Progress in lattice QCD

Tetsuya Onogi
Yukawa Institute for Theoretical Physics, Kyoto, Japan

I review the recent progress in lattice QCD, which will be useful in heavy quark physics near future. Reviewing the theoretical developments in lattice QCD first, I focus our recent unquenched QCD with dynamical overlap fermion by JLQCD collaboration. I also introduce some of our recent studies on the $B^*B\pi$ coupling and on the determination of $|V_{ub}|$ through dispersive bound.

1. Introduction

The lattice computation of weak matrix elements can be defined as

$$\langle \mathcal{O}^{cont}(\mu)\rangle \equiv \lim_{a\to 0, m_i \to m_i^{\rm phys}} Z(a\mu, g_0(a)^2)\langle \mathcal{O}^{lat}(a)\rangle, \tag{1}$$

where

$$\langle \mathcal{O}^{lat}(a)\rangle \equiv \frac{\int [dU]\prod_{i=u,d,s} det[D(m_i)]e^{-S_g(U)}\mathcal{O}^{lat}(a)}{\int [dU]\prod_{i=u,d,s} det[D(m_i)]e^{-S_g(U)}}. \tag{2}$$

In order to extract physical matrix elements from the lattice the sea quark effects, the renormalization factor, and the continuum limit and chiral limit should be incorporated. In addition, the heavy quark should be treated either by the effective theory or by extrapolation of in the heavy quark mass from smaller mass regime. These steps have been quite nontrivial tasks. Recently, there are three major progresses in lattice QCD which can drastically reduce the systematic errors in lattice QCD computation: (1) unquenched lattice QCD simulations in chiral regime, (2) nonperturbative renormalization, and (3) new approach to heavy quarks on the lattice. In the following I explain these developments in some detail.

1.1. Unquenched QCD simulations in chiral regime

A few years ago the only available large scale dynamical QCD simulations with lightest pion mass $m_\pi \simeq 300$ MeV, was by MILC collaboration with the staggered quark [1]. This is because of the smallest numerical cost owing to the small degrees of freedom of the fermion and numerical stability from the exact partial chiral symmetry. In contrast, it has been thought that light dynamical fermion simulations in other fermion formalism would be difficult by Tflops machines.

In recent years, O(10) Tflops machines are available in many places. Also the new preconditioners for Dirac operator inversion algorithm such as 'domain decomposition' [2] and 'mass preconditioning' [3] enables us to treat the high- and low-mode contributions to the Dirac operator separately. Combining this method with the 'multi-time scale' in molecular dynamics step [4] gives significantly efficient algorithms for updating the gauge configurations in hybrid Monte-Calro method. Owing to theses developments, there are now may unquenched simulations [5–9] as shown in Table 1.1. Although the staggered fermion simulations is going much ahead, new results in other approaches will give important numerical and theoretical cross-checks with different advantages and disadvantages.

1.2. Nonperturbative renormalization

Another major development is the proposal of renormalization schemes with which lattice simulation can give the renormalization factors nonperturbatively.

One such scheme is the Schrodinger functional (SF-) scheme which is defined by amplitudes in a finite box with physical size L with Dirchlet boundary conditions in the temporal direction [10] . The physical amplitudes to define this scheme are computable both perturbatively and nonperturbatively. The renormalization scale is defined as $\mu = 1/L$. In this scheme both the renormalization constant of any local operators and their running can be obtained. To obtain the running, additional simulations with different box sizes are need.

Another scheme is the regularization independent momentum scheme (RI-MOM) scheme defined by off-shell quark/gluon amplitudes in Landau gauge [11]. The amplitudes are also computable both perturbatively and nonperturbatively. The renormalization scale is defined by the momentum scale.

