Testing the standard model and beyond with lattice QCD

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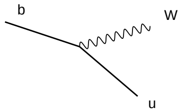
(FLAG EPJC71 (2011), arXiv:1310.8555v2 [hep-lat],
LL in "Modern perspectives in lattice QCD" OUP 2011,
and references therein)

Quark flavor mixing constraints in the SM and beyond

Test SM paradigm of quark flavor mixing and CP violation and look for new physics

Unitary CKM matrix



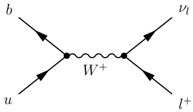
$$\sim V_{ub} \rightarrow V = \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} 1-\frac{\lambda}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \end{array} + \mathcal{O}(\lambda^4)$$

Test CKM unitarity/quark-lepton universality and constrain NP using, e.g.

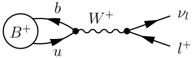
1st row unitarity: $\frac{G_q^2}{G_\mu^2}|V_{ud}|^2\left[1+|V_{us}/V_{ud}|^2+|V_{ub}/V_{ud}|^2\right] = 1 + O\left(\frac{M_W^2}{\Lambda_{NP}^2}\right)$

Unitarity triangle: $\frac{G_q^2}{G_\mu^2}\left(V_{cd}V_{cb}^*\right)\left[1+\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}+\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] = O\left(\frac{M_W^2}{\Lambda_{NP}^2}\right)$

Challenge:



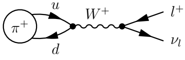
$\sim V_{ub}^* \longrightarrow$



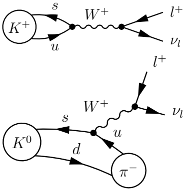
$\sim V_{ub}^* f_B$

Lattice QCD CKM golden modes

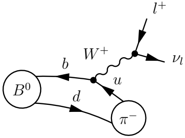
$|V_{ud}|$



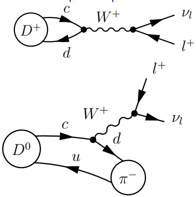
$|V_{us}|$



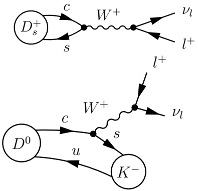
$|V_{ub}|$



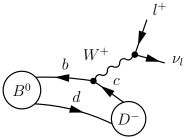
$|V_{cd}|$



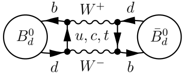
$|V_{cs}|$



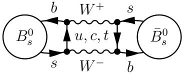
$|V_{cb}|$



$|V_{ct}|$



$|V_{ts}|$

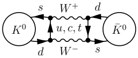


$|V_{tb}|$

Other important modes for the lattice

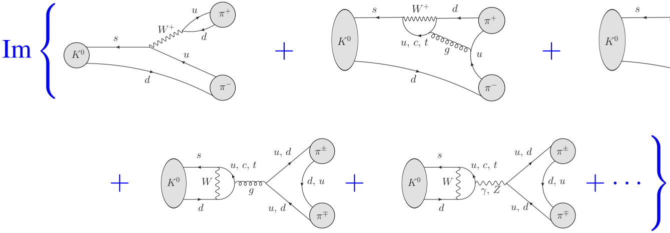
FCNC's are particularly sensitive to possible new physics

- Exclusive $b \to s\ell^+\ell^-$ ($\longrightarrow$ Meinel's talk)
- Indirect CPV in $K \to \pi\pi$

Im [

+ ⋯]

- Direct CPV in $K \to \pi\pi$



- CPV in $K_L \to \pi^0\ell^+\ell^-$
- …

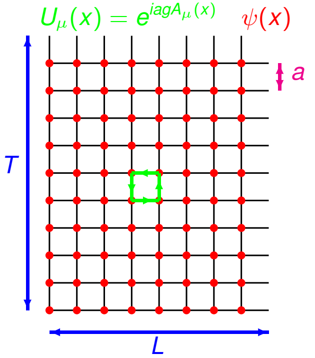
What is lattice QCD (LQCD)?

