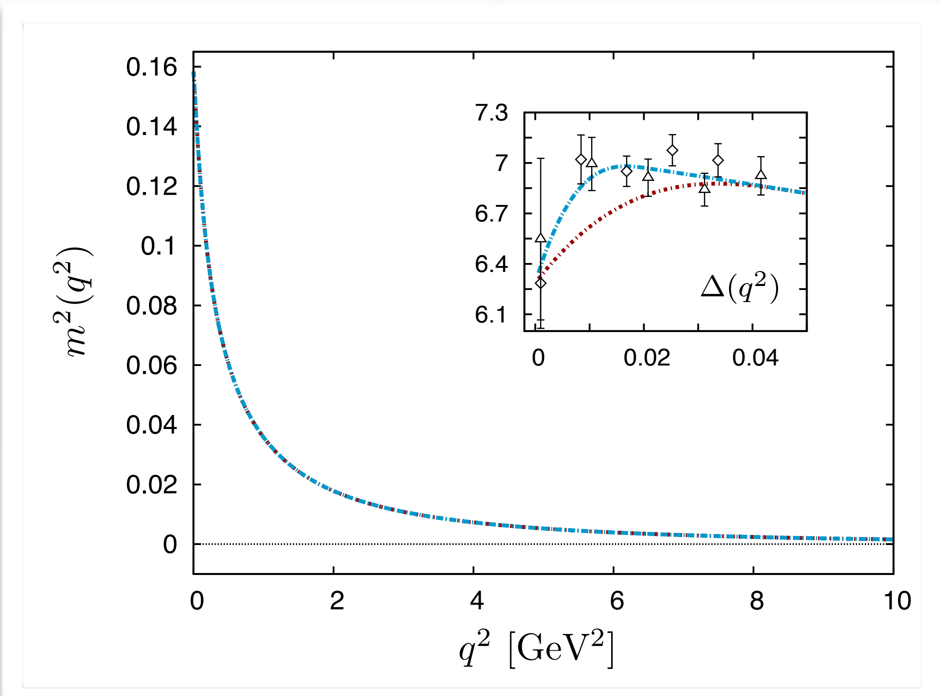
non perturbative calculations-II

- The gluon propagator *must* show a maximum
- We are however unable to directly determine the gluonic contributions to $q^2J(q^2)$
  - Two-loop gluon contribution are practically unknown
- Use an indirect method: $q^2J(q^2) = \underbrace{\Delta^{-1}(q^2)}_{\text{lattice}} - \underbrace{m^2(q^2)}_{\text{mass eq.}}$
- The mass in insensitive to the presence of the IR maximum in the propagator



