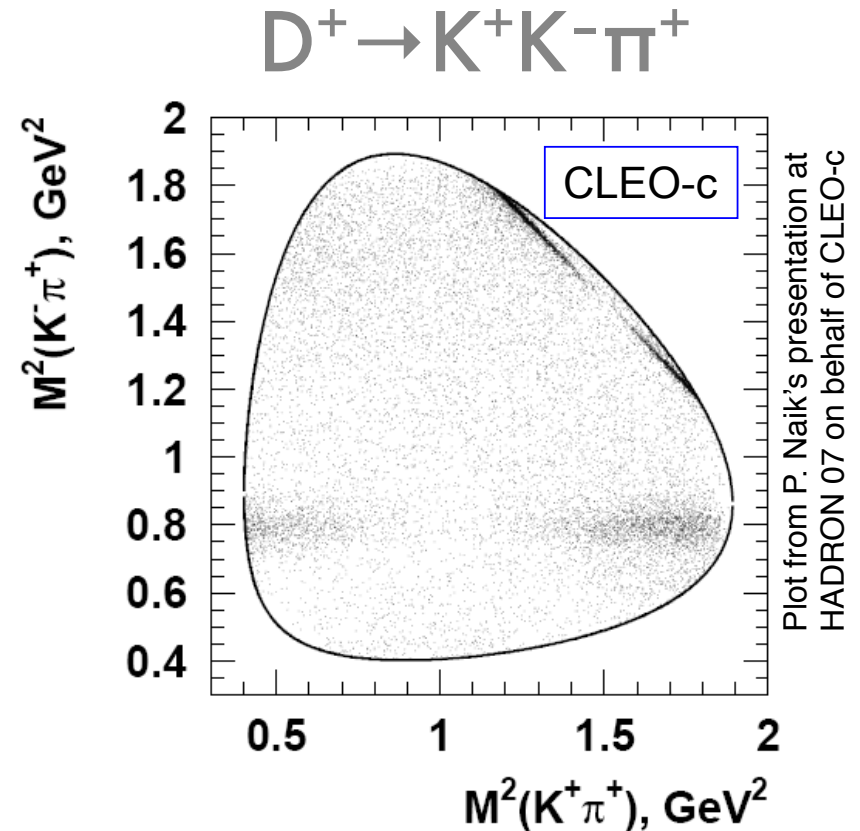


# Dalitz Decays in Charm

Jonas Rademacker on behalf of the CLEO-c collaboration

# Dalitz Plots

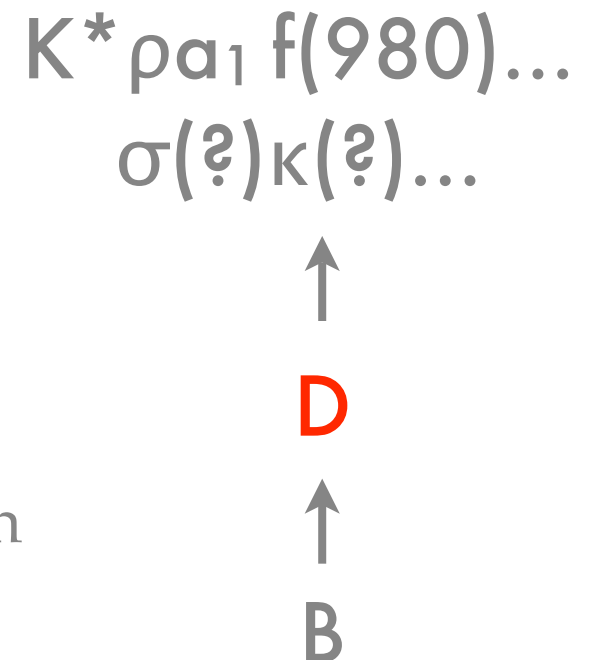
- Kinematics of 3-body decay  
 $D \rightarrow A, B, C$  fully described by 2 parameters.
- Usually chose  $m_{AB}^2 \equiv (p_A + p_B)^2$  and  $m_{BC}^2 \equiv (p_B + p_C)^2$ .
- Lorentz invariant
- Phase-space is flat in these parameters.