1.3. New approach to heavy quark

The precise computation of weak matrix elements of the B meson is one of the most important topics in lattice QCD. However since the typical lattice cutoff used in practical simulations is smaller than the bottom quark mass, naive lattice methods suffer from large discretization error. For this reason lattice nonrelativistic QCD action has been widely used. Unfortunately, due to the nonrenormalizability of the action, one cannot take the continuum limit in this approach

Group	Fermion Action	n_f	a(fm)	L(fm)	m_π(GeV)
MILC [1]	Improved Staggered	2+1	0.09, 0.12	3	≥ 300
CERN [5]	Wilson, O(a)-imp Wilson	2	0.052-0.075	3	≥ 300
PACS-CS [6]	O(a)-imp Wilson	2+1	0.07, 0.10, 0.12	3	≥ 210
ETMC [7]	twisted mass Wilson	2	0.075, 0.096	3	≥ 270
RBC/UKQCD [8]	Domain-wall	2+1	0.12	3	≥ 330
JLQCD [9]	Overlap	2,2+1	0.12	2	≥ 300

Table I Projects of large scale lattice QCD simulations with light dynamical quarks.

so that one suffers from sizable systematic errors from discretization error and perturbative renormalization error. Recently new methods to treat the heavy-light meson in the continuum limit with nonperturbative accuracy have been proposed.

The first approach is proposed by Alpha collaboration [12]. They used the lattice HQET which is matched to QCD with nonperturbative accuracy by Schrodinger functional method in small volume with sufficiently fine lattice. Then, they evolve the lattice HQET action to coarser lattice by step scaling. They can also include $1/M_b$ corrections into the action and operators.

The second approach is the step scaling method proposed by the Rome II group [13, 14]. The compute the physical observable (e.g. f_B) in small volume with $L_0 = 0.4$ fm with sufficiently fine lattice using relativistic quark action. Then, they compute the finite size corrections with larger volumes $L = 2L_0, 4L_0$ by extrapolations from smaller heavy quark masses as

$$f_{B_s}(L_\infty) = f_{B_s}(L_0)\sigma(L_0)\sigma(L_1), \tag{3}$$

where $\sigma(2L_0) = \frac{f_{B_s}(2L_0)}{f_{B_s}(L_0)}$ and $\sigma(L_\infty) = \frac{f_{B_s}(L_\infty)}{f_{B_s}(2L_0)}$. Guazzini et al. [16] combined the above two methods, i.e. they use the static limit result to interpolate the finite volume corrections.

Table 1.3 shows the quenched QCD results of f_B with nonperturbative accuracy. It is remarkable that all three approaches give consistent results with high accuracy.

Method	f_{B_s} (MeV)
HQET with $1/M$ (Alpha) [15]	193(7)
Step scaling (Rome II) [14]	195(11)
Combination [16]	191(6)

Table II Quenched QCD results of f_B with nonperturbative accuracy.

2. Recent results with dynamical overlap fermion

The fermion action satisfying the Ginsparg-Wilson relation [17]

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D \tag{4}$$

realizes the exact chiral symmetry on the lattice [18]

$$\delta\psi = \gamma_5(1 - aD)\psi, \delta\bar{\psi} = \bar{\psi}\gamma_5. \tag{5}$$

An explicit construction of the Ginsparg-Wilson fermion called as the overlap fermion was proposed by Neuberger, which is defined as

$$D = \frac{1}{a}\left[1 + \gamma_5\epsilon(H_W)\right], \tag{6}$$

where $H_W = \gamma_5(D_W + M_0)$ is the Wilson Hamiltonian with negative mass term M_0 at the cutoff scale [19] . The domain-wall fermion is another realization of the Ginsparg-Wilson fermion which introduces 5-th dimension [20]. However, with finite extent in the 5-th dimension, the chiral symmetry becomes approximate with an exponentially suppressed symmetry violation.

