Nucleon Σ Term in the Chiral Mixing Approach

V. Dmitrašinović
(Institute of Physics, Belgrade, Serbia)
collaboration with Hua-Xing Chen and Atsushi Hosaka

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MOTIVATION

- The nucleon Σ term, together with the flavor-singlet nucleon axial coupling constant, i.e., the "nucleon spin problem", has long been viewed as a measure of the "strangeness content", or $(s\bar{s})$ in the nucleon $y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{u}u|N\rangle}$. [T. P. Cheng, Phys. Rev. D13, 2161 (1976)]
- A number of experiments (SAMPLE, HAPPEX, G0, etc.) have measured the strangeness content in nucleon observables (other than the Σ term), but found no significant signal. [A. Acha et al. (HAPPEX Collaboration), Phys. Rev. Lett. 98, 032301 (2007). D. Androic et al. (G0 Collaboration), Phys. Rev. Lett. 104, 012001 (2010)]

FACTS

- The pion-nucleon Σ term

$$\Sigma_{\pi N} = \frac{1}{3}\delta^{ab}\langle N|[Q_5^a, [Q_5^b, H_{\chi\mathrm{SB}}]]|N\rangle$$

  can be extracted from measured πN elastic scattering partial wave amplitudes; most extracted values lie in the range 55 -75 MeV.
- Roughly, $\Sigma_{\pi N}$ ought to equal the number of quarks and antiquarks in the nucleon ("three") times the isospin-averaged current quark mass $\Sigma_{\pi N} \simeq \frac{3}{2}\left(m_u^0 + m_d^0\right) \simeq 26$ MeV, (values from around a.d. 1976).
- Any deviation of $\Sigma_{\pi N}$ from 26 MeV was interpreted as an increase of Zweig-rule-breaking in the nucleon, i.e., as an increased $s\bar{s}$ content of the nucleon.

QUESTIONS

- Q: Why is the nucleon $\Sigma_{\pi N}$ term so large?
- Q: Can we reconcile the large nucleon $\Sigma_{\pi N}$ term (and the small spin content) of the nucleon with zero observed (hidden) strangeness content, and how?

(ONE POSSIBLE) ANSWER

- A: The method of baryon chiral multiplet mixing rather naturally gives a large Σ term ($\geq$ 55 MeV) with zero (hidden) strangeness content and a low (observed) value of the "spin content" of the nucleon.

METHOD:Mixing of baryon chiral multiplets

- Chiral symmetry of QCD is spontaneously broken; consequently the physical nucleon is a linear superposition ("mixture") of different non-exotic chiral multiplets:

$$|N\rangle = \sin\theta|(6,3)\rangle + \cos\theta(\cos\varphi|(3,\bar{3})\rangle + \sin\varphi|(\bar{3},3)\rangle)$$

- Here $(6,3)$ stands for $[(6,3)+(3,6)]$, $(3,\bar{3}) = [(3,\bar{3})+(\bar{3},3)]$ and $(3,\bar{3}) = [(3,\bar{3})+(\bar{3},3)]$ are chiral $SU_L(3)\times SU_R(3)$ multiplets.
- The $(8,1) = [(8,1)+(1,8)]$ and $(1,8) = [(1,8)+(8,1)]$ chiral $SU_L(3)\times SU_R(3)$ multiplets lead to wrong anomalous magnetic moments - hence phenomenologically forbidden.
- Mixing angles θ, φ "parametrize" the effects of QCD dynamical chiral symmetry breaking on the nucleon.

Baryon chiral multiplets from three-quark interpolators

- The q^3 interpolators fall into $(6,3)$, $(3,\bar{3})$, $(3,\bar{3})$, $(8,1)$ and $(1,8)$ chiral $SU_L(3) \times SU_R(3)$ multiplets (T. D. Cohen, X. D. Ji, PRD 55, 6870 (1997)).
- We used the following, and many other non-local three-quark interpolators (PRD.81.054002):

$$
\begin{aligned}
N_1 &= (\tilde{q}q)q, \\
N_2 &= (\tilde{q}\gamma_5 q)\gamma_5 q, \\
N_3 &= (\tilde{q}\gamma_\mu q)\gamma^\mu q, \\
N_4 &= (\tilde{q}\gamma_\mu \gamma_5 \tau^i q)\gamma^\mu \gamma_5 \tau^i q, \\
N_5 &= (\tilde{q}\sigma_{\mu\nu} \tau^i q)\sigma^{\mu\nu} \tau^i q,
\end{aligned}
$$

- (here $\tilde{q} = q^T C \gamma_5 (i\tau_2)$) to explicitly calculate their chiral $SU_L(3) \times SU_R(3)$ transformation properties, i.e., chiral commutators.

