QCD Calculations of Radiative B Decays

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The current status of $\bar{B} \to X_s\gamma$ decay rate calculations is summarized. Missing ingredients at the NNLO level are listed. The global normalization factor and non-perturbative effects are discussed. Arguments are presented that results for the cutoff-enhanced perturbative corrections have been misused in Ref. [15] by applying them in the region $E_\gamma \in [1.0, 1.6]\,\mathrm{GeV}$, which means that the corresponding numerical effect on $\mathcal{B}\left(\bar{B} \to X_s\gamma,\ E_\gamma > 1.6\,\mathrm{GeV}\right)$ is unreliable.

I. INTRODUCTION

The motivation for precision studies of radiative B decays is well known. First, they are sensitive to new physics loop effects that often arise at the same order in the electroweak couplings as the leading Standard Model (SM) contributions. Moreover, the inclusive $\bar{B} \to X_s\gamma$ rate is well approximated by the perturbatively calculable radiative decay rate of the b-quark. The CLEO [1], BELLE [2] and BABAR [3] measurements have been combined by HFAG [4] to get

$$\mathcal{B}\big(\bar{B} \to X_s\gamma\big)_{\rm exp} = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4} \quad (1)$$

for $E_\gamma > 1.6\,\mathrm{GeV}$. The corresponding SM prediction that was published two years ago[39] reads [5]

$$\mathcal{B}\big(\bar{B} \to X_s\gamma\big)_{\rm SM} = (3.15 \pm 0.23) \times 10^{-4}. \quad (2)$$

Its consistency with Eq. (1) provides strong constrains on many extensions of the SM (see, e.g., Ref. [6]).

Resummation of large logarithms $\left(\alpha_s \ln M_W^2/m_b^2\right)^n$ in the calculation of the decay rate is most conveniently performed after decoupling the electroweak bosons and the top quark. In the resulting effective theory, the relevant flavour-changing weak interactions are given by a linear combination of dimension-five and -six operators[40]

$$\begin{aligned}
O_{1,2} &= (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), && \text{(current-current operators)}\\
O_{3,4,5,6} &= (\bar{s}\Gamma_i b){\textstyle\sum_q}(\bar{q}\Gamma'_i q), && \text{(four-quark penguin operators)}\\
O_7 &= \frac{e m_b}{16\pi^2}\,\bar{s}_L\sigma^{\mu\nu} b_R F_{\mu\nu}, && \text{(photonic dipole operator)}\\
O_8 &= \frac{g m_b}{16\pi^2}\,\bar{s}_L\sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}. && \text{(gluonic dipole operator)} \quad (3)
\end{aligned}$$

One begins with perturbatively calculating their Wilson coefficients C_i at the renormalization scale $\mu_0 \sim (M_W, m_t)$. Next, the Renormalization Group Equations (RGE) are used for the evolution of C_i down to the scale $\mu_b \sim m_b/2$. Finally, the operator on-shell matrix elements are calculated at μ_b.

The Wilson coefficient RGE are governed by Anomalous Dimension Matrices (ADM's) that are derived from ultraviolet divergences in the Feynman diagrams with operator insertions. Around 20000 four-loop diagrams with $O_{1,2}$ insertions have been calculated in Ref. [8] to make the large logarithm resummation complete up to $\mathcal{O}\left[\alpha_s^2\left(\alpha_s \ln M_W^2/m_b^2\right)^n\right]$, i.e. at the Next-to-Next-to-Leading-Order (NNLO) in QCD. Including such corrections is necessary to suppress the theoretical uncertainty in Eq. (2) down to the level of the experimental error in Eq. (1). The numerical effect of the four-loop ADM's on the branching ratio amounts to around -4% for $\mu_b = 2.5\,\mathrm{GeV}$. At present, all the relevant Wilson coefficients $C_i(\mu_b)$ are known at the NNLO [8, 9]. However, evaluation of the matrix elements at this order is still in progress — see Sec. IV.