JLQCD collaboration succeeded in the first large scale lattice QCD simulation with dynamical overlap fermion. $N_f = 2$ unquenched simulation on a $16^3 \times 32$ lattice at $a = 0.12$ fm was carried out for 6 quark masses covering the range $m_s/6 \sim m_s$ and various measurements of physical observables were made [23, 25–28, 30, 31]

2.1. Chiral behavior of m_π and f_π

JLQCD collaboration studied the quark mass dependence of m_π and f_π for $n_f = 2$ QCD with dynamical overlap fermion [9]. They found that the lattice data for $m_\pi \leq 450$ MeV are well fitted with NLO ChPT formula

$$m_\pi^2/m_q = 2B(1 + x\ln x) + c_3 x \tag{7}$$

$$f_\pi = f(1 - 2x\ln x) + c_4 x, \tag{8}$$

with $x \equiv m_\pi^2/(4\pi f)^2$ as shown in Figs. 1. They also studied the convergence of the ChPT by replacing the

expansion parameter x by $\hat{x} \equiv 2m_\pi^2/(4\pi f)^2$ or $\xi \equiv m_\pi^2/(4\pi f_\pi)^2$. They find that NNLO ChPT with ξ-expansion can nicely describe the lattice data in the pion mass region of $290 \sim 750$ MeV.

2.2. B_K

Indirect CP violation in the K meson system ϵ_K is one of the most crucial quantities to test the standard model and the physics beyond. The experimental value is determined with high accuracy as

$$|\epsilon_K| = (2.233 \pm 0.015) \times 10^{-3}. \quad (9)$$

Theoretically this quantity is described as $|\epsilon_K| = f(\rho,\eta) \times C(\mu) \times B_K(\mu)$. Here, $f(\rho,\eta)$ is a factor which depends on the CKM matrix elements, $C(\mu)$ is the Wilson coefficient from short-distance QCD corrections and $B_K(\mu)$ is the bag parameter defined as

$$B_K(\mu) = \frac{\langle K^0| \left[\bar{d}\gamma^\mu(1-\gamma_5)s\bar{d}\gamma_\mu(1-\gamma_5)s\right](\mu)|\bar{K}^0\rangle}{\frac{8}{3}f_K^2 m_K^2} \quad (10)$$

The main problem in unquenched lattice calculations is the possible operator mixing in wrong chiralities or tastes. The overlap fermion is free from operator mixing owing to the exact chiral symmetry. JLQCD collaborations study B_K with overlap fermion in 2 flavor QCD at lattice spacing $a = 0.12$ fm on physical volume with $L = 2fm$ [22]. They have 4 points for the sea quark and 10 combinations of valence quark masses (m_1, m_2). They fit the data with NLO PQChPT. The renormalization factor is determined nonperturbatively by RI-MOM scheme. They obtain $\hat{B} = 0.734(5)_{\rm stat.}(50)_{\rm sys.}$, where the dominant error comes from the finite size effect of order 5%.

It should be noted that the long standing operator mixing problem is solved with the advent of overlap fermion with the exact chiral symmetry. Thus the above study the real start of the precision study of B_K for which significant progress will be expected near future.

3. Some new results in heavy quark physics

3.1. Determination of the $B^*B\pi$ coupling

The $B^*B\pi$ coupling is a fundamental parameter of chiral effective Lagrangian with heavy-light mesons defined as

$$L = -\mathrm{Tr}\left[\bar{H}iv\cdot DH\right] + \hat{g}_b \mathrm{Tr}\left[\bar{H}HA_\mu\cdot\gamma_\mu\gamma_5\right] + O(1/M), \quad (11)$$

where the low energy constant $\hat{g}_b$ is the $B^*B\pi$ coupling, v is the four-velocity of the heavy-light meson B or B^*, and H, D_μ, A_μ are described by the B, B^* and π fields as $H = \frac{1}{2}(1+\gamma_\mu v_\mu)(iB\gamma_5 + B^*_\mu\gamma_\mu)$, $\xi = \exp(i\pi/f)$, $D_\mu = \partial_\mu + \frac{1}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger)$, $A_\mu = \frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)$.

