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Exotic hadronic states with charm and bottom

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JHEP 7,153(2013) – arXiv:1304.0345, with S. Nussinov JHEP 8,96(2013) – arXiv:1305.6457, with Y. Frishman

FPCP 2014, Marseille, 27 May 2014

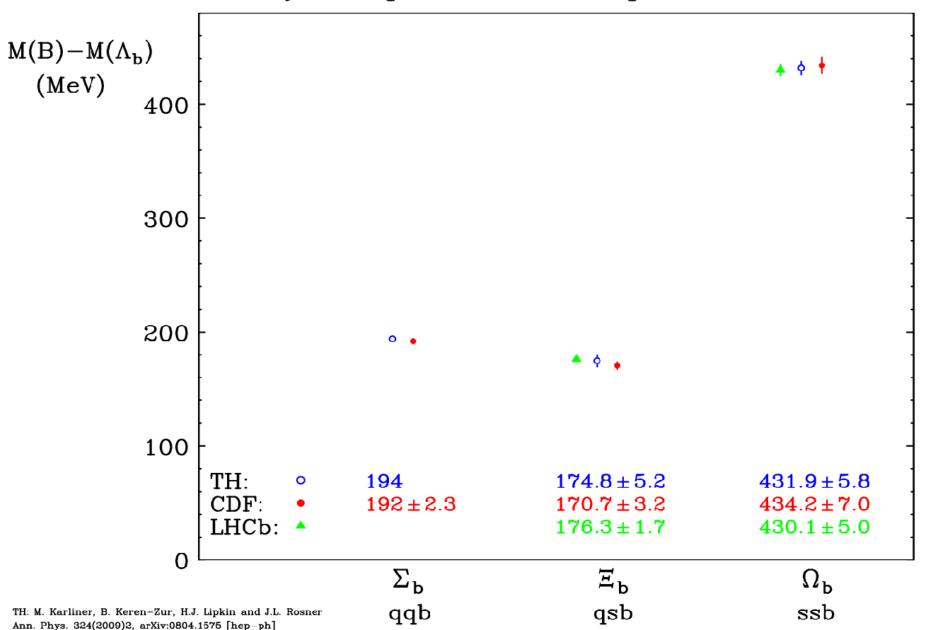
It has been realized early on that quark models and QCD sustain a much richer pattern of different multi-quark and/or color network configurations, beyond the "non-exotic" standard $\bar{q}q$ mesons and qqq baryons. Still, production rates of such particles are often suppressed and the light pions will in most cases allow rapid decays of the exotics into final states with pion(s) – turning them into very broad resonances. This explains why the vast majority of known hadrons are just simple mesons and baryons. The situation is different for exotics which contain a heavy quark-antiquark pair and a light quark-antiquark pair: $\bar{Q}Q\bar{q}q$.

The heavy quarks hardly mix with the light quarks, so such exotics decay into quarkonium and pion(s) or into two heavy-light mesons, providing a clear signature of their exotic nature.

Hadrons containing heavy quarks are simpler than hadrons containing light quarks only, because the heavy quarks are almost static and have a very small spin-dependent interaction with other quarks.

This was the key to the accurate prediction of baryons containing the b quark:





Possibility of Exotic States in the Upsilon system

Marek Karliner^{a*}
and
Harry J. Lipkin a,b†

Abstract

Recent data from Belle show unusually large partial widths $\Upsilon(5S) \to \Upsilon(1S) \pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(2S) \pi^+\pi^-$. The Z(4430) narrow resonance also reported by Belle in $\psi'\pi^+$ spectrum has the properties expected of a $\bar{c}cu\bar{d}$ charged isovector tetraquark $T^{\pm}_{\bar{c}c}$. The analogous state $T^{\pm}_{\bar{b}b}$ in the bottom sector might mediate anomalously large cascade decays in the Upsilon system, $\Upsilon(mS) \to T^{\pm}_{\bar{b}b}\pi^{\mp} \to \Upsilon(nS)\pi^+\pi^-$, with a tetraquark-pion intermediate state. We suggest looking for the $\bar{b}bu\bar{d}$ tetraquark in these decays as peaks in the invariant mass of $\Upsilon(1S)\pi$ or $\Upsilon(2S)\pi$ systems. The $\bar{b}bu\bar{s}$ tetraquark can appear in the observed decays $\Upsilon(5S) \to \Upsilon(1S)K^+K^-$ as a peak in the invariant mass of $\Upsilon(1S)K$ system. We review the model showing that these tetraquarks are below the two heavy meson threshold, but respectively above the $\Upsilon\pi\pi$ and $\Upsilon K\bar{K}$ thresholds.

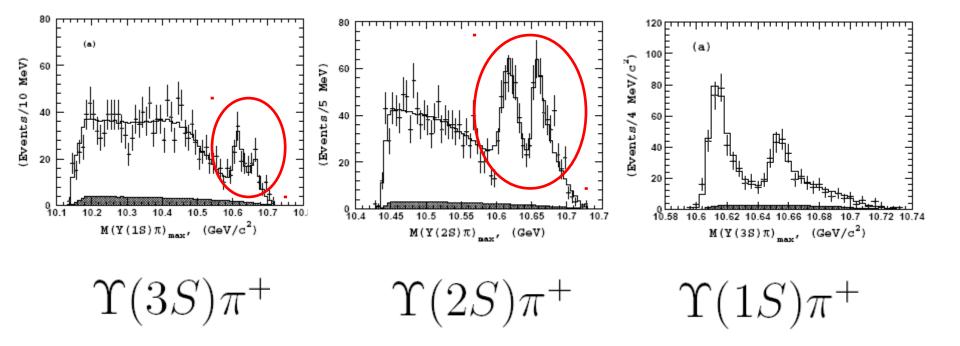
Observation of two charged bottomonium-like resonances