To describe ordinary matter, QCD requires $\geq$ 104 numbers at every point of spacetime
$\rightarrow \infty$ number of numbers in our continuous spacetime
$\rightarrow$ must temporarily "simplify" the theory to be able to calculate *(regularization)*
$\Rightarrow$ Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (& IR) cutoff $\rightarrow$ well defined path integral in Euclidean spacetime:

$$\langle O \rangle = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G - \int \bar{\psi}D[M]\psi}\, O[U,\psi,\bar{\psi}]$$
$$= \int \mathcal{D}U\, e^{-S_G} \det(D[M])\, O[U]_{\mathrm{Wick}}$$

- $\mathcal{D}Ue^{-S_G}\det(D[M]) \geq 0$ & finite # of dofs $\rightarrow$ evaluate numerically using stochastic methods



LQCD is QCD when $m_q \rightarrow m_q^{\mathrm{phys}}$, $a \rightarrow 0$, $L \rightarrow \infty$ (and stats $\rightarrow \infty$)

HUGE conceptual and numerical ($\sim 10^9$ dofs) challenge

State-of-the-art 2014

- $N_f = 2+1$ (u, d & s in isospin limit) or $N_f = 2+1+1$(u, d, s & c in isospin limit)

$\Rightarrow$ precision limit: $O(\alpha, m_d - m_u, 1/(N_c m_c^2), \alpha_s(m_c))$ or $O(\alpha, m_d - m_u,$ $1/(N_c m_b^2), \alpha_s(m_b)) \longrightarrow O(1\%)$

- $M_\pi \searrow 120\,\mathrm{MeV} < M_\pi^{\mathrm{phys}} \Rightarrow$ interpolation to m_{ud}^{phys}
- $a \searrow 0.05\,\mathrm{fm}$ and $L \nearrow 6\,\mathrm{fm} \Rightarrow$ fully controlled continuum and infinite-volume limits
- Full nonperturbative renormalization and running $\Rightarrow$ fully controlled matching in experimental rates
- Improved discretizations & $a \sim 0.05\,\mathrm{fm} \Rightarrow$ good control over charm
- b quark through interpolation between c & HQET or directly in EFTs, e.g. HQET + $O(1/m_b)$, NRQCD, Fermilab, . . .

$\Rightarrow$ tools to perform % level QCD calculations . . . of "simple" quantities

$\Rightarrow$ need large number of simulations over large range of relevant parameters to control all systematics

What is FLAG?

FLAG = Flavor Lattice Averaging Group
= members from major lattice groups collaboration worldwide

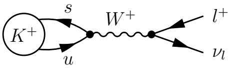
Review lattice computations of important phenomenological quantities and answer questions:

- What is the current lattice value for X?
- What is a reliable estimate of its uncertainty?

For each quantity we provide:

- complete list of references
- summary of relevant formulae and notation
- summary of the essential aspects of each calculation in a unified and easy to read (color coding) manner
  For each source of systematic error
  - ★ satisfactory estimate, convincingly under control
  - ○ reasonable, but improvable, estimate
  - ■ no or unsatisfactory estimate → not included in averages
- original discussions of the relevant phenomenology

(Semi)-leptonic kaon decays and $|V_{us}|$

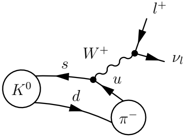


- $\langle 0|\bar{s}\gamma_\mu\gamma_5 u|K^+(p)\rangle = ip_\mu f_{K^+}$
- $\frac{\Gamma(K^\pm \to \mu\nu(\gamma))}{\Gamma(\pi^\pm \to \mu\nu(\gamma))} \longrightarrow$

  $\frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2758(5)$ [0.18%]

  (Antonelli et al '10)
- $\frac{f_{K^\pm}}{f_{\pi^\pm}} = 1 + O(m_s - m_{ud})$
- Need $f_{K^\pm}/f_{\pi^\pm} - 1$ to 1.1%