II. THE GLOBAL NORMALIZATION FACTOR

In order to reduce parametric uncertainties stemming from the CKM angles as well as from the c- and b-quark masses, one writes the branching ratio as follows [10]

$$\mathcal{B}\big(\bar{B} \to X_s\gamma\big)_{E_\gamma > E_0} = \mathcal{B}\big(\bar{B} \to X_c e\bar{\nu}\big)_{\rm exp}\left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \times \times \frac{6\alpha_{\rm em}}{\pi\, C}\left[P(E_0) + N(E_0)\right], \quad (4)$$

where $\alpha_{\rm em} = \alpha_{\rm em}^{\rm on\ shell}$, and $N(E_0)$ denotes the non-perturbative correction (see Sec. V). The m_c-dependence of $\bar{B} \to X_c e\bar{\nu}$ is accounted for by

$$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma\big(\bar{B} \to X_c e\bar{\nu}\big)}{\Gamma\big(\bar{B} \to X_u e\bar{\nu}\big)}, \quad (5)$$

while $P(E_0)$ is defined by the perturbative ratio

$$\frac{\Gamma(b \to X_s\gamma)_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2\, \Gamma(b \to X_u e\bar{\nu})} = \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\rm em}}{\pi}\, P(E_0). \quad (6)$$

The NNLO expression for the phase-space factor C (5) is a known function of m_c/m_b as well as non-perturbative parameters that affect $N(E_0)$, too. All these quantities are determined in a single fit from the measured spectrum of the inclusive semileptonic decay $\bar{B} \to X_c e\bar{\nu}$. The fits are performed using either the 1S or the kinetic renormalization schemes. The corresponding results for C and m_c read

$$\begin{pmatrix} C \\ m_c(m_c) \end{pmatrix} = \begin{cases} \begin{pmatrix} 0.582 \pm 0.016 \\ 1.224 \pm 0.057 \end{pmatrix}, & \text{1S [11]}, \\ \begin{pmatrix} 0.546^{+0.023}_{-0.033} \\ 1.267 \end{pmatrix}, & \text{kin. [12, 13]}. \end{cases} \quad (7)$$

The above $\overline{\rm MS}$-scheme values of m_c have been obtained from the 1S- and kinetic-scheme ones using the three-loop and two-loop relations, respectively. The three-loop relation for the kinetic scheme is not yet known. The ratio C is scheme-independent, but it is affected by the so-called weak annihilation contribution $B_{\rm WA}$ that remains unknown. Since $B_{\rm WA}$ cancels out in Eq. (4), fixing its value is a matter of convention in the present context. Here, we follow the convention of Ref. [12], namely $B_{\rm WA}(\mu = m_b/2) = 0$.

The difference between the two determinations of C amounts to 1.6σ when counted in terms of the upper error of the very recent kinetic-scheme result [12]. It is a consequence of using different experimental data sets, methodology and renormalization schemes. Fortunately, the effects of changing C and m_c partially compensate each other in Eq. (4) because $\partial/\partial m_c\, P(E_0) < 0$. For $E_0 = 1.6\,\text{GeV}$, I find

$$\mathcal{B}(\bar{B} \to X_s\gamma) = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \text{1S}\ , \\ (3.25 \pm 0.24) \times 10^{-4}, & \text{kin.}\ , \end{cases} \quad (8)$$

where the first result is just that of Ref. [5], while the second one has been obtained using the same code but with the input parameters from Refs. [12, 13].

The actual value of $\mathcal{B}\left(\bar{B} \to X_s\gamma\right)$ in Ref. [12] is somewhat larger than 3.25×10^{-4} because $P(E_0)$ was calculated there using the one-loop rather than two-loop determination of $m_c(m_c)$ from $m_c^{\rm kin}$. In principle, using the one-loop relation is allowed at $\mathcal{O}(\alpha_s^2)$ because $P(E_0)$ becomes m_c-dependent only at $\mathcal{O}(\alpha_s)$.

