

Dalitz Analyses in Charm

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Dalitz analyses in charm give access to magnitudes *and* phases of the charm decay amplitudes. They play a significant role in charm-mixing measurements, in the measurement of the CP-violating phase γ in B decays to charm, and in the analysis of light meson resonances. We review recent results in all three categories.

1. Dalitz Analyses

Dalitz analyses give access to the full complex decay amplitudes, allowing the measurement of magnitudes *and* phases. This can be exploited in several ways:

- Explore the properties of light meson resonances such as the ρ , a_1 , $f(980)$ and more controversial ones such as σ and κ .
- Investigate charm itself, in particular mixing and CP violation in charm.
- B mesons decay to charm most of the time. The phases of the B decay amplitude can in certain circumstances be measured by analysing the charm Dalitz plot. This provides the theoretically cleanest and statistically most powerful direct constraint on the CKM phase γ that is currently known.

In the following we will consider each of these items in turn.

2. Dalitz Plots

The kinematics of a 3 body decay $D \rightarrow A, B, C$ (such as $D^+ \rightarrow K^+ K^- \pi^+$) can be fully described by 2 parameters. In terms of the four momenta of the three decay products, which we will denote as p_A, p_B, p_C , one usually picks the following invariant-mass-squared parameters:

$$m_{AB}^2 \equiv (p_A + p_B)^2 \quad (1)$$

$$m_{BC}^2 \equiv (p_B + p_C)^2 \quad (2)$$

These parameters are Lorentz invariant, and the phase space density in terms of these parameters, $\frac{d^2\Phi}{d(m_{AB}^2) d(m_{BC}^2)}$ is flat, i.e. constant inside the kinematically allowed limits, and zero outside. A Dalitz plot [2] is the decay rate in terms of these or equivalent variables, displayed in a 2-dimensional plot. The full decay rate is given by [1]:

$$\begin{aligned} & \frac{d^2\Gamma}{d(m_{AB}^2) d(m_{BC}^2)} \\ &= |a_1 e^{i\delta_1} + a_2 e^{i\delta_2} + \dots|^2 \frac{\pi\sqrt{\lambda}}{2m_D^2} \end{aligned} \quad (3)$$

with $\lambda = (m_D^2 - m_A^2 - m_B^2)^2 - 4m_A^2 m_B^2$ within the kinematically allowed limits, and $\lambda = 0$ outside. In the above expression, $a_i e^{i\delta_i}$ describe complex contributions to the total decay amplitude. In the simplest case, $a_i e^{i\delta_i}$ are complex Breit-Wigner distributions (or similar e.g. the Flatté distribution [3]) describing individual particle resonances, with additional factors taking into account angular momentum conservation, and form factors (Blatt-Weisskopf penetration factors [4]). This so-called isobar model has some shortcomings, the most severe one being that it violates unitarity, especially in the case of wide, overlapping resonances. More complicated models such as the K-matrix formalism [5, 6], which respects unitarity, may therefore be necessary to adequately describe the observed data, and to provide a theoretically satisfactory model. The general consensus - at least amongst experimentalists - appears to be that the isobar description is adequate for P and D wave resonances, but not for wide S wave resonances. The adequate description of $L = 0$ decays is one of the main topics of interest in the next section.

3. Dalitz Analyses and Light Meson Resonance

3.1. The κ, σ problem

The S-wave resonances $\sigma \rightarrow \pi^+ \pi^-$ and $\kappa \rightarrow K^+ \pi^-$ are needed to describe the data in isobar fits to $D^+ \rightarrow \pi^+ \pi^- \pi^+$, $D^0 \rightarrow K_S \pi^- \pi^+$ and $D^+ \rightarrow K^- \pi^+ \pi^+$. However, it is unclear if this is compatible with LASS scattering data, and fits to $D^0 \rightarrow K^- \pi^+ \pi^0$, $D^0 \rightarrow \pi^+ \pi^- \pi^0$ do not require the addition of σ or κ . K-matrix models do not explicitly require σ or κ .

