Nonzero θ_{13} and CP Violation from Broken $\mu-\tau$ Symmetry

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Outline

- **Introduction**
- **Broken** $\mu-\tau$ **Symmetry**
- **Nonzero** θ_{13} **and Jarlskog rephasing invariant**
- **Conclusions**

Introduction

- Recently, there are two unsolved major problems in neutrino physics (Beyond the Standard Model):
  - Underlying symmetry of neutrino mass matrix
  - Neutrino mixing matrix (recently: TBM, BM, DC)
- One of the interesting underlying symmetries is the $\mu-\tau$ symmetry:
  - Reduce the number of parameters from 6 to 4
  - Predict mixing angle $\theta_{13}=0$ which is enough to accommodate the experimental data before the measurement of the Double Chooz, MINOS, T2K, Daya Bay, and RENO collaborations that $\theta_{13}\neq 0$.

- The neutrino mass matrix with $\mu-\tau$ symmetry can also be obtained trivially from three well-known mixing matrices: TBM, TB, and DC
- After the exp. facts, that mixing angle $\theta_{13} \neq 0$ and relatively large, the $\mu-\tau$ symmetry and the mixing matrices TBM, TB, and DC should be ruled out or modified, because it cannot predict $\theta_{13} \neq 0$ any more as dictated by experimental facts.
- In this talk, I modify the neutrino mass matrix that obey $\mu-\tau$ symmetry by introducing a small parameter to perturb the neutrino mass matrix with constraint:
  - The trace of the perturbed neutrino mass matrix remain constant or equal with the trace of unperturbed $\mu-\tau$ symmetry.

Broken $\mu-\tau$ Symmetry

- Many authors have made effort to modify the neutrino mass matrix of $\mu-\tau$ symmetry (see refs. 7-9).
- As the first step, we begin from the standard parameterization of neutrino mixing matrix:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \qquad (1)$$

where c_{ij} is the $\cos\theta_{ij}$, s_{ij} is the $\sin\theta_{ij}$, and θ_{ij} are the mixing angles. In the basis where the charged lepton mass matrix is diagonal, the neutrino mass matrix M_ν can be diagonalized by mixing matrix V as follow:

$$M_\nu = VMV^T, \tag{2}$$

where the diagonal neutrino mass matrix M is given by:

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{3}$$

If we put the mixing angles $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ and consequently: $c_{23} = s_{23} = 1/\sqrt{2}$ and $s_{13} = 0$ and $c_{13} = 1$ in Eq. (1), then the neutrino mass matrix M_ν in Eq. (2) read:

$$M_\nu = \begin{pmatrix} P & Q & -Q \\ Q & R & S \\ -Q & S & R \end{pmatrix}, \tag{4}$$

where:

$$P = m_1 c_{12}^2 + m_2 s_{12}^2, \tag{5}$$

$$Q = \frac{\sqrt{2}}{2}(m_2 - m_1)c_{12}s_{12}, \tag{6}$$

$$R = \frac{1}{2}(m_1 s_{12}^2 + m_2 c_{12}^2 + m_3), \tag{7}$$

$$S = \frac{1}{2}(-m_1 s_{12}^2 - m_2 c_{12}^2 + m_3). \tag{8}$$

It is also possible to obtain the neutrino mass matrix of $\mu - \tau$ symmetry by choosing the mixing angle $\theta_{23} = \pi/4$ with $\delta = \pm\pi/2$ of neutrino mixing matrix in Eq. (1). But, as dictated from the experimental results, the mixing angle $\theta_{13} \neq 0$ and relatively large [1, 2, 3, 4, 5] which imply that the assumption: $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ in formulating the neutrino mixing matrix must be rule out and hence the exact $\mu - \tau$ symmetry as the underlying symmetry of neutrino mass matrix is no longer adequate to accommodate the recent experimental results.

