Cosmological consequences of the relativistic theory of gravitation.

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It is pointed out that according to RTG based on the Minkowski space the presence of the quintessence is necessary to explain the Universe accelerated expansion. RTG based on the deSitter background space predicts acceleration without any quintessence.

Friedmann cosmological solution is one of the most important consequences of any classical theory of gravitation. We'll consider such solutions in relativistic theory of gravitation (RTG) [1], developed by the group of A.A.Logunov [2-13], paying more attention to the dark energy problem.

RTG starts from the assumption that the gravitational field, as all other fields, develops in the Minkowski space and that the tensor of the energy-momentum of all matter fields, including the gravitational field, is the source of this field. Such approach is concordant with modern gauge theories of the electroweak interactions and QCD, where conserving charges and their currents are the source of the vector fields. As the energy-momentum tensor is chosen to be the source of the gravitational field, the gravitational field itself should be described by symmetrical tensor of the second rank, $\varphi^{\alpha\beta}$. Further this gives rise to a "geometrization" of the theory. It is emphasized there that the main requirement imposed on the background space is that it be a maximally symmetric space, i.e., admit the existence of ten Killing vectors. This, in its turn, leads to the existence of all ten integral conservation laws for the matter and the gravitational field (maximum for 4d space-time). The spaces that satisfy this requirement are called spaces of constant curvature[1],[14]. In particular, Minkowski space is one such spaces and that is why this simplest space was chosen as the background in [1]. However, nothing prevents us from using some other constant-curvature space.

Let's consider first Minkowski case. The initial set of the RTG equations, based on the Minkiwski space-time has the form ($G = \hbar = c = 1$):

$$\left(\gamma^{\alpha\beta} D_\alpha D_\beta + m^2\right)\tilde{\varphi}^{\mu\nu} = 16\pi\, t^{\mu\nu} \tag{1}$$

$$D_\beta \sqrt{-\gamma}\varphi^{\alpha\beta} = 0 \tag{2}$$

where D_β is the covariant derivative in the Minkowski space with metric tensor $\gamma_{\mu\nu}$ and $t^{\mu\nu}$ are the densities of the gravitational field and total energy-momentum tensor, correspondingly:

$$\tilde{\varphi}^{\mu\nu} = \sqrt{-\gamma}\varphi^{\mu\nu}, \sqrt{-\gamma} = det\left(\gamma_{\alpha\beta}\right), \quad t^{\mu\nu} = \frac{\delta L}{\delta\gamma_{\mu\nu}}$$

where L is the density of the matter and gravitational field Lagrangian. Eq. (1) guarantees the conservation of the total energy-momentum tensor, singles out polarization states corresponding to the gravitons with the spin 2 and 0, and excludes the states with the spin 1, 0′ (analogously to the Lorentz condition, which excludes the photon with the spin 0). For equation set (1)-(2) to follow from the minimal action principle, i.e. for it to result from the Euler equations:

$$\frac{\delta L}{\delta\tilde{\varphi}^{\mu\nu}} = 0, \qquad \frac{\delta L}{\delta\phi_A} = 0$$

it is necessary and sufficient that the density of the $\tilde{\varphi}^{\mu\nu}$_ tensor and density of the Minkowski space metric tensor $\tilde{\gamma}^{\mu\nu}$ should enter into the matter Lagrangian in combination:

$$\sqrt{-g}g^{\alpha\beta} = \tilde{g}^{\alpha\beta} = \sqrt{-\gamma}\left(\gamma^{\alpha\beta} + \varphi^{\alpha\beta}\right), g = det\left(g_{\alpha\beta}\right)$$

Thus

$$L = L_g\left(\gamma_{\alpha\beta}, \sqrt{-g}g^{\alpha\beta}\right) + L_M\left(\sqrt{-g}g_{\alpha\beta}, \phi_A\right)$$

$$L_g = \sqrt{-g}g^{\alpha\beta}\left(G^{\sigma}_{\alpha\beta}G^{\mu}_{\sigma\mu} - G^{\mu}_{\alpha\nu}G^{\nu}_{\beta\mu}\right) - \frac{m^2}{16\pi}\left(\frac{1}{2}\sqrt{-g}g^{\alpha\beta}\eta_{\alpha\beta} - \sqrt{-g} - \sqrt{-\gamma}\right)$$

$$G^{\sigma}_{\alpha\beta} = \frac{1}{2}g^{\sigma\nu}\left(D_\alpha g_{\beta\nu} + D_\beta g_{\alpha\nu} - D_\nu g_{\alpha\beta}\right)$$

and the matter motion in the gravitational field looks like, as it would appear in the effective Riemann space with the metrics $g_{\mu\nu}$. It should be noted that all the crucial changes as compared with the Einstein general theory of relativity appear in the RTG due to the fact that the gravitational field is considered as the physical field in the Minkowski space. RTG takes into account the fact that in the Minkowski space one can use any frames of reference, including accelerated ones, in which metric coefficients $\gamma_{\alpha\beta}$ form the tensor with respect to arbitrary coordinate transformation. That is why Eqs. (2) and (3) are generally covariant. The structure of the term in the gravitational field Lagrangian, which breaks gauge arbitrariness of the gravitational field by introducing a nonvanishing graviton mass, was unambiguously derived in [1]. As the result, the equations of the gravitational field and matter takes the form

