AVERAGE THERMAL EVOLUTION OF THE UNIVERSE[1]

Natacha Leite *(U. Coimbra, Portugal and U. Hamburg, Germany)*

Alex H. Blin *(U. Coimbra, Portugal)*

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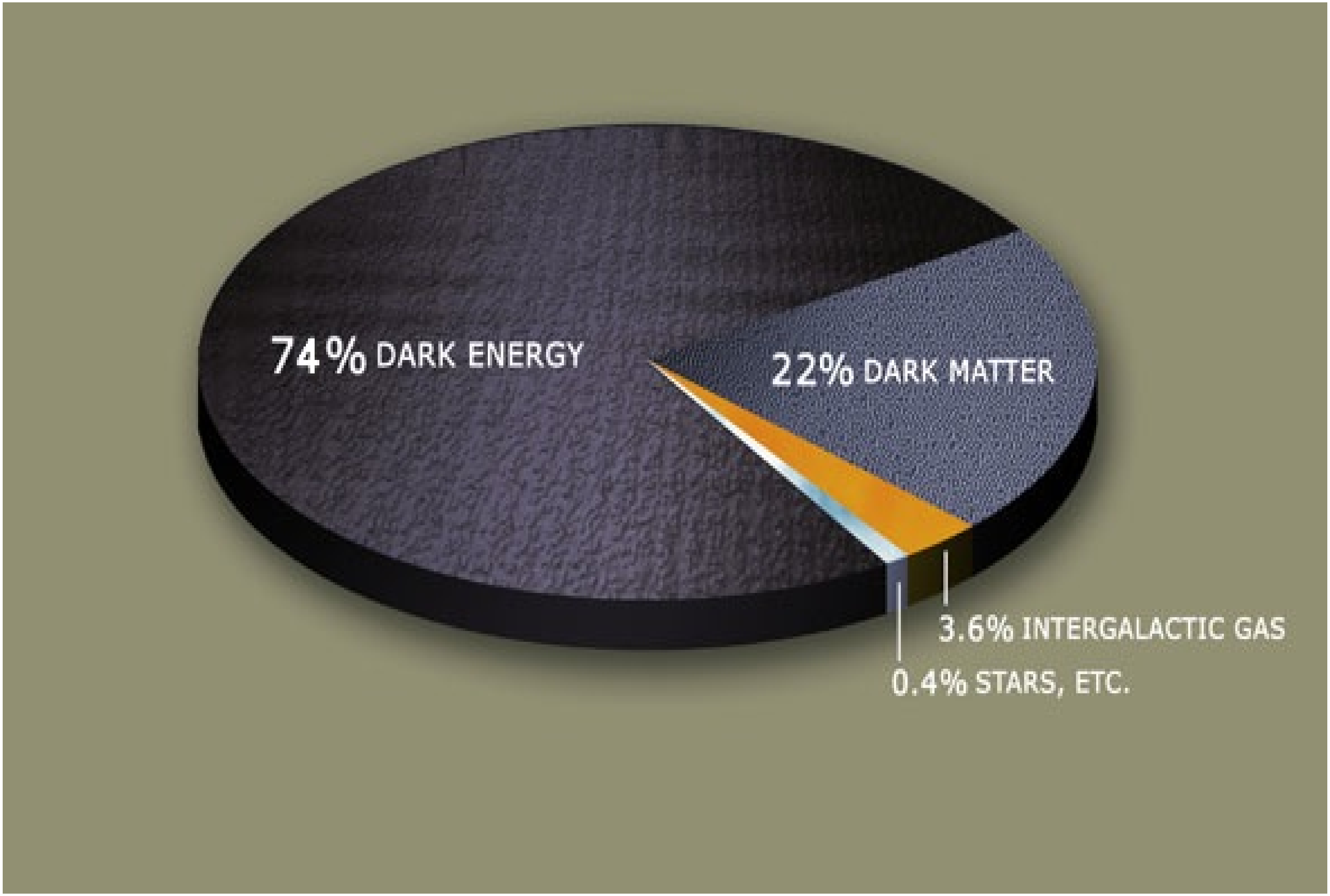
Introduction

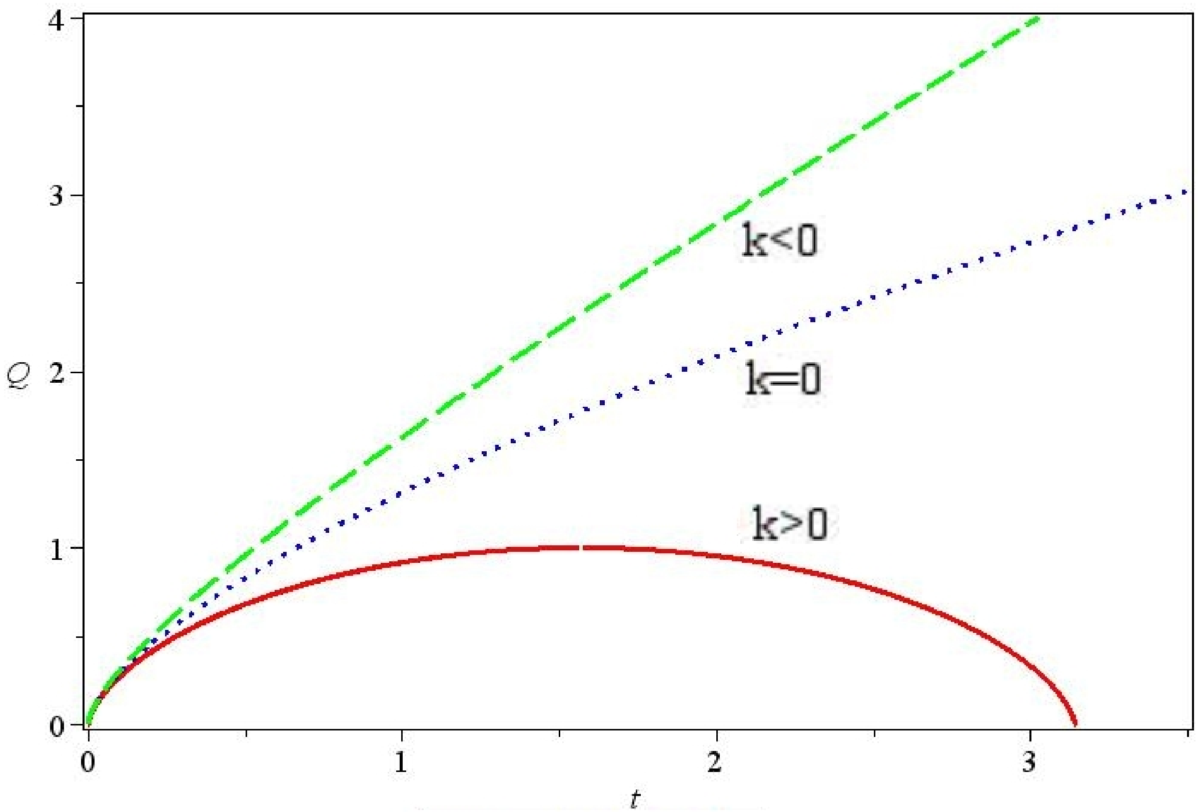


What does this pie chart mean?

Einstein's original equation ($\hbar = c = k_B = 1$ in most eqs.)

$$R_{mn} - \frac{1}{2} g_{mn} R = 8\pi G T_{mn}$$

has no static solution. Depending on the amount of matter: open or closed.



