early thermalization of quark-gluon matter in heavy ion collisions

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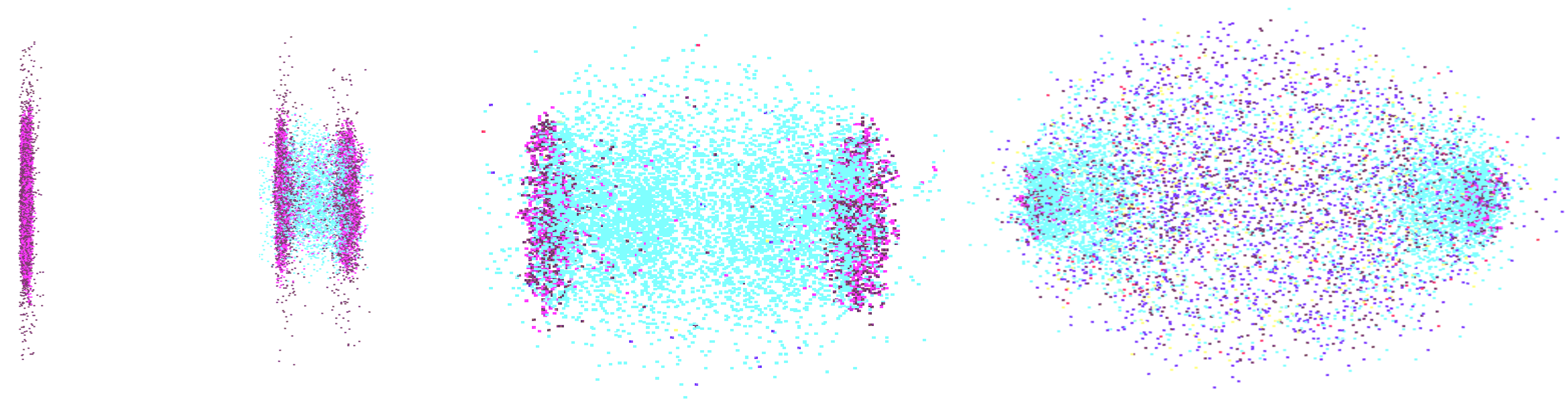
history of high-energy nucleus-nucleus collisions



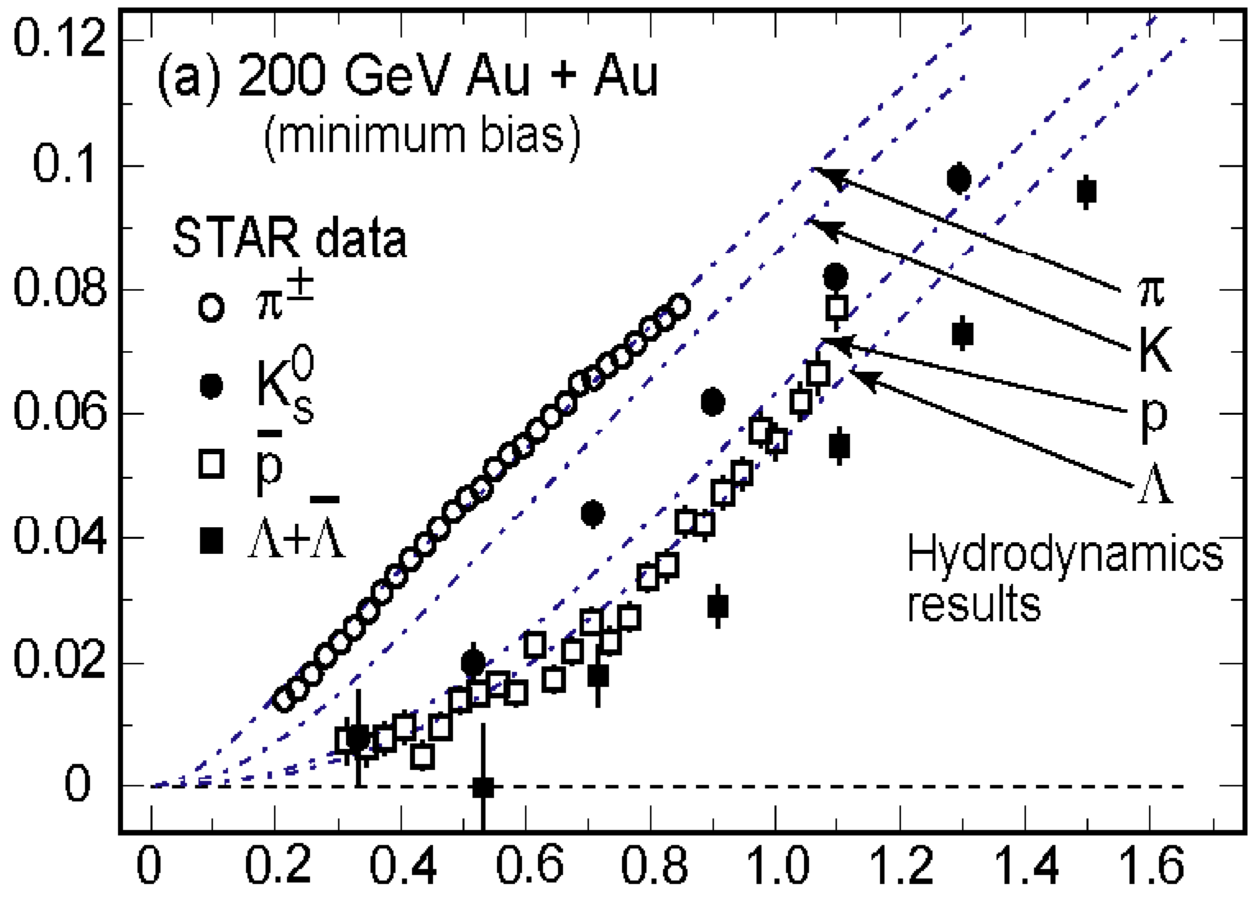
initial AA collisions	thermalization of quark-gluon matter	evolution of quark-gluon plasma	evolution of hadronic matter
	(no T)	(with T)	

K. H. Ackermann, et al., STAR Collaboration, nucl-ex/0009011; Phys. Rev. Lett. 86 (2001) 402.
P. F. Kolb, P. Huovinen, U. Heinz, H. Heiselberg, hep-ph/0012137; Phys. Lett. B500 (2001) 232.