A wealth of measurements and interesting information has been published on this topic. Here, we will only consider two decay channels for which we have recent results, one for the $\pi^+ \pi^-$ S wave (σ), and one for the $K^+ \pi^-$ S wave (κ).

3.2. $D^+ \rightarrow \pi^+ \pi^- \pi^+$

Recent analyses of this channel include E791's analysis using an isobar fit with a σ resonance [7], and FOCUS, who pioneered the K-matrix approach in

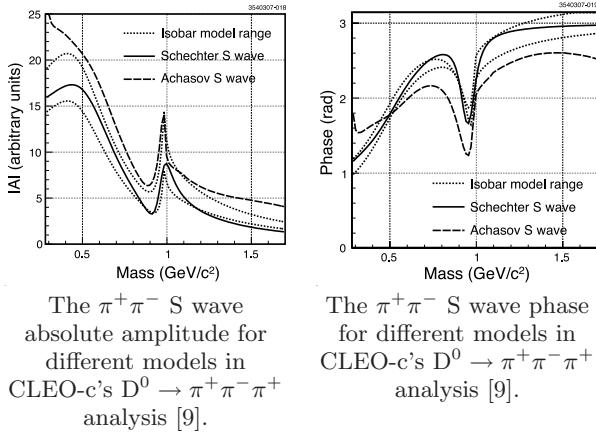


Figure 1: The plots show the magnitude and phase of the S wave component for the different models considered in CLEO-c's $D^0 \rightarrow \pi^+\pi^-\pi^+$ analysis [9]. The dotted lines represent the upper and lower limit of the range of values obtained for the various isobar models. The broken line corresponds to Achasov's model, the solid line to Schechter's model. There is good agreement between the models.

this channel, and also analyse $D_s \rightarrow \pi^+\pi^-\pi^+$ in their study [8]. The most recent result is by CLEO-c [9], using ~ 2600 signal events. CLEO consider isobar models with different descriptions of the $f_0(980)$ and σ , and Two models that respect unitarity and chirality, one according to Schechter [10] and another developed by Achasov [11]. All models considered agree with each other and the results are consistent with previous fits. The amplitude and phase of the S wave contribution as function of $\pi^+\pi^-$ invariant mass is reproduced in figure Fig 1.

3.3. $D^+ \rightarrow K^-\pi^+\pi^+$

The branching fraction of $D^+ \rightarrow K^-\pi^+\pi^+$ is comparably large, with 9.51% 0.34% [14]. Over 60% of its decay rate proceeds via a $K\pi$ S -wave, as has been observed in several experiments. In 2002 E791 [13], using an isobar model, found a large κ contribution. In 2006, E791 re-analysed the same data with a model-independent description of the S wave, using a binned amplitude and phase [15]. In 2007, FOCUS [16] applied the K-matrix formalism, constrained by LASS scattering data [17], to 54k events. The most recent result, which also has the largest data set, is from CLEO-c in 2008, using 140k events, with very little background (1.1%) [12]. CLEO-c fit the data using both the isobar and the model-independent approach, and compare their result to models used by other experiments. For both types of model, CLEO-c get a significantly improved fit if they allow for an isospin=2 $\pi^+\pi^-$ S wave contribution, where the model-independent approach gives the better χ^2 per degree of freedom.

3.4. Four-body “Dalitz” Analysis

Essentially the same formalism as for 3 body decays can be applied to to 4 body decay. Such analyses are challenging as the equivalent of the Dalitz plot now has 5 dimensions instead of 2, phase space is not flat in the usual invariant-mass squared variables, and the amplitude structure is more complex. A recent example of such an analysis, using an isobar model, is given by FOCUS for the decay channel $D^0 \rightarrow \pi^+\pi^+\pi^-\pi^-$ [18]. FOCUS observe that $D^0 \rightarrow a_1(1260)\pi$ is the dominant decay channel, followed by $D^0 \rightarrow \rho\rho$. The authors find that the a_1 predominantly decays to $\sigma\pi$. Many more results can be found in the paper, including the $\rho\rho$ polarisation. Four body amplitude analyses of D^0 decays also play as significant role in extracting γ from $B^\pm \rightarrow D^0K^\pm$ decays, which is discussed in section 5.