Einstein's "biggest blunder": force static solution by including an extra term, *"cosmological constant"* Λ ("dark energy"):

$$R_{mn} - \frac{1}{2} g_{mn} R - \Lambda g_{mn} = 8\pi G T_{mn} \ .$$

Cosmological principle of homogeneity and isotropy:
Friedmann (Фридман)-Lemaître-Robertson-Walker (FLRW) metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - Q^2(t)[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)] \ .$$

$Q(t)$ is size parameter of universe, sign of k determines universe open or closed $\Rightarrow$
Friedmann eq. from 00 component

$$\dot{Q}^2 = \frac{8\pi}{3} G\rho(Q) Q^2 - k + \frac{1}{3}\Lambda Q^2$$

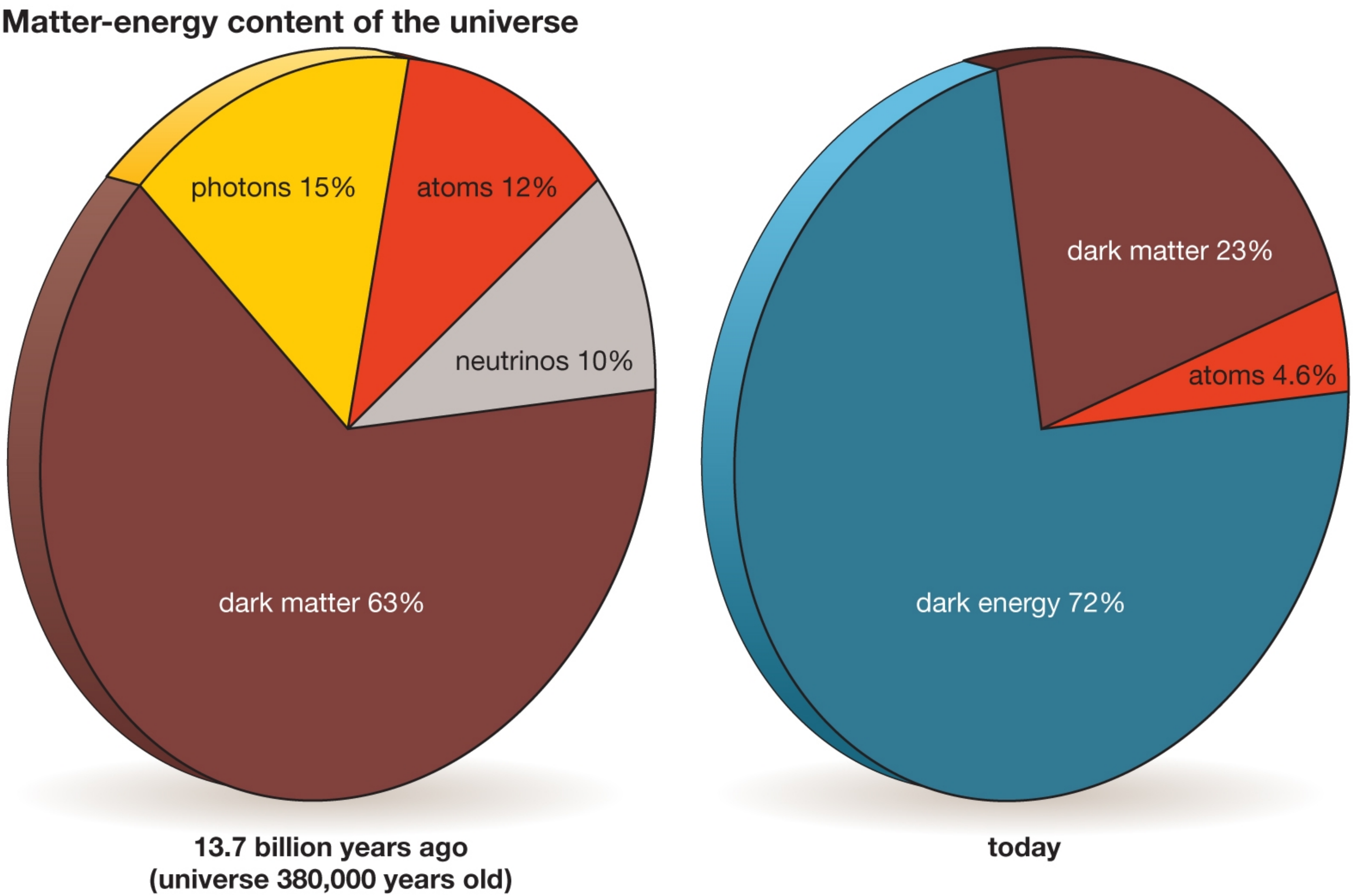
Divide by $H_0^2 = (\dot{Q}_0/Q_0)^2$ and set the scale parameter $Q_0 = 1$:

$$1 = \Omega_\rho + \Omega_k + \Omega_\Lambda$$

Ω_ρ : dark matter, baryonic matter, neutrinos, radiation
Ω_k compatible with 0
Ω_Λ explains *accelerated* expansion[2],[3]

[2] S. Perlmutter et al., Nature 391, 51 (1998)
[3] A.G. Riess et al., Astron. J. 116, 1009 (1998)

The proportions vary with the epoch:



WMAP 7 yr. observations[4]
$\rightarrow$ Newer data for reference, from Planck 2013[5] & WMAP 9 yr.[6]

Dark energy: $\Omega_\Lambda = .725 \pm .016 \quad \rightarrow .683$

Dark matter: $\Omega_c = .229 \pm .0025 \quad \rightarrow .268$

Baryonic matter: $\Omega_b = .0458 \pm .0016 \quad \rightarrow .049$

Curvature: $\Omega_k = -.0024 \pm .0055 \quad \rightarrow$ still compatible with 0 (flat)

Age of the universe: $\tau_0 = (13.76 \pm .11)$ Gyr $\quad \rightarrow 13.817$

Hubble constant: $H_0 = (70.2 \pm 1.4)$ km s^{-1} Mpc^{-1} $\quad \rightarrow 67.3$

$= 2.28 \times 10^{-18}$ s^{-1}

[4] E. Komatsu et al.(WMAP), Astrophys. J.Suppl. 192:18 (2011)
[5] Planck Collaboration, arXiv:1303.5062 (2013)
[6] G. Hinshaw et al. (WMAP), Astrophys. J. Suppl. 208,19 (2013)

Equations of state

$$P_i = \omega_i \rho_i$$

Allowed states per phase space cell:

$$n_i = \frac{g_i}{h^3} \int \mathrm{d}^3 p f_i(p)$$

$$\rho = \frac{g_i}{h^3} \int \mathrm{d}^3 p f_i(p) E_i$$

$$P = \frac{g_i}{h^3} \int \mathrm{d}^3 p f_i(p) \frac{p}{3E_i}$$

$$f_i(p) = (\exp\{(E_i - \mu_i)/kT_i\} \pm 1)^{-1}$$

(+) fermions, (-) bosons, g_i degeneracy, μ_i chemcal potential (0 in good approx.)