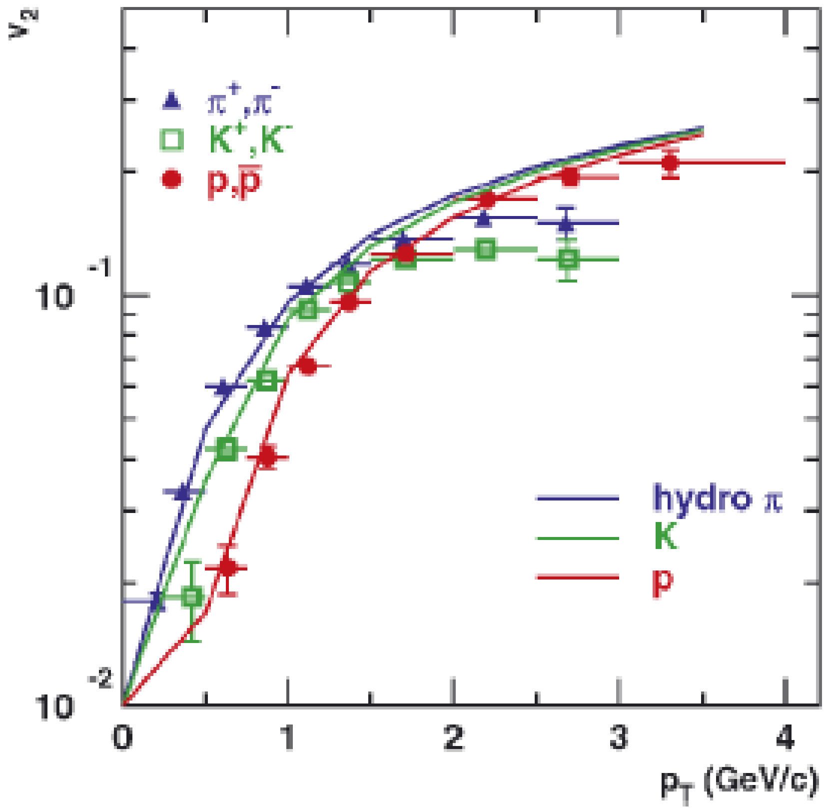
The experimental and theoretical studies reveal the early thermalization that is a thermal state is achieved with a time less than 1 fm/c from the moment when quark-gluon matter is initially created in heavy-ion collisions.

Early thermalization (rapid equilibration) exists at RHIC.

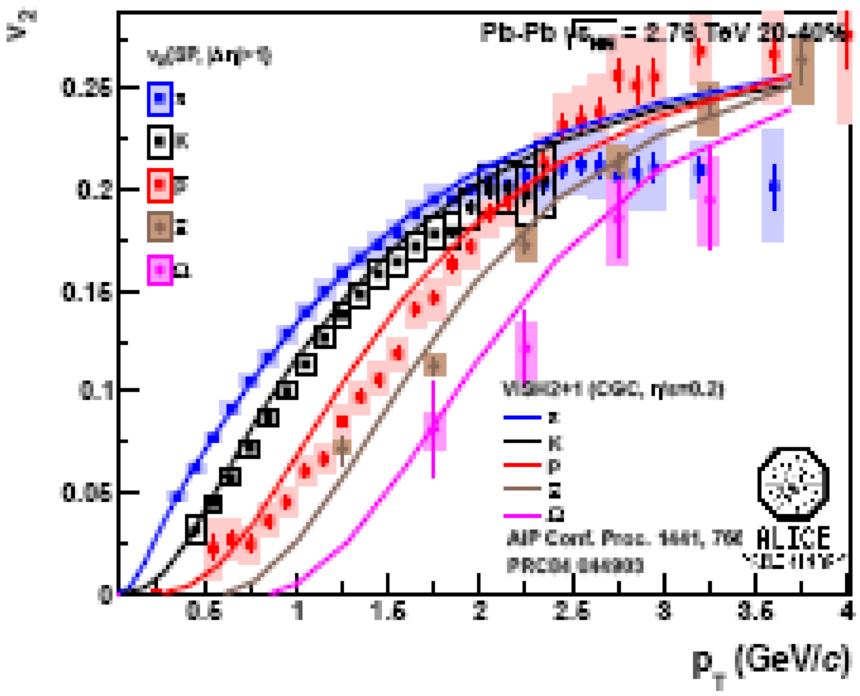
STAR experimental results (Nucl. Phys. A 757 (2005) 102) + hydrodynamic calculations

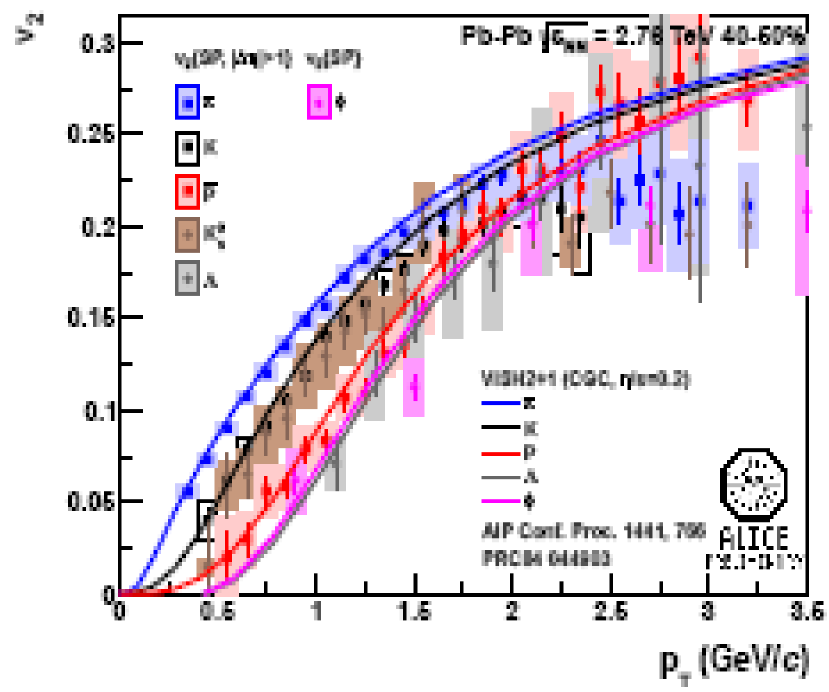


PHENIX experimental results (Nucl. Phys. A 757 (2005) 184) + hydrodynamic calculations



F. Noferini, for the ALICE Collaboration, arXiv, 1212.1292
U. W. Heinz, C. Shen, H. Song, AIP Conf. Proc. 1441, 766 (2012)





Early thermalization (rapid equilibration) also exists at LHC.