III. CUTOFF-ENHANCED CORRECTIONS

The perturbative ratio $P(E_0)$ in Eq. (6) depends on the cutoff energy E_0 via the dimensionless parameter

$$\delta = 1 - \frac{2E_0}{m_b}. \quad (9)$$

For very small δ, i.e. close to the kinematical endpoint $E_0 = m_b/2$, the usual ("fixed-order") perturbative expansion breaks down because the corrections behave like powers of $\ln\delta$. In that region, one needs to resum large logarithms of δ. Such a resummation of

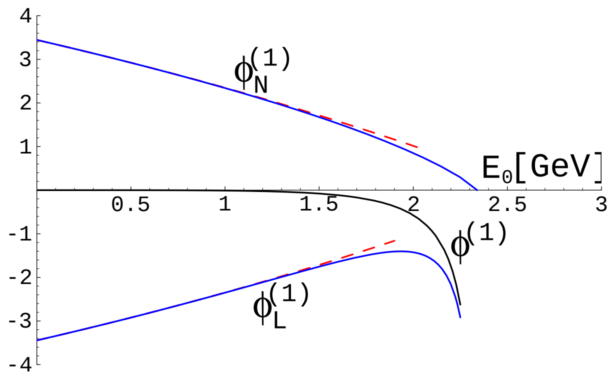

FIG. 1: Approximate cancellation of the logarithmic ($\phi_L^{(1)}$) and non-logarithmic ($\phi_N^{(1)}$) terms in $\phi^{(1)}$ away from the endpoint. The exact expressions from Eq. (11) are represented by the solid lines. The Taylor expansions of $\phi_L^{(1)}$ and $\phi_N^{(1)}$ around $E_0 = 0$ up to $\mathcal{O}(E_0^3)$ are shown by the dashed lines.

the cutoff-enhanced corrections (i.e. corrections enhanced by powers of $\ln\delta$) has been performed up to the NNLO in Refs. [14, 15]. These results constitute a valuable contribution to our knowledge of the photon energy spectrum in the endpoint region. However, they need to be treated with extreme care further from the endpoint, where logarithms of δ no longer dominate. Naively, one might expect that resummation of small logarithms does not hurt, even if it is not an improvement. Unfortunately, this is not the case because the logarithmic and non-logarithmic terms undergo a strong cancellation away from the endpoint.

In order to illustrate this issue in a simple manner, let us consider only the dominant photonic dipole operator O_7 in Eq. (3). When all the other operators are neglected, the fixed-order expression for the cutoff-dependence of $P(E_0)$ is given by

$$\frac{P(E_0)}{P(0)} = 1 + \frac{\alpha_s}{\pi}\phi^{(1)}(\delta) + \frac{\alpha_s^2}{\pi^2}\phi^{(2)}(\delta) + \ldots \quad (10)$$

Each of the functions $\phi^{(k)}$ can be split into two parts: $\phi_L^{(k)}$ that is polynomial in $\ln\delta$, and $\phi_N^{(k)}$ that vanishes at the endpoint. The explicit expressions for $k = 1$ read [16]

$$\begin{aligned} \phi^{(1)} &= \phi_L^{(1)} + \phi_N^{(1)}, \\ \phi_L^{(1)}(\delta) &= -\frac{2}{3}\ln^2\delta - \frac{7}{3}\ln\delta - \frac{31}{9}, \\ \phi_N^{(1)}(\delta) &= \frac{10}{3}\delta + \frac{1}{3}\delta^2 - \frac{2}{9}\delta^3 + \frac{1}{3}\delta(\delta-4)\ln\delta. \end{aligned} \quad (11)$$

In Fig. 1, $\phi_L^{(1)}$, $\phi_N^{(1)}$ and their sum are plotted as functions of E_0. The endpoint is located at $m_b/2 \simeq 2.34\,\text{GeV}$. One can see that $\phi_L^{(1)}$ begins to dominate around $E_0 = 2\,\text{GeV}$. On the other hand, already in

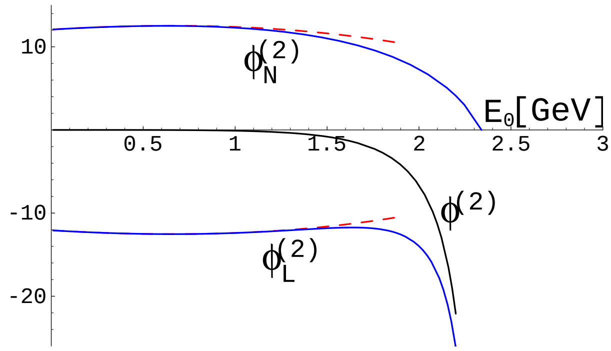

FIG. 2: Same as Fig. 1 but for $\phi^{(2)}$ in the case when $\alpha_s = \alpha_s^{n_f=3}(m_b)$ in Eq. (10).