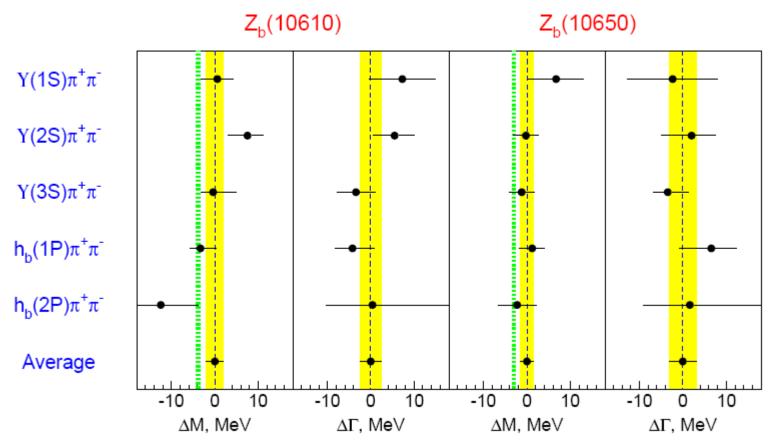
The Belle Collaboration

(Dated: May 24, 2011)

Abstract

We report the observation of two narrow structures at $10610 \,\mathrm{MeV}/c^2$ and $10650 \,\mathrm{MeV}/c^2$ in the $\pi^{\pm}\Upsilon(nS)$ (n=1,2,3) and $\pi^{\pm}h_b(mP)$ (m=1,2) mass spectra that are produced in association with a single charged pion in $\Upsilon(5S)$ decays. The measured masses and widths of the two structures averaged over the five final states are $M_1 = 10608.4 \pm 2.0 \,\mathrm{MeV}/c^2$, $\Gamma_1 = 15.6 \pm 2.5 \,\mathrm{MeV}$ and $M_2 = 10653.2 \pm 1.5 \,\mathrm{MeV}/c^2$, $\Gamma_2 = 14.4 \pm 3.2 \,\mathrm{MeV}$. Analysis favors quantum numbers of $I^G(J^P) = 1^+(1^+)$ for both states. The results are obtained with a $121.4 \,\mathrm{fb}^{-1}$ data sample collected with the Belle detector near the $\Upsilon(5S)$ resonance at the KEKB asymmetric-energy e^+e^- collider.





Comparison of $Z_b(10610)$ and $Z_b(10650)$ parameters obtained from different decay channels. The vertical dotted lines indicate $B^*\overline{B}$ and $B^*\overline{B}^*$ thresholds.

$$J^P = 1^+$$
 for both $Z_b(10610)$ and $Z_b(10650)$

The Z_b resonances decay into

 $\Upsilon(nS)$ and a charged pion

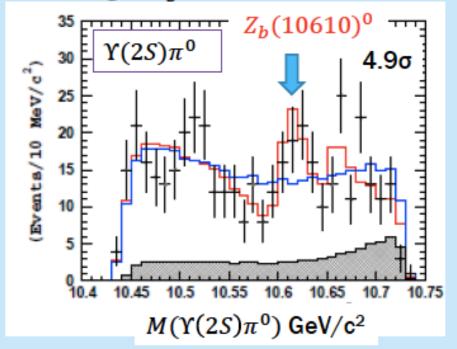
 \implies must contain both $\bar{b}b$ and $\bar{d}u$

manifestly exotic

Neutral member of the I=1 multiplet very recently also observed by Belle in Dalitz plot analysis

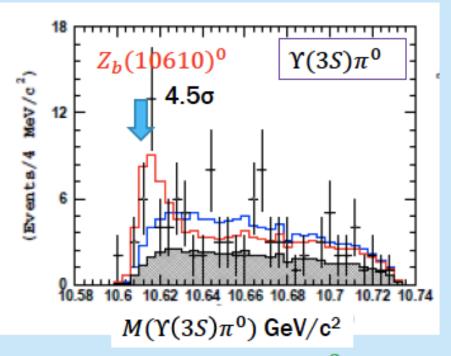
$\Upsilon(5S) \to \Upsilon(nS)\pi^0\pi^0$ decay

In this fit mass and width are fixed from the charged Z_b result.



— fit result with Z_b

— fit result without Z_b



Simultaneous fit gives 6.3 σ for $Z_b(10610)^0$

After the discovery of Z_b -s by Belle, natural to expect analogous states in the charm system

one caveat:

a priori unknown whether charmed quarks are heavy enough to allow for binding

encouraging indications from toy model in D=1+1, Y.Frishman & MK

[JHEP 8,96(2013) - arXiv:1305.6457]

in March 2013 BES in Beijing, followed by Belle in KEK provided

the answer for the question if charm is heavy enough:

BESIII Collaboration

PRL **110,** 252001 (2013)

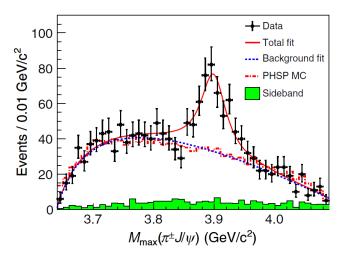
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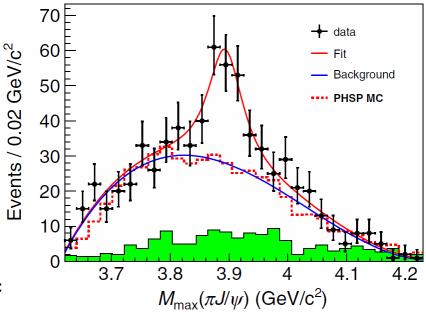
Observation of a Charged Charmoniumlike Structure in $e^+e^- \to \pi^+\pi^- J/\psi$ at $\sqrt{s} = 4.26$ GeV

We study the process $e^+e^- \to \pi^+\pi^- J/\psi$ at a center-of-mass energy of 4.260 GeV using a 525 pb⁻¹ data sample collected with the BESIII detector operating at the Beijing Electron Positron Collider. The Born cross section is measured to be $(62.9 \pm 1.9 \pm 3.7)$ pb, consistent with the production of the Y(4260). We observe a structure at around 3.9 GeV/ c^2 in the $\pi^\pm J/\psi$ mass spectrum, which we refer to as the $Z_c(3900)$. If interpreted as a new particle, it is unusual in that it carries an electric charge and couples to charmonium. A fit to the $\pi^\pm J/\psi$ invariant mass spectrum, neglecting interference, results in a mass of $(3899.0 \pm 3.6 \pm 4.9) \text{ MeV}/c^2$ and a width of $(46 \pm 10 \pm 20) \text{ MeV}$. Its production ratio is measured to be $R = (\sigma(e^+e^- \to \pi^\pm Z_c(3900)^\mp \to \pi^+\pi^- J/\psi)/\sigma(e^+e^- \to \pi^+\pi^- J/\psi)) = (21.5 \pm 3.3 \pm 7.5)\%$. In all measurements the first errors are statistical and the second are systematic.