$\Rightarrow$ isospin breaking and sea charm



- $\langle\pi^-(p')|\bar{s}\gamma_\mu\gamma_5 u|K^0(p)\rangle$

  $\longrightarrow f_+(q^2), \quad f_0(q^2)$
- $\Gamma(K \to \pi\ell\nu) \longrightarrow$

  $|V_{us}|f_+(0) = 0.2163(5)$ [0.23%]

  (Antonelli et al '10)
- $f_+(0) = 1 + O(m_s - m_{ud})^2$
- Need $f_+(0) - 1$ to 6.6%

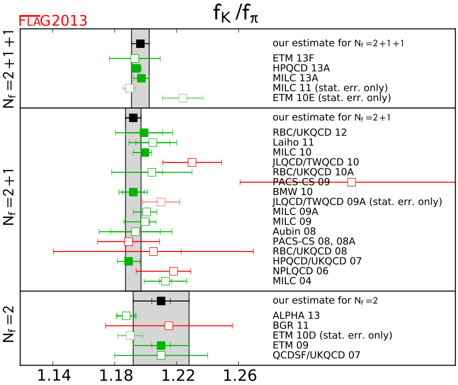
$\Rightarrow$ isospin limit still sufficient

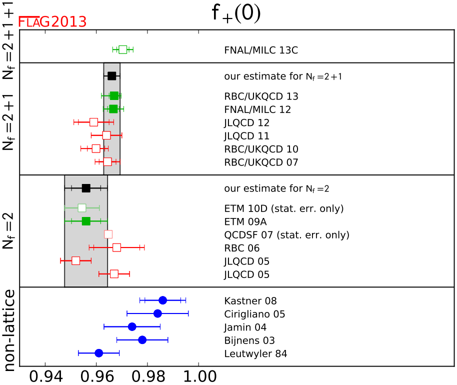
f_K/f_π lattice calculations

Collaboration	N_f	pub.	$m_{ud} \to m_{ud}^{\rm phys}$	$a \to 0$	$L \to \infty$	f_K/f_π	$f_{K^\pm}/f_{\pi^\pm}$
ETM 13F	2+1+1	C	○	★	○	1.193(13)(10)	1.183(14)(10)
HPQCD 13A	2+1+1	A	★	○	★		1.1916(15)(16)
MILC 13A	2+1+1	A	★	○	★		1.1947(26)(37)
MILC 11	2+1+1	C	○	○	○		$1.1872(42)^{\dagger}_{\rm stat}$.
ETM 10E	2+1+1	C	○	○	○	$1.224(13)_{\rm stat}$	
RBC/UKQCD 12	2+1	A	★	○	★	1.199(12)(14)	
Laiho 11	2+1	C	○	○	○		$1.202(11)(9)(2)(5)^{\dagger\dagger}$
MILC 10	2+1	C	○	★	★		$1.197(2)(^{+3}_{-7})$
JLQCD/TWQCD 10	2+1	C	○	■	★	1.230(19)	
RBC/UKQCD 10A	2+1	A	○	○	★	1.204(7)(25)	
PACS-CS 09	2+1	A	★	■	■	1.333(72)	
BMW 10	2+1	A	★	★	★	1.192(7)(6)	
JLQCD/TWQCD 09A	2+1	C	○	■	■	$1.210(12)_{\rm stat}$	
MILC 09A	2+1	C	○	★	★		$1.198(2)(^{+6}_{-8})$
MILC 09	2+1	A	○	★	★		$1.197(3)(^{+6}_{-13})$
Aubin 08	2+1	C	○	○	○		1.191(16)(17)
PACS-CS 08, 08	2+1	A	★	■	■	1.189(20)	
RBC/UKQCD 08	2+1	A	○	■	★	1.205(18)(62)	
HPQCD/UKQCD 07	2+1	A	○	★	○	1.189(2)(7)	
NPLQCD 06	2+1	A	○	■	■	$1.218(2)(^{+11}_{-24})$	
MILC 04	2+1	A	○	○	○		1.210(4)(13)
ALPHA 13	2	C	★	★	★	1.1874(57)(30)	
BGR 11	2	A	★	■	■	1.215(41)	
ETM 10D	2	C	○	★	○	$1.190(8)_{\rm stat}$	
ETM 09	2	A	○	★	○	1.210(6)(15)(9)	
QCDSF/UKQCD 07	2	C	○	○	★	1.21(3)	

† Result with statistical error only from polynomial interpolation to the physical point. †† This work is the continuation of Aubin 08.