Once the $B^*B\pi$ coupling is determined, the heavy meson effective theory can predict various quantities which are important for CKM phenomenology [32]. For example the light quark mass dependence of the B meson decay constant can be determined as

$$f_{B_d} = F\left(1 + \frac{3}{4}(1+3\hat{g}_b^2)\frac{m_\pi^2}{(4\pi f_\pi)^2}\log(m_\pi^2/\Lambda^2)\right) + \cdots, \quad (12)$$

F is the low energy constant associated with the heavy-light axial-vector current. The form factor $f^+(q^2)$ for the semileptonic decay $B \to \pi l\nu$ can also be expressed in terms of the B^* meson decay constant f_{B^*} and $\hat{g}_b$ as

$$f^+(q^2) = -\frac{f_{B^*}}{2f_\pi}\left[\hat{g}_b\left(\frac{m_{B^*}}{v\cdot k - \Delta} - \frac{m_{B^*}}{m_B}\right) + \frac{f_B}{f_{B^*}}\right], \quad (13)$$

where v is the velocity of the B meson, k is the pion momentum, and $\Delta = m_{B^*} - m_B$. Therefore, the precise determination of the $B^*B\pi$ coupling is crucial for determining $|V_{ub}|$, $|V_{td}|$ accurate. Despite its importance the $B^*B\pi$ has been known not so accurately due to the large statistical error of the heavy-light meson in static limit [33–35].

We carry out precise determination of the $B^*B\pi$ coupling in $n_f = 2$ QCD using 100 to 150 gauge configurations provided by CP-PACS collaboration [38] through JLDG (Japan Lattice DataGrid), which $12^3\times 24$ lattices at $\beta = 1.80$ and $16^3\times 32$ lattices at $\beta = 1.95$ with two flavors of $O(a)$-improved Wilson quarks and Iwasaki gauge action. In order to improve the statistical signal the static quark action with the HYP smeared links [39] with the smearing parameter values $\mathrm{low}(\alpha_1,\alpha_2,\alpha_3) = (0.75, 0.6, 0.3)$. We also employ the all-to-all propagator [40] using the low-mode averaging technique [41, 42]. Based on the previous quenched study [43], we take 200 low-eigenmodes.

The physical value of the $B^*B\pi$ coupling is obtained by multiplying the bare value by the renormalization constant at one-loop. The chiral extrapolation is made in three ways: (a) the linear extrapolation, (b) the quadratic extrapolation, and (c) the quadratic plus chiral log extrapolation where the log coefficient is determined from ChPT [44]. We take the result at $\beta = 1.95$ as our best estimate for the physical value of $\hat{g}_\infty$, and estimate the discretization error of $O((a\Lambda)^2)$ by order counting with $\Lambda \sim 0.3$ GeV. Including the perturbative error of $O(\alpha^2)$ also by order counting, our results for $\hat{g}_\infty$ is

$$\hat{g}_\infty^{n_f=2} = 0.516(5)_{\rm stat}(31)_{\rm chiral}(28)_{\rm pert}(28)_{\rm disc}. \quad (14)$$

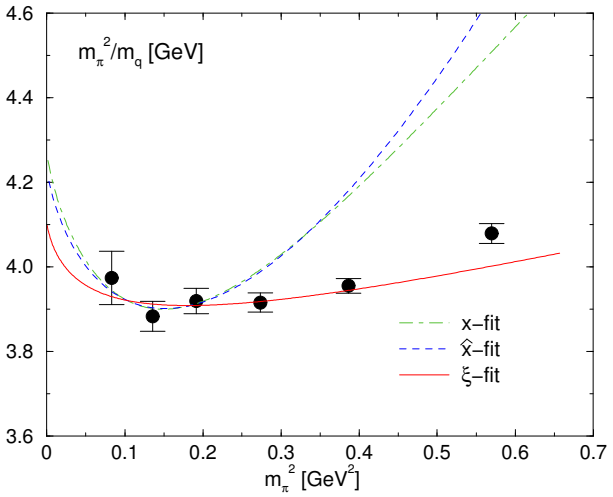

Figure 1: ChPT fit with x-expansion and xi-expansion. x-expansion makes the convergence of the ChPT fits better.