Study of $e^+e^- \to \pi^+\pi^- J/\psi$ and Observation of a Charged Charmoniumlike State at Belle

The cross section for $e^+e^- \to \pi^+\pi^- J/\psi$ between 3.8 and 5.5 GeV is measured with a 967 fb⁻¹ data sample collected by the Belle detector at or near the Y(nS) ($n=1,2,\ldots,5$) resonances. The Y(4260) state is observed, and its resonance parameters are determined. In addition, an excess of $\pi^+\pi^- J/\psi$ production around 4 GeV is observed. This feature can be described by a Breit-Wigner parametrization with properties that are consistent with the Y(4008) state that was previously reported by Belle. In a study of $Y(4260) \to \pi^+\pi^- J/\psi$ decays, a structure is observed in the $M(\pi^\pm J/\psi)$ mass spectrum with 5.2 σ significance, with mass $M=(3894.5\pm6.6\pm4.5)~{\rm MeV}/c^2$ and width $\Gamma=(63\pm24\pm26)~{\rm MeV}/c^2$, where the errors are statistical and systematic, respectively. This structure can be interpreted as a new charged charmoniumlike state.



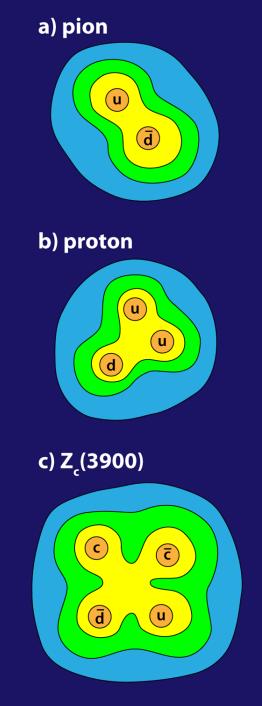
14 FPCP2014, 27/5/2014

$$M_{Z_c} = 3899.0 \pm 3.6 \pm 4.9 \text{ MeV}$$

$$\Gamma_{Z_c} = 46 \pm 10 \pm 20 \text{ MeV}$$

$$Z_c^+(3900)$$
 decays to $J/\psi \pi^+$

should also be seen in $\bar{D}D^*$



tetraquark or a "molecule"?

The molecule idea has a long history: Voloshin & Okun 1976, de Rujula, Georgi & Glashow 1977 Tornqvist, Z. Phys. C61,525 (1993)

 Z_b -s sit 3 MeV above the $\bar{B}B^*$ and \bar{B}^*B^* thresholds

X(3872) sits at $\bar{D}D^*$ threshold

narrow widths of the resonances in decay into quarkonium and pion, despite the large phase space imply very small overlap of wave functions

strong hints in favor of the molecular interpretation

what about $Z_c(3900)$?

- Heavy-light $Q\bar{q}$ mesons have I=1/2
- → they couple to pions
- → deuteron-like meson-meson bound states, "deusons"

via pion exchange – no $\bar{D}D$, only $\bar{D}D^*$

$$D\bar{D}$$
* (I=0) at threshold \longleftrightarrow X(3872)!
S-wave \to $J^P=1^+$ confirmed by BESIII

- I=1 attraction x3 weaker than I=0
- → I=1 expected well above threshold

What about $\bar{B}B^*$ analogue ?...

B B* vs D D*:

- -- same attractive potential
- -- much heavier, so smaller kinetic energy
- \Rightarrow expect $B\bar{B}^*$ and $B^*\bar{B}^*$ I=1 states near threshold
- $\rightarrow Z_b(10610)$ and $Z_b(10650)$ seen by Belle!!!

I=0 binding much stronger

→I=0 states expected well below threshold

EXP signature:

$$X_b(I=0) \longrightarrow \Upsilon(nS)\omega, \quad \chi_b \pi^+ \pi^-$$

perhaps also

$$X_b'(I=0) \longrightarrow \Upsilon(nS)\bar{B}^*B\gamma \text{ via EM } B^* \to B\gamma$$



in the $M_Q \longrightarrow \infty$ limit attractive potential between the two heavy mesons becomes universal, as kinetic energy vanishes:

Kinetic
$$E \sim \frac{p^2}{M_Q} \longrightarrow 0$$
 as $M_Q \to \infty$

→ treat kinetic E as perturbation:

$$H = a \cdot p^2 + V(r)$$
 where $a \equiv 1/2\mu_{\rm red}$

convert the parameter $a \sim 1/M_Q$ into a dimensionless parameter \tilde{a}

"natural" unit of $\sim 0.8 \text{ Fermi} \sim 4.0 \text{ GeV}^{-1}$

With $m_D \sim 2\,\mathrm{GeV}$ and $m_B \sim 5.3\,\mathrm{GeV}$

$$\tilde{a}(D) = 1/8 \qquad \qquad \tilde{a}(B) = 1/21$$

→ small: can use 1-st order P.T.

for I=1 potential have 2 data points:

 $Z_c(3900)$ at $\tilde{a}(D)$ approximately 27 MeV above $\bar{D}D^*$ threshold

 $Z_b(10610)$ at $\tilde{a}(B)$ approximately 3 MeV above $\bar{B}B^*$ threshold

PCP2014, 27/5/2014

Linear extrapolation to
$$\tilde{a} = 0$$
 yields $E_b^{I=1}(\tilde{a}=0) \approx -11.7 \,\mathrm{MeV}$

In view of the convexity, the actual binding energy likely to slightly exceed this linear extrapolation

 \rightarrow use this result for the isovector channel to estimate the $\bar{B}B^*$ binding in the isoscalar channel

Assuming that the isoscalar binding energy in the $m_Q \to \infty$ limit is 3 times larger than for the isovector,

$$E_b^{I=0}(\tilde{a}=0) \approx 3 \cdot (-11.7) = -35 \,\text{MeV}$$

$$X(3872)$$
 at $\bar{D}D^*$ threshold $\rightarrow E_b^{I=0}(\tilde{a}(D)) \approx 0$

Linear extrapolation to $\tilde{a}(B)$ yields $\bar{B}B^*$ binding energy in the isoscalar channel $\approx -20\,\mathrm{MeV}$

Heavy Quark Nuclear Physics!