4. Charm Mixing and CP violation with Dalitz Plots

4.1. The neutral D^0 system

The neutral D system is the only neutral meson system that mixes and consists of up-type quarks. It therefore provides a unique window on Flavour Changing Neutral Currents (FCNC's) between up-type quarks, which can be affected by new physics in a very different way than those in down-type quarks, such as investigated in the $B_{s,d}^0$ systems. Particularly interesting is CP violation in the D system, which is essentially zero in the Standard Model, but could be significant in many New Physics scenarios. The discovery of CP violation in charm decays would be a clear signal for New Physics.

A. Mixing Parameters

The physical mass eigenstates D_1 , D_2 of the neutral D meson are superpositions of the flavour-specific states D^0 and \bar{D}^0 :

$$D_1 = pD^0 + q\bar{D}^0 \quad (4)$$

$$D_2 = pD^0 - q\bar{D}^0 \quad (5)$$

where p and q are complex numbers satisfying $|p|^2 + |q|^2 = 1$. The mass and the width difference between D_1 and D_2 are Δm and $\Delta\Gamma$. Mixing in the neutral D system is conventionally parametrised by the parameters x and y given by

$$x = \frac{\Delta m}{\bar{\Gamma}} \quad (6)$$

$$y = \frac{\Delta\Gamma}{\bar{\Gamma}} \quad (7)$$

In the absence of mixing, both parameters are zero. The Standard model expectations for x and y are $\sim 10^{-3} - 10^{-2}$ [19].

B. CP Violation in Charm

If $\left| \frac{q}{p} \right| \neq 1$, CP is violated (CP violation in mixing). CP violation in the interference between mixing and decay is parametrised by the phase ϕ . The phase ϕ is the exact equivalent in the neutral D^0 system of the parameter -2β in the neutral B_d^0 system measured in $B^0 \rightarrow J/\psi K_s$ by the B factories. The phase of the ratio $\frac{q}{p}$ is convention dependent and without fixing the convention, its value does not say anything about CP violation. Usually, however, a convention is chosen where $\frac{q}{p} = \left| \frac{q}{p} \right| e^{i\phi}$, so that both kinds of CP violation discussed here are encoded in the same complex ratio q/p .

In the SM, CP violation in charm is zero for all practical purposes, i.e. relative to the sensitivities of current experiments and those planned for the foreseeable future. However, many NP models predict CP violation in charm at a level that would be experimentally accessible. A review with further details on charm mixing, CP violation and its sensitivity to physics beyond the Standard Model can be found in [14].

4.2. D mixing using Dalitz Plots

Evidence for charm mixing has been observed first using the “wrong-sign” decay $D^0 \rightarrow K^+ \pi^-$ [20–22]. The final decay rate has contributions from the DCS amplitude $D^0 \rightarrow K^+ \pi^-$ and a combination of mixing and a CF amplitude, $D^0 \rightarrow \bar{D}^0 \rightarrow K^+ \pi^-$. A time-dependent measurement is sensitive both to CP violation and to the mixing parameters x'^2 and y' . The primed parameters are related to x, y :

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \delta_{K\pi} & \sin \delta_{K\pi} \\ -\sin \delta_{K\pi} & \cos \delta_{K\pi} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (8)$$

where $\delta_{K\pi}$ is the phase difference between the amplitude $D^0 \rightarrow K^+ \pi^-$ and $\bar{D}^0 \rightarrow K^+ \pi^-$, measured at CLEO-c to be $\cos \delta_{K\pi} = 0.9 \pm 0.3$ [23].