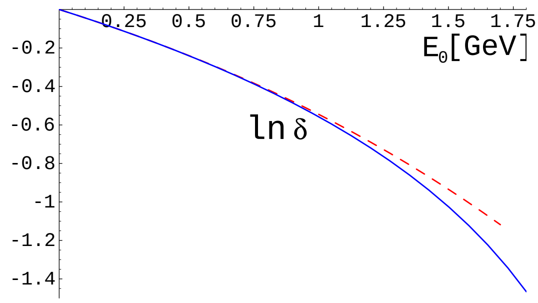

FIG. 3: $\ln\delta$ (solid) and its expansion (dashed) up to $\mathcal{O}(E_0^3)$.

the vicinity of $E_0 = 1.6\,\text{GeV}$, the cancellation of the two components of $\phi^{(1)}$ is very strong.

One may wonder whether similar cancellations occur at higher orders, too. A positive answer concerning $\phi^{(2)}$ is immediate because this function can easily be derived from the results of Ref. [17]. The corresponding plot is presented in Fig. 2 for the case when $\alpha_s = \alpha_s^{n_f=3}(m_b)$ in Eq. (10).

No explicit results at order $\mathcal{O}(\alpha_s^3)$ are available. However, we know on general grounds that all the $\phi^{(k)}$ behave like $(2E_0/m_b)^4$ at small E_0. Two powers of E_0 originate from $F_{\mu\nu}$ in the vertex O_7 in Eq. (3), and two additional powers come from the phase-space measure $E_\gamma dE_\gamma$. Consequently, the Taylor expansions of $\phi_L^{(k)}$ and $\phi_N^{(k)}$ at small E_0 up to $\mathcal{O}(E_0^3)$ must exactly cancel each other. These Taylor expansions are shown by the dashed lines in Figs. 1 and 2. One can see that both $\phi_L^{(1)}$ and $\phi_L^{(2)}$ are well approximated by the dashed lines in the region below 1.6 GeV. It must also be the case at higher orders because $\phi_L^{(k)}$ are polynomial in $\ln\delta$ that is well approximated by the same expansion in the considered region (see Fig. 3). Thus, cancellations like those shown in Figs. 1 and 2 are expected to occur at any order in α_s.

In the approach of Refs. [14, 15], logarithms of δ have been resummed at the NNLO in $\phi_L^{(k)}$, while $\phi_N^{(k)}$ have been retained in the fixed order. More precisely, Eq. (10) has been effectively re-expressed as

$$\frac{P(E_0)}{P(0)} = X + \frac{\alpha_s(\mu_b)}{\pi}\phi_N^{(1)} + \frac{\alpha_s^2(\mu_b)}{\pi^2}\phi_N^{(2)}, \qquad (12)$$

and X has been calculated up to $\mathcal{O}\left(\alpha_s^2 \times \alpha_s^n \ln^m \delta\right)$, with $n = 0, 1, 2, \ldots$, and $m = n, n+1, \ldots, m_{\max}(n)$. This is a reasonable approximation only in the region very close to the endpoint where no cancellation between the two components takes place. Elsewhere, it leads to overestimating the numerical effect of the $\mathcal{O}(\alpha_s^3)$ terms in Eq. (10) by a factor of order $\left|\phi_L^{(3)}/\phi^{(3)}\right| \sim (m_b/(2E_0))^4$ that amounts to around 4.6 for $E_0 = 1.6\,\text{GeV}$, and around 30 for $E_0 = 1.0\,\text{GeV}$.