The newly discovered $Z_c(3900)$ isovector resonance confirms and refines the estimates for the masses of the putative $\bar{B}B^*$ and \bar{B}^*B^* bound states.

it immediately leads to several predictions:

- two I = 0 narrow resonances in bottomonium system
- $\sim 23 \text{ MeV below } Z_b(10610) \text{ and } Z_b(10650), \text{ i.e.}$
- $\sim 20 \text{ MeV}$ below $\bar{B}B^*$ and \bar{B}^*B^* thresholds

- I = 0 narrow resonance very close to \bar{D}^*D^* threshold
- I = 1 narrow resonance slightly above D^*D^* threshold

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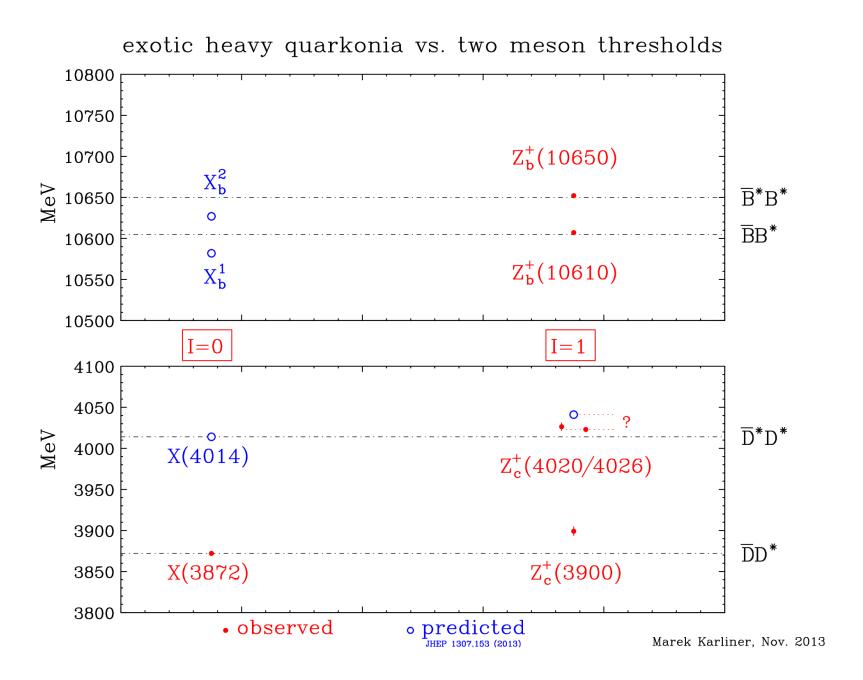
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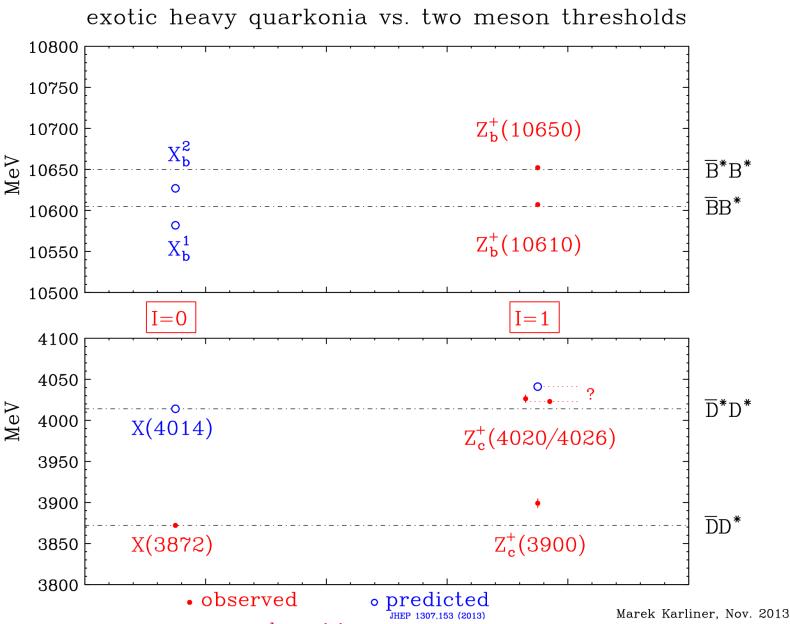
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M. Karliner, exotics with b and c

- $\sim 20~{\rm MeV}$ below $\bar{B}B^*$ and \bar{B}^*B^* thresholds
- I = 0 narrow resonance very close to \bar{D}^*D^* threshold
- I = 1 narrow resonance slightly above \bar{D}^*D^* threshold