$f_{K^\pm}/f_{\pi^\pm}$ & $f_+(0)$ from the lattice





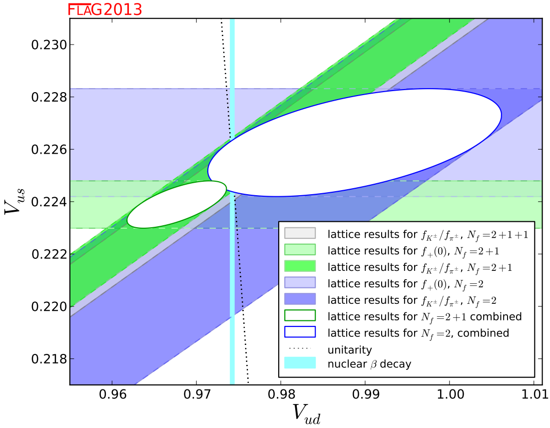
$$f_{K^\pm}/f_{\pi^\pm} - 1 = 0.194(5)\ [2.6\%] \quad (N_f = 2+1+1)$$

$$= 0.192(5)\ [2.6\%] \quad (N_f = 2+1)$$

$$1 - f_+(0) = 0.0339(32)\ [9.4\%] \quad (N_f = 2+1)$$

Must still improve to match experiment

Consequences for $|V_{ud}|$ and $|V_{us}|$



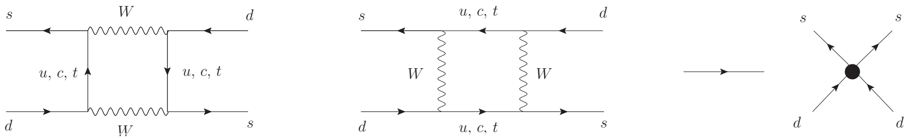
Tests of 1st row unitarity (N_f=2+1 results)

$$\sum_{q=d,s,b} |V_{uq}|^2 - 1 = -0.013(10)$$ lattice $f_{K^\pm}/f_{\pi^\pm}$ & $f_+(0)$ $\rightarrow$ allows $\Lambda_{NP} \gtrsim 500$ GeV @ 1 σ

$$= 0.0000(6)$$ lattice $f_{K^\pm}/f_{\pi^\pm}$ & $|V_{ub}|$ from nuclear β-decay

$$= -0.0007(5)$$ lattice $f_+(0)$ & $|V_{ub}|$ from nuclear β-decay

K^0-$\bar{K}^0$ mixing in the SM



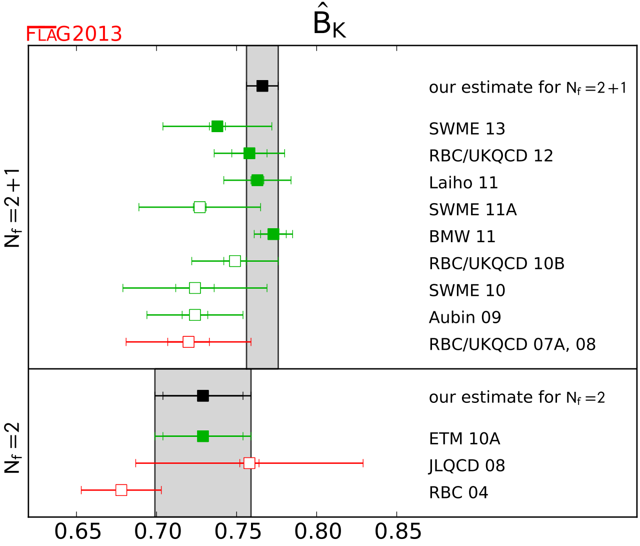
$$M_{12} - \frac{i}{2}\Gamma_{12} = \frac{\langle K^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|\bar{K}^0\rangle}{2M_K} - \underbrace{\frac{i}{2M_K}\int d^4x \langle K^0|\mathcal{T}\{\mathcal{H}_{\rm eff}^{\Delta S=1}(x)\mathcal{H}_{\rm eff}^{\Delta S=1}(0)\}|\bar{K}^0\rangle}_{\text{gives } \Gamma_{12} \text{ and LD contributions to } M_{12}} + O(G_F^3)$$