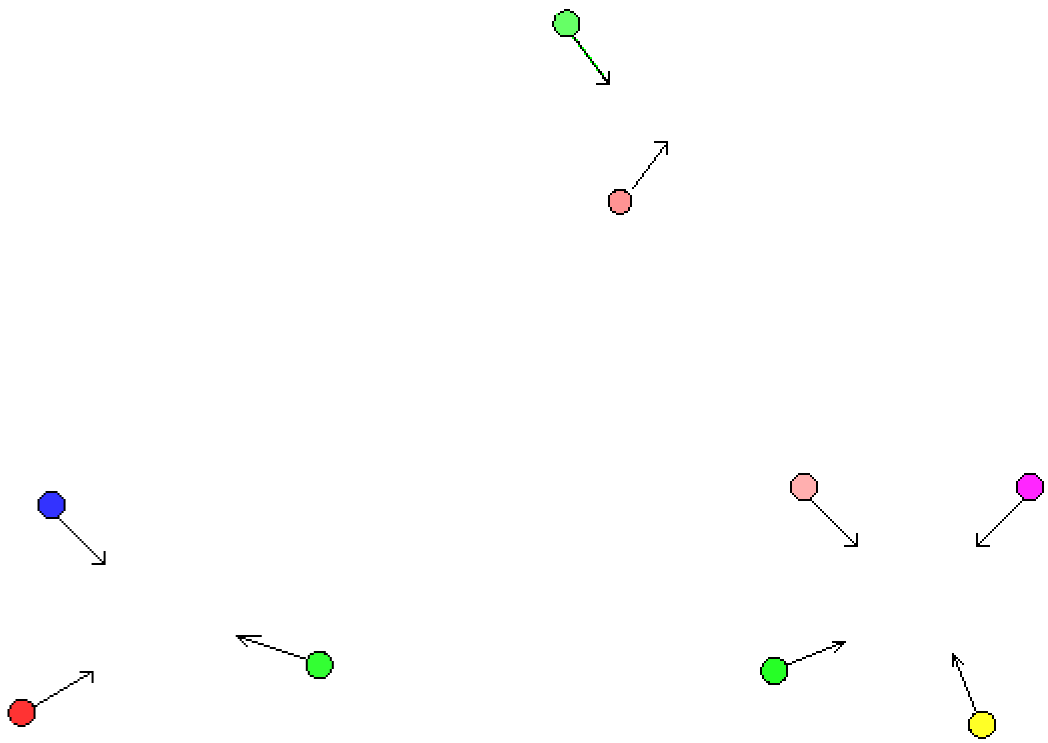
Quark-gluon matter initially created in nucleus-nucleus collisions does not has a temperature since the gluon and quark distributions are anisotropic in momentum space.

When does initially producd quark-gluon matter establish a thermal state?

How does initially producd quark-gluon matter establish a thermal state?

The two problems are crucial to the perfect liquid of quark-gluon plasma. The quark-gluon plasma formed by the early thermalization is a perfect liquid.

Thermalization by elastic two-body scattering is conventional.



How about multi-gluon scattering in thermalization of gluon matter?

gluon number density at τ=0.2fm/c: ~38 fm^{-3} at RHIC,~140 fm^{-3} at LHC

⇒ thermalization of gluon matter

$$gg \to gg \qquad ggg \to ggg$$

X.-M. Xu, *et al*., Nucl. Phys. A744(2004)347

⇒ thermalization of quark matter

$$qq \to qq \qquad qqq \to qqq$$

X.-M. Xu, R. Peng, H.J. Weber, Phys. Lett. B629(2005)68

⇒ thermalization of quark matter and antiquark matter

$$qq \to qq \qquad q\bar{q} \to q\bar{q} \qquad \bar{q}\bar{q} \to \bar{q}\bar{q}$$

$$qq\bar{q} \to qq\bar{q} \qquad q\bar{q}\bar{q} \to q\bar{q}\bar{q}$$

X.-M. Xu, *et al*., Phys. Lett. B645(2007)146

thermalization of quark–gluon matter

$$gg \to gg$$
$$qq \to qq$$
$$\bar{q}\bar{q} \to \bar{q}\bar{q}$$
$$q\bar{q} \to q\bar{q}$$
$$gq \to gq$$
$$g\bar{q} \to g\bar{q}$$

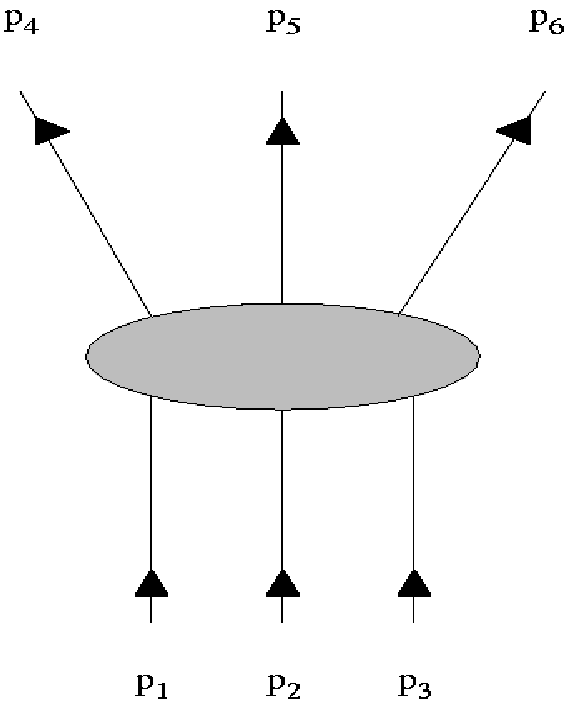
$$ggg \to ggg$$
$$qqq \to qqq$$
$$\bar{q}\bar{q}\bar{q} \to \bar{q}\bar{q}\bar{q}$$
$$qq\bar{q} \to qq\bar{q}$$
$$q\bar{q}\bar{q} \to q\bar{q}\bar{q}$$
$$gqq \to gqq$$
$$g\bar{q}\bar{q} \to g\bar{q}\bar{q}$$
$$gq\bar{q} \to gq\bar{q}$$

X.-M. Xu, L.-S. Xu, J. Phys. G37 (2010) 115003

$$ggq \to ggq$$
$$gg\bar{q} \to gg\bar{q}$$

X.-M. Xu, *et al.*, Phys. Rev. C87 (2013) 054904

Conclusion: the three-gluon scattering is important due to the high gluon number density.



Effect of the elastic triple-gluon scattering:

early thermalization of gluon matter initially produced in central Au-Au collisions.

EARLY THERMALIZATION IS AN EFFECT OF MANY-BODY INTERACTION!

elastic two-body scattering

$$g(p_1)+g(p_2)\rightarrow g(p_3)+g(p_4)$$
$$g(p_1)+q(p_2)\rightarrow g(p_3)+q(p_4)$$
$$q(p_1)+q(p_2)\rightarrow q(p_3)+q(p_4)$$

•••

Squared amplitudes are derived by hand in perturbative QCD and are expressed in terms of the Mandelstam variables.