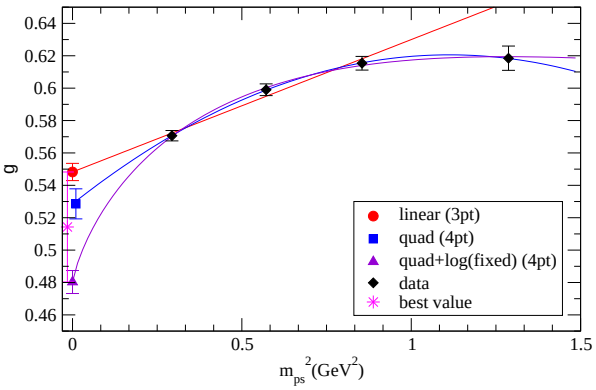

Figure 2: The chiral extrapolation of the physical $B^*B\pi$ coupling at $\beta = 1.95$.

3.2. Dispersive bounds on form factor for $B \to \pi l \nu$ decay

The momentum range of $B \to \pi l \nu$ form factors computed from lattice QCD is limited by the small recoil or large q^2 region. This leads to a big disadvantage because most of the experimental data lies in large recoil region. While one can extrapolate in q^2 with a fit ansatz, this will always introduce some model dependence. Dispersive bounds is one possible way to constrain the q^2 dependence in model independent fashion using unitarity. Consider the imaginary part of the vacuum polarization amplitude for the current $V(x) = \bar{u}\gamma_\mu b(x)$ and a map as in Fig. 3.2

$$Pi^{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|T\left\{V^\mu(x)V^{\nu\dagger}(0)\right\}|0\rangle \qquad (15)$$

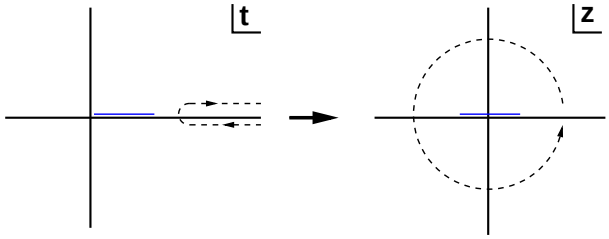

Figure 3: A map from t plane to z plane

$$= (q^\mu q^\nu - g^{\mu\nu}q^2)\Pi_1(q^2) + q^\mu q^\nu \Pi_0(q^2) \qquad (16)$$

Then, from dispersion relations one obtains

$$\chi_{F_+}(Q^2) = \frac{1}{2}\frac{\partial^2}{\partial(q^2)^2}\left[q^2\Pi_1\right] = \frac{1}{\pi}\int_0^\infty dt\, \frac{t\mathrm{Im}\Pi_1(t)}{(t+Q^2)^3}\,,$$
$$\chi_{F_0}(Q^2) = \frac{\partial}{\partial q^2}\left[q^2\Pi_0\right] = \frac{1}{\pi}\int_0^\infty dt\, \frac{t\mathrm{Im}\Pi_0(t)}{(t+Q^2)^2}\,. \qquad (17)$$

with $Q^2 = -q^2$ and η an isospin factor, while χ 's can be computed using OPE and perturbative QCD. Unitarity tells us that this is equal to the sum over all the hadronic states. and dropping all the excited states and leaving only $B\pi$ and B^* states gives an exact bound.

$$\frac{\eta}{48\pi}\frac{[(t-t_+)(t-t_-)]^{3/2}}{t^3}|F_+(t)|^2 \le \mathrm{Im}\Pi_1(t)\,,$$
$$\frac{\eta t_+ t_-}{16\pi}\frac{[(t-t_+)(t-t_-)]^{1/2}}{t^3}|F_0(t)|^2 \le \mathrm{Im}\Pi_0(t) \qquad (18)$$

Combining Eqs. 17, 18 and making change of variables in the integration from t to z