Chiral multiplets' properties

TABLE I. The Abelian and the non-Abelian axial charges (+ sign indicates naive, − sign mirror transformation properties) and the non-Abelian chiral multiplets of $J^P = \frac{1}{2}$, Lorentz representation $(\frac{1}{2}, 0)$ nucleon and Δ fields; see Refs. [15–18].

Case	Field	$g_A^{(0)}$	$g_A^{(1)}$	F	D	$SU_L(3) \times SU_R(3)$
I	$N_1 - N_2$	-1	$+1$	0	$+1$	$(3, \bar{3}) \oplus (\bar{3}, 3)$
II	$N_1 + N_2$	$+3$	$+1$	$+1$	0	$(8, 1) \oplus (1, 8)$
III	$N_1' - N_2'$	$+1$	-1	0	-1	$(\bar{3}, 3) \oplus (3, \bar{3})$
IV	$N_1' + N_2'$	-3	-1	-1	0	$(1, 8) \oplus (8, 1)$
0	$\partial_\mu(N_3^\mu + \frac{1}{3}N_4^\mu)$	$+1$	$+\frac{5}{3}$	$+\frac{2}{3}$	$+1$	$(6, 3) \oplus (3, 6)$

- Table shows the isovector $g_A^{(1)}$, the flavor singlet $g_A^{(0)}$, and SU(3) octet F, D, axial couplings. Use them in mixing

$$\frac{5}{3}\sin^2\theta + \cos^2\theta \left(g_A^{(1)}\cos^2\varphi + g_A^{(1)\prime}\sin^2\varphi\right) = 1.267$$

$$\sin^2\theta + \cos^2\theta \left(g_A^{(0)}\cos^2\varphi + g_A^{(0)\prime}\sin^2\varphi\right) = 0.33 \pm 0.08$$

Explicit breaking of chiral symmetry

- $SU_L(3) \times SU_R(3)$ symmetry is not exact: it is broken by both the current quark mass terms and the EM interactions.
- How can we separate out this explicit chiral symmetry breaking from the spontaneous symmetry breaking?
- The commutator of the QCD axial charge Q_5^a and the total Hamiltonian $H = H_{\chi conserv.} + H_{\chi SB}$ is only sensitive to the *explicit* chiral symmetry breaking part $H_{\chi SB}$

$$[Q_5^b, H] = [Q_5^b, H_{\chi \mathrm{SB}}].$$

- Chiral symmetry breaking ("current") nucleon mass term can be deduced from the current quark mass term:

$$\mathcal{H}^{\mathrm{N}}_{\chi SB} = \sum_{i=1}^{3} \bar{N}_i M^0_{N_i} N_i$$

- The (current/bare) nucleon mass equals three times the isospin-averaged current quark mass for three-quark interpolators, or more for "higher" interpolators:

$$M^0_{N_i} \geq 3\bar{m}^0_q = \frac{3}{2}\left(m^0_u + m^0_d\right)$$

Dashen's double commutator

- Double commutator of axial charges Q_5^a and Hamiltonian $H_{\chi SB}$ measures the explicit chiral symmetry breaking!

$$\Sigma = \frac{1}{3}\delta^{ab}[Q_5^a, [Q_5^b, H_{\chi \mathrm{SB}}]]$$

- From this point on we shall work with two light flavors (u, d) only - no strange quarks. Therefore we shall use $SU_L(2) \times SU_R(2)$ multiplets instead of $SU_L(3) \times SU_R(3)$ multiplets: $(1, \frac{1}{2}) \leftrightarrow (6, 3)$, $(\frac{1}{2}, 0) \leftrightarrow (3, \bar{3})$, $(0, \frac{1}{2}) \leftrightarrow (\bar{3}, 3)$.
- Must evaluate this double commutator in each chiral multiplet.