4.2.1. D mixing and CPV in $D \rightarrow K_s \pi \pi$

CLEO-c pioneered the Dalitz plot analysis of the self-conjugate decay $D^0 \rightarrow K_s \pi^+ \pi^-$ for the D^0 mixing analysis [24]. In this case, CF modes such as $D^0 \rightarrow K^{*-} \pi^+$ and DCF modes like $D^0 \rightarrow K^{*+} \pi^-$, which contribute to the mixing measurement in a similar way as in the two-body case, are in the same Dalitz plot, so their relative phase can be measured. The method gives direct access to x and y , and is also sensitive to the CP violation parameters $|p/q|$ and ϕ . The

BELLE collaboration analysed approximately 0.5 M $D^0, \bar{D}^0 \rightarrow K_s \pi^+ \pi^-$ events, with the result [25]:

$$\begin{aligned} x &= (0.81 \pm 0.20^{+0.13}_{-0.17}) \% \\ y &= (0.37 \pm 0.25^{+0.10}_{-0.15}) \% \\ |p/q| &= 0.86 \pm 0.30^{+0.10}_{-0.09} \\ \phi &= -14^\circ \pm 18^\circ \pm 5^\circ \end{aligned}$$

where the first error is statistical and the second error is systematic. This measurement dominates the world precision on x [26].

4.2.2. CPV measurements in SCS decays

The statistically dominant measurement of CP violation in charm from $D^0 \rightarrow K\pi$ and the Dalitz plot method with $D^0 \rightarrow K_s \pi\pi$ study CP violation in amplitudes where doubly Cabibbo suppressed amplitudes interfere with D^0 mixing and a subsequent Cabibbo favoured decay. However, decays involving singly Cabibbo suppressed decays could in principle be affected by New Physics in a different way and hence show CP violation even though CF and DCS modes do not.

A. $D^0 \rightarrow \pi^+ \pi^- \pi^0$ and $D^0 \rightarrow K^+ K^- \pi^0$

BaBar measure several CP violation observable the 3-body decays $D^0/\bar{D}^0 \rightarrow \pi^+ \pi^- \pi^0$ and $D^0/\bar{D}^0 \rightarrow K^+ K^- \pi^0$ [27]. BaBar perform a model independent binned analysis of the rate of $D^0 \rightarrow \pi^+ \pi^- \pi^0$ vs $\bar{D}^0 \rightarrow \pi^+ \pi^- \pi^0$ across the Dalitz plot, as well as a model-dependent one, comparing magnitudes and phases of the CP-conjugate amplitudes. No significant CP asymmetry has been found. The authors conclude that any CP violation in the singly Cabibbo-suppressed charm decays occurs at a rate which is not larger than a few percent.

BELLE consider the asymmetry of the total decay rate in $D^0/\bar{D}^0 \rightarrow \pi^+ \pi^- \pi^0$, integrated across the whole Dalitz space). Their result [28], $A_{CP} = (0.43 \pm 1.30)\%$, also shows no evidence for CP violation in SCS decays.

B. $D^+ \rightarrow K^+ K^- \pi^+$

CLEO-c fit an isobar amplitude model to the Dalitz plot of the SCS decay $D^+ \rightarrow K^+ K^- \pi^+$ and its CP conjugate, $D^- \rightarrow K^- K^+ \pi^-$, and form the asymmetry of the decay fractions ($\propto |A|^2$) for each of the amplitude contributions. No evidence for CP violation is observed.

5. Dalitz Analysis in Charm for Precision B physics

5.1. Introduction

A central aim of current and future flavour physics experiments is the precision determination of the CP-violating phase γ . In terms of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, γ is defined as $\arg(-V_{ub}^* V_{ud}/V_{cb}^* V_{cd})$.

A theoretically clean and statistically powerful method exploits the interference of $B^\pm \rightarrow D^0 K^\pm$ and $B^\pm \rightarrow \bar{D}^0 K^\pm$ decays, where the D^0 and \bar{D}^0 decay to a common final state f [29–33]. Suitable final states f include 2-body states such as KK , $\pi\pi$ [29, 30], $K\pi$ [31], 3-body final states such as $K_S\pi^+\pi^-$ and $K\pi\pi^0$ [32, 33] and 4-body final states such as $K\pi\pi\pi$ [31, 34] and $KK\pi\pi$ [35].