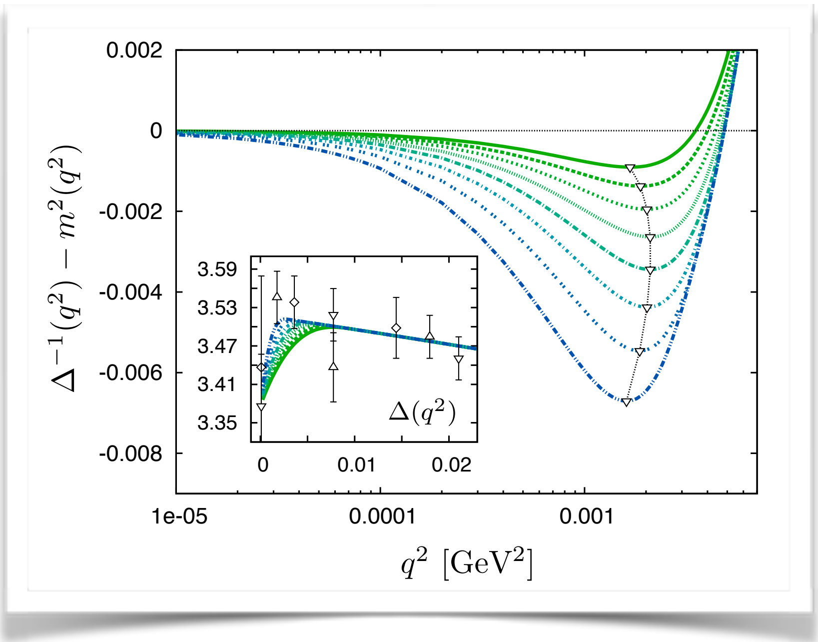


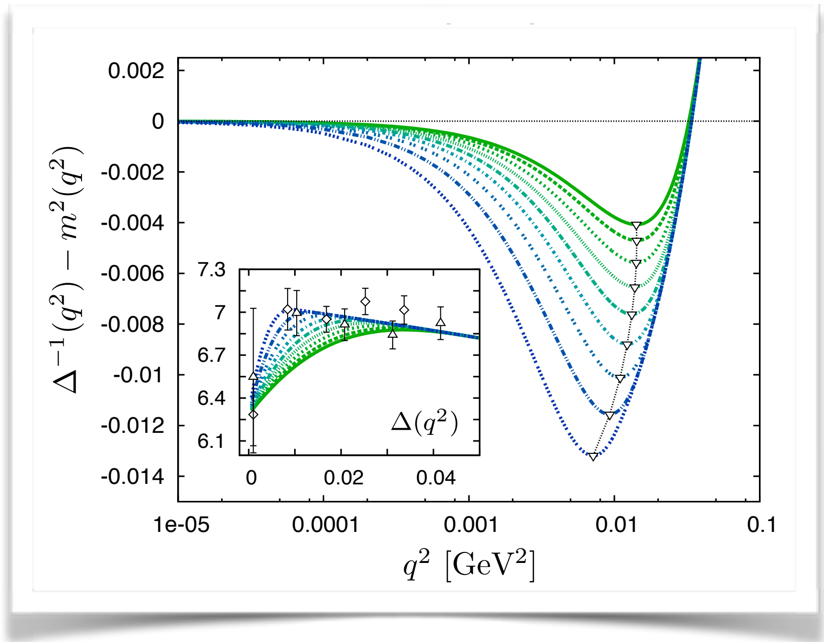


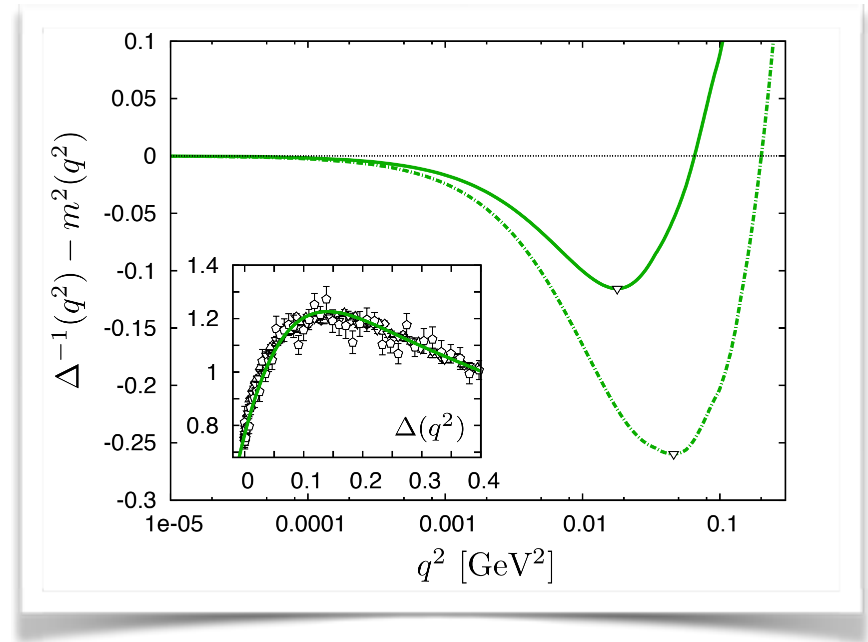


non perturbative calculations-III

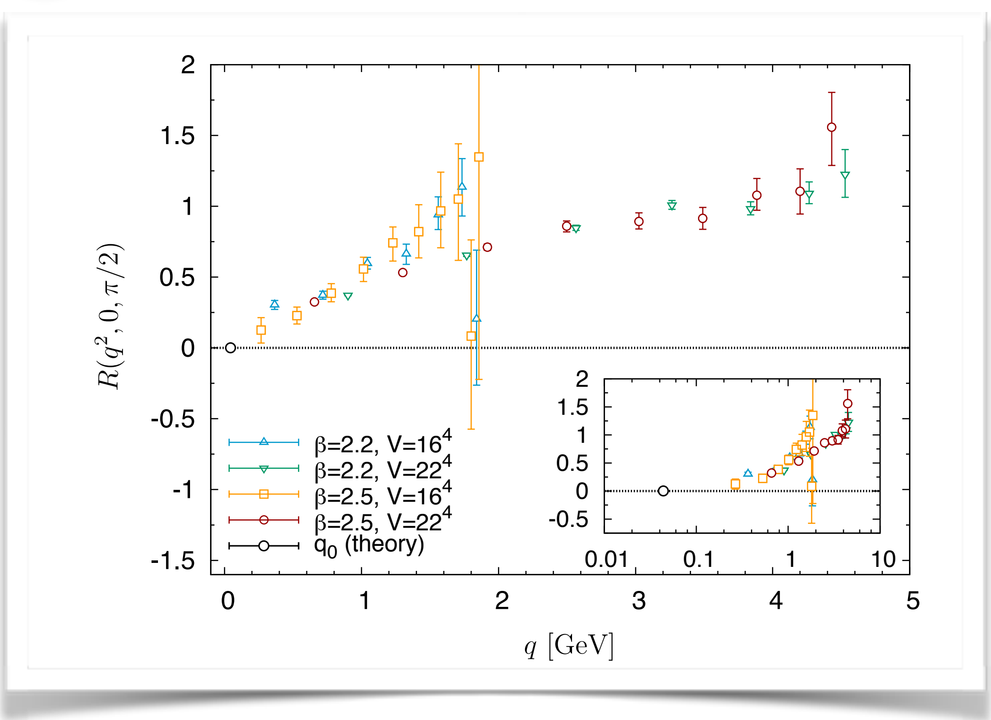
- The full inverse dressing function displays a minimum

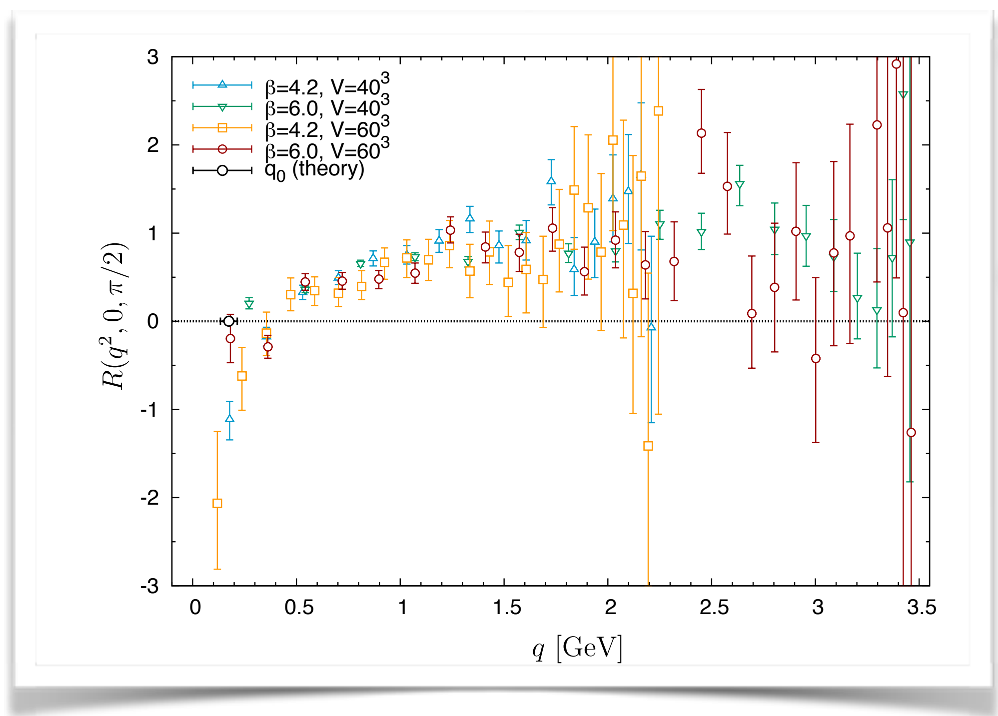






- The **position** of the minimum determines the **zero crossing** of the R projector





- d=3
  - SU(2): $q_0 \sim 200$ MeV
- d=4
  - SU(2): $q_0 \sim 44$ MeV, $L \sim 130$ (22)
  - SU(3): $q_0 \sim 132$ MeV, $L \sim 60$ (0)

Epimithion

conclusions & outlook

- Massless ghosts imply a negative IR divergence in the kinetic part of the gluon propagator
  - Maximum in the gluon propagator
  - Divergence in the three gluon vertex
- Estimates of where the divergence of the vertex happens can be given without computing the full vertex
- We expect this to happen for all Green's functions containing a close ghost loop