Neglecting long-distance (LD) corrections

$$2M_K M_{12}^* \stackrel{\rm SD}{=} \langle \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle = C_1^{\rm SM}(\mu)\langle \bar{K}^0|O_1(\mu)|K^0\rangle$$

$$O_1 = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A} \qquad \langle \bar{K}^0|O_1(\mu)|K^0\rangle = \frac{16}{3}M_K^2 F_K^2 B_K(\mu)$$

Lattice results for $\hat{B}_K$



$$\hat{B}_K = 0.7661(99)\ [1.3\%] \qquad (N_f = 2+1)$$

Indirect CPV in $K \to \pi\pi$

Parametrized by

$$\epsilon \equiv \frac{T[K_L \to (\pi\pi)_0]}{T[K_S \to (\pi\pi)_0]} = \frac{i}{2}\left[\frac{\mathrm{Im}M_{12} - \frac{i}{2}\mathrm{Im}\Gamma_{12}}{\mathrm{Re}M_{12} - \frac{i}{2}\mathrm{Re}\Gamma_{12}}\right] + i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}$$

$$\to \kappa_\epsilon \frac{e^{i\phi_\epsilon}}{\sqrt{2}} \frac{\mathrm{Im}M_{12}}{\Delta M_K} = 2.228(11)\cdot 10^{-3} \times e^{i\phi_\epsilon}\ [0.5\%] \qquad \text{(PDG 14)}$$

w/ $\phi_\epsilon = \tan^{-1}(2\Delta M_K/\Delta\Gamma_K)$, $2\,\mathrm{Re}\,M_{12} \simeq \Delta M_K$, $2\,\mathrm{Re}\,\Gamma_{12} \simeq \Delta\Gamma_K$

$$\longrightarrow \kappa_\epsilon = 0.96(2)\ [2\%] \qquad \text{(Buras et al 10)}$$

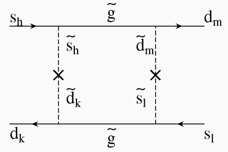
To NNLO in α_s (Brod et al 10, 11)

$$|\epsilon| = \kappa_\epsilon C_\epsilon |V_{cb}|^2 \lambda^2 \bar{\eta} \left[|V_{cb}|^2 (1-\bar{\rho})\eta_{tt} S_{tt} + \eta_{ct} S_{ct} - \eta_{cc} S_{cc}\right] \hat{B}_K$$

$$= 1.96(15)_{\eta_{cc}}(8)_{\eta_{ct}}(2)_{\eta_{tt}}(25)_{\{|V_{cb}|\ \text{et al}\}}(2)_{\kappa_\epsilon}(1)_{\hat{B}_K} \times 10^{-3}$$

- $\Rightarrow$ No point in further reducing error on $\hat{B}_K$
- $\Rightarrow$ Even precise lattice computation of κ_ϵ (e.g. $\mathrm{Im}A_0$ (RBC/UKQCD 11) & long-distance contributions) not sufficient
- $\Rightarrow$ Must re-instate c as dynamical d.o.f. (RBC/UKQCD 12) to improve precision