R. Cutler, D. Sivers, Phys. Rev. D17(1978)196.

elastic three-body scattering

$$g(p_1)+g(p_2)+q(p_3) \rightarrow g(p_4)+g(p_5)+q(p_6)$$

•••

Squared amplitudes are calculated in perturbative QCD,

have to be derived with fortran code, and are expressed in terms of nine Lorentz-invariant variables

$$s_{12}=(p_1+p_2)^2, \quad s_{23}=(p_2+p_3)^2, \quad s_{31}=(p_3+p_1)^2$$

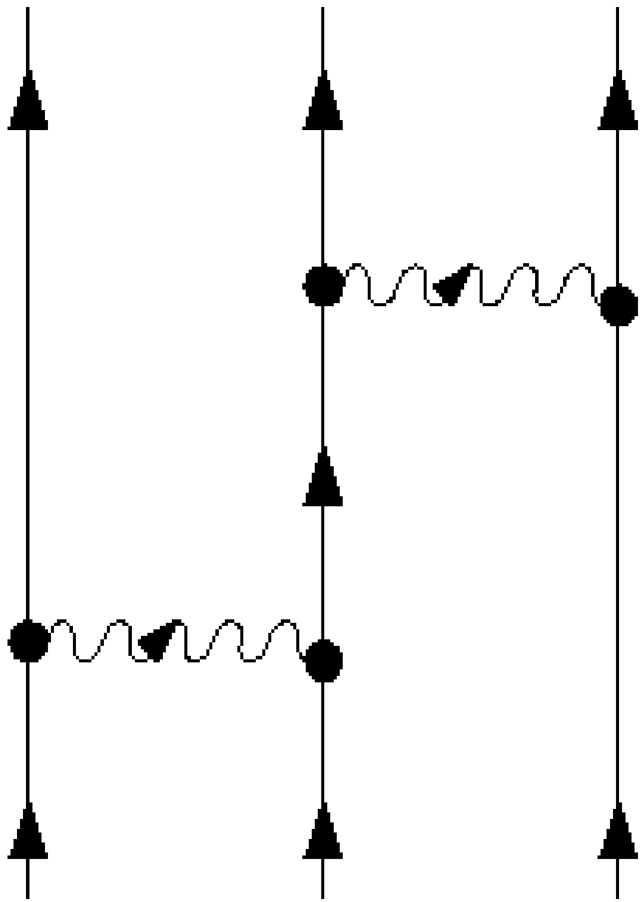
$$u_{15}=(p_1-p_5)^2, \quad u_{16}=(p_1-p_6)^2$$

$$u_{24}=(p_2-p_4)^2, \quad u_{26}=(p_2-p_6)^2$$

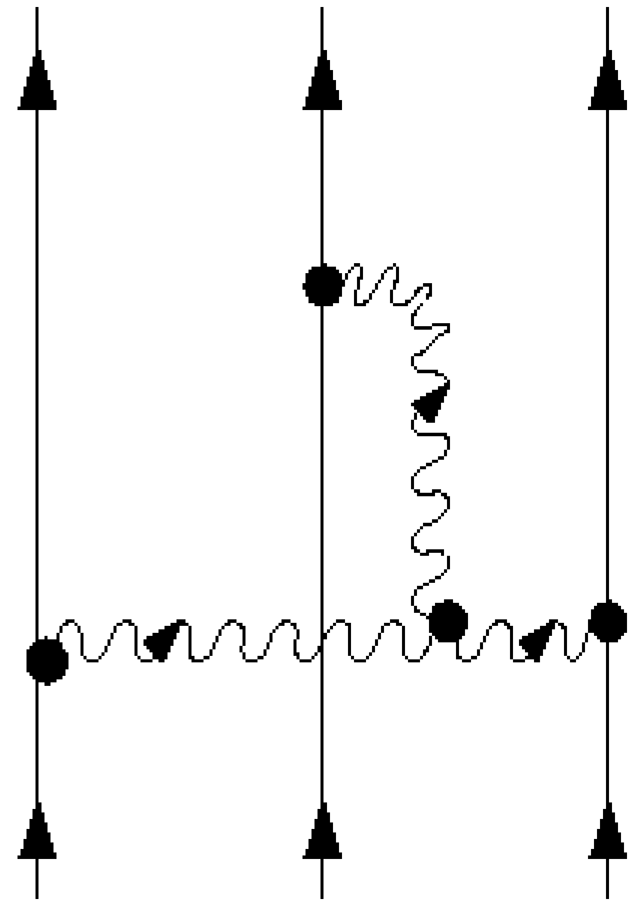
$$u_{34}=(p_3-p_4)^2, \quad u_{35}=(p_3-p_5)^2$$

Elastic quark-quark-quark scattering

The three quarks are identical	42 diagrams
Only two quarks are identical	14 diagrams
The three quarks are different	7 diagrams



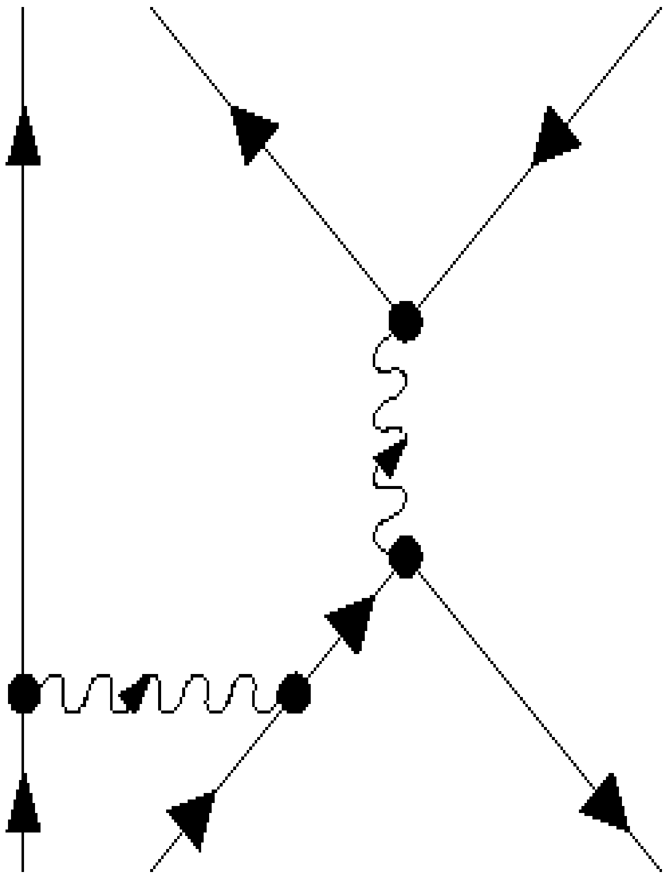
A_-



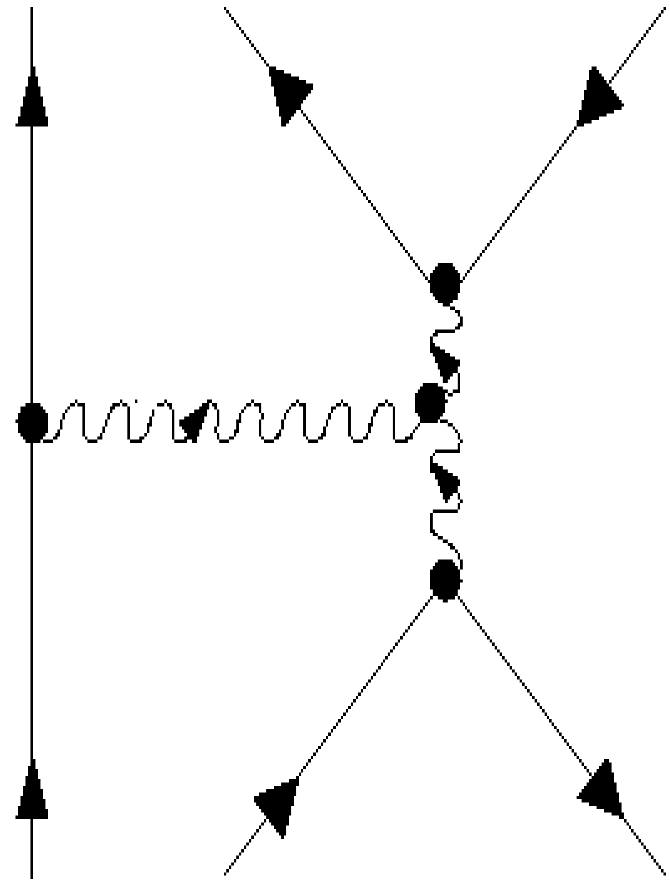
A_+

Elastic quark-quark-antiquark scattering

The two quarks and the antiquark have the same flavor	58 diagrams
Only the two quarks have the same flavor	14 diagrams
Only one quark has the same flavor as the antiquark	29 diagrams
The three flavors of the two quarks and the antiquark are not identical	7 diagrams



