The interaction region of high energy protons

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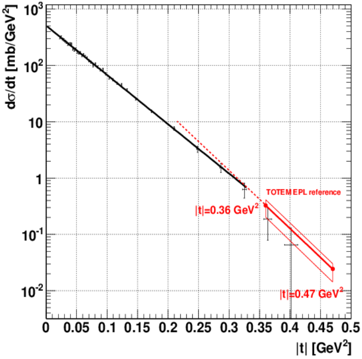
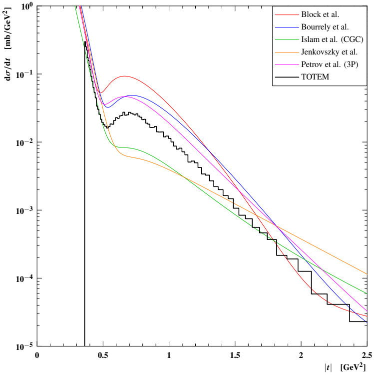
24.06.2014; Protvino, HEPFT2014

arXiv:1406.2153

Kinematically simplest process with two variables
$s = 4E^2$ and $t = -2p^2(1-\cos\theta) \approx -p^2\theta^2$.

The only measurable characteristics
$\frac{d\sigma}{dt} = |f|^2 = (\mathrm{Im} f(s,t))^2 + (\mathrm{Re} f(s,t))^2$ - modulus!
and
$\rho(s, t=0) = \frac{\mathrm{Re} f(s,t=0)}{\mathrm{Im} f(s,t=0)}$ with $\mathrm{Im} f(s,0) = \sigma_t / 4\sqrt{\pi}$





The diffraction cone (left) and beyond it (right) - TOTEM
The predictions of five models are demonstrated.

The geometry of the collision – the impact parameter b

The impact parameter b is a transverse distance between the trajectories of colliding particles

$$i\Gamma(s,b) = \frac{1}{2\sqrt{\pi}} \int_0^{\infty} d|t| f(s,t) J_0(b\sqrt{|t|}).$$

Two functions $\mathrm{Im} f(s,t)$, $\mathrm{Re} f(s,t)$ as parts of a single analytic function $f(s,t)$ are related by the dispersion relations and the unitarity condition

$$G(s,b) = 2\mathrm{Re}\Gamma(s,b) - |\Gamma(s,b)|^2.$$

$G(s,b)$ is the impact parameter profile of inelastic interactions (the overlap function) showing full absorption if $G = 1$ or complete transparency if $G = 0$. It describes the shape of the inelastic interaction region of colliding particles. Elastic profile is given by the subtrahend.
If integrated over all b, the relation leads to $\sigma_{in} = \sigma_t - \sigma_{el}$.

The diffraction cone approximation

$$\frac{d\sigma}{dt} \propto e^{-B|t|},$$

$$i\Gamma(s,b) = \frac{\sigma_t}{8\pi}\int_0^\infty d|t| e^{-B|t|/2}(i+\rho(s,t))J_0(b\sqrt{|t|}),$$

$$\mathrm{Re}\Gamma(s,b) = \frac{1}{Z}e^{-\frac{b^2}{2B}},$$

where $Z = 4\pi B/\sigma_t$.

NOTE! The value of Z is defined by the ratio of the diffraction cone slope to the total cross section.

If $\rho(s,t) \ll 1$

$$G(s,b) = \frac{2}{Z}e^{-\frac{b^2}{2B}} - \frac{1}{Z^2}e^{-\frac{b^2}{B}}.$$

The energy dependence of the overlap function for central collisions with $b = 0$

$$G(s, b = 0) = \frac{2Z - 1}{Z^2}$$

The opacity at $b = 0$ is fully determined by Z!
Accuracy is better than $\rho^2(s, 0)$ (< 0.02 at high energies) and $(\Delta Z)^2$ at $Z = 1$ if Z is measured with precision ΔZ.

Table. The energy behavior of Z and $G(s, 0)$

$\sqrt{s}$, GeV	2.70	4.11	7.62	13.8	62.5	546	1800	7000
Z	0.64	1.02	1.34	1.45	1.50	1.20	1.08	1.00
$G(s, 0)$	0.68	1.00	0.94	0.904	0.89	0.97	0.995	1.00

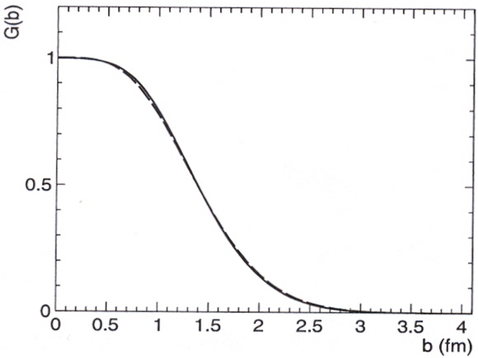
NOTE! Z decreases from ISR to LHC!