Unfortunately, it was precisely the range $E_\gamma \in [1.0, 1.6]\,\text{GeV}$ where the authors of Ref. [15] applied their results to calculate the effect on $\mathcal{B}\big(\bar{B} \to X_s\gamma,\ E_\gamma > 1.6\,\text{GeV}\big)$. They adopted the fixed-order result at $E_0 = 1.0$ GeV from Ref. [5]

$$\mathcal{B}\big(\bar{B} \to X_s\gamma,\ E_\gamma > 1.0\,\text{GeV}\big) = 3.27 \times 10^{-4}, \qquad (13)$$

and supplemented it with their own numerical value of $[P(1.0) - P(1.6)]/P(0)$ that follows from Eq. (12). That value is almost twice larger than in the fixed-order NNLO calculation. In the end, their result for the branching ratio with a cutoff at $E_0 = 1.6\,\text{GeV}$ was considerably lower than the one in Eq. (2). In view of the above remarks, their prediction should be considered unreliable.

IV. STATUS OF THE NNLO QCD CALCULATIONS OF THE MATRIX ELEMENTS

On the l.h.s. of Eq. (6) that defines $P(E_0)$, the denominator is already known at the NNLO [18, 19]. In the expression for the numerator

$$\Gamma(b \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_b^5}{32\pi^4} |V_{tb}V_{ts}^*|^2 \times \times \sum_{i,j=1}^{8} C_i^{\rm eff}\, C_j^{\rm eff}\, G_{ij}(E_0), \qquad (14)$$

the quantities G_{ij} are determined by the matrix elements of $O_1, \ldots, O_8$.

So far, only G_{77} has been evaluated up to $\mathcal{O}(\alpha_s^2)$ in a complete manner [17, 20, 21]. The remaining G_{ij} are fully known at the Next-to-Leading Order (NLO), i.e. up to $\mathcal{O}(\alpha_s)$ (see Ref. [22] for a complete list of references). At the NNLO, it is practically sufficient to restrict considerations to G_{ij} with $i, j \in \{1, 2, 7, 8\}$ because the four-quark penguin operators have small

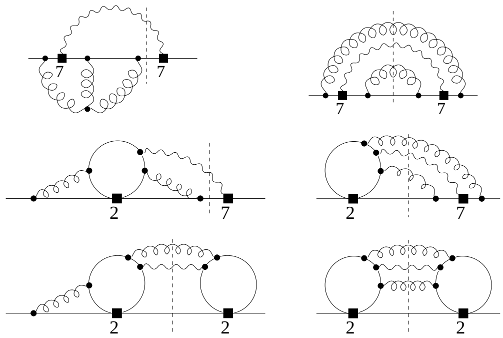

FIG. 4: Examples of Feynman diagrams that contribute to G_{77}, G_{27} and G_{22} at the NNLO. The dashed vertical lines mark the unitarity cuts. Black squares represent the operators O_2 and O_7 in Eq. (3).

Wilson coefficients. It is often convenient to apply the optical theorem, and calculate G_{ij} by summing imaginary parts of the b-quark propagator Feynman diagrams. One can see in Fig. 4 that imaginary parts of three-, four- and five-loop diagrams occur at the NNLO in G_{77}, G_{27} and G_{22}, respectively.

A relatively simple set of the NNLO contributions is given by diagrams with quark loops on the gluon lines. The quark in the loop is either massive (charm and bottom) or treated as massless (up, down and strange). Such contributions are already known [21, 23–25] for all the G_{ij} with $i, j \in \{1, 2, 7, 8\}$, except for the massless case in G_{18} and G_{28}. The BLM [26] (or large-β_0) approximation for the complete NNLO correction is derived from the massless quark results [23, 24].

In Ref. [27], the asymptotic behaviour for $m_c \gg m_b/2$ was calculated for all the non-BLM NNLO corrections to G_{ij} with $i, j \in \{1, 2, 7, 8\}$, except for G_{78} and G_{88}. Next, an interpolation in m_c of these corrections was performed assuming that the interpolated quantities vanish at $m_c = 0$. The result of that procedure was an essential input for the NNLO estimate in Eq. (2). The overall error of around 7% in the branching ratio was obtained by combining in quadrature four types of uncertainties: 5% non-perturbative, 3% parametric, 3% higher-order, and 3% due to the m_c-interpolation ambiguity.

The results in Eqs. (2) and (8) do not include several contributions to the branching ratio that are known at present. These additional effects are summarized in Tab. I. They sum up to around +1.6%, which is small when compared to the overall uncertainty of around 7%. Therefore, Eq. (8) can still be treated as an up-to-date SM prediction.