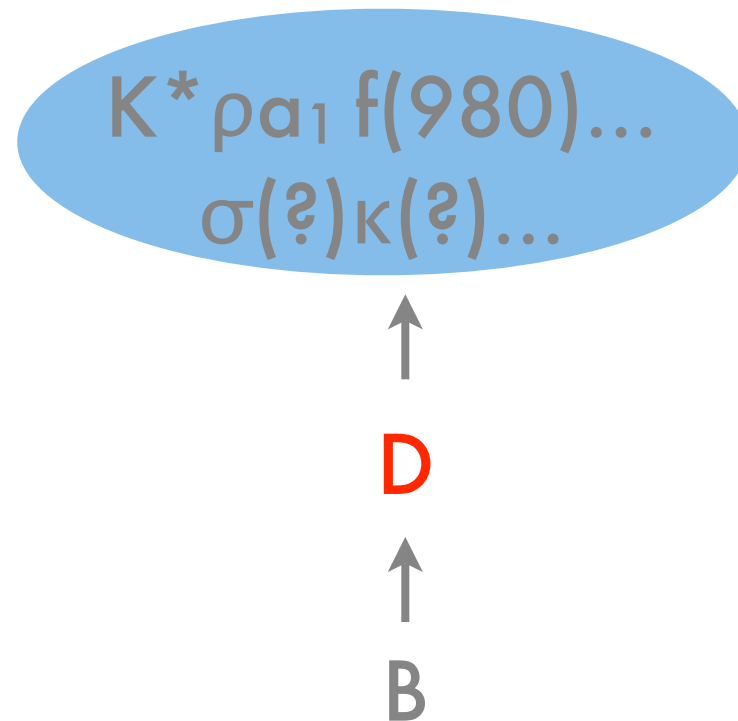
- Decay rates:
$$\frac{d\Gamma(m_{ab}^2, m_{bc}^2)}{dm_{ab}^2 dm_{bc}^2} = \left| a_1 e^{i\delta_1} + a_2 e^{i\delta_2} + a_3 e^{i\delta_3} + \dots \right|^2 \frac{d\Phi(m_{ab}^2, m_{bc}^2)}{dm_{ab}^2 dm_{bc}^2}$$

# Dalitz analyses can...

- Main strength of Dalitz analyses: Access to complex amplitudes, incl phases.
- Use for...
  - Understanding properties the light meson resonances.
  - Understand charm itself - mixing, CPV
  - Measure properties (phases) of B-meson decays to charm (CPV)



# Charm Dalitz for the resonances

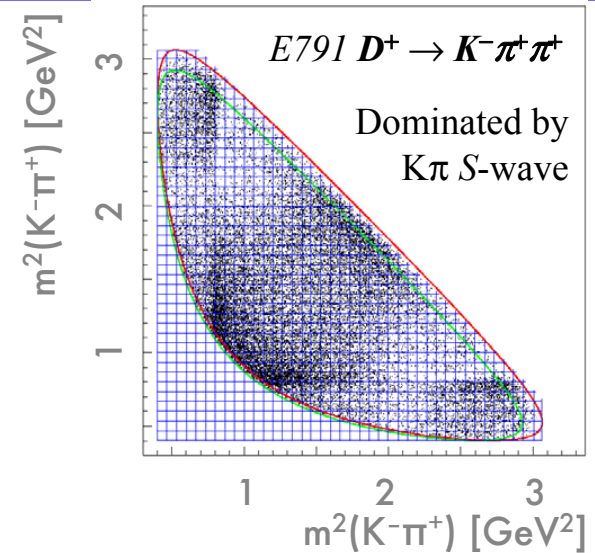


# “Isobar” Model

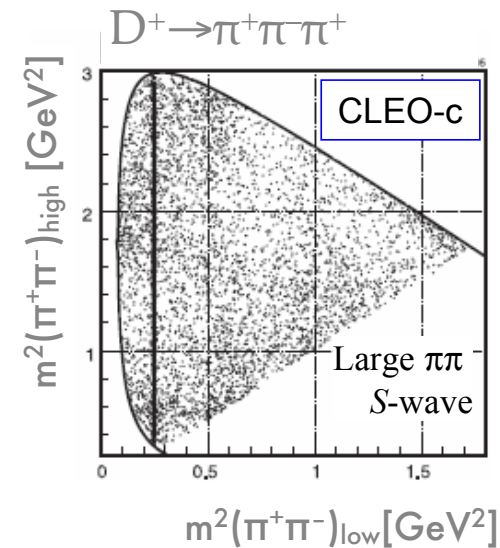
- Each resonance = Breit Wigner lineshape (or similar) times factors accounting for spin.
- Popular amongst experimentalists, but less so amongst theorists: violates unitarity. But not much as long as resonances are reasonably narrow, don't overlap too much.
- General consensus: Isobar OK for P, D wave, but problematic for S-wave.
- Alternatives exist, e.g. K-matrix formalism, which respects unitarity.