F_{QD}

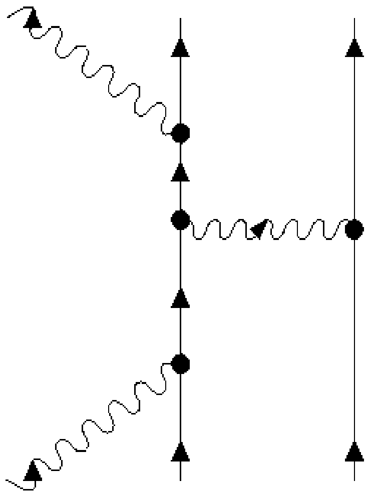


F_{++}

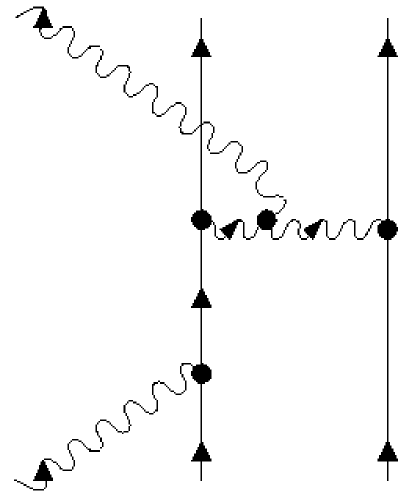
Elastic gluon-quark-quark scattering

The two quarks have the same flavor	72 diagrams
The two quarks take different flavors	36 diagrams

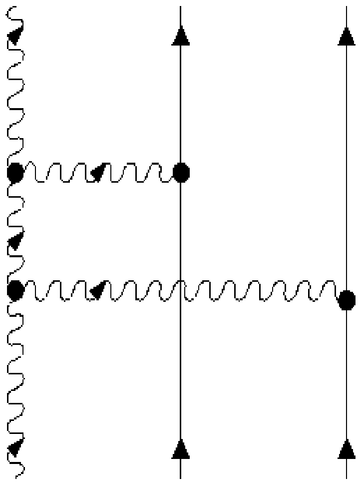
Elastic gluon-quark-quark scattering



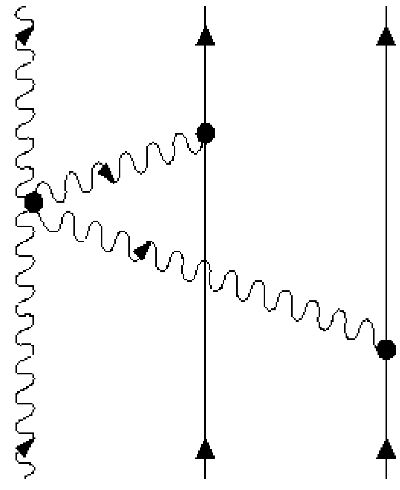
$D_{\sim LM}$



D_{GUML}



D_{GG+}

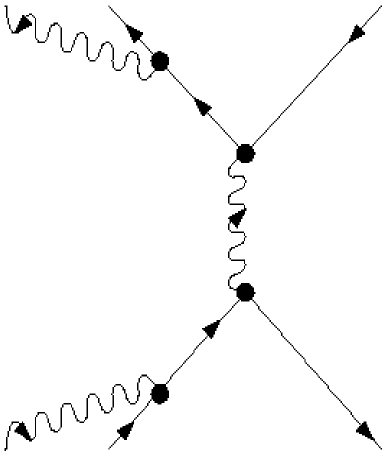


D_{+}

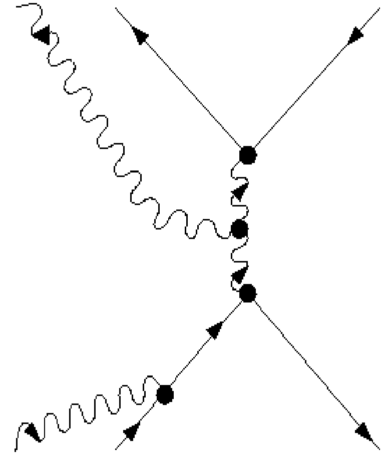
Elastic gluon-quark-antiquark scattering

The quark and the antiquark have the same flavor	76 diagrams
The quark and the antiquark take different flavors	36 diagrams

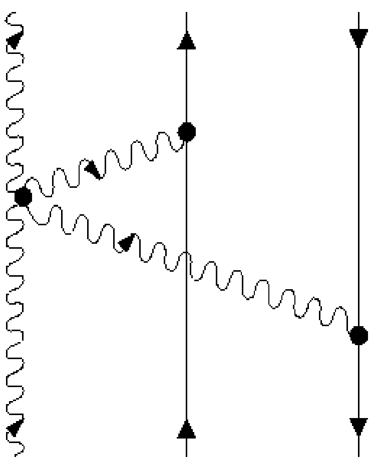
Elastic gluon-quark-antiquark scattering