Now, we are in position to study the effect of neutrino mass matrix that obey the $\mu - \tau$ symmetry breaking in relation to the nonzero θ_{13} and Jarlskog rephasing invariant $J_{\rm CP}$ by breaking the neutrino mass matrix in Eq. (4). If we assume $\theta_{13} \neq 0$ the value of the mixing angle $\theta_{23} = \pi/4$ is no longer valid and consequently the parameters: P, Q, R, and S of Eqs. (5)-(8) can not be used furthermore. We can obtain the neutrino mass matrix with $\theta_{23} \neq \pi/4$ and $\theta_{13} \neq 0$ from Eqs. (1) and (3). To simplify the problem, we take the value of $s_{13}e^{i\delta} \approx 0$ and then we break the resulted neutrino mass matrix by introducing a complex paramater ix with the constraint that the trace of the broken neutrino mass matrix is remain constant or equal to the trace of the unbroken one. This scenario of breaking has been applied by Damanik [13] to break the neutrino mass matrix invariant under a cyclic permutation. In this breaking scenario, the broken neutrino mass matrix reads:

$$M_\nu \approx \begin{pmatrix} A & B & -B \\ B & C - ix & D \\ -B & D & E + ix \end{pmatrix}, \tag{9}$$

where:

$$A = m_1 c_{12}^2 + m_2 s_{12}^2, \tag{10}$$

$$B = (m_2 - m_1) c_{12} s_{12} c_{23}, \tag{11}$$

$$C = (m_1 s_{12}^2 + m_2 c_{12}^2) c_{23}^2 + m_3 s_{23}^2, \tag{12}$$

$$D = (-m_1 s_{12}^2 - m_2 c_{12}^2 + m_3) c_{23} s_{23}, \tag{13}$$

$$E = (m_1 s_{12}^2 + m_2 c_{12}^2) s_{23}^2 + m_3 c_{23}^2. \tag{14}$$

- From Eq. (9) we determine the Jarlskog rephasing invariant J_{CP} as a parameter for the existence of the CP violation, where:

$$J_{\rm CP} = -\frac{{\rm Im}\left[(M'_\nu)_{e\mu}(M'_\nu)_{\mu\tau}(M'_\nu)_{\tau e}\right]}{\Delta m^2_{21}\Delta m^2_{32}\Delta m^2_{31}}, \quad (15)$$

where $(M'_\nu)_{ij} = (M_\nu M^\dagger_\nu)_{ij}$ with $i,j = e,\nu,\tau$, and $\Delta m^2_{kl} = m^2_k - m^2_l$ with $k = 2,3,$ and $l = 1,2$. From Eq. (9), we have:

$$M'_\nu = \begin{pmatrix} A^2+2B^2 & B(A-D+C+ix) & B(-A+D-E+ix) \\ B(A-D+C-ix) & B^2+C^2+D^2+x^2 & -B^2+D(C+E-2ix) \\ B(-A+D-E-ix) & -B^2+D(C+E+2ix) & B^2+D^2+E^2+x^2 \end{pmatrix}. \quad (16)$$

Nonzero θ_{13} and Jarlskog rephasing invariant

From Eqs. (15) and (16) we have the Jarlskog rephasing invariant as follow:

$$J_{\rm CP} = \frac{B^2\left[(B^2(2D - C - 2A - E) + D(E^2 - 2D^2 + C^2 + 4AD))\,x + 2Dx^3\right]}{\Delta m^2_{21}\Delta m^2_{32}\Delta m^2_{31}}. \tag{17}$$

If we insert Eqs. (10)-(14) into Eq. (17), then we have the $J_{\rm CP}$ as follow:

$$\begin{aligned} J_{\rm CP} = \frac{2(m_2-m_1)^2 c^2_{12}s^2_{12}c^2_{23}}{\Delta m^2_{21}\Delta m^2_{32}\Delta m^2_{31}} &[[(-m_1 s^2_{12} - m_2 c^2_{12} + m_3)s_{23}c_{23} \\ &\times((-m_1 s^2_{12} - m_2 c^2_{12} + m_3)s_{23}c_{23} + m_1 c^2_{12} + m_2 s^2_{12})^2 \\ &-(m_2-m_1)^2 c^2_{12}s^2_{12}c^2_{23}((m_1 s^2_{12} + m_2 c^2_{12})c^2_{23} + m_2 s^2_{23} \\ &\quad -(m_1 s^2_{12} + m_2 c^2_{12} - m_3)s_{23}c_{23} + m_1 c^2_{12} + m_2 s^2_{12}) \\ &-((m_1 s^2_{12} + m_2 c^2_{12})c^2_{23} + m_2 s^2_{23})^2(-m_1 s^2_{12} - m_2 c^2_{12} \\ &+m_3)s_{23}c_{23}]x - c_{23}s_{23}[-m_1 s^2_{12} - m_2 c^2_{12} + m_3]x^3]. \end{aligned} \tag{18}$$

- One can see from Eq. (18):
  - if we put: $x = 0$, then we have $J_{CP} = 0$ (exact mu-tau symmetry).
  - Jarlskog rephasing invariant, for our scenario, does not depend on mixing angle θ_{13} explicitly.
- If we neglect contribution of the last term in Eq. (18), *x* is very small compare to first term, then we have:

$$
\begin{aligned}
J_{\rm CP} \approx \frac{2(m_2-m_1)^2 c_{12}^2 s_{12}^2 c_{23}^2}{\Delta m_{21}^2 \Delta m_{32}^2 \Delta m_{31}^2}[(-m_1 s_{12}^2 - m_2 c_{12}^2 + m_3) s_{23} c_{23} \\
\times((-m_1 s_{12}^2 - m_2 c_{12}^2 + m_3) s_{23} c_{23} + m_1 c_{12}^2 + m_2 s_{12}^2)^2 \\
-(m_2 - m_1)^2 c_{12}^2 s_{12}^2 c_{23}^2 ((m_1 s_{12}^2 + m_2 c_{12}^2) c_{23}^2 + m_2 s_{23}^2 \\
-(m_1 s_{12}^2 + m_2 c_{12}^2 - m_3) s_{23} c_{23} + m_1 c_{12}^2 + m_2 s_{12}^2) \\
-((m_1 s_{12}^2 + m_2 c_{12}^2) c_{23}^2 + m_2 s_{23}^2)^2 (-m_1 s_{12}^2 - m_2 c_{12}^2 \\
+m_3) s_{23} c_{23}] x.
\end{aligned}
\tag{14}
$$

- If we insert the experimental value of neutrino oscillations:

$$\Delta m^2_{21} = 7.59 \pm 0.20(^{+0.61}_{-0.69}) \times 10^{-5}\ \mathrm{eV}^2, \tag{19}$$

$$\Delta m^2_{32} = 2.46 \pm 0.12(\pm 0.37) \times 10^{-3}\ \mathrm{eV}^2,\ \text{(for NH)} \tag{20}$$

$$\Delta m^2_{32} = -2.36 \pm 0.11(\pm 0.37) \times 10^{-3}\ \mathrm{eV}^2,\ \text{(for IH)} \tag{21}$$

$$\theta_{12} = 34.5 \pm 1.0(^{3.2}_{-2.8})^o,\quad \theta_{23} = 42.8^{+4.5}_{-2.9}(^{+10.7}_{-7.3})^o,\quad \theta_{13} = 5.1^{+3.0}_{-3.3}(\leq 12.0)^o, \tag{22}$$

and for simplicity we put: $m_1 = 0$, then we have:

$$m^2_2 = \Delta m^2_{21}, \tag{23}$$

$$m^2_3 = \Delta m^2_{32} + \Delta m^2_{21}. \tag{24}$$

By inserting the values of Eqs. (19)-(24), we have:

$$J_{\rm CP} = 0.4644x - 832.9790x^3. \tag{25}$$

From Eq. (25), we can determine the maximum value of $J_{\rm CP}$ by using the relation:

$$\frac{dJ_{\rm CP}}{dx} = 0, \tag{26}$$

which proceed $x = 0.0167$. By substituting the value of $x = 0.0167$ into Eq. (25), we have the maximum value of Jarlskog rephasing invariant:

$$J_{\rm CP} \approx 0.004. \tag{27}$$

It is also possible to determine the Jarlskog rephasing invariant $J_{\rm CP}$ from the neutrino mixing matrix by using the relation:

$$J_{\rm CP} = {\rm Im}(V_{11}^{*}V_{23}^{*}V_{13}V_{21}). \tag{28}$$

From neutrino mixing matrix of Eq. (1), the Jarlskog rephasing invariant $J_{\rm CP}$ reads:

$$\begin{aligned} J_{\rm CP} &= c_{12}s_{12}c_{23}s_{23}c_{13}^{2}s_{13}\sin\delta \\ &= c_{12}s_{12}c_{23}s_{23}\left(s_{13} - s_{13}^{3}\right)\sin\delta. \end{aligned} \tag{29}$$

If we put the experimental values of mixing angles θ_{12} and θ_{23} of Eq. (22) into Eq. (29), then we have:

$$J_{\rm CP} = 0.2327\left(s_{13} - s_{13}^3\right)\sin\delta. \tag{30}$$

Since the value of $s_{13}^3 << s_{13}$, the Eq. (30) can be approximated as follow:

$$J_{\rm CP} \approx 0.2327 s_{13}\sin\delta. \tag{31}$$

If we insert the value of Jarlskog rephasing invariant of Eq. (27) into Eq. (31), then we have the Dirac phase δ as follow:

$$\delta \approx \arcsin\left(\frac{0.0172}{\sin\theta_{13}}\right). \tag{32}$$

By inserting the value of mixing angle θ_{13} as shown in Eq. (22), we have:

$$\delta \approx 11.2^{\rm o} \tag{33}$$

Conclusions

We have studied systematically the effect of breaking on neutrino mass matrix that obey $\mu - \tau$ symmetry by introducing a complex parameter ix with the requirement that the trace of the broken $\mu - \tau$ symmetry is remain constant. By using the experimental data of mixing angles θ_{12} and θ_{23} as input, we can obtain the Jarlskog rephasing invariant $J_{\mathrm{CP}} \neq 0$ which indicate the existence of CP violation in neutrino sector and hence the Dirac phase δ which also depend on the mixing angle θ_{13} for neutrino mass in normal hierarchy for the case: $m_1 = 0$.

References

[1] Apollonio M *et al.* [Double Chooz Collab.] 1999 *Phys. Lett.* **B 466** 415

[2] Adamson P *et al.* [MINOS Collab.] 2011 *Phys. Rev. Lett.* **107** 181802

[3] Abe K *et al.* [T2K Collab.] 2011 *Phys. Rev. Lett.* **107** 041801

[4] An F P *et al.* 2012 *Phys. Rev. Lett.* **108** 171803

[5] Ahn J K *et al.* [RENO Collab.] 2012 *Phys. Rev. Lett.* **108** 191802

[6] Mohapatra R N 2004 *JHEP* **0410** 027

[7] Aizawa I and Yasue M 2006 *Phys. Rev.* **D 73** 015002

[8] Fuki F and Yasue M 2006 *Phys. Rev.* **D 73** 055014

[9] Mohapatra R N and Rodejohann W 2005 *Phys. Rev.* **D 72** 053001

[10] Baba T and Yasue M 2007 *Phys. Rev.* **D 75** 055001

[11] He H -J and Xu X -J 2012 Phys. Rev. **D 86** 111301 (R)

[12] Damanik A 2013 Nonzero θ_{13}, CP violation, and $\mu - \tau$ symmetry *Preprint* arXiv:1305.6900 [hep-ph]

[13] Damanik A 2011 *Mod. Phys. Lett.* **A 26** 567

[14] Branco C G, Gonzalez Felipe R, Joaquim F R, Masina I, Rebelo M N, and Savoy C A 2003 *Phys. Rev.* **D 67** 073025

[15] Gonzales-Carcia M, Maltoni M and Salvado J 2010 *JHEP* **056** 1004

[16] Fogli G *et al.* 2010 *J. Phys. Con. Ser.* **203** 012103

I thank you !!!