$$R_\nu^\mu - \frac{1}{2}\delta_\nu^\mu R + \frac{m^2}{2}\delta_\nu^\mu + \frac{m^2}{2}\left(g^{\mu\varepsilon} - \frac{1}{2}\delta_\nu^\mu g^{\varepsilon\tau}\right)\gamma_{\varepsilon\tau} = 8\pi T_\nu^\mu \tag{3}$$

$$D_\beta\sqrt{-g}g^{\alpha\beta} = 0 \tag{4}$$

where R_ν^μ and R are the corresponding curvatures in the effective Riemann space, and T_ν^μ is the energy-momentum tensor of the matter in the effective Riemann space

$$\sqrt{-g}T_{\mu\nu} = -2\frac{\delta L_M}{\delta g^{\mu\nu}}$$

Eqs. (3-4) are generally covariant with respect to arbitrary coordinate transformations and form-invariant with respect to the Lorentz transformations. Due to $m \neq 0$, the connection of the effective Riemann space with the metrics of the original Minkowsky space $\gamma_{\alpha\beta}$ in Eq. (3) is retained. For the uniform and isotropic Universe the interval between events in the effective Riemann space can be represented in the Freedman-Robertson-Walker metrics:

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = U(t)dt^2 - V(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin\theta^2 d\varphi^2)\right], \tag{5}$$

$$d\sigma^2 = \gamma_{\alpha\beta}dx^\alpha dx^\beta = dt^2 - dr^2 - r^2(d\theta^2 + \sin\theta^2 d\varphi^2)$$

where $k = 1, -1, 0$ for the closed (elliptic), open (hyperbolic), and flat (parabolic) Universe, correspondingly. Eqs. (4) for metrics (5) takes the form:

$$\frac{d}{dt}\sqrt{\frac{V^{3}(t)}{U(t)}} = 0 \tag{6}$$

$$\frac{d}{dr}\left(\frac{R^2(r)\sqrt{1-kR^2(r)}}{R'(r)}\right) - \frac{2rR'(r)}{\sqrt{1-kR^2(r)}} = 0 \tag{7}$$

It follows from Eq. (8) that $\frac{V^{3}(t)}{U(t)} = \text{const}$, or

$$V(t) = \beta^4 U(t)^{1/3}, \quad \beta = const > 0. \tag{8}$$

Eq. (7) is valid only for $k = 0$. Thus, from (4) it follows immediately that space geometry of the Universe has to be flat (at that the initial inflationary expansion is not required).

Introducing the proper time

$$d\tau = \sqrt{U}dt$$

and, as usually, notation of scaling factor

$$a^2(\tau) = U(t)^{1/3}$$

one can rewrite interval (5) in the following form

$$ds^2 = d\tau^2 - \beta^4 a^2(\tau)\left(dr^2 + r^2(d\theta^2 + \sin\theta^2\, d\varphi^2)\right) \tag{9}$$

When expressions of (9) are used the equations of gravity (3) for uniform and isotropic Universe take the form

$$\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 = \frac{8\pi}{3}\rho - \frac{m^2}{6}\left[1 - \frac{3}{2\beta^4 a^2} + \frac{1}{2a^6}\right] \tag{10}$$

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\frac{4\pi}{3}(\rho + 3p) - \frac{m^2}{6}\left[1 - \frac{1}{a^6}\right] \tag{11}$$

where ρ and p are the total density of all types of matter and the pressure caused by the matter.
The β constant, which is determined by Eq. (8) and enters in Eq. (10), has a simple physical meaning. Considering the gravitational field $\varphi^{\alpha\beta}$ as a physical field in the Minlowski space it is necessary to require the fulfillment of the *causality principle*, which lies in the fact that the trajectory of the particle subjected to the gravitational field action should not leave the limits of the light cone in the Minkowsky space. An analytic formulation of the causality principle for all background spaces is the following: For any four-vector v^α that is isotropic in the background space,

$$\gamma_{\alpha\beta}v^\alpha v^\beta = 0,$$

the following inequality must hold:

$$g_{\alpha\beta}v^\alpha v^\beta \leq 0$$

For interval (9) this condition leads to inequality

$$a^4(\tau) - \beta^4 \leq 0$$

Thus, the β constant determines the maximal value of the scale multiplier

$$\beta = a_{max}$$

It means that according to RTG, the unlimited increase of the scale factor $a(\tau)$ is not possible, i.e. the unlimited expansion of the Universe is not possible. The mass of the graviton drastically changes the low of evolution at high energies: there is no Big Bang singularity and there is maximal density ρ_{max}.