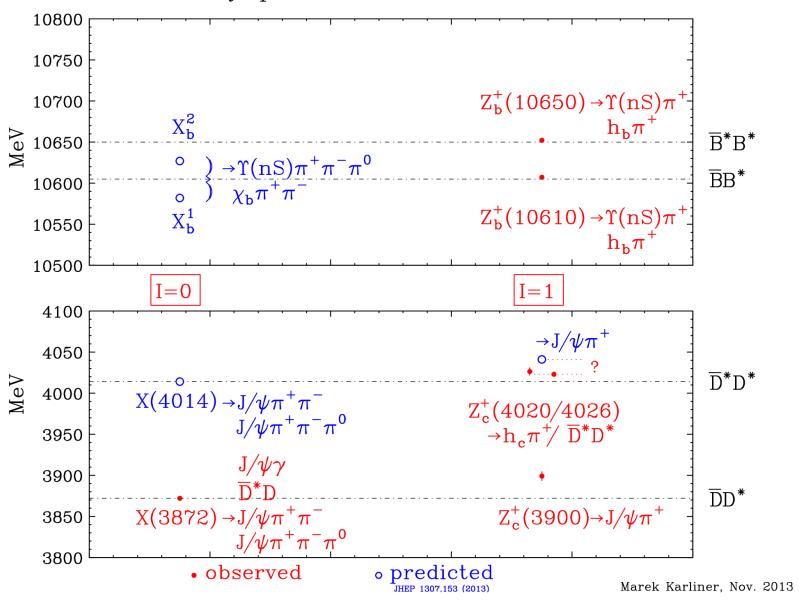
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reported by BES: Aug 13, arXiv:1308.2760 Z_c^+(4025): M{=}4026.3 \pm 2.6 + 3.7, \ \Gamma = 24.8 \pm 5.6 \pm 7.7 \ \text{MeV} \quad \text{seen in } D^*D^* \\ \text{\& Sep 10 arXiv:1309.1896} \\ Z_c^+(4020): M{=}4022.9 \pm 0.8 + 2.7, \ \Gamma = 7.9 \pm 2.7 \pm 2.6 \ \text{MeV} \quad \text{(mass a bit low)} \\ \text{seen in } h_c\pi
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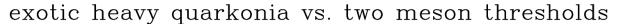


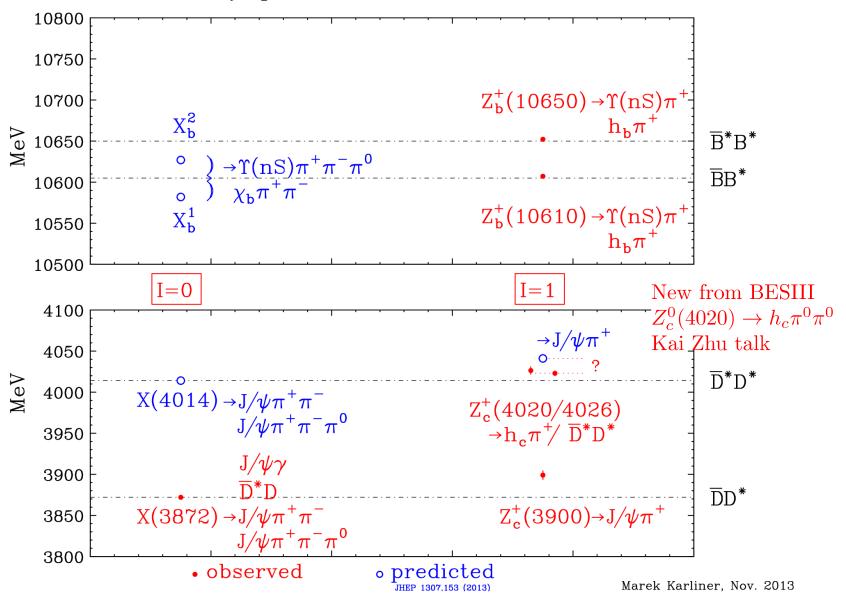


caveat: some masses = peak positions, with interference \neq pole mass

exotic heavy quarkonia vs. two meson thresholds







Caveat about mass predictions:

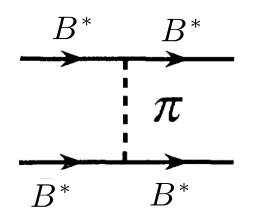
$$m(D^*) - [m(D) + m(\pi)] \approx 0^{\pm},$$

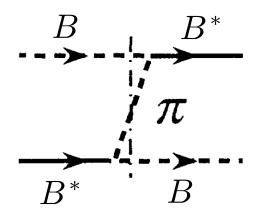
dep. on D^* and π charge, affecting $D^* \to D\pi$ (strong decay)

VS.

$$B^* \to B\gamma$$
 (EM decay)

so $\bar{D}D^*$ and \bar{D}^*D^* potential due to π exchange might be slightly different from $\bar{B}B^*$ and \bar{B}^*B^*





Likely observable at LHC and Tevatron:

Guo, Meißner & Wang, arXiv:1308.0193

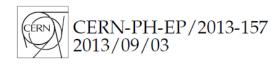
 \sim nb x-section for Z_b(10610) and Z_b(10650)

x-section for Z_c(3900) and Z_c(4020) larger by a factor of 20-30

large enough to be observed

x-section for neutral exotic states?

Null result from CMS:





CMS-BPH-11-016

Search for a new bottomonium state decaying to $Y(1S)\pi^+\pi^-$ in pp collisions at $\sqrt{s}=8\,\text{TeV}$

The CMS Collaboration* Abstract

The results of a search for the bottomonium counterpart, denoted as X_b , of the exotic charmonium state X(3872) is presented. The analysis is based on a sample of pp collisions at $\sqrt{s} = 8$ TeV collected by the CMS experiment at the LHC, corresponding to an integrated luminosity of 20.7 fb⁻¹. The search looks for the exclusive decay channel $X_b \to Y(1S)\pi^+\pi^-$ followed by $Y(1S) \to \mu^+\mu^-$. No evidence for an X_b signal is observed. Upper limits are set at the 95% confidence level on the ratio of the inclusive production cross sections times the branching fractions to $Y(1S)\pi^+\pi^-$ of the X_b and the Y(2S). The upper limits on the ratio are in the range 0.9–5.4% for X_b masses between 10 and 11 GeV. These are the first upper limits on the production of a possible X_b at a hadron collider.