RESULTS: Chiral commutators

1. In PRD.81.054002 we derived the $(1, \frac{1}{2})$ commutators:

$$[Q_5^a, N_{(1,\frac{1}{2})}] = \gamma_5 \left(\frac{5}{3}\frac{\tau^a}{2} N_{(1,\frac{1}{2})} + \frac{2}{\sqrt{3}} T^a \Delta_{(1,\frac{1}{2})} \right),$$

$$[Q_5^a, \Delta_{(1,\frac{1}{2})}] = \gamma_5 \left(\frac{2}{\sqrt{3}} T^{\dagger a} N_{(1,\frac{1}{2})} + \frac{1}{3} t^a_{(3/2)} \Delta_{(1,\frac{1}{2})} \right)$$

2. The $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ chiral multiplets:

$$[Q_5^a, N_{(\frac{1}{2},0)}] = \gamma_5 \frac{\tau^a}{2} N_{(\frac{1}{2},0)},$$

$$[Q_5^a, N_{(0,\frac{1}{2})}] = -\gamma_5 \frac{\tau^a}{2} N_{(0,\frac{1}{2})}$$

RESULTS: Chiral double commutators

1. The $(1, \frac{1}{2})$ chiral multiplet:

$$
\begin{aligned}
\left[Q_5^b, [Q_5^a, \bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})}]\right] &= \frac{41}{9} \delta^{ab} \bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})} \\
+\bar{\Delta}_{(1,\frac{1}{2})} \Big(2\delta^{ab} &- \frac{4}{9} \left\{ t^a_{(3/2)}, t^b_{(3/2)} \right\} \Big) \Delta_{(1,\frac{1}{2})} + \dots \\
\left[Q_5^b, [Q_5^a, \bar{\Delta}_{(1,\frac{1}{2})} \Delta_{(1,\frac{1}{2})}]\right] &= \frac{16}{9} \delta^{ab} \bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})} \\
+\bar{\Delta}_{(1,\frac{1}{2})} \Big(2\delta^{ab} &- \frac{2}{9} \left\{ t^a_{(3/2)}, t^b_{(3/2)} \right\} \Big) \Delta_{(1,\frac{1}{2})} + \dots
\end{aligned}
$$

2. The $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ chiral multiplets:

$$
\begin{aligned}
\left[Q_5^b, [Q_5^a, \bar{N}_{(\frac{1}{2},0)} N_{(\frac{1}{2},0)}]\right] &= \delta^{ab} \bar{N}_{(\frac{1}{2},0)} N_{(\frac{1}{2},0)} \\
\left[Q_5^b, [Q_5^a, \bar{N}_{(0,\frac{1}{2})} N_{(0,\frac{1}{2})}]\right] &= \delta^{ab} \bar{N}_{(0,\frac{1}{2})} N_{(0,\frac{1}{2})} .
\end{aligned}
$$

RESULTS: Sigma terms for different chiral multiplets

1. The $(1, \frac{1}{2})$ chiral multiplet Σ term is enhanced:

$$\Sigma_{\pi N}(1, \frac{1}{2}) \;=\; \frac{41}{9} M^0_{N(1,\frac{1}{2})} + \frac{16}{9} M^0_{\Delta(1,\frac{1}{2})}$$

2. The $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ Σ terms are “trivial” (i.e. no enhancement)

$$\begin{aligned}\Sigma_{\pi N}(\frac{1}{2}, 0) &= \langle N(\frac{1}{2}, 0)|\Sigma(\frac{1}{2}, 0)|N(\frac{1}{2}, 0)\rangle \\ &= M^0_{(\frac{1}{2},0)} = \Sigma_{\pi N}(0, \frac{1}{2})\end{aligned}$$

RESULTS: Nucleon Σ term

- Nucleon Σ term

$$\begin{aligned}\Sigma_{\pi N} &= \sin^2\theta\left(\frac{41}{9}M^0_{N(1,\frac{1}{2})}+\frac{16}{9}M^0_{\Delta(1,\frac{1}{2})}\right)\\ &+ \cos^2\theta\left(\cos^2\varphi M^0_{N(\frac{1}{2},0)}+\sin^2\varphi M^0_{N(\frac{1}{2},0)}\right)\ ,\end{aligned}$$