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}, \qquad (19)$$

with $t_\pm = (m_B \pm m_\pi)^2$, one obtains

$$\langle \phi f_0 | \phi f_0 \rangle < \chi_0, \ \langle P\phi f_+ | P\phi f_+ \rangle < \chi_+, \qquad (20)$$

where J is a quantity which can be obtained using OPE and perturbative QCD. The inner product $\langle g|h\rangle$ for arbitrary functions $g(z)$ and$h(z)$ is defined by the integral along the unit circle in z plane as

$$\langle g|h\rangle \equiv \int \frac{dz}{2\pi i}(g(z))^*. \qquad (21)$$

$P(z) = z(t, m_B^*)$ is multiplied to f_+ in order to remove B^* pole inside the unit circle. Cauchy's theorem tells that if we know additional integrated quantity $\langle g_i | P\phi_+ f_+ \rangle$ with a set of known functions $\{g_i(z), i = 1, ..., N\}$ one can make the bound stronger as

$$\det \begin{pmatrix} \chi & \langle P\phi f_+|g_1\rangle & \dots & \langle P\phi f_+|g_N\rangle \\ \langle g_1|P\phi f_+\rangle & \langle g_1|g_1\rangle & \dots & \langle g_1|g_N\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle g_N|P\phi f_+\rangle & \langle g_N|g_1\rangle & \dots & \langle g_N|g_N\rangle \end{pmatrix} > 0. \qquad (22)$$

Choosing $g_n(z) = \frac{1}{z - z(t)}$, Lellouch [45] obtained stronger form factor bounds with statistical analysis. We improved the bound using also the experimental q^2 spectrum from CLEO as additional inputs [46]. After the BABAR measurement of the q^2 spectrum of $B \to \pi l \nu$ decay [47], Arnesen et al. [48] set $g_n(z) = z^n$ to obtain a simple bound on the coefficients of the z-polynomial parameterization, which was further improved imposing HQET power counting by Becher and Hill [49], [50].

Since the BABAR measurement of the q^2 spectrum allows for the form factor shape determination, we also updated our determination of the $|V_{ub}|$ using dispersive bound [51]. Using the form factor from HPQCD collaboration [52] and the CLEO data, we obtain our preliminary estimate

$$|V_{ub}| = \left[3.4^{+4}_{-6}\right] \times 10^{-3} \qquad (23)$$

4. Summary

There have been major progresses in the unquenched QCD simulation in chiral regime, the renormalization schemes which allows for the nonperturbative determinations of the renormalization factors, and the new approach to the heavy quarks on the lattice. These developments have been tested in light hadron physics or in quenched QCD and are promising for improving the lattice calculation for B physics in near future.

I reviewed recent results with $n_f = 2$ dynamical overlap fermion by JLQCD collaboration. It was found that the chiral behavior of m_π and f_π are consistent the Next-to-next-leading order Chiral Perturbation theory. With the advent of the exact chiral symmetry precise determination of B_K.

I also explained some new results in heavy quark physics such as $B^*B\pi$ coupling and the model independent determination of $|V_{ub}|$ from dispersive bound.

Acknowledgments

I would like to thank my colleagues in JLQCD collaboration. Work is supported in part by the Grant-in-Aid of the Ministry of Education (Nos. 19540286, 20039005).