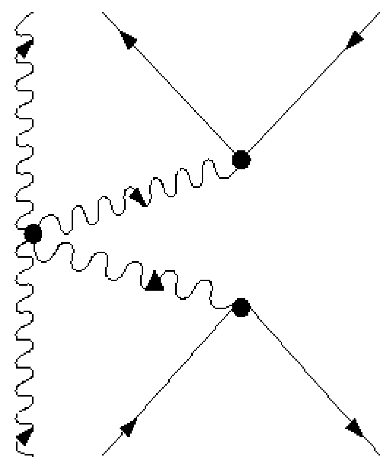
$E_{\sim LMA}$



E_{GUMLA}

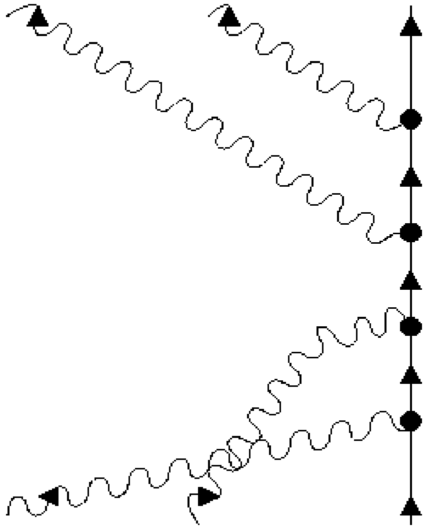


E_{*}

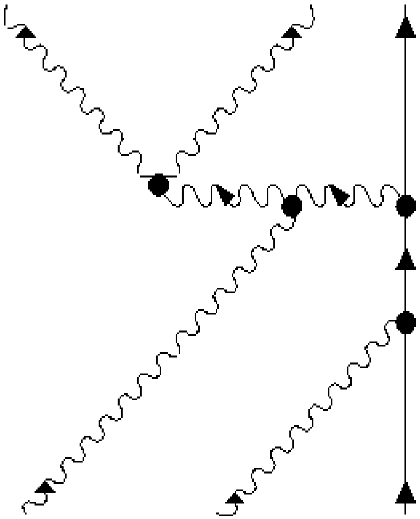


$E_{\sim}$

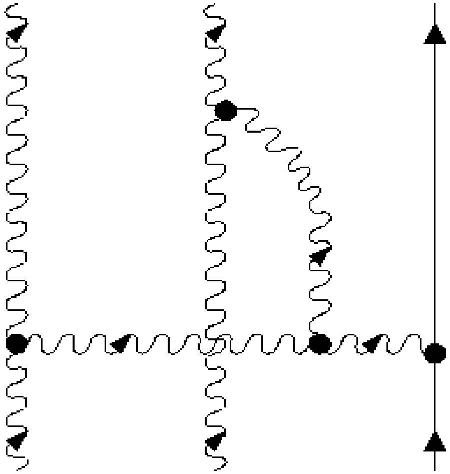
123 Feynman diagrams in elastic gluon-gluon-quark scattering



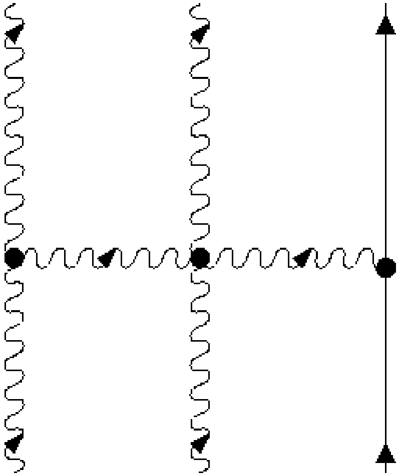
C_{1245}



$C_{2(451)}$

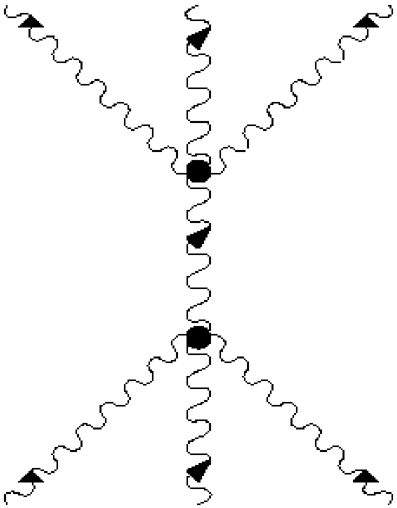


$C_{(14)(25)M}$

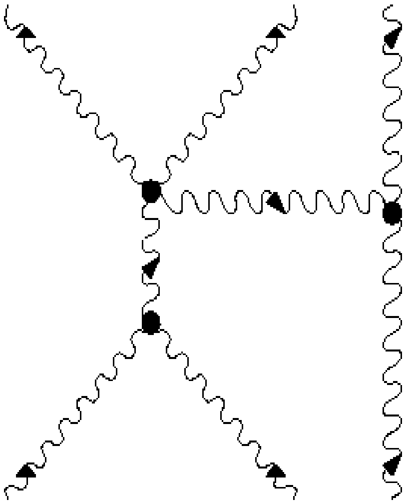


$C_{25\sim}$

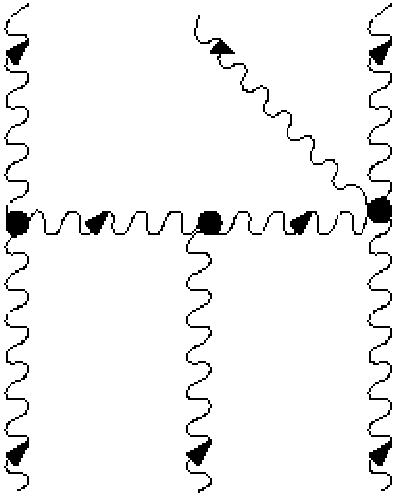
220 Feynman diagrams in elastic gluon-gluon-gluon scattering



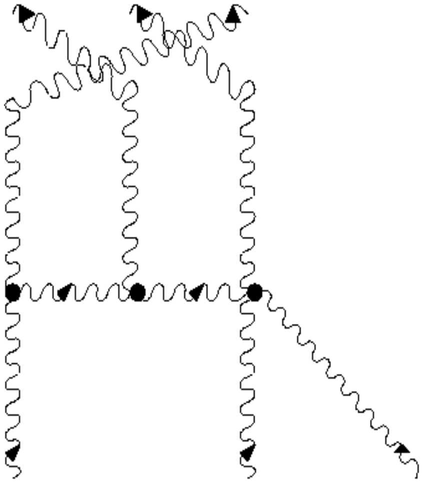
B_{--}



B_{U}



B_{-+}



$B_{--1(23)}$

transport equation for gluon matter

$$\frac{\partial f_{g1}}{\partial t} + \vec{\mathrm{v}}_1 \cdot \vec{\nabla}_{\vec{r}} f_{g1}$$

$$= -\frac{1}{2E_1}\int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4 (p_1 + p_2 - p_3 - p_4)$$

$$\times \{ \frac{g_G}{2} \left| M_{gg\to gg} \right|^2 [f_{g1} f_{g2} (1+ f_{g3})(1+ f_{g4}) - f_{g3} f_{g4} (1+ f_{g1})(1+ f_{g2})]$$

$$+ g_Q (\left| M_{gu\to gu} \right|^2 + \left| M_{gd\to gd} \right|^2 + \left| M_{g\bar{u}\to g\bar{u}} \right|^2 + \left| M_{g\bar{d}\to g\bar{d}} \right|^2)$$

$$\times [f_{g1} f_{q2} (1+ f_{g3})(1- f_{q4}) - f_{g3} f_{q4} (1+ f_{g1})(1- f_{q2})]\}$$