In the relativistic limit for boson (B) and fermion (F) species:

$$\rho_r = \frac{\pi^2}{30} \left(\sum_B g_B + \frac{7}{8} \sum_F g_F \right) T^4 \equiv \frac{\pi^2}{30} N(T) T^4$$

Assume perfect fluid ($u_k = (1,0,0,0)$ in co-moving coordinates) with pressure P:

- $T_{kl} = (\rho + P)u_k u_l - P g_{kl}$

With conservation of energy

$$T^{mn}{}_{;n} = 0$$

$\Rightarrow$ Q, T-dependence:

radiation and ultrarelativistic matter
$\omega_r = 1/3, \quad \rho_r \propto 1/Q^4, \quad T_r \propto 1/Q$

nonrelativistic matter (dust)
$\omega_m = 0, \quad \rho_m \propto 1/Q^3, \quad T_m \propto 1/Q^2$

dark energy
$\omega_\Lambda = -1, \quad \rho_\Lambda \propto const.$

curvature
$\omega_k = -1/3, \quad \rho_k \propto 1/Q^2$

Dark energy

The Planck scale

QM combined with GR, Schwarzschild radius = Compton wavelength $\Rightarrow$

$E_{Pl} = \sqrt{\hbar c^5/G}$ Planck energy

$l_{Pl} = \sqrt{G\hbar/c^3} \cong 10^{-35}$m Planck length

At the level of the Planck scale 10^{-35}m, GR and QM are fundamentally interwoven. The metric g_{mn} itself has to be regarded a quantum variable.

Conformal fluctuations

keep light cone structure of space-time intact, important for *causality*:

- $g_{mn} = \Phi^2 \bar{g}_{mn} = (1 + \varphi)^2 \bar{g}_{mn} \quad ,$

$\bar{g}_{mn}$ is classical or background metric about which the fluctuations occur,

fluctuation average is $< \varphi > = < \varphi_{,m} > = 0$.

The scalar field φ represents an additional degree of freedom.

The Einstein eq. can be derived from the *variation of the Hilbert action* ($\hbar = c = 1$):

- $$S = S_{\mathrm{g}} + S_{\mathrm{m}} = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}R + S_{\mathrm{m}}$$

S_{g} = gravitational action without Λ (i.e. **not** $S_{\mathrm{g}} \propto \int \mathrm{d}^4x\sqrt{-g}(R + 2\Lambda)$),

S_{m} = matter part,

- $$\delta S = 0 \quad .$$

Example: vacuum ($S_{\mathrm{m}} = 0$) in flat background spacetime $\bar{R} = 0$

$\Rightarrow$ *Fluctuation average of the squared four-distance*

$$< x^2 > = x^2 + l^2/3\pi \quad .$$

Spacetime is *fuzzy* at the level of the Planck scale.

Variation of the action with fluctuations

- $$\delta S = \frac{\delta S}{\delta g^{ik}}\delta g^{ik} = \frac{\delta S}{\delta \bar{g}^{ik}}\delta \bar{g}^{ik} + \frac{\delta S}{\delta \varphi}\delta \varphi$$

The variations are independent $\Rightarrow$ two equations, one $\propto \delta\bar{g}^{ik}$, the other $\propto \delta\varphi$, the φ-dependence can then be eliminated.

At this point the field φ, which represents the quantum fluctuations, is treated to lowest order as an effective classical field, since the variational principle is applied instead of a full Feynman path integral approach.

$\Rightarrow$ *Einstein equation*

$$\bar{R}_{ik} - \frac{1}{2}\bar{g}_{ik}\bar{R} - \bar{g}_{ik}\Lambda = 8\pi G T_{ik}$$

where a cosmological constant arises in the form[7]

$$\Rightarrow \quad \Lambda = -\frac{1}{4}(8\pi G \bar{g}^{mn} T_{mn} + \bar{R}) \ .$$

[7] A.H. Blin, arXiv:astro-ph/0107503 (2001)& Af. J. Math. Phys. 3, 121 (2006)

This result does not change if starting with a some *prior* $\bar{\Lambda}$
(e.g. due to QFT vacuum fluctuations, the old 120-order-of-magnitude problem):

$$S_{\mathrm{g}} = \frac{1}{16\pi G} \int \mathrm{d}^4 x \sqrt{-g}(R + 2\bar{\Lambda})$$

The contribution of $\bar{\Lambda}$ cancels in the present approach.[8]

[8] A.H. Blin, Int. J. Theor. Math. Phys. 2, 61 (2012)

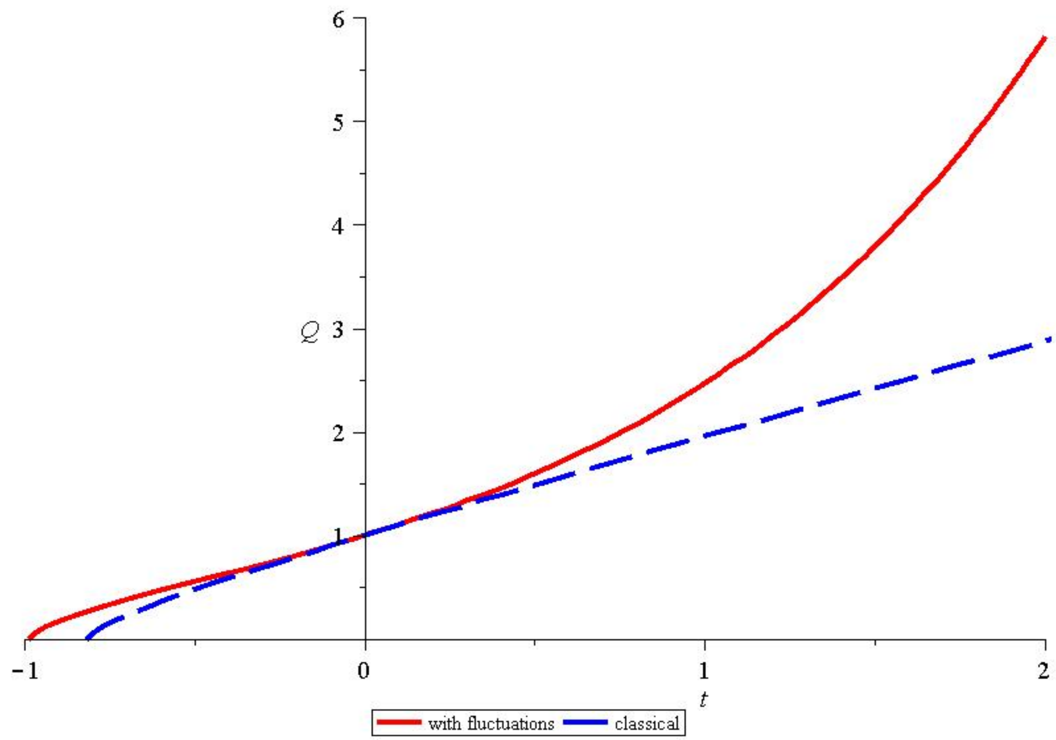
Matter part $\delta S_m = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\delta g_{mn}T^{mn}$.

For perfect fluid:

$$\Lambda = \frac{3(\dot{Q}^2 + k)}{Q^2} - 8\pi G\rho_0\frac{Q_0^3}{Q^3}$$

It comes out *constant* and is not quintessence (scalar field coupled via potential).

Allows accelerated expansion (fig. in dust approx.):



Dark matter

Hypothesis: cold dark matter (WIMP) neutralinos (lightest superymmetric particle, linear combination of the super-partners of the gauge and Higgs fields), since properties quantitatively studied [9, 10], $M_{\tilde{\chi}} \simeq 10$ GeV ... 1 TeV.