All such measurements are sensitive to the amplitude ratios

$$\frac{A(B^\pm \rightarrow \bar{D}^0 K^\pm)}{A(B^\pm \rightarrow D^0 K^\pm)} = r_B e^{i(\delta_B \pm \gamma)}. \quad (9)$$

In all cases, the measurement is affected by the properties (especially phases) of the D^0 decay amplitudes. This is where charm physics can make a significant contribution to precision B physics. By measuring the phases of the charm decay amplitudes, the uncertainties in the γ extraction in B physics experiments can be significantly reduced.

5.2. CLEO-c and $B^\pm \rightarrow D(K_S\pi\pi)K^\pm$

The best direct constraints on γ come from measurements in $B^\pm \rightarrow D(K_S\pi\pi)K^\pm$ and related modes, at the B factories [39, 40]. The combined result is $\gamma = 67^\circ_{-25^\circ}^{+32^\circ}$ [41]. The dominant systematic uncertainty in these measurements is the model uncertainties in the description of the D^0 decay amplitude, currently between 5° and 9° , which would soon limit the precision of this measurement at the next-generation flavour physics experiment, LHCb.

CLEO-c's quantum correlated $D\bar{D}$ pairs give model-independent access to both magnitude *and* phase information of the decay amplitude across the Dalitz plot. This additional information can be used as input for a model-independent extraction of γ from a binned Dalitz plot analysis [32, 34, 36, 37], thus eliminating the model-uncertainty.

As Dalitz plot variables, we use the invariant-mass squared of the $K_S\pi^-$ and the $K_S\pi^+$ pairs, which we denote as s_- and s_+ respectively. The phase of the D decay amplitude at a given point in Dalitz space is $\delta^{K_S\pi\pi}(s_-, s_+)$. For the phase-difference between the $D \rightarrow K_S\pi\pi$ amplitude and the $\bar{D} \rightarrow K_S\pi\pi$ amplitude at the same point in Dalitz space, we define

$$\Delta_\delta(s_-, s_+) \equiv \delta^{K_S\pi\pi}(s_-, s_+) - \delta^{K_S\pi\pi}(s_+, s_-) \quad (10)$$

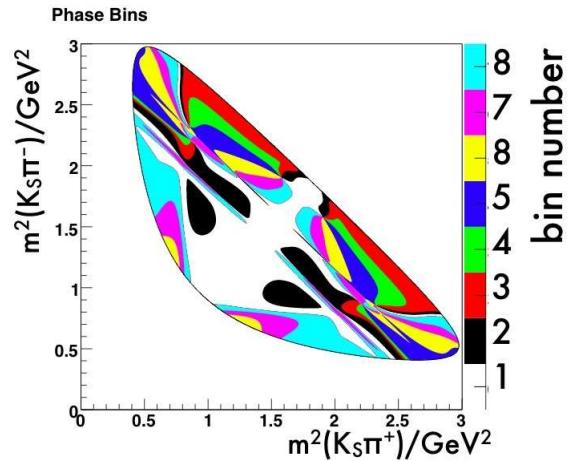


Figure 2: Binning used for this preliminary CLEO-c result. The binning is uniform in $|\Delta_\delta|$, with bin 1 centred at 0° .

The quantities measured by CLEO-c that provide the input to the γ analyses of the B-factories and LHCb, are the averages of $\cos \Delta_\delta^{K_S\pi\pi}$ and $\sin \Delta_\delta^{K_S\pi\pi}$ for each bin, c_i and s_i :