$$
\begin{aligned}
&-\frac{1}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\frac{d^3p_5}{(2\pi)^3 2E_5}\frac{d^3p_6}{(2\pi)^3 2E_6}\\
&\times(2\pi)^4\delta^4(p_1+p_2+p_3-p_4-p_5-p_6)\{\frac{g_G^2}{12}\left|M_{ggg\to ggg}\right|^2\\
&\times[f_{g1}f_{g2}f_{g3}(1+f_{g4})(1+f_{g5})(1+f_{g6})-f_{g4}f_{g5}f_{g6}(1+f_{g1})(1+f_{g2})(1+f_{g3})]\\
&+\frac{g_G g_Q}{2}(\left|M_{ggu\to ggu}\right|^2+\left|M_{ggd\to ggd}\right|^2+\left|M_{gg\bar{u}\to gg\bar{u}}\right|^2+\left|M_{gg\bar{d}\to gg\bar{d}}\right|^2)\\
&\times[f_{g1}f_{g2}f_{q3}(1+f_{g4})(1+f_{q5})(1-f_{q6})-f_{g4}f_{g5}f_{q6}(1+f_{g1})(1+f_{g2})(1-f_{q3})]\\
&+g_Q^2[\frac{1}{4}\left|M_{guu\to guu}\right|^2+\frac{1}{2}(\left|M_{gud\to gud}\right|^2+\left|M_{gdu\to gdu}\right|^2)+\frac{1}{4}\left|M_{gdd\to gdd}\right|^2\\
&+\left|M_{gu\bar{u}\to gu\bar{u}}\right|^2+\left|M_{gu\bar{d}\to gu\bar{d}}\right|^2+\left|M_{gd\bar{u}\to gd\bar{u}}\right|^2+\left|M_{gd\bar{d}\to gd\bar{d}}\right|^2+\frac{1}{4}\left|M_{gu\bar{u}\to gu\bar{u}}\right|^2\\
&+\frac{1}{2}(\left|M_{g\bar{u}\bar{d}\to g\bar{u}\bar{d}}\right|^2+\left|M_{g\bar{d}\bar{u}\to g\bar{d}\bar{u}}\right|^2)+\frac{1}{4}\left|M_{gd\bar{d}\to gd\bar{d}}\right|^2]\\
&\times[f_{g1}f_{q2}f_{q3}(1+f_{g4})(1-f_{q5})(1-f_{q6})-f_{g4}f_{q5}f_{q6}(1+f_{g1})(1-f_{q2})(1-f_{q3})]\}
\end{aligned}
$$

transport equation for up-quark matter

$$\frac{\partial f_{q1}}{\partial t} + \vec{\mathrm{v}}_1 \cdot \vec{\nabla}_{\vec{r}} f_{q1}$$

$$= -\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$

$$\times\{g_G \left|M_{ug\to ug}\right|^2 [f_{q1} f_{g2}(1-f_{q3})(1+f_{g4}) - f_{q3} f_{g4}(1-f_{q1})(1+f_{g2})]$$

$$+ g_G (\frac{1}{2}\left|M_{uu\to uu}\right|^2 + \left|M_{ud\to ud}\right|^2 + \left|M_{u\bar{u}\to u\bar{u}}\right|^2 + \left|M_{u\bar{d}\to u\bar{d}}\right|^2)$$

$$\times[f_{q1} f_{q2}(1-f_{q3})(1-f_{q4}) - f_{q3} f_{q4}(1-f_{q1})(1-f_{q2})]\}$$

$$
-\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\frac{d^3p_5}{(2\pi)^3 2E_5}\frac{d^3p_6}{(2\pi)^3 2E_6}
$$

$$
\times(2\pi)^4\delta^4(p_1+p_2+p_3-p_4-p_5-p_6)\{\frac{g_G^2}{4}\left|M_{ugg\to ugg}\right|^2
$$

$$
\times[f_{q1}f_{g2}f_{g3}(1-f_{q4})(1+f_{g5})(1+f_{g6})-f_{q4}f_{g5}f_{g6}(1-f_{q1})(1+f_{g2})(1+f_{g3})]
$$

$$
+g_Q g_G(\frac{1}{2}\left|M_{uug\to uug}\right|^2+\left|M_{udg\to udg}\right|^2+\left|M_{u\bar{u}g\to u\bar{u}g}\right|^2+\left|M_{u\bar{d}g\to u\bar{d}g}\right|^2)
$$

$$
\times[f_{q1}f_{q2}f_{g3}(1-f_{q4})(1-f_{q5})(1+f_{g6})-f_{q4}f_{q5}f_{g6}(1-f_{q1})(1-f_{q2})(1+f_{g3})]
$$

$$
+g_Q^2\left[\frac{1}{12}\left|M_{uuu\to uuu}\right|^2+\frac{1}{4}(\left|M_{uud\to uud}\right|^2+\left|M_{udu\to udu}\right|^2)+\frac{1}{4}\left|M_{udd\to udd}\right|^2\right.
$$

$$
+\frac{1}{2}\left|M_{uu\bar{u}\to uu\bar{u}}\right|^2+\frac{1}{2}\left|M_{ud\bar{d}\to ud\bar{d}}\right|^2+\left|M_{ud\bar{u}\to ud\bar{u}}\right|^2+\left|M_{ud\bar{d}\to ud\bar{d}}\right|^2
$$

$$
\left.+\frac{1}{4}\left|M_{u\bar{u}\bar{u}\to u\bar{u}\bar{u}}\right|^2+\frac{1}{2}(\left|M_{u\bar{u}\bar{d}\to u\bar{u}\bar{d}}\right|^2+\left|M_{u\bar{d}\bar{u}\to u\bar{d}\bar{u}}\right|^2)+\frac{1}{4}\left|M_{u\bar{d}\bar{d}\to u\bar{d}\bar{d}}\right|^2\right]
$$

$$
\times[f_{q1}f_{q2}f_{q3}(1-f_{q4})(1-f_{q5})(1-f_{q6})-f_{q4}f_{q5}f_{q6}(1-f_{q1})(1-f_{q2})(1-f_{q3})]\}
$$

gluon distribution at τ=0.2 fm/c created in central Au-Au collisions

$$f(k_\perp, y, r, z, t) = \frac{1}{16\pi R_A^2} g(k_\perp, y) \frac{e^{-(z - t\tanh y)^2/2\Delta_k^2}}{\sqrt{2\pi}\Delta_k},$$