From the covariant conservation low of the matter stress-energy tensor

$$\nabla_\mu \sqrt{-g} T^{\mu\nu} = 0$$

where ∇_μ- covariant derivative with respect to the Riemannian metrics $g_{\mu\nu}$, it follows

$$\frac{\dot{a}}{a} = -\frac{\dot{\rho}}{3(\rho+p)} \tag{12}$$

Therefore the matter density for the equation of state $p = \omega\rho$ equals to, as in GR,

$$\rho = \frac{A_\omega}{a^{3(1+\omega)}} \tag{13}$$

In hot Universe the ultrarelativistic stage, with very small values of the scaling factor, is dominated by the radiation

$$\omega = 1/3, \qquad \rho = \frac{A_r}{a^4}$$

In this case, as follows from the Eq.(10), the negative term in RHS monotonically increases as $1/a^6$ when a goes to zero. The LHS is strictly positive, hence there should exist the minimal value a_{min}, and, correspondingly the maximal value of the matter density ρ_{max}:

$$a_{min} = \left(\frac{m^2}{32\pi\rho_{max}}\right)^{1/6} > 0$$

The causality principle constraints the scaling factor from above $a \leq a_{max}$, therefore the theory describes the infinite Universe, oscillating between a_{min} and a_{max}. It helps to solve homogeneity and isotropy problem as well as concentration of the relic monopoles without inflation.

One of most prominent discovery of the recent time is that the expansion of the Universe is accelerated

$$\frac{1}{a}\frac{d^2a}{d\tau^2} > 0$$

In our case

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\frac{4\pi}{3}\rho(1+3\omega) - \frac{m^2}{6} + \frac{m^2}{a^6}$$

For normal, e.g. barionic, matter $\omega(>0)$ means simply squared sound velocity, therefore the only positive term here is the last one: a^{-6}. It's of the same order of magnitude with ρ in vicinity of a_{min}, but it is rapidly decreasing when the scaling factor goes up. At the present time m^2/a^6 much less than 1. Therefore one has to add a new matter of quintessence (in analogy with GR). This substance is not interacting with ordinary matter and has equation of state with negative $\omega < 0$, which provides acceleration. In GR exists the second recipe –to add to the Lagrangian the cosmological constant corresponding to

$$\omega = -1$$

But it's impossible to do this in RTG based on the Minkowski space, cause the vacuum solution would be nonzero $\left(\varphi^{\alpha\beta} \neq 0\right)$, and, secondly, the causality principle would be violated. To ensure acceleration, Λ term should be greater than $m^2/6$ (other m^2 terms in (10) at the contemporary epoch are negligibly small). In this case the scaling factor will unboundedly increased as $\exp(\Lambda - m^2/6)\tau$ in contradiction with condition $a \leq a_{max}$.

Consequently in RTG based on the Minkowski space one has only one option –quintessence. Usually it is a model of cosmological scalar field Φ. We're also propose such model [10], with having strictly $\omega = const < 0$:

$$L = -\sqrt{-g}V(\Phi)(I^2)^q, \qquad I^2 = g^{\alpha\beta}\partial_\alpha \Phi \partial_\beta \Phi$$

Equations of field Φ and energy-momentum tensor $T_{\mu\nu}$ are

$$(4q-1)\frac{dlnV(\Phi)}{d\Phi} I^2 + 8q(2q-1)g^{\alpha\beta}g^{\mu\nu}\nabla_\alpha \Phi \nabla_\beta \Phi \nabla_\mu \nabla_\mu \Phi + 4qI\nabla_\mu \nabla_\mu \Phi = 0$$

$$T_{\mu\nu} = \frac{\delta L}{\delta g^{\mu\nu}} = \sqrt{-g}V(\Phi)(I^2)^{q-1}\left[g_{\mu\nu}\ I^2 - 4qI\partial_\mu \Phi \partial_\nu \Phi\right]$$

When $\Phi = \Phi(\tau)$ tensor $T_{\mu\nu}$ will have the form of perfect fluid one

$$\rho = (1-4q)V(\Phi)[(\partial_0 \Phi)^2]^{2q}$$

$$p = -V(\Phi)[(\partial_0 \Phi)^2]^{2q}$$

For any $V(\Phi)$ we get

$$p = \omega_\nu\, \rho \qquad \omega_\nu = -\frac{1}{(1-4q)} \equiv -(1-\nu) = const \qquad 0 < \nu < \frac{2}{3}$$

Having solved scalar field equation, one obtains finally

$$\rho_\nu = \frac{A_\nu}{a^{3\nu}}, \qquad p_\nu = -(1-\nu)\frac{A_\nu}{a^{3\nu}}, \quad A_\nu = const > 0, \nu = -\frac{4q}{1-4q}$$

Then the total mass density equals to

$$\rho = \rho_m + \rho_r + \rho_\nu = \frac{A_{CDM}}{a^3} + \frac{A_r}{a^4} + \frac{A_\nu}{a^{3\nu}}$$

where ρ_m is a mass density at the barion domination phase, ρ_r is a mass density at the radiation phase and ρ_ν is a mass density at the last, quintessence period; A_{CDM},A_r, A_ν –constants; equation of state $\omega_{CDM} = 0$ determines the equation of state of the cold dark matter, including the dark mass and the mass of barions; equation of state $\omega_r = 1/3$ labels the matter of radiation, and $\omega_\nu = -1 + \nu$ is equation of state for quintessence [9], [10].