- For simplicity, assume $M^0_{N(1,\frac{1}{2})} = M^0_{\Delta(1,\frac{1}{2})} = M^0_{N(\frac{1}{2},0)} = M^0_N$.

$$\Sigma_{\pi N} = \left(1+\frac{16}{3}\sin^2\theta\right)M^0_N\ .$$

- Note that the enhancement term $\frac{16}{3}\sin^2\theta$ is due to the factor $\frac{41+16}{9}=\frac{19}{3}\approx 6.33$ appearing in the $[(1,\frac{1}{2})\oplus(\frac{1}{2},1)]$ chiral multiplet.

RESULTS: Nucleon Σ term II

- Chiral mixing tells us

$$\frac{8}{3}\sin^2\theta = g_A^{(0)} + g_A^{(3)}$$

- and the π-nucleon Σ term becomes

$$\Sigma_{\pi N} = \left(1 + 2\left(g_A^{(0)} + g_A^{(3)}\right)\right)\frac{3}{2}\left(m_u^0 + m_d^0\right)$$

- PDG2012 has $m_u^0 = 2.3 \times 1.35$ MeV and $m_d^0 = 4.8 \times 1.35$ MeV, yielding $\frac{1}{2}\left(m_u^0 + m_d^0\right) \approx 4.73$ MeV (substantially lower than before - cf. 7.6 MeV in PDG1998)

RESULTS: Discussion I

- Inserting $g_A^{(3)} = 1.267$ and the quark masses we find

$$\Sigma_{\pi N} = 59.5 \pm 2.3\mathrm{MeV},$$

with $g_A^{(0)} = 0.33 \pm 0.03 \pm 0.05$ [W. Vogelsang, J. Phys. G **34**, S149 (2007)]

- or

$$\Sigma_{\pi N} = 58.0 \pm 4.5\mathrm{MeV},$$

with $g_A^{(0)} = 0.28 \pm 0.16$ [B. W. Filippone and X. -D. Ji, Adv. Nucl. Phys. **26**, 1 (2001)]

RESULTS: Discussion II

- This is actually an inequality

$$\Sigma_{\pi N} \geq \left(1 + 2\left(g_A^{(0)} + g_A^{(3)}\right)\right)\frac{3}{2}\left(m_u^0 + m_d^0\right)$$

when we realize that

$$M^0_{N(1,\frac{1}{2})}, M^0_{\Delta(1,\frac{1}{2})} \geq M^0_{N(\frac{1}{2},0)} = M^0_N = \frac{3}{2}\left(m_u^0 + m_d^0\right) = 14.2\mathrm{MeV}$$

RESULTS: Discussion III

- Roughly one half of the total value (≈30 MeV) of $\Sigma_{\pi N}$ can be attributed to the Δ d.o.f.: $\frac{16}{9}\ M_N^0 \sin^2\theta \approx 15$ MeV is the "direct" Δ contribution
- and the same amount (≈ 15 MeV) comes about from the "virtual" Δ states.
- Similar results appear in a recent baryon chiral perturbation calculation of Alarcon et al., arXiv:1209.2870 [hep-ph].

SUMMARY

- We calculated $\Sigma_{\pi N}$ in the chiral mixing formalism and found $\Sigma_{\pi N} \geq \left(1 + 2\left(g_A^{(0)} + g_A^{(3)}\right)\right) \frac{3}{2}\left(m_u^0 + m_d^0\right) \simeq 58$ MeV, four times larger than naively expected.
- Do not need any $\bar{s}s$ content in the nucleon: entire $\Sigma_{\pi N}$ enhancement is due to $(1, \frac{1}{2})$ chiral multiplet, which is also responsible for $g_A^{(3)} = 1.267$.
- The result also depends on the "spin content of nucleon" $g_A^{(0)} = 0.28 \pm 0.16$, which is correctly reproduced (not a prediction) in this approach.
- Roughly one half ($\approx$30 MeV) of the total value of $\Sigma_{\pi N}$ can be attributed to the Δ degrees-of-freedom.