As the reader has already noticed, even in the m_c-interpolation approach of Ref. [27], there are still some missing NNLO ingredients, namely:

- the BLM contributions to G_{18} and G_{28},
- the large-m_c results for G_{78} and G_{88}.

TABLE I: Additional known effects not included in Eq. (8).

The BLM terms from Ref. [24]	+2.0%
O_8 in the 4-loop ADM's [8]	−0.3%
b and c loops on gluon lines [19, 21, 25]	+1.6%
Non-perturbative $\mathcal{O}(\alpha_s \Lambda/m_b)$ effects [28]	−1.5%
Non-perturbative collinear effects [29]	−0.2%
Total	+1.6%

Their numerical effect on the branching ratio is expected to remain within the estimated higher-order uncertainty of around 3%. The calculation of the most interesting G_{78} is very advanced [30], and the results should become available soon (for any value of m_c). As far as G_{88} is concerned, its calculation in the large-m_c limit will automatically give the result for any value of m_c.

For the full NNLO calculation, the currently missing ingredients (apart from the ones listed above) are the non-BLM corrections to

(i) G_{17} and G_{27},

(ii) G_{11}, G_{12} and G_{22},

(iii) G_{18} and G_{28}.

The calculation of *(i)* in the $m_c = 0$ limit is quite advanced, but also extremely difficult and time-consuming — see Ref. [31] for the status reports. Once it is finished, the main challenge will amount to finding *(ii)*, even for $m_c = 0$. The corrections *(iii)* are expected to be numerically less important.

When the non-BLM corrections are known at $m_c = 0$ sometime in the future, the interpolation in m_c is still going to be necessary. However, our error estimates should become more solid once the BLM approximation is no longer used at the boundary.

Finding all the non-BLM corrections for the actual value of $m_c \simeq m_b/4$ is even more difficult, but it must be considered at some point, too. A calculation of the IR-divergent two-particle-cut contributions to *(i)* is being currently performed for arbitrary m_c [31]. The IR-convergent two-particle-cut contributions to *(ii)* for arbitrary m_c are already known because they are given by products of the NLO corrections. The ($n \geq 3$)-particle-cut contributions to *(ii)* vanish at the endpoint, so their numerical relevance should be diminished by the high photon energy cutoff.

Apart from the NNLO corrections, there are other perturbative contributions to $\Gamma(b \to X_s\gamma)_{E_\gamma > E_0}$ that have been neglected so far, namely tree-level diagrams with the u-quark analogues of $O_{1,2}$ and the four-quark penguin operators $O_{3,4,5,6}$. Such contributions are suppressed with respect to the leading term by $|(C^u_{1,2}, C_{3,4,5,6})/C_7|^2 \leq 0.2$, where $C^u_{1,2} =$

$(V_{us}^* V_{ub})/(V_{ts}^* V_{tb}) C_{1,2}$, as well as by the high photon energy cutoff. A quantitative verification of how small they really are should become available soon [32].

V. NON-PERTURBATIVE EFFECTS

Let us begin with considering a simplified world where $m_c = m_b$. There, in the decay of the $\bar{B}$ meson, a high-energy photon ($E_\gamma \sim m_b/2$) can be produced in four different ways:

1. Hard: The photon is emitted directly from the hard process of the b-quark decay.
2. Conversion: The b-quark decays in a hard way into quarks and gluons only. Next, one of the decay products scatters in a non-soft radiative manner with the remnants of the $\bar{B}$ meson. This can be viewed as a parton-to-photon conversion in the QCD medium.
3. Collinear: In the process of hadronization, a collinear photon is emitted.
4. Annihilation: An energetic $q\bar{q}$ state produced in the $\bar{B}$ meson decay disintegrates radiatively.