Introducing the standard relative densities and new variable x

$$\Omega_r = \frac{\rho_r^0}{\rho_c^0}, \qquad \Omega_m = \frac{\rho_{CDM}^0}{\rho_c^0}, \qquad \Omega_\nu = \frac{\rho_\nu^0}{\rho_c^0}, \qquad \rho_{\mathrm{c}}^0 = \frac{3}{8\pi} H_0^2, \qquad H_0 = \left(\frac{1}{a}\frac{da}{d\tau}\right)_0^2, \qquad x = \frac{a}{a_0}$$

where “0” labels present moment of evolution, one can get from (10-11)

$$\left(\frac{1}{x}\frac{dx}{d\tau}\right)^2 = H_0^2\left[\frac{\Omega_r}{x^4}+\frac{\Omega_m}{x^3}+\frac{\Omega_\nu}{x^{3\nu}}-\frac{m^2}{6H_0^2}\left(1-\frac{3}{2\beta^4 a_0^2 x^2}+\frac{1}{2a_0^6x^6}\right)\right] \tag{14}$$

$$\frac{1}{x}\frac{d^2x}{d\tau^2} = -\frac{H_0^2}{2}\left[\frac{2\Omega_r}{x^4}+\frac{\Omega_m}{x^3}-2\left(1-\frac{3\nu}{2}\right)\frac{\Omega_\nu}{x^{3\nu}}+\frac{m^2}{3H_0^2}\left(1-\frac{1}{a_0^6x^6}\right)\right] \tag{15}$$

Our model differs from the ΛCDM model of GR only by the m^2terms.

Radiation density Ω_r has been calculated: $\Omega_r = 2{,}5\cdot 10^{-5}/\,h^2$, $h = H_0/100$ km/s/Mps, but the parameters $H_0, \Omega_m, \Omega_\nu, m, \nu$ should be fitted from the observations. Assuming $a_0 \gg 1$, one can conclude $\beta = a_{max} \gg 1$. Therefore, given value of a_0 by virtue of integration of Eq.(14) one can establish full cosmological scenario. Maximal density ρ_{max} is determined by a_0 and the mass of graviton m:

$$\rho_{max} = 9\pi\frac{H_0^4}{m^2}\Omega_r^3 a_0^{12}.$$

If ρ_{max} corresponds electroweak scale $\rho_{max} = 10^{31}\, g/cm^3$, $kT \approx 1\, Tev$, then $a_0 = 5\cdot 10^5$.

In GUT case $\rho_{max} = 10^{79} g/cm^3$,$kT \approx 10^{15}\ Gev$ one has to take $a_0 = 6\cdot 10^9$.

In that case we should choose $a_0 \gg 1$ and at the present time a_0^{-6} −term in Eq.(14) is at least 30 orders of magnitude less than 1. The a_0^{-2}-term is always $(a/a_{max})^4$ times smaller the former one and is also negligible, confirming our assumption. Thus at the present moment of quintessence domination, only constant m^2 term survives, with formally having the form of negative cosmological constant. Contrary to GR, where the dark energy is represented either by quintessence or by cosmological constant, in RTG on the base of Minkowski space one has both of them:

$$\left(\frac{1}{x}\frac{dx}{d\tau}\right)^2 = H_0^2\left[\frac{\Omega_r}{x^4}+\frac{\Omega_m}{x^3}+\frac{\Omega_\nu}{x^{3\nu}}-\frac{m^2}{6H_0^2}\right] \tag{16}$$

where

$$\Omega_r+\Omega_m+\Omega_\nu-\frac{m^2}{6H_0^2}=1$$

Deceleration parameter q in that case equals to

$$q \equiv -\frac{\left(\frac{1}{a}\frac{da}{d\tau}\right)^2}{\left(\frac{1}{a}\frac{d^2a}{d\tau^2}\right)} = \frac{\Omega_m}{2}-\left(1-\frac{3\nu}{2}\right)\Omega_\nu+\frac{m^2}{6H_0^2}$$

consequently $0<\nu<\frac{2}{3}$.

In the quintessence-domination period the time τ is determined from (14-16) as

$$\tau \approx \tau_1+\frac{\sqrt{6}}{H_0 m}\int_{x_\nu}^{x(\tau)}\frac{x^{\frac{3\nu}{2}-1}dx}{\sqrt{\frac{6\Omega_\nu}{m^2}-x^{3\nu}}}$$

where

$$\tau_1 = \frac{1}{H_0}\int_{x_{min}}^{x_\nu} \frac{dx}{x\left[\frac{\Omega_r}{x^4}+\frac{\Omega_m}{x^3}+\frac{\Omega_\nu}{x^{3\nu}}-\frac{m^2}{6H_0^2}\left(1-\frac{3}{2\beta^4 a_0^2 x^2}+\frac{1}{2a_0^6 x^6}\right)\right]^{\frac{1}{2}}}$$

— the moment of start of this domination [11]. Therefore at this stage one can get $a(\tau)$

$$\left(\frac{a(\tau)}{a_0}\right)^{3\nu} \approx \frac{3\Omega_\nu H_0^2}{m^2}\left[1-\cos\left(\sqrt{\frac{3}{2}}m\left(\nu(\tau-\tau_1)+\frac{2}{3H_0}\left(\frac{a_\nu}{a_0}\right)^{3\nu}\frac{1}{\sqrt{\Omega_\nu}}\right)\right)\right]$$

where the period of oscillating evolution τ_{max} and a_{max} are equal to

$$\tau_{max} = \sqrt{\frac{2}{3}}\frac{\pi}{m\nu}, \quad \left(\frac{a_{max}}{a_0}\right)^{3\nu} = \frac{6\Omega_\nu H_0^2}{m^2}$$

Let's notice that the τ_{max} depends not only on graviton mass, but also on the power of quintessence ν. In paper [8] the upper limit on the graviton mass was calculated

$$m \leq 1.6 \cdot 10^{-66} g \text{ (95\% c.l.)}$$

This quantity determines the size of the observed Universe by the present time. Qualitatively (without exact scales) the temporal dependence of the scale factor, its velocity $\dot{a}$ and $\ddot{a}$ is given in the Figure.