Special cases:
Full absorption $Z = 1$; $G(s, 0) = 1$.
Complete transparency $Z = 0.5$; $G(s, 0) = 0$.

The impact parameter dependence of the overlap function

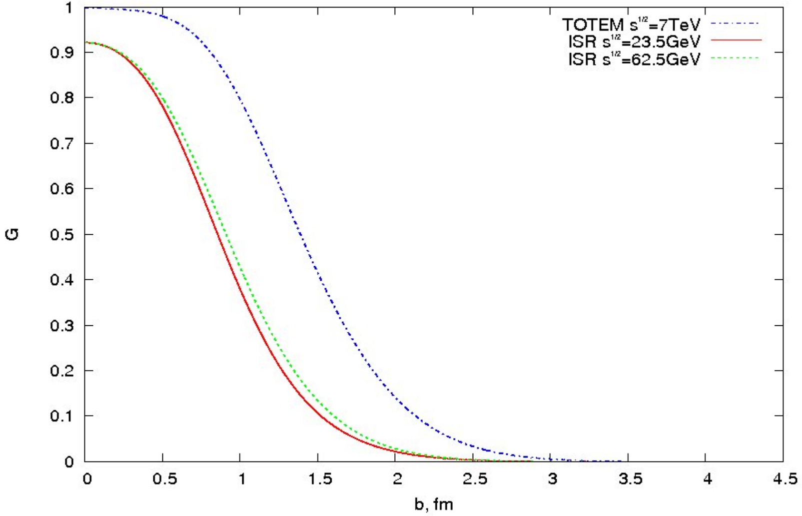
$$G(s,b) = \frac{1}{Z^2}\left[2Z - 1 - \frac{b^2}{B}(Z-1) - \frac{b^4}{4B^2}(2-Z)\right] \text{ at } b^2 \ll B$$

PLATEAU at $Z = 1$ since the second term vanishes!



The impact parameter dependence of the overlap function $G(b)$ at 7 TeV according to the direct computation from experimental data (solid line) and to the diffraction cone approximation (dashed line). Both curves practically coincide.

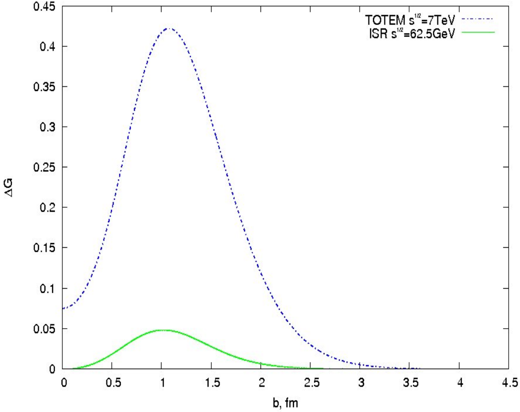
The energy evolution of the impact parameter picture



The overlap functions at 23.5 GeV (solid curve), 62.5 GeV (dotted curve) and 7 TeV (dash-dotted curve)

Difference between the overlap functions at different energies

$$\Delta G(b) = G(s_1, b) - G(s_2, b)$$



Dash-dotted curve is for 7 TeV and 23.5 GeV energies, solid curve is for 62.5 GeV and 23.5 GeV energies.

Conclusion: The parton density at the periphery strongly increases!

Some consequences for inelastic processes

The two-scale picture

(black plateau + more transparent gray tail)

The probability of inelastic collision with an impact parameter smaller than b is $P_{in}(b) = \int_0^b d^2bG(s,b)/\sigma_{in}(s)$. The cross section increases mostly due to the growth in the gray region. The dark region constitutes only about 8% at 7 TeV. However, namely this region is responsible for rare high multiplicity events with jet production.