$\Delta S = 2$ processes beyond the SM



$$+\cdots \xrightarrow{OPE} \begin{cases} O_1 = [\bar{s}d]_{V-A}[\bar{s}d]_{V-A} & \\ O_{2,3} = [\bar{s}d]_{S-P}[\bar{s}d]_{S-P} & (\text{unmix}, \text{mix}) \\ O_{4,5} = [\bar{s}d]_{S-P}[\bar{s}d]_{S+P} & (\text{unmix}, \text{mix}) \end{cases}$$

$$\langle K^0|\mathcal{H}_{\text{eff,BSM}}^{\Delta S=2}|\bar{K}^0\rangle = \sum_{i=1}^{5} C_i^{\text{BSM}}(\mu)Q_i(\mu) = \sum_{i=1}^{5} C_i^{\text{BSM}}(\mu)\langle K^0|O_i(\mu)|\bar{K}^0\rangle$$

- $C_i^{\text{BSM}}(\mu)$ short distance coefficients $\supset$ flavor mixing parameters of BSM model
- $Q_i(\mu)$ are chirally enhanced

$$\frac{Q_i(\mu)}{Q_1(\mu)} \sim \frac{\langle\bar{q}q\rangle(\mu)}{F_\pi^2(m_s+m_{ud})(\mu)} \sim 10, \qquad i = 2,\cdots,5$$

$\Rightarrow$ ϵ and ΔM_K impose strong constraints on Im and Re of BSM parameters

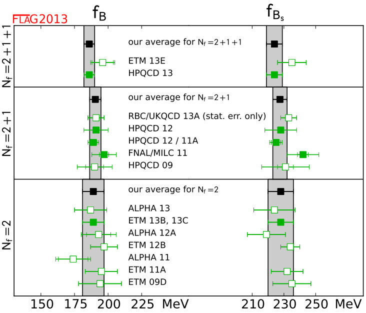
- 2 old $N_f = 0$ calculations of $Q_i(\mu)$, $i = 2,\cdots,5$ (Donini et al 99, Babich et al 06) and 3 $N_f \geq 2$ recent ones (RBC/UKQCD 12,13, ETM 12, SWME 13)

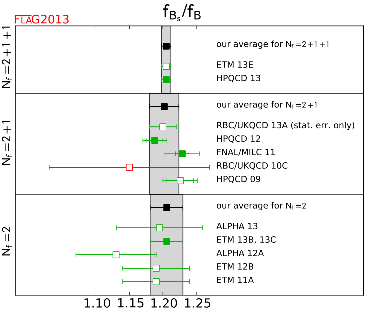
$\rightarrow$ poor agreement on $Q_i(\mu)$, $i = 4, 5$

B_q decay constants ($q = u, d, s$)

First evidence of $B(B_s \to \mu^+\mu^-) = (3.2^{+1.5}_{-1.2}) \times 10^{-9}$ by LHCb 13, and in SM

$$B(B_q^0 \to \ell^+\ell^-) = \tau_{B_q} \frac{G_F^2}{\pi} Y \left(\frac{\alpha}{4\pi \sin^2 \Theta_W}\right)^2 m_\ell^2 \sqrt{1 - 4\frac{m_\ell^2}{m_{B_q}^2}} |V_{tb}^* V_{tq}|^2 m_{B_q} f_{B_q}^2$$





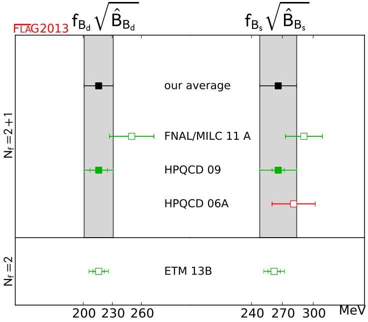
$$N_f = 2+1: \; f_B = (190.5 \pm 4.2)\,\mathrm{MeV}\ [2.2\%], \; f_{B_s} = (227.7 \pm 4.5)\,\mathrm{MeV}\ [2.0\%]$$