$$c_i \equiv \langle \cos \Delta_\delta \rangle_i \quad (11)$$

$$s_i \equiv \langle \sin \Delta_\delta \rangle_i \quad (12)$$

where the index i denotes the i th bin. The analysis of $K_L\pi\pi$ events in the similar way to $K_S\pi\pi$, provides further input to the c_i and s_i measurement in the $K_S\pi\pi$ Dalitz plot. The clean environment at CLEO-c allows the K_L reconstruction from kinematic constraints with high purity. The choice of binning affects the statistical precision of the analysis - it is beneficial to choose the bins such that the phase difference Δ_δ , defined in Eq 10, varies as little as possible across each bin [37]. The binning used for the preliminary CLEO-c results presented here is based on the BaBar isobar model [38]. A uniform binning in Δ_δ , with eight pairs of bins (arranged symmetrically with respect to the diagonal axis defined by $s_- = s_+$) is chosen. This binning is shown in Fig 2. The latest preliminary CLEO results for c_i and s_i from the combined analysis in both $K_S\pi\pi$ and $K_L\pi\pi$ Dalitz plots are shown in Table I. When used as input to the γ extraction in the $K_S\pi\pi$ mode at the B factories and LHCb, this is expected to replace the current model uncertainty of $7^\circ - 9^\circ$ with an uncertainty due to the statistically dominated error on c_i and s_i of about $1^\circ - 2^\circ$ [42].

5.3. Coherence Factor

The decay rates $B^\pm \rightarrow D(hh')K^\pm$, where hh' stands for any two-body final state accessible to both D^0 and \bar{D}^0 , are sensitive to γ [29–31]. For example for $D^0 \rightarrow K^+\pi^-$:

$$\Gamma(B^- \rightarrow (K^+\pi^-)_D K^-) \propto$$

bin number	c_i	s_i
1	$0.742 \pm 0.041 \pm 0.022$	$-0.022 \pm 0.168 \pm 0.096$
2	$0.607 \pm 0.073 \pm 0.038$	$0.009 \pm 0.220 \pm 0.054$
3	$0.064 \pm 0.077 \pm 0.052$	$0.548 \pm 0.198 \pm 0.096$
4	$-0.492 \pm 0.133 \pm 0.056$	$0.124 \pm 0.227 \pm 0.074$
5	$-0.918 \pm 0.053 \pm 0.039$	$-0.118 \pm 0.194 \pm 0.057$
6	$-0.743 \pm 0.071 \pm 0.033$	$-0.296 \pm 0.203 \pm 0.070$
7	$0.156 \pm 0.092 \pm 0.050$	$-0.870 \pm 0.183 \pm 0.062$
8	$0.398 \pm 0.047 \pm 0.020$	$-0.438 \pm 0.146 \pm 0.041$

Table I Preliminary CLEO-c results for the measurement of c_i and s_i in the $D \rightarrow K_S\pi\pi$ Dalitz plot, using input from both $D \rightarrow K_S\pi\pi$ events and $D \rightarrow K_L\pi\pi$ events in 818 pb^{-1} . The first error is the combination of the statistical error and the uncertainty that arises from the use of $K_L\pi\pi$ results for the $K_S\pi\pi$ c_i and s_i determination. The 2nd error is the remaining systematic uncertainty.

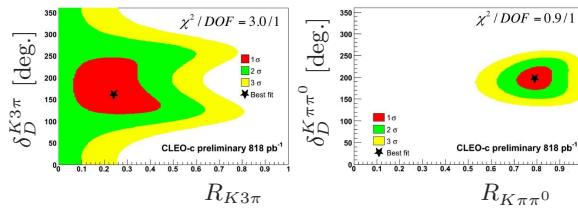


Figure 3: 1, 2 and 3 σ confidence regions in $R_f - \delta_D^f$ space for $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$ and $D^0 \rightarrow K^+\pi^-\pi^0$. The star represents the central value. Preliminary CLEO-c result, first presented at CKM 2008.

$$r_B^2 + (r_D^{K\pi})^2 + 2r_B r_D^{K\pi} \cos(\delta_B + \delta_D^{K\pi} - \gamma) \quad (13)$$

where we used, in analogy to Eq 9

$$\frac{A(D^0 \rightarrow K^+\pi^-)}{A(\bar{D}^0 \rightarrow K^+\pi^-)} = r_D^{K\pi} e^{i(\delta_D^{K\pi})}. \quad (14)$$