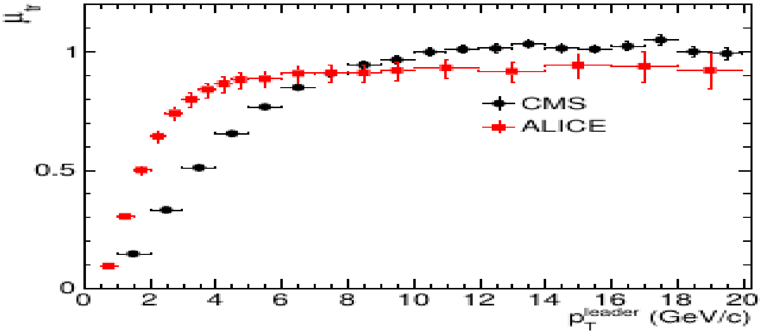
M.Yu. Azarkin, I.M. Dremin, M. Strikman, arXiv:1401.1973.

The analysis of CMS data at 7 TeV has revealed some deficit of jet production in experiment at very high multiplicities vs MC predictions. The discrepancy can be explained by further increase of the parton density beside the purely geometrical extension of the interaction region in combination with the increasing role of multiparton interactions. These features should be accounted in new modifications of MC models.

The black plateau in the central part of the interaction region with $b < 0.4 - 0.5$ fm results in the corresponding plateau of the charged particle density in the transverse region $60^0 < |\Delta\phi| < 120^0$

$$\mu_{tr} = \frac{N_{ch}^{tr}}{\Delta\eta\Delta(\Delta\phi)},$$

N_{ch}^{tr} - the charged particle multiplicity in the transverse region, $\Delta\eta$ - the pseudorapidity range studied, $\Delta(\Delta\phi)$ - the azimuthal width of the region. This is really the case - see Figure (interpretation!).



What can happen at higher energies?

The tendency of Z to decrease from ISR to LHC!
If it persists, and Z will be smaller than 1 at 13-100 TeV?

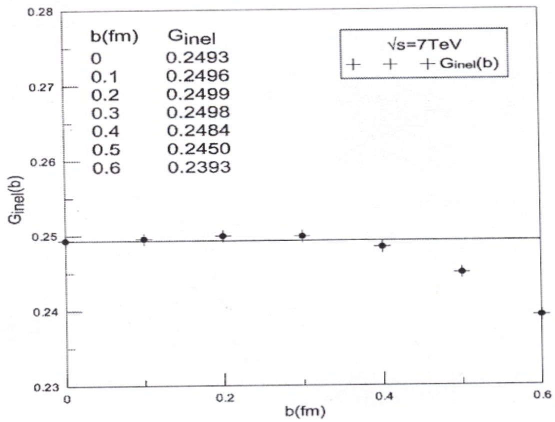
The limiting behavior at $Z = 0.5$: $\sigma_{el} = \sigma_{in} = 0.5\sigma_t$
This limit is usually called as the black disk. However, in this case $G(s,b) = 4[e^{-\frac{b^2}{2B}} - e^{-\frac{b^2}{B}}]$ and $G(s,0) = 0$, i.e. complete transparency at the very central collisions and full absorption $G(s,b_m) = 1$ at $b_m = R\sqrt{0.5\ln 2} \approx 0.59R$. Only at $b > b_m$ inelastic profile is higher than elastic! Inelastic periphery!

TORUS! or RING (in two dimensions), not black disk.
Three dimensional TUBE! Large longitudinal distances.

Paradox? Protons at $b = 0$ pass through each other unnoticed!
Physics: Coherence of partons? High parton density?
Analogy: light scattering in water (coherent) and air (decoh.)

Implications for inelastic processes: Decreasing role of central interactions - changing shape of multiplicity distributions with low tails? Lower share of jets? or their new features? ...

Another analysis of the TOTEM data at 7 TeV.
A. Alkin, E. Martynov, O. Kovalenko, S.M. Troshin, arXiv:1403.8036



The impact parameter dependence of the function $G_{inel}(b) = 0.25G(b)$ at 7 TeV according to the direct computation from experimental data

All model predictions failed beyond the diffraction cone (Fig). Namely here the approach of Z to 1 is very important.

The *linear* integral equation in Orear region $\mathrm{Im} f(p,\theta) =$

$$\frac{p\sigma_t}{4\pi\sqrt{2\pi B}}\int_{-\infty}^{+\infty} d\theta_1 \exp(-Bp^2(\theta-\theta_1)^2/2) g_\rho \mathrm{Im} f(p,\theta_1) + G(p,\theta),$$

where $g_\rho = 1+\rho_0\rho(\theta_1)$ and $G(p,\theta)$ is the overlap function.