- $T > M_{\tilde{\chi}}$ thermal equilibrium due to annihilations and production processes
- $T < M_{\tilde{\chi}} \ldots T \simeq M_{\tilde{\chi}}/25$, annihilating until annihilation rate drops below Hubble rate
- **but** still maintain heat bath temperature via elastic collisions with fermions
- $T < T_{kd} = [1.2 \times 10^{-2} M_{Pl}/M_{\tilde{\chi}}(M_{\tilde{L}}^2 - M_{\tilde{\chi}}^2)^2]^{-\frac{1}{4}}$, kinetic decoupling

  ($M_{\tilde{L}} \simeq 200$ GeV slepton mass, M_{Pl} = Planck mass)

  From now on $T \propto 1/Q^2$.

[9] S. Hofmann, D.J. Schwarz, H. Stöcker, Phys.Rev.D64:083507 (2001)
[10] T. Bringmann, S. Hoffmann, JCAP 0407:016 (2007)

Standard model particles

Relativistic components

$$\rho_r = \frac{\pi^2}{30} N(T) T^4$$

Annihilation thresholds (adapted from PDG[11]):

<table>
<tr><th></th><th></th><th colspan="2">$4N^{Maj}(T)$</th><th colspan="2">$4N^{Dir}(T)$</th></tr>
<tr><th>T</th><th>New particles</th><th>$4N_r(T)$</th><th>$4N_\nu(T)$</th><th>$4N_r(T)$</th><th>$4N_\nu(T)$</th></tr>
<tr><td>$T < m_e$</td><td>γ's</td><td>8</td><td rowspan="2">21</td><td>8</td><td rowspan="2">42</td></tr>
<tr><td>$m_e < T < T_{D_\nu}$</td><td>$e^\pm$</td><td>22</td><td>22</td></tr>
<tr><td>$T_{D_\nu} < T < m_\mu$</td><td>ν's</td><td colspan="2">43</td><td colspan="2">64</td></tr>
<tr><td>$m_\mu < T < m_\pi$</td><td>$\mu^\pm$</td><td colspan="2">57</td><td colspan="2">78</td></tr>
<tr><td>$m_\pi < T < T_c$</td><td>π's</td><td colspan="2">69</td><td colspan="2">90</td></tr>
<tr><td>$T_c < T < m_s$</td><td>$u, \bar{u}, d, \bar{d}$ + gluons $-\pi$'s</td><td colspan="2">205</td><td colspan="2">226</td></tr>
<tr><td>$m_s < T < m_c$</td><td>s and $\bar{s}$</td><td colspan="2">247</td><td colspan="2">268</td></tr>
<tr><td>$m_c < T < m_\tau$</td><td>c and $\bar{c}$</td><td colspan="2">289</td><td colspan="2">310</td></tr>
<tr><td>$m_\tau < T < m_b$</td><td>$\tau^\pm$</td><td colspan="2">303</td><td colspan="2">324</td></tr>
<tr><td>$m_b < T < m_{W,Z}$</td><td>b and $\bar{b}$</td><td colspan="2">345</td><td colspan="2">366</td></tr>
<tr><td>$m_{W,Z} < T < m_H$</td><td>$W^\pm$ and Z^0</td><td colspan="2">381</td><td colspan="2">402</td></tr>
<tr><td>$m_H < T < m_t$</td><td>H^0</td><td colspan="2">385</td><td colspan="2">406</td></tr>
<tr><td>$m_t < T$</td><td>t and $\bar{t}$</td><td colspan="2">427</td><td colspan="2">448</td></tr>
</table>

[11] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86:010001 (2012)

Entropy conservation:[12] $N_b(QT)_b^3 = N_a(QT)_a^3$

- Assume latent heat makes the annihilations proceed isothermically $\Rightarrow$ step function of $T(t)$ at each threshold.
- Between two thresholds $T_r \propto 1/Q$

Neutrinos

- Decouple when interaction rate Γ less than Hubble rate[12] at $T_{D_\nu} = 10^{10}$K: $\Gamma/H \simeq G_F^2 T^5 M_{Pl}/T^2$ (G_F= Fermi coupling constant)
- Follow radiation temperature $T_\nu \propto 1/Q$ until electrons annihilate
- Latent heat of electron annihilation makes photons hotter: above eq. at fixed Q $\Rightarrow$ & jump in N_r:

  $T_\gamma = (\frac{4}{11})^{\frac{1}{3}} T_\nu$ Today: $T_\gamma = 2.725$K, $T_\nu = 1.945$K

Baryons

- Not in the table since nonrelativistic when they form from quarks at $T_c = 2.3 \times 10^{12}$K
- Stay in thermal equilibrium with radiation until recombination $T = 3000$K
- Then follow $T_m \propto 1/Q^2$

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[12] E. Kolb, M. Turner, *The Early Universe*, Addison-Wesley (1989)

Results

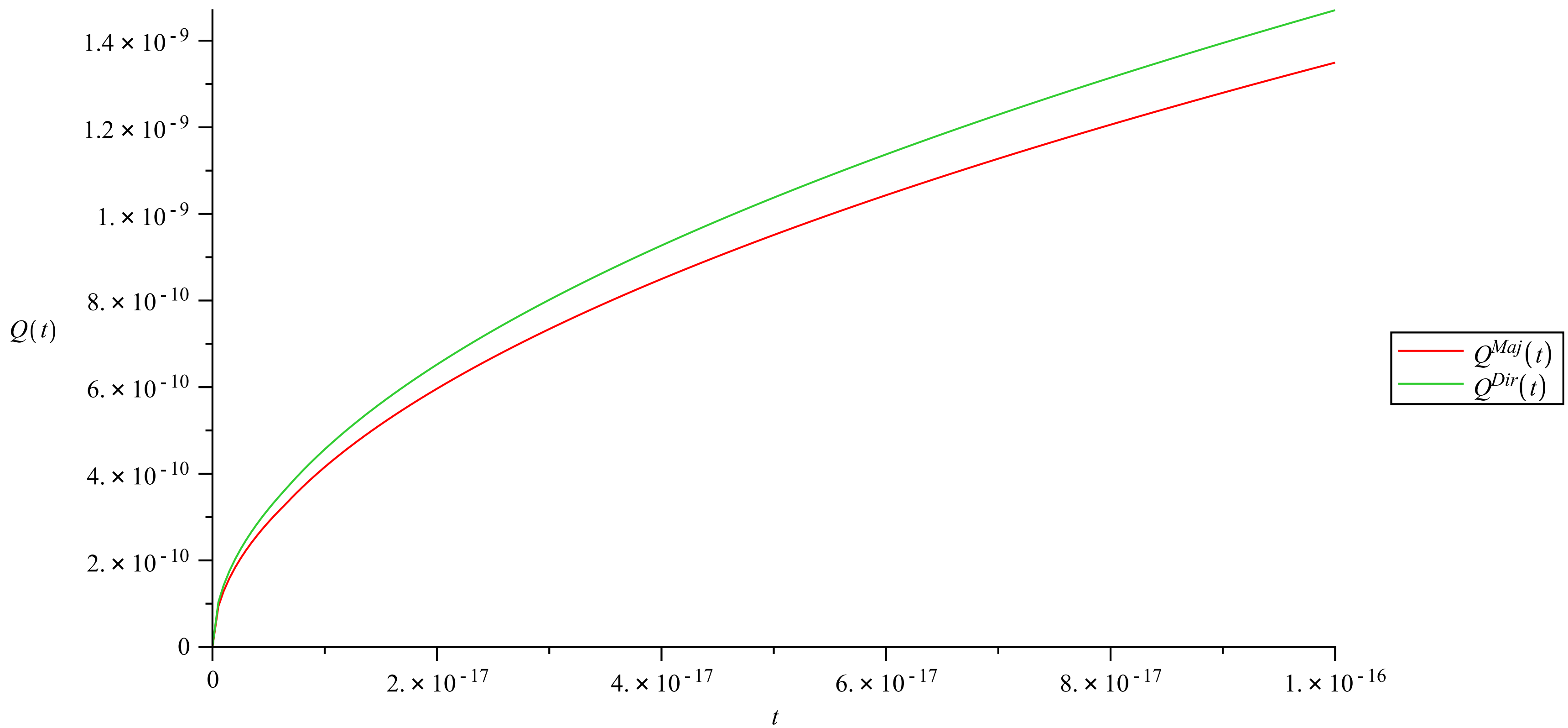
Study evolution *after inflation* ($t \sim 10^{-34}...10^{-32}$s)