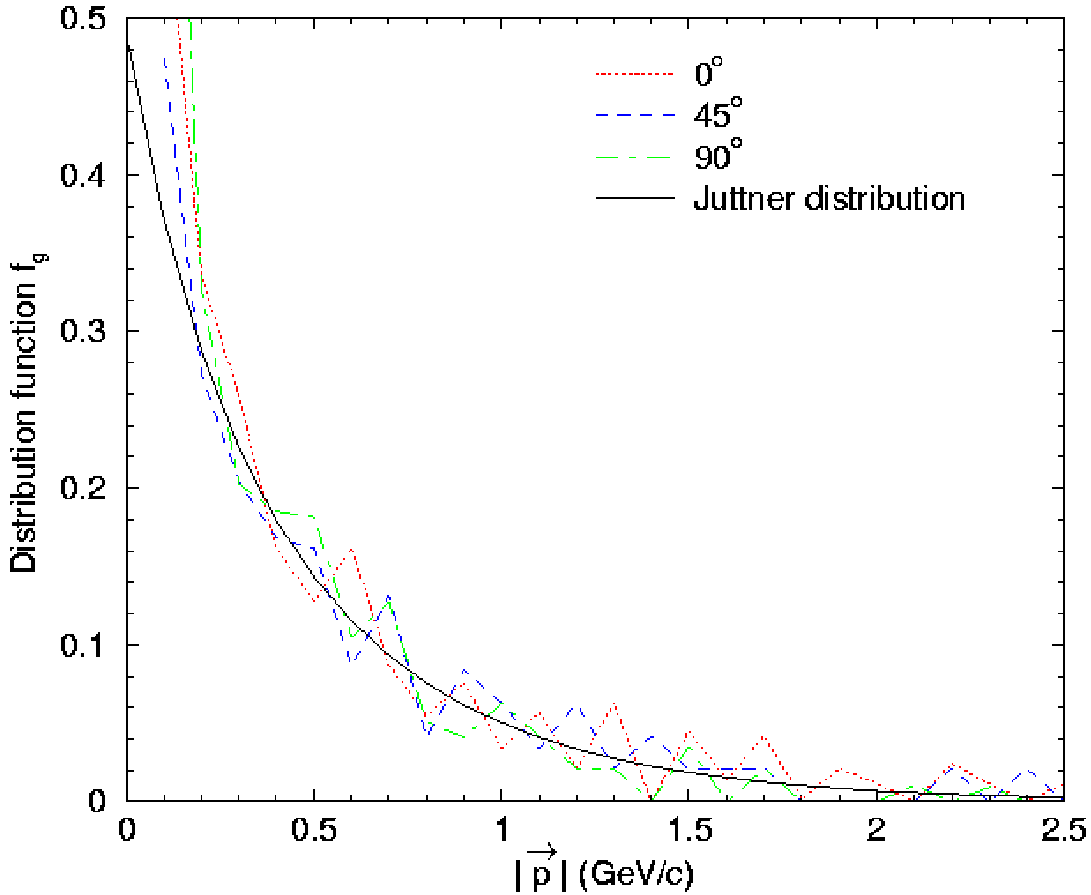
$$\Delta_k \approx \frac{2}{k_\perp \cosh y},$$

$$g(k_\perp, y) = \frac{(2\pi)^3}{k_\perp \cosh y} \frac{dN}{dy d^2 k_\perp}$$

The distribution is anisotropic in momentum space because it has different dependence on $k_\perp$ and cosh y.

Local momentum isotropy of gluon matter is established by the elastic 2-to-2 scattering and the elastic 3-to-3 scattering first at t=0.52 fm/c.

Distribution functions at three angles relative to the incoming beam direction:



the Juttner distribution for gluon matter at t=0.52 fm/c

$$f_g(\vec{p}) = \frac{\lambda_g}{e^{|\vec{p}|/T} - \lambda_g}$$

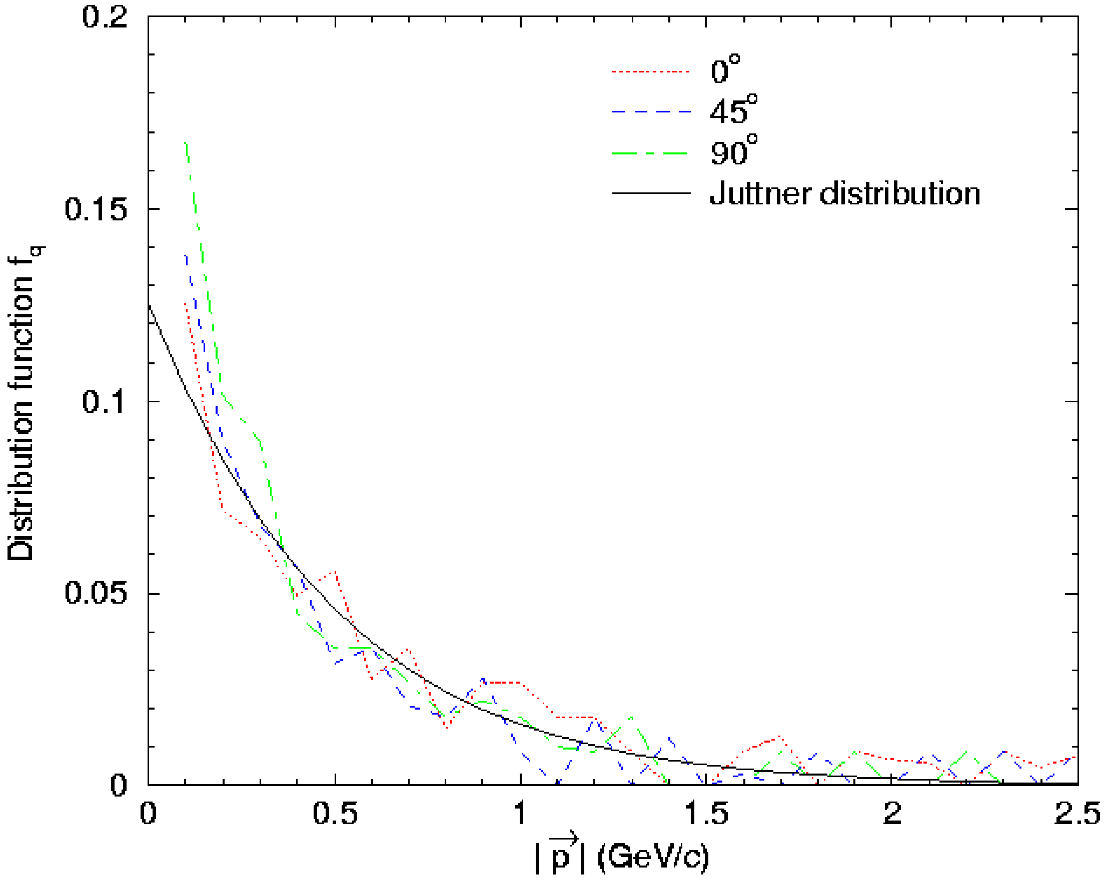
λ_g=0.328 T=0.52 GeV

Thermalization time of initial gluon matter is 0.32 fm/c.

early thermalization!

Local momentum isotropy of quark matter or antiquark matter is established by the elastic 2-to-2 scattering and the elastic 3-to-3 scattering first at t=0.86 fm/c.

Distribution functions at three angles relative to the incoming beam direction:



the Juttner distribution for quark matter or antiquark matter at t=0.86fm/c

$$f_q(\vec{p}) = \frac{\lambda_q}{e^{|\vec{p}|/T} + \lambda_q}$$

λ_q=0.143 T=0.46 GeV

Thermalization time of initial quark matter or antiquark matter is 0.66 fm/c.

early thermalization!

Conclusions

(1) The elastic 3-to-3 scattering is important and yields the early thermalization of initially created quark-gluon matter at a high number density.

(2) The early thermalization is an effect of many-body scattering.

(3) Different thermalization times of gluon matter and quark matter are found.

(4) The early thermalization is crucial to the perfect liquid the quark-gluon plasma.

(5) Two-body scattering, three-body scattering and other many-body scattering are the origin of temperature.