The hard way would be the only one if no other operators but O_7 were present in the effective theory. Non-perturbative effects in such a case were first analyzed in Ref. [33]. They arise as corrections of order Λ^2/m_b^2 to $\Gamma(b \to X_s\gamma)_{E_\gamma > E_0}$ when $(m_b - 2E_0) \sim m_b$. Moreover, these $\mathcal{O}(\Lambda^2/m_b^2)$ terms cancel out in $N(E_0)$ in Eq. (4) with the analogous non-perturbative corrections to the charmless semileptonic rate. Thus, we are left with the small $\mathcal{O}(\Lambda^3/m_b^3)$ effects [34]. The $\mathcal{O}\left(\Lambda^2/(m_b - 2E_0)^2\right)$ corrections are also small [35] for $E_0 \leq 1.6\,\text{GeV}$. All these terms are included in Eq. (8), and affect the branching ratio by around -0.7%.

The photon production via conversion is suppressed both by α_s (due to the non-soft scattering) and by Λ/m_b (due to dilution of the target). The analysis in Ref. [28] shows that no other suppression factors occur. An effect on the branching ratio of roughly $-1.5 \pm 1.5\%$ was found in that paper (see Tab. I).

In our simplified case ($m_c = m_b$), the tree-level decay $b \to (\text{quarks \& gluons})_s$ is possible only via the operators $O_{3,4,5,6,8}$, and by the u-quark analogues of $O_{1,2}$. Consequently, the collinear photon emission is suppressed either by $\alpha_s |C_8/C_7|^2 \leq 0.08$, or by $|(C_{1,2}^u, C_{3,4,5,6})/C_7|^2 \leq 0.2$. Moreover, there is an additional suppression by products of the quark electric charges and, most importantly, by the high photon energy cutoff. The results of Ref. [29] lead to an estimate that the non-perturbative collinear effects due to O_8 amount to around -0.2% in the branching ratio for $E_0 = 1.6\,\text{GeV}$ (see Tab. I).

As far as annihilation is concerned, photons originating from decays of π^0, η, η' and ω are removed on the experimental side as (huge) background. Contributions from the other established $q\bar{q}$ mesons are negligible. The corresponding perturbative diagrams are responsible for only around 0.1% of the total rate for $E_0 = 1.6\,\text{GeV}$.

Once the assumption $m_c = m_b$ is relaxed, the $\mathcal{O}(\Lambda^2/m_b^2)$ hard effects from $O_{1,2}$ get replaced by a series of the form [36]

$$\frac{\Lambda^2}{m_c^2} \sum_{n=0}^{\infty} b_n \left(\frac{\Lambda m_b}{m_c^2} \right)^n , \tag{15}$$

with quickly decreasing coefficients b_n. The calculable leading term has been included in Eq. (8). It affects the branching ratio by around $+3.1\%$.

All the quantitatively estimated non-perturbative effects that have been mentioned so far sum up to

$$(-0.7 - 0.2 - 1.5 + 3.1)\% = +0.7\%. \tag{16}$$

However, since their evaluation is often very uncertain, and the knowledge of $\mathcal{O}(\alpha_s \Lambda/m_b)$ contributions is by no means complete, a non-perturbative error of $\pm 5\%$ has been assumed in Eq. (8), as already mentioned Sec. IV. Probably the most interesting of all the unknown $\mathcal{O}(\alpha_s \Lambda/m_b)$ effects originate from charm annihilation in the massive $(\bar{c}s)(\bar{q}c)$ intermediate states ($q = u$ or d), for the actual value of $m_c \simeq m_b/4$.

One should remember that the error in Eq. (1) is affected by a non-perturbative theoretical uncertainty, too. It follows from the fact that the actual measurements are not performed with $E_0 \simeq 1.6\,\text{GeV}$. The most precise experimental results correspond to significantly higher photon energy cutoffs for which the $\mathcal{O}\left(\Lambda^n/(m_b - 2E_0)^n\right)$ effects are no longer small. These effects are described by a non-perturbative shape function [37] that is constrained by the semileptonic data. The very recent analysis [38] of this function and its effects on the $\bar{B} \to X_s\gamma$ photon spectrum can hopefully provide input for the future HFAG averages.

VI. CONCLUSIONS

Thanks to the efforts of the past years, the uncertainties in $\mathcal{B}(\bar{B} \to X_s\gamma)$ have reached the level of around $\pm 7\%$ on both the experimental and theoretical sides. A significant progress in the perturbative calculations is expected in the near future. However, understanding the $\mathcal{O}(\alpha_s \Lambda/m_b)$ non-perturbative effects remains the key issue.

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