Time t in units H_0^{-1}, densities Ω in units of critical density $3H_0^2/8\pi G$

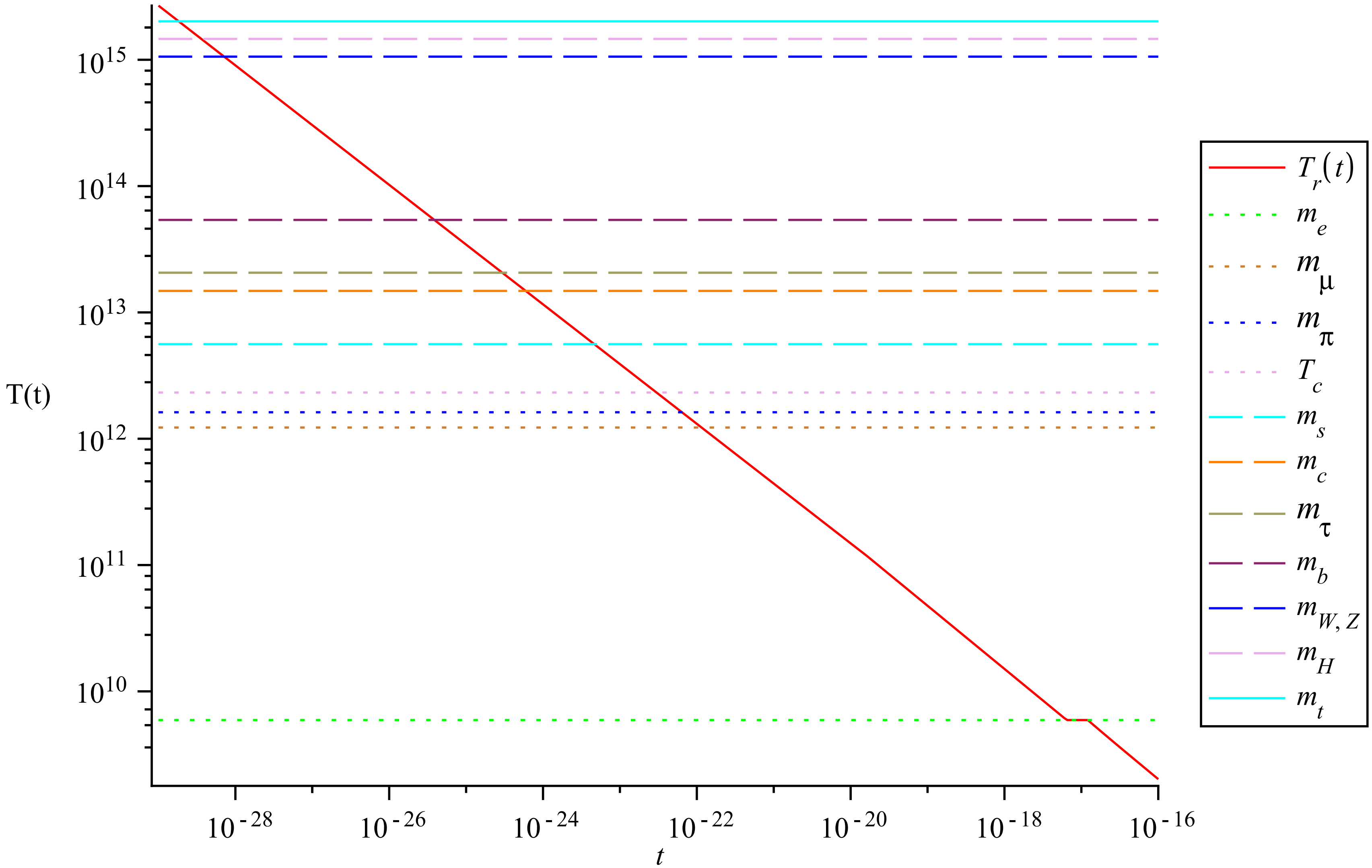
Equation of motion - Friedmann eq. rewritten:

$$\dot{Q}(t) = \sqrt{[\Omega_\gamma(t) + \Omega_\nu(t) + \Omega_b(t) + \Omega_c(t)]\ Q(t)^2 + \Omega_k + \Omega_\Lambda Q(t)^2}$$

Scale factor if starting with the same value in past:

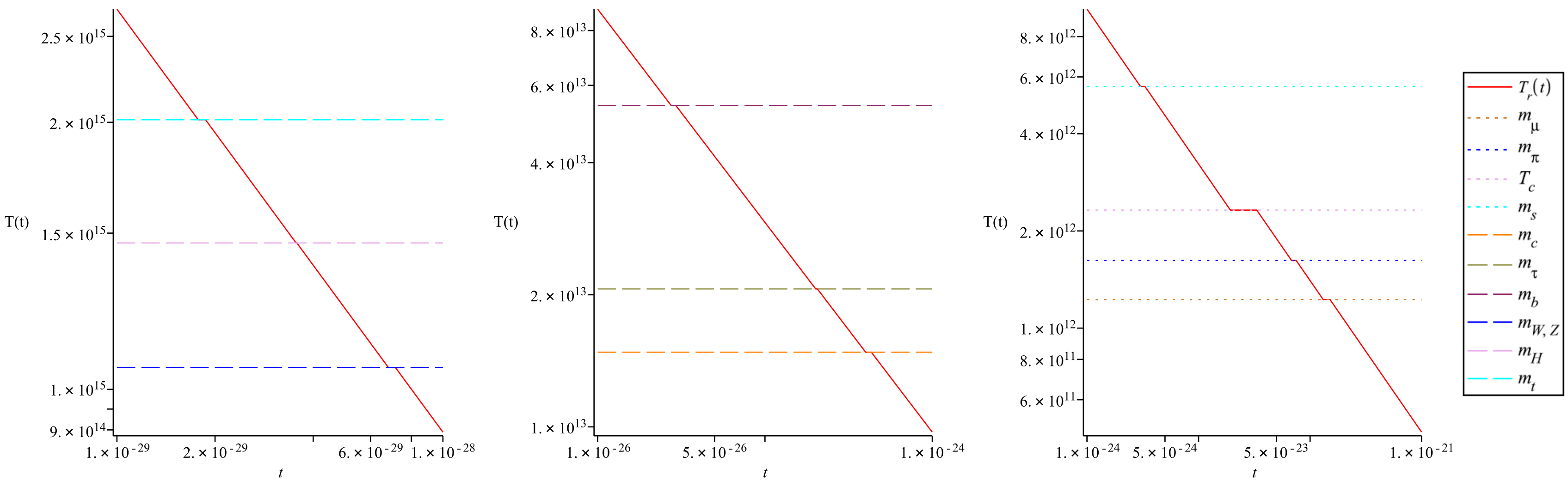


Radiation temperature assuming Majorana neutrinos:



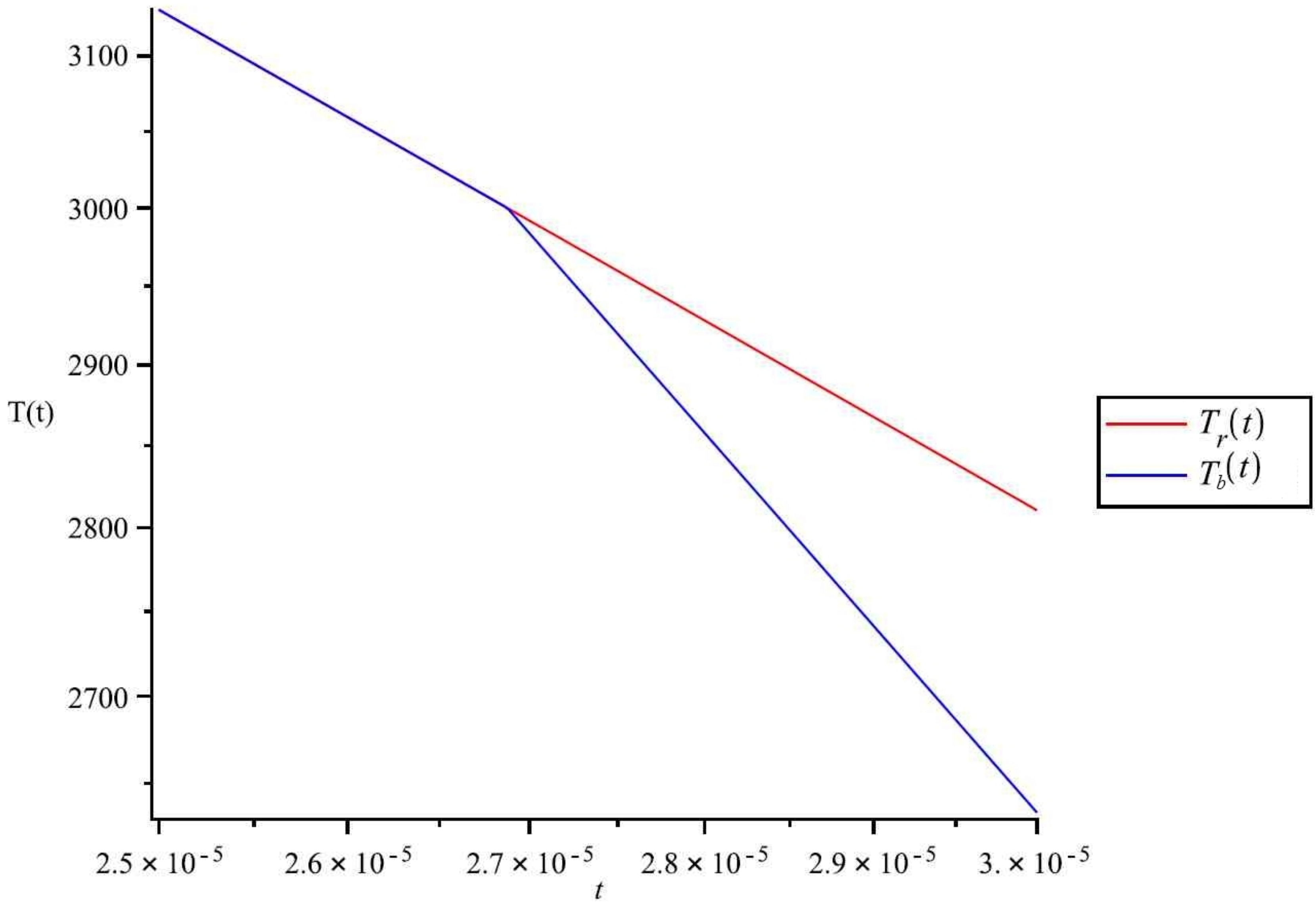
Only electron-positron threshold appreciable.

Zoom:

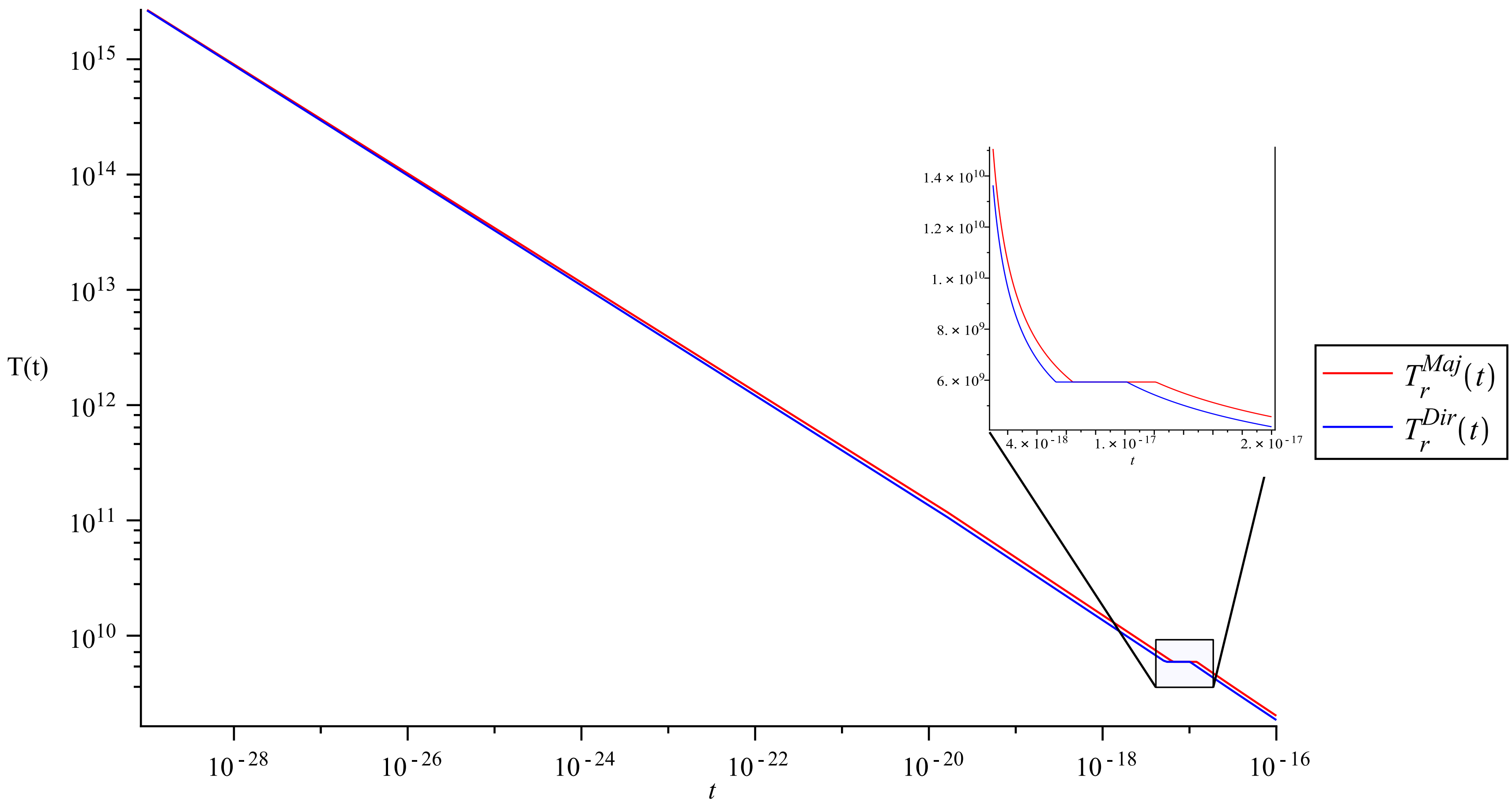


2.5 × 10^15
2. × 10^15
1.5 × 10^15
1. × 10^15
9. × 10^14
T(t)
1. × 10^-29
2. × 10^-29
6. × 10^-29
1. × 10^-28
t
8. × 10^13
6. × 10^13
4. × 10^13
2. × 10^13
1. × 10^13
T(t)
1. × 10^-26
5. × 10^-26
1. × 10^-24
t
8. × 10^12
6. × 10^12
4. × 10^12
2. × 10^12
1. × 10^12
8. × 10^11
6. × 10^11
T(t)
1. × 10^-24
5. × 10^-24
5. × 10^-23
1. × 10^-21
t
$T_r(t)$
m_μ
m_π
T_c
m_s
m_c
m_τ
m_b
$m_{W,Z}$
m_H
m_t

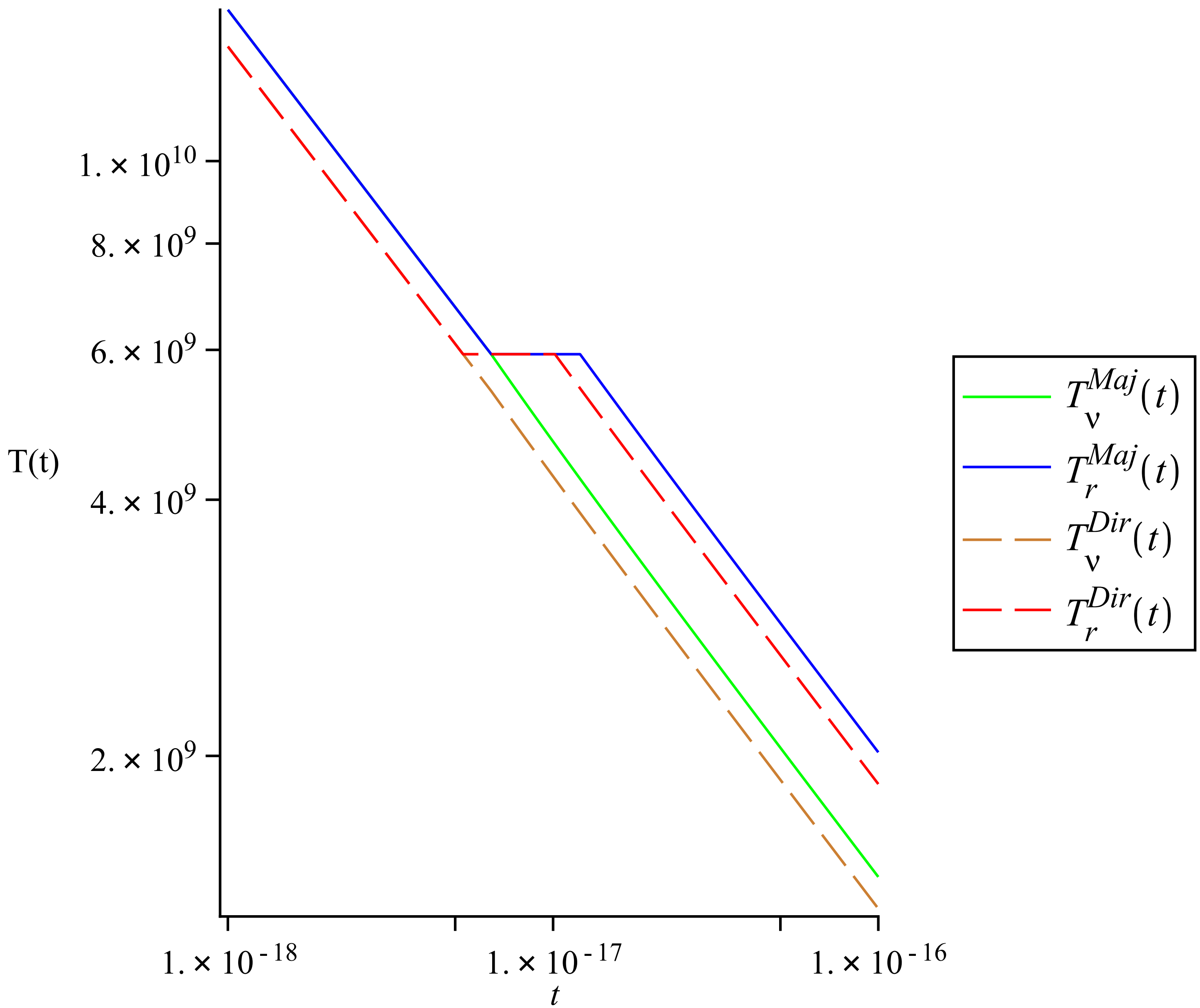
Radiation and baryonic matter:



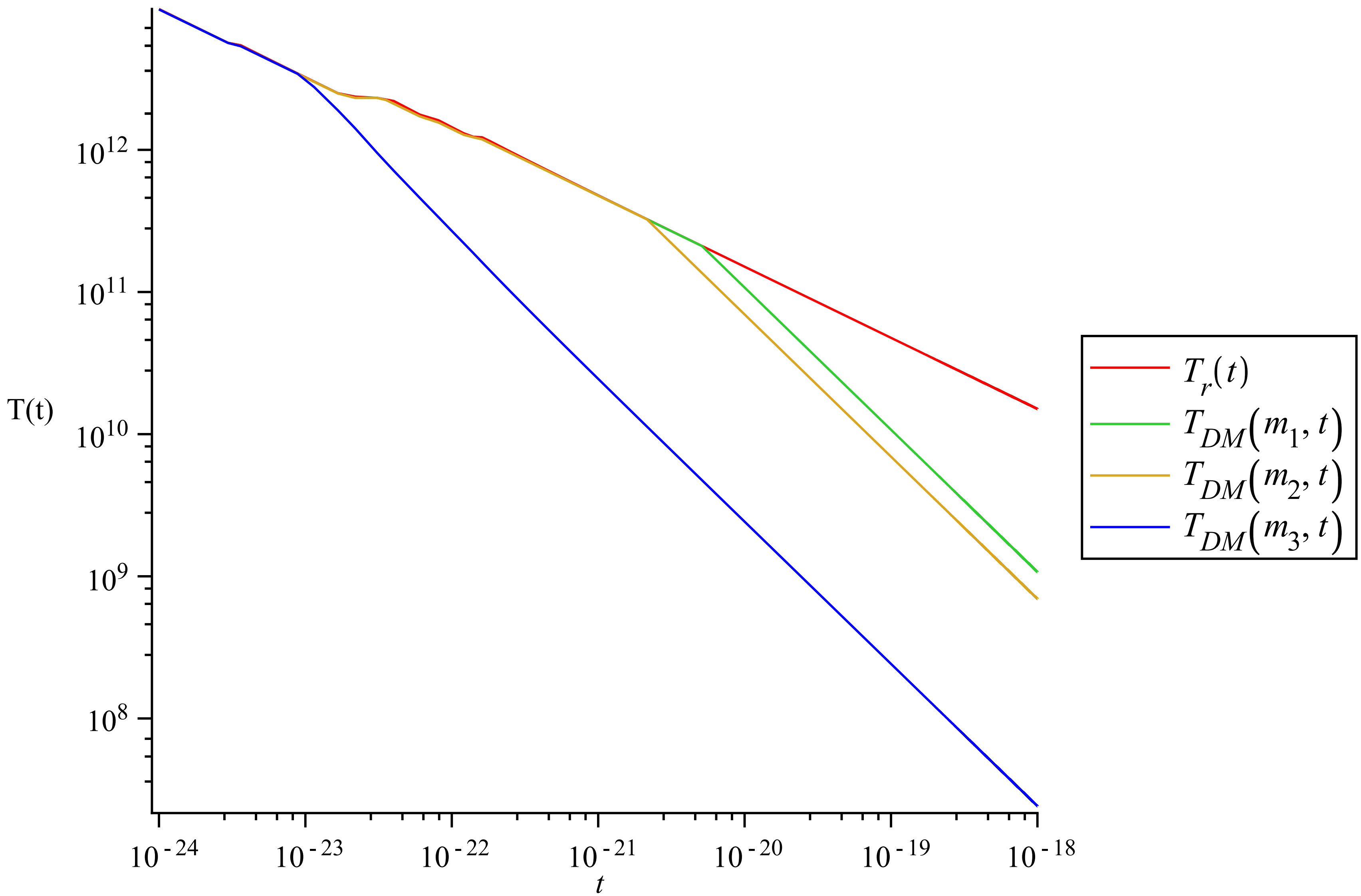
Compare radiation temperature when neutrinos are Majorana or Dirac:



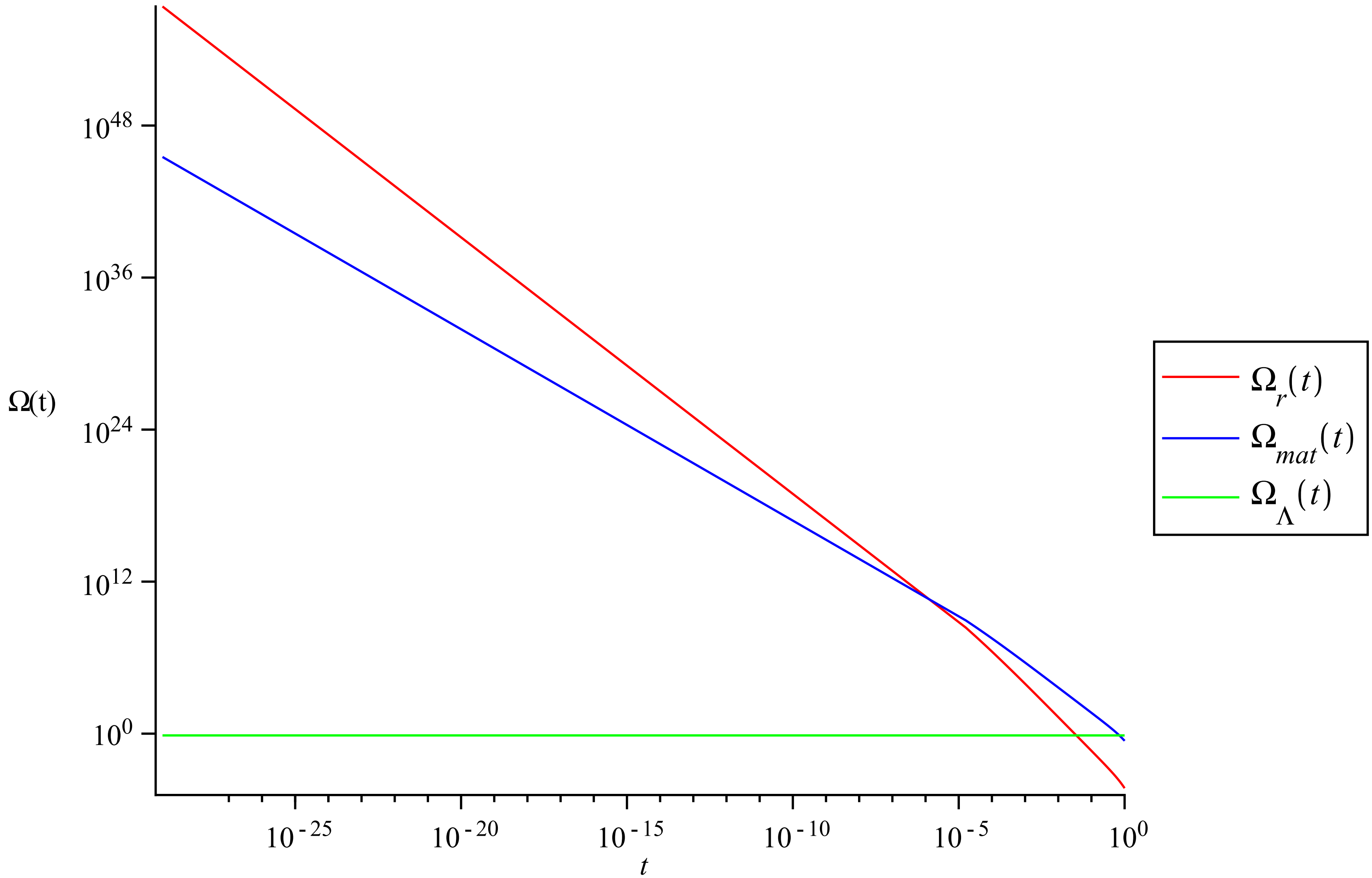
Radiation and neutrino temperatures:



Neutralinos with mass values $m_1 = 10$ GeV, $m_2 = 100$ GeV, $m_3 = 1$ TeV:



Overall density parameters:



Summary

- Universe as perfect fluid
- Dark energy from fluctuations of metric
- Dark matter as neutralinos
- Calculated $Q(t)$, $\Omega_i(t)$, $T_i(t)$
- **Average** for baryonic and dark matter:
  $T_b(t_0) = 2.5 \times 10^{-3}$K, $T_c(t_0) = 4.7 \times 10^{-13}$K ... 1.8×10^{-11}K
- Radiation-matter equality at 22 ky
  Baryonic matter decoupling at 373 ky
  Dark_energy-matter equalty at 9.6 Gy
- Age of universe depends slightly on neutrino character:
  Majorana: 13.87 Gy, Dirac: 13.86 Gy. Observable in future?
- $\Omega_\nu(t_0)$ differs for Majorana Dirac neutrinos. Probably hopeless to observe.