The addition of 3 and 4-body decay modes of the D such as $D^0 \rightarrow K^+\pi^-\pi^0$ and $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$ can significantly improve this measurement. In multibody decays $D^0 \rightarrow f$, the resonant substructure needs to be taken into account, which can be achieved by adding only one additional parameter to describe the decay rate [34]. For a generic final state f :

$$\Gamma(B^- \rightarrow (\bar{f})_D K^-) \propto \bar{A}_f^2 + r_B^2 A_f^2 + 2r_B R_f A_f \bar{A}_f \cos(\delta_B + \delta_D^f - \gamma) \quad (15)$$

where R_f is the coherence factor which satisfies $0 \leq R_f \leq 1$; δ_D^f is the average strong phase difference. The larger R_f , the higher the sensitivity to γ in a given mode. Decays of quantum-correlated $D\bar{D}$ pairs at CLEO-c can be used to measure both R_f and δ_D^f . CLEO-c measure the following values for the coherence factors in the $K^+\pi^- \pi^+\pi^-$ and $K^+\pi^- \pi^0$ mode,

as presented for the first time at CKM 2008 [44]:

$$\begin{aligned} R_{K3\pi} &= 0.24^{+0.21}_{-0.17} & R_{K\pi\pi^0} &= 0.79 \pm 0.08 \\ &< 0.57 \text{ at } 95\% \text{ CL} & \\ \delta_D^{K3\pi} &= 161^{+85^\circ}_{-48^\circ} & \delta_D^{K\pi\pi^0} &= 197^{+28^\circ}_{-27^\circ} \end{aligned}$$

The confidence regions in R_f and δ_D^f space for $f = K3\pi$ and $f = K\pi\pi^0$, as presented at CKM 2008 [44], are shown in Fig 3. The small coherence factor in $K3\pi$ implies that this mode on its own would not be very sensitive to γ . However, this coherence factor measurement still provides a significant constraint in a combined measurement with two-body decays of the D . The precise effect depends on the exact value of the strong phase $\delta^{K\pi}$; typically, without the $K3\pi$ constraint from CLEO-c (but including the $\delta^{K\pi}$ constraint from CLEO-c), the precision for 2 fb^{-1} data at LHCb from $D^0 \rightarrow KK, \pi\pi, K\pi, K3\pi$ modes is $\sim 10^\circ$, which improves to $\sim 8^\circ$ with the additional information from CLEO-c's coherence factor measurement in $K3\pi$ [43]. An equivalent study for $K\pi\pi^0$ has not yet been performed, but the large value of $R_{K\pi\pi^0}$ suggests that we can expect a further significant improvement.

6. Conclusion

Charm physics has recently undergone a remarkable renaissance. Dalitz analyses in charm give access to magnitudes *and* phases of the charm decay amplitudes. This can be used to analyse light-meson resonance, where an improved description of broad S-wave resonances is of particular interest. Dalitz plots analyses in charm also play a significant role in charm mixing measurements, where Dalitz analyses provide the best constraints on x and important phase information that allows the translation of the x', y' measurements in $D \rightarrow K\pi$ into the unprimed parameters. And finally, in the decay chain $B^\pm \rightarrow D^0 K^\pm$, and equivalently $B^0 \rightarrow D^0 K^*$, phase information from the B decay is encoded in the D Dalitz plot, allowing a theoretically clean measurement of γ . This method provides currently the best constraint on γ . Input from the analysis of quantum-correlated $D\bar{D}$ pairs at CLEO-c will significantly reduce the model uncertainty in this group of measurements, in the $D^0 \rightarrow K_S\pi\pi$ mode as well as other 2, 3 and 4-body decay modes of the D^0 . This is of particular importance for future facilities such as the proposed Super Flavour Factory, and LHCb, which is due to start data taking in 2009, and where these uncertainties would soon be the limiting factor in the precision on γ .

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