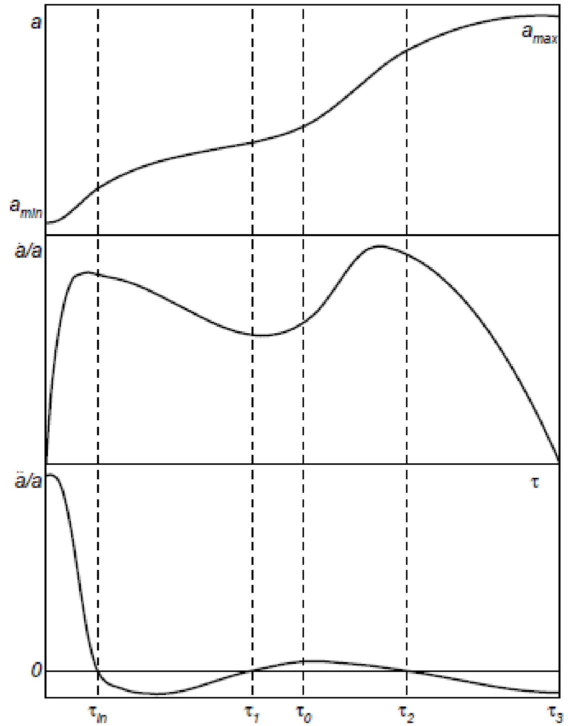


Figure. *Qualitative curves of the dependence of the scale factor (upper part) velocity and acceleration (lower part) dependent on time* τ. *Here* $\tau_{in} = 1{,}15\tau_r$. *The present moment of time is designated* τ_0. *In the beginning the scale factor grows from its minimum value* a_{min} *with a very high acceleration which in a short time enough,* τ_{in}*, becomes zero. The velocity in this period of time increases from the zero value up to the maximum one. The scale factor during this period of time changes insignificantly:* $a(\tau_{in}) = \sqrt{2}a_{min}$. *Further on the expansion occurs with negative acceleration which becomes zero at some moment of time* τ_1. *The value of the velocity drops and somewhat later than* τ_1 *it achieves its minimum value. The scale factor in this period of time continues to rise (expansion continues). The motion with positive acceleration continues till the moment* τ_2. *The velocity and the scale factor increase. At* $\tau > \tau_2$ *the expansion occurs again with negative acceleration until the time when at the momentum* τ_3 *the expansion stops. The scale factor achieves its maximum value. On this the half-cycle is completed and everything repeats in the opposite order: the expansion epoch is changed by the compression epoch. For the quantity* $\dot{a}/a$ *the first maximum is situated at* $a = \sqrt{3/2}\ a_{min}$ $(\tau \sim 0{,}76\ \tau_r)$ *somewhat earlier than* τ_{in}*, in the some way as the second maximum happens prior to* τ_2*. The minimum of* $\dot{a}/a$*, contrary to this, is situated later than* τ_1*. This follows from the fact that the quantity* $(d/d\tau)(\dot{a}/a) = (\ddot{a}/a) - (\dot{a}^2/a^2)$ *at* $\ddot{a} = 0$ *is negative.*

If in the future some evidences of strict condition $\nu = 0$ will appear, then this opportunity forces us to think about searching possible alternative of quintessence in RTG. As we see, one is unable to put Λ term straightforwardly into the gravitational field Lagrangian, or as a part of L_M. But in that case we may consider relativistic theory of gravitation, based on the constant curvature space [14], hoping it provides new mechanism for dark energy.

Such space with the metric tensor $\eta_{\alpha\beta}$ has the following curvature tensor

$$K_{\mu\nu\alpha\beta} = -\frac{P}{12}\left(\eta_{\mu\nu}\eta_{\alpha\beta} - \eta_{\mu\alpha}\eta_{\nu\beta}\right), \qquad P = K \equiv \eta^{\mu\nu}K_{\mu\nu}, \qquad K_{\mu\nu} \equiv \eta^{\alpha\beta}K_{\alpha\mu\nu\beta}$$

$P = 0$ — Minkowski space $\left(\eta_{\alpha\beta} \equiv \gamma_{\alpha\beta}\right)$,

$P < 0$ — de Sitter space, (dS)

$P > 0$ — anti de Sitter space(adS).