-

The null result from CMS in search for

$$X_b \rightarrow Y(1S)\pi^+\pi^-$$

is excellent news for the molecular picture,

since isoscalar
$$X_b$$
 with $J^{PC} = 1^{++}$

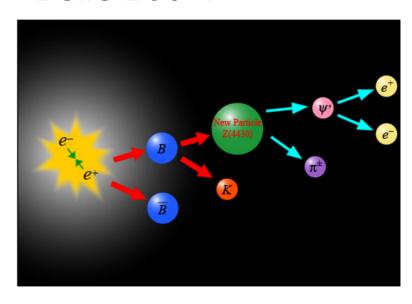
cannot decay into
$$\Upsilon(1S)\pi^+\pi^-$$

It can decay into
$$\Upsilon(1S) \omega$$
 or $\chi_b \pi^+ \pi^-$

Z(4430)

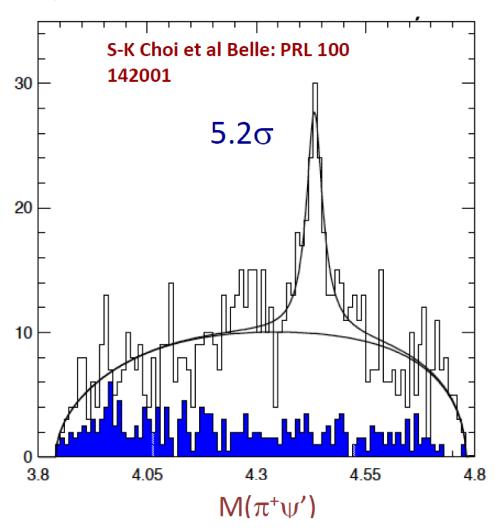
also manifestly exotic, but odd man out

Belle 2007:

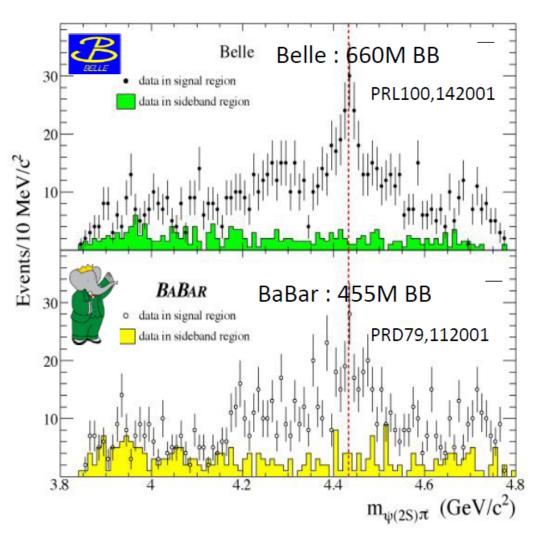


$$M = 4433 \pm 4 \pm 2 \text{ MeV}$$

$$\Gamma = 45^{+18+30}_{-13-13} \text{ MeV}$$



Z(4430) not seen by BaBar



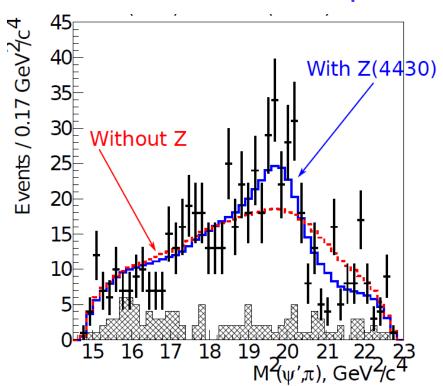
$$Z(4430)^{\pm} \rightarrow \psi' \pi^{\pm}$$

Significant signal at Belle v.s.

Only hint with 1.9σ at BaBar

Statistically, both are not contradictory, answer requires higher statistics data.

2013: Belle 4-dim amplitude analysis -



$$M = 4485^{+22+28}_{-22-11} \text{ MeV}/c^2$$

 $\Gamma = 200^{+41+26}_{-46-35} \text{ MeV}.$

$$6.4\sigma$$

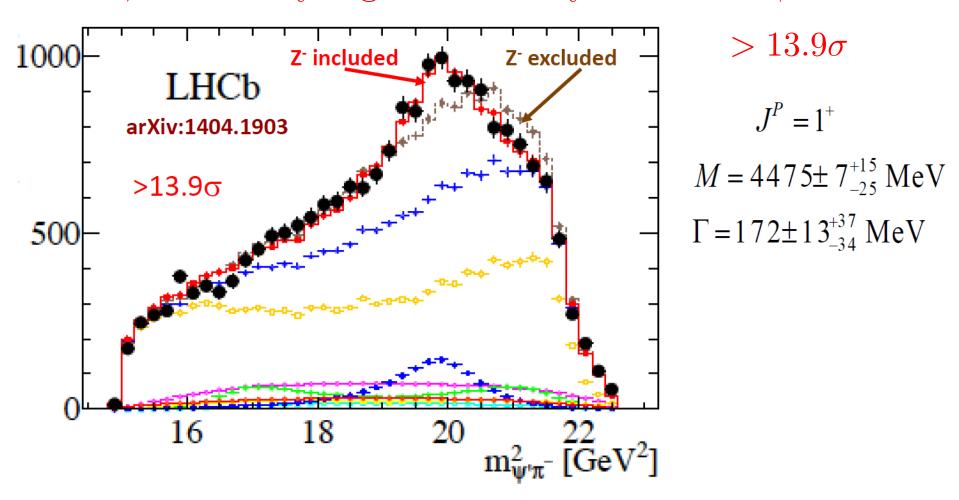
$$J^P = 1^+$$

Belle preliminary, K. Chilikin, Moriond QCD 2014:

$$\frac{BR(Z(4430) \to \psi' \pi^{-})}{BR(Z(4430) \to J/\psi \pi^{-})} \approx 10$$

natural if Z(4430) radius is large, as then w.f. overlap with ψ' larger than with J/ψ

LHCb, 2014: very high stats analysis of $B \to \psi' \pi^- K^+$



large width: $\Gamma(Z(4430)) \gg \Gamma(Z_b)$, $\Gamma(Z_c)$.