References

[1] C. Bernard *et al.*, arXiv:0710.1118[hep-lat].
[2] M. Lüscher, JHEP **0305**, 052 (2003); Comput. Phys. Commun. **165**, 199 (2005).
[3] M. Hasenbusch, Phys. Lett. **B519**, 177 (2001);. M. Hasenbusch and K. Jansen, Nucl. Phys. **B659**, 299 (2003).
[4] J. C. Sexton and D. H. Weingarten, Nucl. Phys. **B380**, 665 (1992).
[5] L. Del Debbio *et al.*, JHEP **0602**, 011 (2006); L. Del Debbio *et al.*, JHEP **0702**, 056 (2007); JHEP **0702**, 082 (2007).
[6] S. Aoki *et al.* [PACS-CS Collaboration], arXiv:0807.1661 [hep-lat].
[7] C. Urbach, PoS **LATTICE2007**, 022 (2007).
[8] RBC and UKQCD Collaborations, C. Allton *et al.*, arXiv:0804.0473[hep-lat].
[9] J. Noaki *et al.* [JLQCD and TWQCD Collaborations], Phys. Rev. Lett. **101**, 202004 (2008) [arXiv:0806.0894 [hep-lat]].
[10] M. Luscher *et al.*, Nucl. Phys. B384, 168 (1992); M. Luscher *et al.*, Nucl. Phys. B389, 247 (1993); M. Luscher *et al.*, Nucl. Phys. B413, 481 (1994);
[11] G. Martinelli, C. Pittori, C. T. Sachrajda, M. Testa and A. Vladikas, Nucl. Phys. B **445**, 81 (1995) [arXiv:hep-lat/9411010].
[12] J. Heitger and R. Sommer [ALPHA Collaboration], JHEP **0402**, 022 (2004) [arXiv:hep-lat/0310035].
[13] G.M. de Divitiis *et al.*, M. Guagnelli, R. Petronzio, N. Tantalo and F. Palombi, Nucl. Phys. B **675**, 309 (2003) [arXiv: hep-lat/0305018].
[14] G. M. de Divitiis, M. Guagnelli, F. Palombi, R. Petronzio and N. Tantalo, Nucl. Phys. B **672**, 372 (2003) [arXiv:hep-lat/0307005].
[15] J. Rolf *et al.* [ALPHA collaboration], Nucl. Phys. Proc. Suppl. **129**, 322 (2004) [arXiv:hep-lat/0309072].