Introducing gravitational field in close analogy with Minkowski case

$$\sqrt{-g}g^{\alpha\beta} = \sqrt{-\eta}\left(\eta^{\alpha\beta} + \varphi^{\alpha\beta}\right), \quad g = det\left(g_{\alpha\beta}\right), \quad \sqrt{-\eta} = det\left(\eta_{\alpha\beta}\right)$$

and again put forward geometrization of the matter Lagrangian, we, following [14], finally come to the generalized field equations, containing mass of the graviton:

$$R_\nu^\mu - \frac{1}{2}\delta_\nu^\mu R + \frac{m^2}{2}\delta_\nu^\mu + \left(\frac{m^2}{2} - \frac{P}{2}\right)\left(g^{\mu\varepsilon} - \frac{1}{2}\delta_\nu^\mu g^{\varepsilon\tau}\right)\eta_{\varepsilon\tau} = 8\pi T_\nu^\mu \tag{17}$$

$$D_\beta\sqrt{-g}g^{\alpha\beta} = 0 \tag{18}$$

Comparing with (3),(4), we see the full identity in form with only exception - P-term in Eq.(17).

Field equations (17), (18) obtained from the Lagrangian

$$L = L_g\left(\eta_{\alpha\beta}, \sqrt{-g}g^{\alpha\beta}\right) + L_M\left(g_{\alpha\beta}, \phi_A\right)$$

$$L_g = \sqrt{-g}g^{\alpha\beta}\left(G^{\sigma}_{\alpha\beta}G^{\mu}_{\sigma\mu} - G^{\mu}_{\alpha\nu}G^{\nu}_{\beta\mu}\right) - \frac{m^2}{16\pi}\left(\frac{1}{2}\sqrt{-g}g^{\alpha\beta}\eta_{\alpha\beta} - \sqrt{-g} - \sqrt{-\eta}\right)$$

$$G^{\sigma}_{\alpha\beta} = \frac{1}{2}g^{\sigma\nu}\left(D_\alpha g_{\beta\nu} + D_\beta g_{\alpha\nu} - D_\nu g_{\alpha\beta}\right)$$

To simplify the following calculations, we introduce the proper time τ:

$$d\tau = \sqrt{U}dt, \qquad U > 0$$

We can now rewrite the intervals of the background space and the Riemannian space in the form

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = d\tau^2 - a^2(\tau)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin\theta^2\, d\varphi^2)\right] \tag{19}$$

$$d\sigma^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta = \frac{1}{U(\tau)}d\tau^2 - b^2(\tau)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin\theta^2\, d\varphi^2)\right] \tag{20}$$

Substituting $g_{\alpha\beta}$ and $\eta_{\alpha\beta}$ given by (19) and (20) in the RTG equations (17) and (18), we obtain the equations

$$\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 = \frac{8\pi}{3}\rho - \frac{m^2}{6} - \left(\frac{m^2}{6} - \frac{P}{4}\right)\left[\frac{1}{6U} - \frac{b^2}{2a^2}\right] - \frac{k}{a^2} \tag{21}$$

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\frac{4\pi}{3}(\rho + 3p) - \frac{m^2}{6} + \left(\frac{m^2}{2} - \frac{P}{4}\right)\frac{1}{3U} \tag{22}$$

$$\frac{d}{d\tau}\frac{a^3}{\sqrt{U}} = \frac{3}{2}a\sqrt{U}\frac{db^2}{d\tau} \tag{23}$$

Consider the second derivative of the scaling factor a which is responsible for cosmological acceleration. In the absence of quintessence the only positive term in the RHS of Eq.(22) is $\frac{m^2}{2} - \frac{P}{4}$ with $P < 0$. Henceforth, only deSitter background space potentially can be the “new dark matter”. And this is really the case.
For dS space the homogeneous and isotropic intervals have the form

$$k < 0 \qquad d\sigma^2 = dt^2 - \frac{k_0^2}{H_0^2}\sinh^2 H_0 t\left[\frac{dr^2}{1+k_0^2r^2} + r^2(d\theta^2 + \sin\theta^2\, d\varphi^2)\right]$$

$$k = 0 \qquad d\sigma^2 = dt^2 - \frac{k_0^2}{H_0^2}e^{2H_0t}[dr^2 + r^2(d\theta^2 + \sin\theta^2\, d\varphi^2)]$$

$$k > 0 \qquad d\sigma^2 = dt^2 - \frac{k_0^2}{H_0^2}\cosh^2 H_0 t\left[\frac{dr^2}{1-k_0^2r^2} + r^2(d\theta^2 + \sin\theta^2\, d\varphi^2)\right]$$

$$H_0 \equiv \sqrt{\frac{|P|}{12}}\,, \qquad k_0 \equiv \sqrt{|k|}$$

The previous calculations enable us to describe in general features the evolution of the universe that is predicted by the RTG for different choices of the background space. Naturally, the question arises of which of the possible models is closest to the real universe. Unfortunately, present experimental data do not permit the selection of one definite model.
Due to the cosmological acceleration rejects adS background, we now describe the possible cosmological scenarios corresponding to different de Sitter background spaces. There exist three types of these spaces.

a)Open dS space ($P < 0, k < 0$*).*

For this choice of the background, the evolution of the universe can be divided into three stages. In the first stage, there is contraction: The scale factor $a(\tau)$ decreases exponentially, and the proper time τ is (up to an additive constant) equal to the background time t. The Hubble constant H is related to the curvature P of the background space:

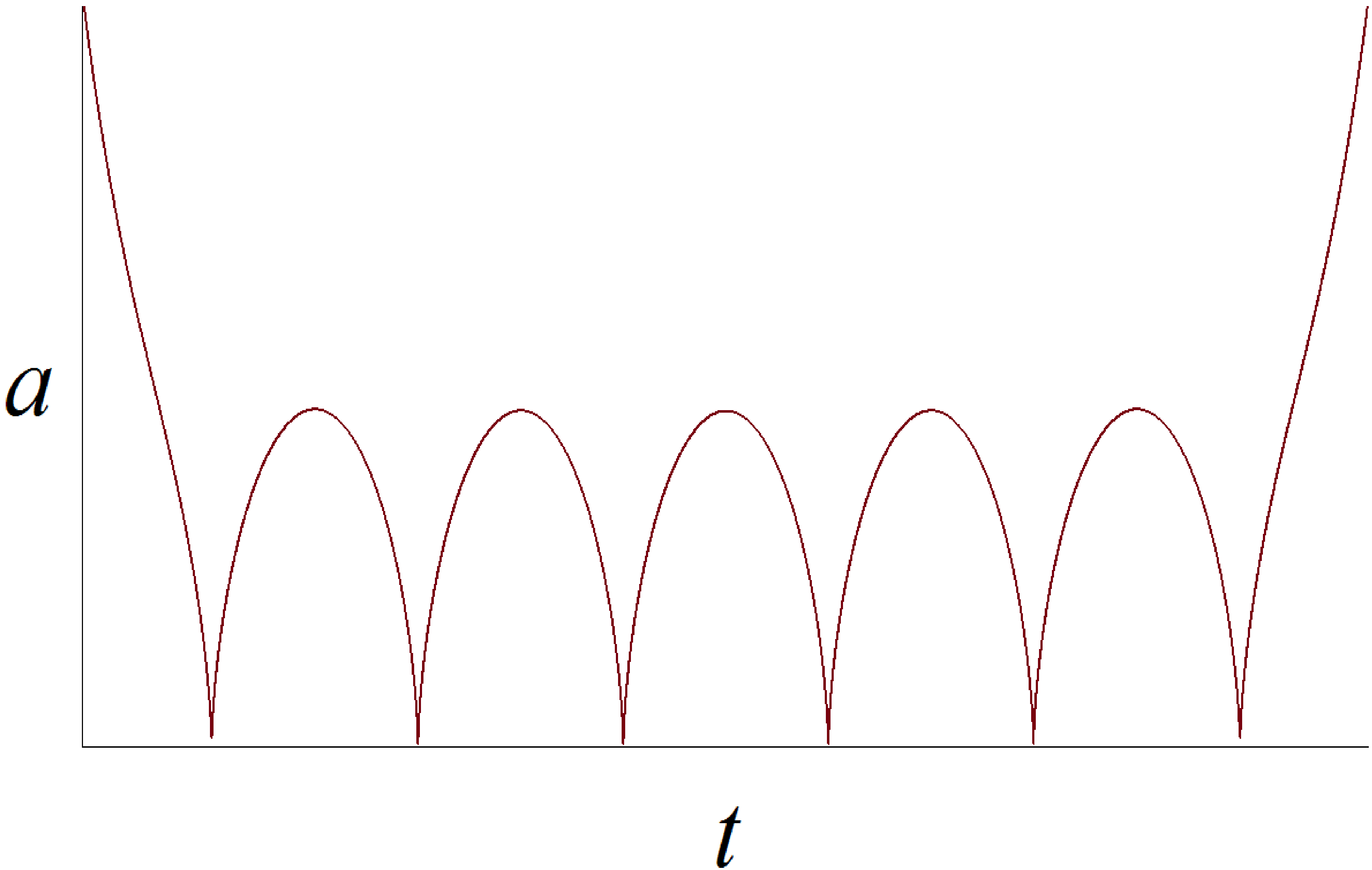
$$H \approx -Ho = -\sqrt{-\frac{P}{12}} < O.$$

Since we observe $H > 0$, this stage cannot be associated with the present state of the universe. In the second stage some oscillating behavior of the functions $a(\tau)$ and $U(\tau)$ is evidently possible. At the same time, H can sometimes be positive, but we can say nothing about its numerical value. To the third stage, there corresponds exponential expansion of the universe, and τ is again equal to t (up to a constant). If we assume that it is in this epoch that we live, then the value of H enables us to determine the curvature of the background Riemann space: $H \approx \sqrt{-P/12}$. The deceleration parameter q must have the value close to -1 (see below).

b) *Closed dS space (*$P < 0, k > 0$*).*

In this case, the scenario of the evolution of the universe is practically identical to the previous case; certain differences are possible only in the "central region" (around $t \sim 0$) .

$P < 0, k \neq 0$



c). *Flat dS space* ($P < 0, k = 0$).