Assume that $g_\rho =$const and $G(p,\theta) \ll 1$.
Then the eigensolution of this equation is $\mathrm{Im} f(p,\theta) =$

$$C_0 \exp\left(-\sqrt{2B\ln\frac{Z}{g_\rho}p\theta}\right) + \sum_{n=1}^{\infty} C_n \exp(-(\mathrm{Re} b_n)p\theta)\cos(|\mathrm{Im} b_n|p\theta - \phi)$$

with

$$b_n \approx \sqrt{2\pi B|n|}(1+i\,\mathrm{sign}\,n) \qquad n = \pm 1, \pm 2, \ldots$$

(I.V. Andreev, I.M. Dremin JETP Lett. 6 (**1967**) 262)

The crucial role of Z again!

The elastic differential cross-section **outside** the diffraction cone contains the exponentially decreasing with θ (or $\sqrt{|t|}$) term (Orear regime!) with imposed on it damped oscillations:

$$\frac{d\sigma}{p_1 dt} = \left(e^{-\sqrt{2B|t| \ln \frac{Z}{g_\rho}}} + p_2 e^{-\sqrt{2\pi B|t|}} \cos(\sqrt{2\pi B|t|} - \phi) \right)^2 .$$

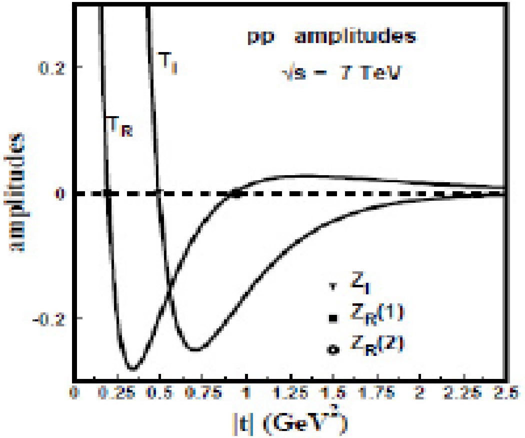
The experimental values of the diffraction cone slope B and the total cross section σ_t determine mostly the shape of the differential cross section in the Orear region. The value of $Z = 4\pi B/\sigma_t$ is so close to 1 that the fit is extremely sensitive to $g_\rho = 1 + \rho(s,0)\rho(s,t)$. Thus, it becomes possible for the first time to estimate the ratio ρ outside the diffraction cone from fits of experimental data.

Its average value is negative, equal to -2.1 at LHC!

No zero of $\mathrm{Im} f(s,t)$ at the dip!

in distinction to all widely used phenomenological models.

Zeros of real and imaginary parts



Real $T_R = \mathrm{Re} f$ and imaginary $T_I = \mathrm{Im} f$ parts of the proton-proton amplitude at 7 TeV according to a particular phenomenological model.

One zero of $\mathrm{Im} f$ at the dip and two zeros of $\mathrm{Re} f$.

Negative ρ at the end of the diffraction cone and at Orear region.

Conclusions

- The black plateau at small impact parameters at 7 TeV. The parameter Z determines the opacity at $b = 0$ which increases at present energies (ISR $\rightarrow$ S$p\bar{p}$S $\rightarrow$ LHC)
- Steady increase of the parton density in the peripheral regions of protons at present energies.
- The two-scale structure of the overlap function is crucial for understanding the experimental data at 7 TeV on inelastic production of jets in very high multiplicity events.
- If $Z \rightarrow 0.5$, inelastic processes become peripheral!
- Most theoretical models describe the diffraction peak ($G(b)$!) but fail outside it. Common - $\mathrm{Im} f(s, t_d) = 0$ at the dip.
- Outside the diffraction cone the unitarity condition predicts the Orear regime with exponential decrease in angles and imposed on it damped oscillations. The slope in this region strongly depends on the values of Z and the ratio ρ there. The last one happens to be about -2 at 7 TeV
- All model fits ask for the pole of $\rho(s, t)$ at the dip! The unitarity condition does not require the pole!

Coherence and Incoherence

1. **Unitarity**
2. **QCD**
3. **Models**

1. 2-particle intermediate states – Coherence.
Multiparticle states ($F(p,\theta)$) – Incoherence.
Large $F(p,\theta)$ in d.c.; $F(p,\theta) \ll 1$ in Orear.
Inelastic diffraction – low masses in $F(p,\theta)$ – slightly incoherent but Pomeron exchange.

2. Coherent Pomeron in proton PDF
(see A.D. Martin, M.G. Ryskin, arXiv:1406.2118)
DGLAP evolution of PDF with Pomeron included?

3. ρ in Orear – consistency with unitarity?