⇒ unlikely to be a simple "hadronic molecule"

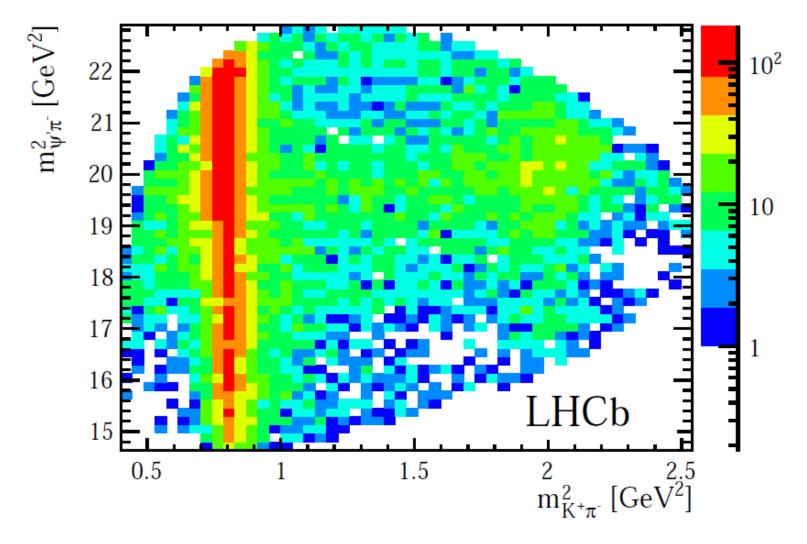


Figure 2: Dalitz plot for $B^0 \to \psi' K^+ \pi^-$ candidates. The background has been subtracted using sWeights determined by the fit shown in Fig. 1. The colors indicate number of signal events per bin. The dominant vertical band is due to the $K^*(892)$ resonance. A faint vertical band at $m_{K^+\pi^-}^2$ around 2 GeV² is due to the $K_2^*(1430)$ peak. A horizontal $Z(4430)^-$ band is also visible $(m_{\psi'\pi^-}^2$ around 20 GeV²).

Argand plot

Breit-Wigner resonant behaviour:

counter-clockwise rotation of the amplitude as function of energy

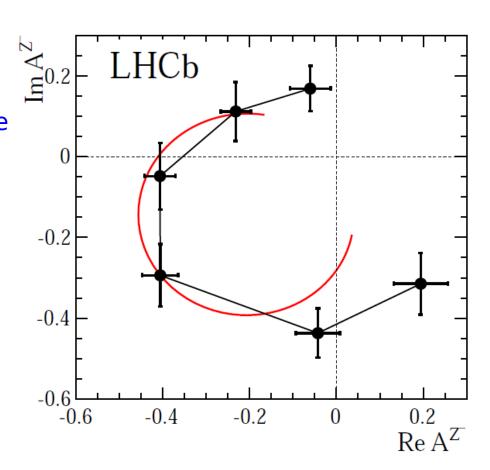


Figure 3: Fitted values of the Z_1^- amplitude in six $m_{\psi'\pi^-}^2$ bins, shown in an Argand diagram (connected points with the error bars, $m_{\psi'\pi^-}^2$ increases counterclockwise). The red curve is the prediction from the Breit-Wigner formula with a resonance mass (width) of 4475 (172) MeV and magnitude scaled to intersect the bin with the largest magnitude centered at (4477 MeV)². Units are arbitrary. The phase convention assumes the helicity-zero $K^*(892)$ amplitude to be real.

Should see the neutral partner, $Z(4430)^0$.

For example in

$$B^- \to \psi' \pi^0 K^-$$

or

$$e^+e^- \rightarrow \psi'\pi^0\pi^0$$

two interesting coincidences related to Z(4430) mass:

$$M(Z(4430) - M(Z_c(3900)) \approx 575 \pm 25 \text{ MeV}$$

 $M(\psi') - M(J/\psi) = 589 \text{ MeV}$

$$\implies Z(4430)$$
 a radial excitation of $Z_c(3900)$?

if so, a node-in the w.f., so much bigger overlap with ψ' w.f. still, hard to understand why $\Gamma(Z(4430)) \gg \Gamma(Z_c(3900))$

If radial excitation, analogue in the bottom system:

$$M(Z_b(10610) + 575 \text{ MeV} \approx 11185 \text{ MeV}$$

On the other hand, $\psi'\rho$ threshold nearby:

$$M(\psi') + M(\rho) = 4456 \text{ MeV}$$

if threshold effect \longrightarrow bottom analogue:

$$M(\Upsilon(2S)) + M(\rho) = 10793 \text{ MeV}$$

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both testable, but not at Belle, as

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 $M(\Upsilon(5S))=10870 \text{ MeV}$

$$\Sigma_b^+ \Sigma_b^-$$
 dibaryon ?

 Σ_b heavier, with $I=1\to {
m stronger}$ binding via π

→ deuteron-like J=1, I=0 bound state: "beautron" electric charges contribute extra ~3 MeV to binding energy

exp. signature:

$$\begin{split} (\Sigma_b^+ \Sigma_b^-) &\to \Lambda_b \, \Lambda_b \, \pi^+ \, \pi^- \\ &\Gamma(\Sigma_b^-) = 4.9 \pm 3 \, \, \mathrm{MeV}, \quad \Gamma(\Sigma_b^+) = 9.7 \pm 3 \, \, \mathrm{MeV} \\ &\text{so might be visible} \end{split}$$

should be seen in lattice QCD

$$(\Sigma_c^0 \Sigma_c^+) \to \Lambda_c \Lambda_c \pi^- \pi^0$$
 as well?

doubly heavy baryons QQq (bbq,ccq, bcq)

- not exotic, must exist
- excellent challenge for EXP (LHCb!) & TH (lattice?) LHCb: many $B_c(\bar{b}c)$ -s $\Longrightarrow bcq, ccq$ baryons $(bbq) \to (\bar{c}cs) (\bar{c}cs)q \to J/\psi J\psi \Xi$ unique signature, w/o background
- -QQq and $QQ\bar{q}\bar{q}$ have the same color structure \implies once QQq mass is known, can immediately predict $QQ\bar{q}\bar{q}$ mass:

$$m(cc\bar{u}\bar{d}) = m(\Xi_{ccu}) + m(\Lambda_c) - m(D^0) - \frac{1}{4}[m(D^*) - m(D)]$$

SUMMARY

a simple and consistent picture emerges from Belle, BES, CLEO and LHCb data:

- the new narrow exotic resonances are loosely bound states of $\bar{D}D^*$, \bar{D}^*D^* , $\bar{B}B^*$ and \bar{B}^*B^*
- $Z(4430)^{\pm}$: odd man out. What about $Z(4430)^{0}$?
- predictions:
- $-\bar{D}^*D^*$ in I=0 and I=1 channels seen!
- new isosinglet $\bar{B}B^*$ and \bar{B}^*B^* states below threshold
- "heavy deuteron": $\Sigma_b^+ \Sigma_b^-$ (and maybe also $\Sigma_c^+ \Sigma_c^-$?)
- challenge for EXP: doubly heavy baryons QQq (LHCb?)
- $QQq \Rightarrow$ accurate prediction for $QQ\bar{q}\bar{q}$ tetraquark
- challenge for TH: derive from QCD

Supplementary transparencies

Bound states in QED_2 , a confining theory

$$\mathcal{L} = \sum_{k} \bar{\psi}_k (i\gamma^{\mu} D_{\mu} - m_k) \psi_k - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- transform to bosonic variables χ_k : $\psi_k \gamma_\mu \psi_k = -\frac{1}{\sqrt{\pi}} \epsilon_{\mu\nu} \partial^\nu \chi_k$
- integrate out F^2

$$\mathcal{L} = \frac{1}{2} \sum_{k} (\partial_{\mu} \chi_{k})^{2} - \frac{e^{2}}{2\pi} (\sum_{k} \chi_{k})^{2} + \sum_{k} m_{k}^{2} \cos \sqrt{4\pi} \chi_{k}$$

 $\alpha \equiv \frac{e^2}{4\pi}$; eqs. of motion in static case:

$$\chi_k'' - 4\alpha(\sum_l \chi_l) - \sqrt{4\pi} m_k^2 \sin\sqrt{4\pi} \chi_k = 0$$

look for nontrivial finite-energy solutions

 \Longrightarrow states composed of solitons and anti-solitons in χ_k

fermions: $\chi_k(\infty) = \sqrt{\pi}$, anti-fermions: $\chi_k(\infty) = -\sqrt{\pi}$

energy density:

$$\epsilon = \frac{1}{2} \sum_{k} \chi_{k}^{'2} + 2\alpha (\sum_{l} \chi_{l})^{2} + \sum_{k} m_{k}^{2} (1 - \cos \sqrt{4\pi} \chi_{k})$$

now focus on 2 flavors:

heavy Q with mass M, and light q with mass m

can solve analytically in the limit M=m

and numerically in the general case $M \neq m$

 $\Longrightarrow Q\bar{q}$ meson as bound state of $\chi_1(x)$ soliton and $\chi_2(x)$ anti-soliton

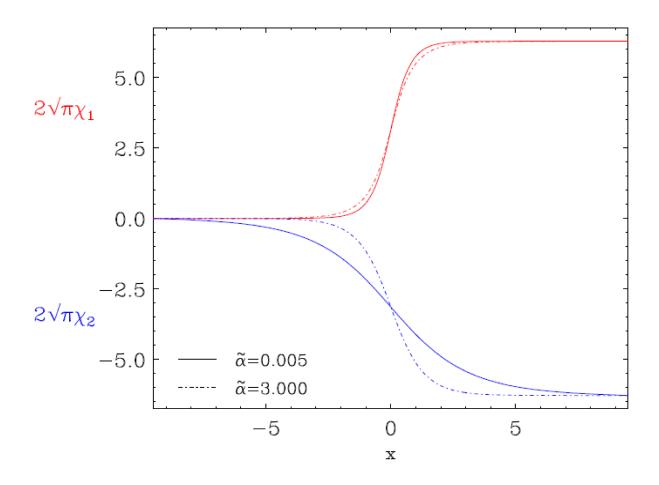


Figure 1. The soliton and anti-solitons profiles $\chi_1(x)$ and and $\chi_2(x)$ in a $(Q\bar{q})$ meson, for $\tilde{m}_1=2.0$ and $\tilde{m}_2=0.5$. Continuous lines: $\tilde{\alpha}=0.005$; dash-dotted lines: $\tilde{\alpha}=3$.

$(Q\bar{Q}q\bar{q})$ tetraquarks:

a Q soliton, a \bar{Q} anti-soliton, a q soliton and a \bar{q} antisoliton

$$Q \Leftrightarrow \bar{Q}, q \Leftrightarrow \bar{q} \text{ symmetry: } \chi_{\bar{Q}}(x) = -\chi_{Q}(x), \chi_{\bar{q}}(x) = -\chi_{q}(x)$$

 \implies interaction term multiplying coupling α vanishes so for any α the mass of the state is

$$\mathcal{M}(Q\bar{Q}q\bar{q},\alpha) = 2M + 2m = \mathcal{M}(Q\bar{q};\alpha=0) + \mathcal{M}(\bar{Q}q;\alpha=0)$$

and
$$\mathcal{M}(Q\bar{Q}) = 2M, \ \mathcal{M}(q\bar{q}) = 2m$$

no phase space for $(Q\bar{Q}q\bar{q}) \to (Q\bar{Q}) + (q\bar{q})$

in addition $\mathcal{M}(Q\bar{q})$ is a monotonically increasing function of α :

$$\frac{\partial}{\partial \alpha} M(Q, \bar{q}; \alpha) > 0$$
 (Feynman-Hellmann theorem)

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Therefore, for any $\alpha \neq 0$ have

$$\mathcal{M}(Q\bar{Q}q\bar{q},\alpha) < \mathcal{M}(Q\bar{q};\alpha) + \mathcal{M}(\bar{Q}q;\alpha)$$

so decay
$$(Q\bar{Q}q\bar{q}) \to (Q\bar{q}) + (\bar{Q}q)$$

is also kinematically forbidden

$$\implies$$
 in QED₂ the $(Q\bar{Q}q\bar{q})$ tetraquark is stable

analogous analysis: in QED₂ ($QQ\bar{q}\bar{q}$) tetraquark can always decay into two ($Q\bar{q}$) heavy-light mesons