In the period $t < 0$ the universe oscillates, and the nature of the oscillations is practically the same it was considered in the framework of the RTG on the basis of flat Minkowski space, but without any acceleration. The only viable for the present time stage is $t \gg 0$. In this case the universe can behave in the same way as in the third stage for the open background dS space described above.
In all these cases background scaling factor $b(t)$ on the $t \to \infty$ stage is not constrained from above

$$b(t) \sim e^{H_0 t} \to \infty$$

In this limit we have asymptotically

$$a = e^{H_0 t}\big(1 + \varphi(t)\big),\ \varphi(t) = \frac{4\pi(\gamma-3)A_\gamma}{H_0^2\gamma(\gamma-3)+m^2} e^{-\gamma H_0 t} \ll 1 \tag{24}$$

$$\frac{1}{\sqrt{U}} = 1 + \psi(t), \qquad \psi(t) = \frac{4\pi(6-\gamma)A_\gamma}{3H_0^2\gamma(\gamma-3)+3m^2} e^{-\gamma H_0 t} \ll 1 \tag{25}$$

where dominating matter has the density

$$\rho = \frac{A_\gamma}{a^{3\gamma}}$$

The deceleration parameter is positive and goes to 1:

$$-q = 1 - 2\varphi + \frac{\dot{\psi}}{H_0} + \frac{\ddot{\varphi}}{H_0^2} = 1 + (\gamma^2 - 2)\varphi - 3\psi$$

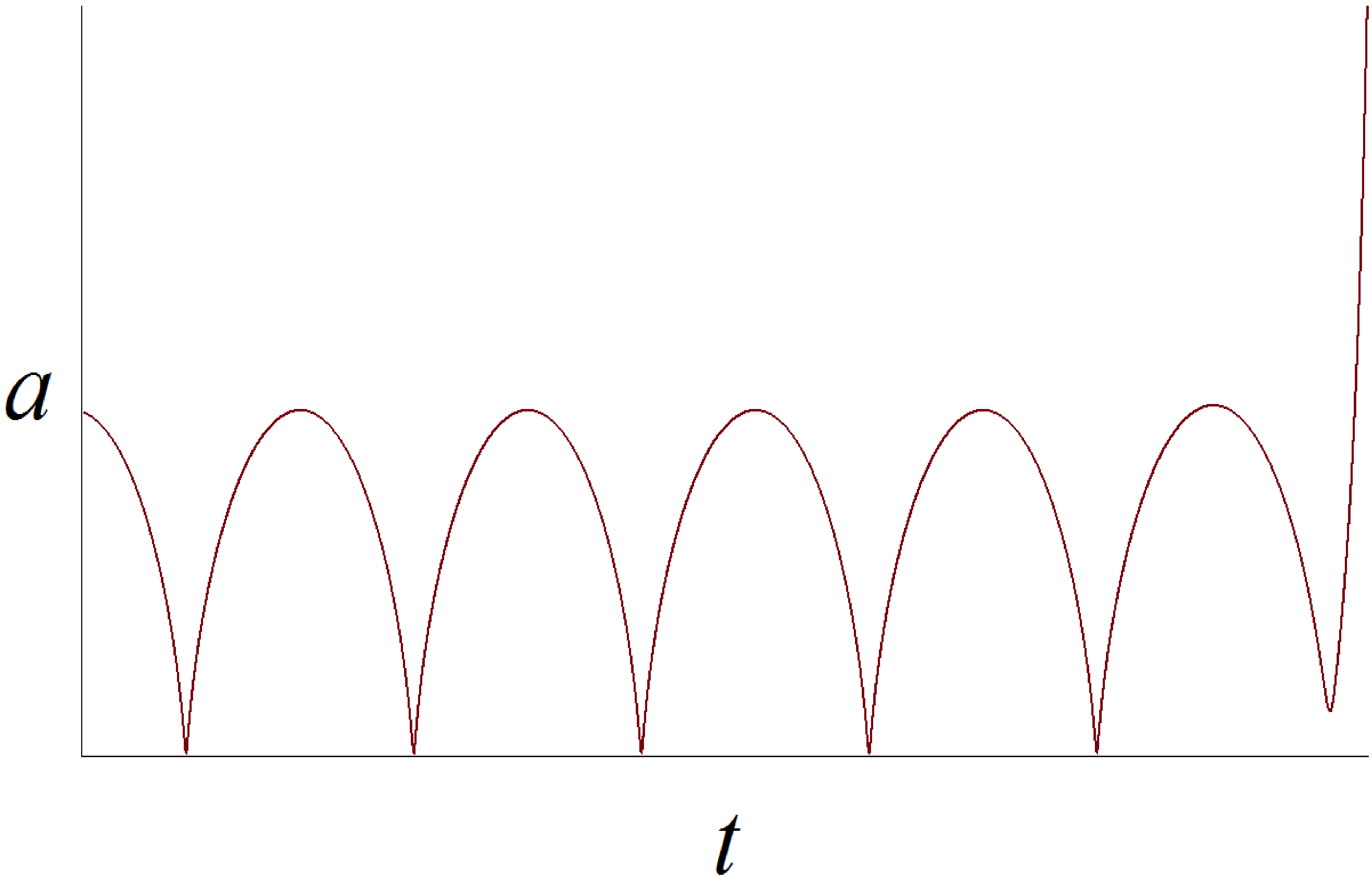
For CDM

$$\gamma = 1 + \omega_m\,, \qquad \omega_m \ll 1\,, \quad \varphi \ll \psi$$

we get finally

$$-q = 1 - \frac{12\pi A_\gamma}{9H_0^2\omega_m + m^2} e^{-3H_0 t}$$

$P < 0, k = 0$



The final viability of the proposed models will depends on the full fit procedure with the observational data.

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