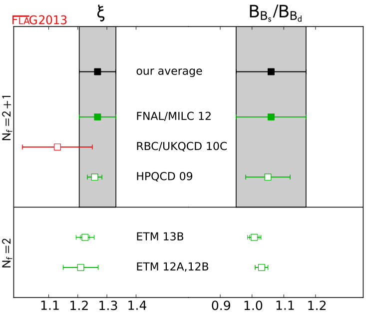
$$f_{B_s}/f_B = 1.202 \pm 0.022\ [1.8\%]$$

B_q mixing ($q = d, s$)

In SM, B_q oscillation frequency is given by:

$$\Delta m_q = \frac{G_F^2 m_W^2 m_{B_q}}{6\pi^2} |V_{tq}^* V_{tb}|^2 S_{tt} \eta_{2B} f_{B_q}^2 \hat{B}_{B_q}$$





$f_{B_d}\sqrt{\hat{B}_{B_d}} = 216(15)$ MeV [7%]
$\hat{B}_{B_d} = 1.27(10)$ [8%]
$\xi - 1 = 0.268(63)$ [24%]

$f_{B_s}\sqrt{\hat{B}_{B_s}} = 266(18)$ MeV [7%]
$\hat{B}_{B_s} = 1.33(6)$ [4.5%]
$B_{B_s}/B_{B_d} - 1 = 0.06(11)$ [183%]

CKM matrix and lattice QCD

FLAG arXiv:1310.8555v2 [hep-lat] – $N_f = 2 + 1$

CKM	process	LQCD	N_{avg}	prec. (%)
$\lvert V_{us}\rvert$	$K \to \ell\nu$	f_K iso.	3	0.6
	$K \to \pi\ell\nu$	$f_+^{K^0\pi^-}(0)$	1	0.4
$\lvert V_{us}\rvert/\lvert V_{ud}\rvert$	$K \to \mu\nu/\pi \to \mu\nu$	f_{K^+}/f_{π^+}	4	0.4
$\lvert V_{cd}\rvert$	$D \to \ell\nu$	f_D	2	1.6
	$D \to \pi\ell\nu$	$f_+^{D\pi}(0)$	1	4.3
$\lvert V_{cs}\rvert$	$D_s \to \ell\nu$	f_{D_s}	2	1.1
	$D \to K\ell\nu$	$f_+^{DK}(0)$	1	2.5
$\lvert V_{ub}\rvert$	$B \to \ell\nu$	f_B	3	2.2
		f_{B_s}/f_B	2	1.8
	$B \to \pi\ell\nu$	$f_+^{B\pi}(q^2)$	2	~ 20
$\lvert V_{cb}\rvert$	$B \to D^{(*)}\ell\nu$	$\mathcal{F}_{B\to D^{(*)}}(1)$	1	1.8
$(\bar{\rho}, \bar{\eta})$	ϵ	$\hat{B}_K$	4	1.3
	ϵ'	$K \to \pi\pi$		
$\lvert V_{tb}^* V_{tq}\rvert$	Δm_d	$f_{B_d}\sqrt{\hat{B}_{B_d}}$	1	6.9
	Δm_d	ξ	1	5.0
	Δm_s	$f_{B_s}\sqrt{\hat{B}_{B_s}}$	1	6.8

Conclusion

- Lattice QCD calculations have made tremendous progress in the last few years
- State-of-the-art is $N_f = 2 + 1$ or $N_f = 2 + 1 + 1$ w/ sufficient ranges of parameters to **control all errors** ... on "simple" quantities
- Typical precision of "golden modes" currently $\sim 1 \div 5\%$
- Smaller errors correspond quantities whose normalization is approximately fixed by symmetry (e.g. $SU(3)$-flavor)
- To break the % barrier, must include EM, $m_d - m_u$ (Duncan et al 96, RBC 07-12, BMW 10-13, RM123 11-13, PACS-CS 12) and sea charm effects...
- ...and in some cases long-distance contributions to effective Hamiltonians (e.g. RBC/UKQCD 12)
- Many other quantities studied, from $(g-2)_\mu$ to light nuclei ...
- So much to do for anyone interested in understanding nonperturbative strong interactions and their effects on quark processes