[16] D. Guazzini, R. Sommer and N. Tantalo, JHEP **0801**, 076 (2008) [arXiv:0710.2229 [hep-lat]].
[17] P. H. Ginsparg and K. G. Wilson, Phys. Rev. D **25**, 2649 (1982).
[18] M. Luscher, relation," Phys. Lett. B **428**, 342 (1998) [arXiv:hep-lat/9802011].
[19] H. Neuberger, Phys. Lett. B **417**, 141 (1998) [arXiv:hep-lat/9707022]; H. Neuberger, Phys. Lett. B **427**, 353 (1998) [arXiv:hep-lat/9801031]; R. Narayanan and H. Neuberger, Phys. Rev. D **62**, 074504 (2000) [arXiv:hep-lat/0005004].
[20] D. B. Kaplan, Phys. Lett. B **288**, 342 (1992) [arXiv:hep-lat/9206013]; Y. Shamir, Nucl. Phys. B **406**, 90 (1993) [arXiv:hep-lat/9303005].
[21] E. Gamiz, S. Collins, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration and UKQCD Collaboration], Phys. Rev. D **73**, 114502 (2006) [arXiv:hep-lat/0603023].
[22] S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. D **77** (2008) 094503 [arXiv:0801.4186 [hep-lat]].
[23] H. Ohki *et al.*, arXiv:0806.4744 [hep-lat].
[24] E. Shintani *et al.* [JLQCD Collaboration and TWQCD Collaboration], arXiv:0807.0556 [hep-lat].
[25] E. Shintani *et al.* [JLQCD Collaboration], arXiv:0806.4222 [hep-lat].
[26] S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. D **78**, 014508 (2008) [arXiv:0803.3197 [hep-lat]].
[27] H. Fukaya *et al.* [JLQCD collaboration], Phys. Rev. D **77**, 074503 (2008) [arXiv:0711.4965 [hep-lat]].
[28] S. Aoki *et al.* [JLQCD and TWQCD Collaborations], Phys. Lett. B **665**, 294 (2008) [arXiv:0710.1130 [hep-lat]].
[29] S. Aoki, H. Fukaya, S. Hashimoto and T. Onogi, Phys. Rev. D **76**, 054508 (2007) [arXiv:0707.0396 [hep-lat]].
[30] H. Fukaya *et al.*, Phys. Rev. D **76**, 054503 (2007) [arXiv:0705.3322 [hep-lat]].
[31] H. Fukaya *et al.* [JLQCD Collaboration], Phys. Rev. Lett. **98**, 172001 (2007) [arXiv:hep-lat/0702003].
[32] C. G. Boyd and B. Grinstein, Nucl. Phys. B **442**, 205 (1995) [arXiv:hep-ph/9402340].
[33] G. M. de Divitiis, L. Del Debbio, M. Di Pierro, J. M. Flynn, C. Michael and J. Peisa [UKQCD Collaboration], JHEP **9810**, 010 (1998) [arXiv:hep-lat/9807032].
[34] A. Abada, D. Becirevic, Ph. Boucaud, G. Herdoiza, J. P. Leroy, A. Le Yaouanc and O. Pene, JHEP **0402**, 016 (2004) [arXiv:hep-lat/0310050].
[35] D. Becirevic, B. Blossier, Ph. Boucaud, J. P. Leroy, A. LeYaouanc and O. Pene, PoS **LAT2005**, 212 (2006) [arXiv:hep-lat/0510017].
[36] M. Della Morte, S. Durr, J. Heitger, H. Molke, J. Rolf, A. Shindler, and R. Sommer [ALPHA Collaboration], Phys. Lett. B **581**, 93 (2004) [Erratum-ibid. B **612**, 313 (2005)] [arXiv:hep-lat/0307021].
[37] M. Della Morte, A. Shindler and R. Sommer, JHEP **0508**, 051 (2005) [arXiv:hep-lat/0506008].
[38] A. Ali Khan *et al.* [CP-PACS Collaboration], Phys. Rev. D **65**, 054505 (2002) [Erratum-ibid. D **67**, 059901 (2003)] [arXiv:hep-lat/0105015].
[39] A. Hasenfratz and F. Knechtli, Phys. Rev. D **64**, 034504 (2001) [arXiv:hep-lat/0103029].
[40] J. Foley, K. Jimmy Juge, A. O'Cais, M. Peardon, S. M. Ryan and J. I. Skullerud, Comput. Phys. Commun. **172**, 145 (2005) [arXiv:hep-lat/0505023].
[41] T. A. DeGrand and U. M. Heller [MILC collaboration], Phys. Rev. D **65**, 114501 (2002) [arXiv:hep-lat/0202001].
[42] L. Giusti, P. Hernandez, M. Laine, P. Weisz and H. Wittig, JHEP **0404**, 013 (2004) [arXiv:hep-lat/0402002].
[43] S. Negishi, H. Matsufuru and T. Onogi, Prog. Theor. Phys. **117**, 275 (2007) [arXiv:hep-lat/0612029].
[44] H. Y. L. Cheng, C. Y. L. Cheung, G. L. L. Lin, Y. C. Lin, T. M. Yan and H. L. Yu, Phys. Rev. D **49**, 5857 (1994) [Erratum-ibid. D **55**, 5851 (1997)] [arXiv:hep-ph/9312304].
[45] L. Lellouch, Nucl. Phys. B **479**, 353 (1996) [arXiv:hep-ph/9509358].
[46] M. Fukunaga and T. Onogi, Phys. Rev. D **71**, 034506 (2005) [arXiv:hep-lat/0408037].
[47] S. B. Athar *et al.* [CLEO Collaboration], Phys. Rev. D **68**, 072003 (2003) [arXiv:hep-ex/0304019].
[48] M. C. Arnesen, B. Grinstein, I. Z. Rothstein and I. W. Stewart, Phys. Rev. Lett. **95**, 071802 (2005) [arXiv:hep-ph/0504209].
[49] T. Becher and R. J. Hill, Phys. Lett. B **633**, 61 (2006) [arXiv:hep-ph/0509090].
[50] R. J. Hill, eConf **C060409**, 027 (2006) [arXiv:hep-ph/0606023].
[51] H. Ohki, H. Matsufuru, T. Onogi, in progress.
[52] E. Dalgic, A. Gray, M. Wingate, C. T. H. Davies, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **73**, 074502 (2006) [Erratum-ibid. D **75**, 119906 (2007)] [arXiv:hep-lat/0601021].