# S-wave

- S-wave resonances  $\sigma \rightarrow \pi^+\pi^-$ ,  $\kappa \rightarrow K^+\pi^-$  are the real?
- needed in isobar fits to  $D^+ \rightarrow \pi^+\pi^-\pi^+$ ,  $D^0 \rightarrow K_s\pi^+\pi^-$ ,  $D^+ \rightarrow K^-\pi^+\pi^+$
- unclear if compatible with LASS scattering data
- not required in  $D^0 \rightarrow K^-\pi^+\pi^0$ ,  $D^0 \rightarrow \pi^+\pi^-\pi^0$  isobar fits
- K-matrix Models don't explicitly require  $\sigma$ ,  $\kappa$ .



Phys. Rev. Lett 89, 121801 (2002)



P. Naik on behalf of  
CLEO at HADRON 07

# CLEO-c's $D^+ \rightarrow \pi^+ \pi^+ \pi^-$ 3-model fit

- Previously on this channel: E791's isobar fit, FOCUS pioneered K-matrix.
- CLEO-c: Isobar (two types: Flatté and complex-pole for  $f_0(980)$  and  $\sigma$ ) and two models that respect unitarity & chirality [CLEO-c: Phys. Rev. D 76, 012001 (2007)]

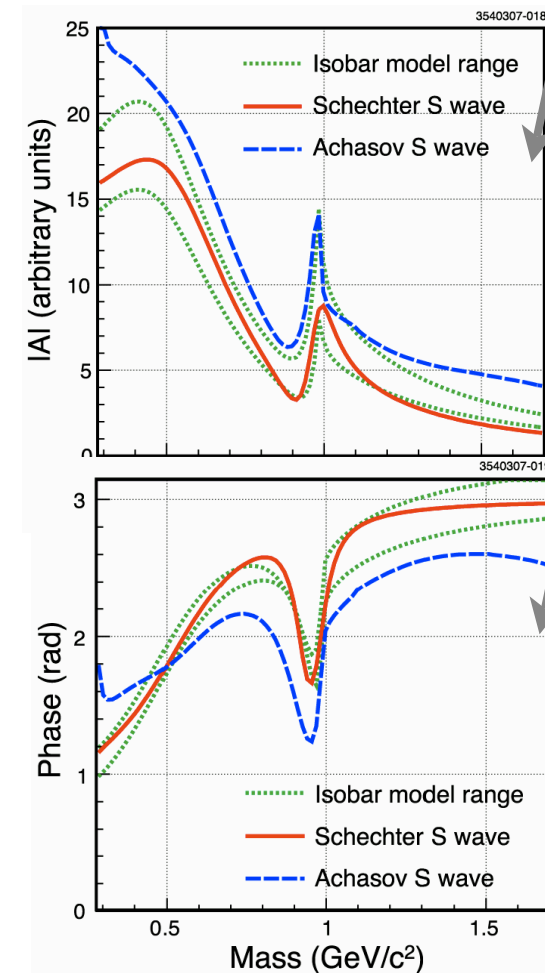
- Schechter [see CLEO paper and Int J. Mod. Phys. A20, 6149 (2005)]

- Achasov [see CLEO paper and N. N. Achasov and G. N. Shestakov, Phys. Rev. D 67, 114018 (2003), and many more -[click here](#).]

- All fit data well. Results compatible with prev experiments and each other.

E791: Phys. Rev. Lett. 86, 770 (2001); FOCUS: Phys. Lett. B 585, 200 (2004); CLEO-c: Phys. Rev. D 76, 012001 (2007)

$$A(\text{S-wave } \pi\pi) = |A| e^{i\delta}$$



CLEO-c: Phys. Rev. D 76, 012001 (2007)

# $D^+ \rightarrow K^- \pi^+ \pi^+$

- Large  $K\pi$  S-wave component, large B.R.
- History
  - 2002, highest-statistics result (15k evts), isobar model fit by E791\* (result not in PDG - model used not compatible with PDG prescription).
  - In 2006, E791\*\* re-analysed their data, describing the S-wave contribution in a model-independent way.
  - CLEO-c repeated this analysis in 2008\*\*\* (140k evts)
  - FOCUS use K-matrix approach (2007)\*\*\*\* (54k evts)

\*Phys. Rev. Lett 89, 121801 (2002);      \*\*Phys. Rev. D73, 032004 (2006);

\*\*\*[arXiv:0802.4214](https://arxiv.org/abs/0802.4214) (submitted to PRD);      \*\*\*\*[arXiv:0705.2248](https://arxiv.org/abs/0705.2248) [hep-ex], to appear in Phys. Lett. B



# $D^+ \rightarrow K^- \pi^+ \pi^+$ Isobar and Model Independent Fit at CLEO-c

- Quasi Model Independent: Replace S-wave Breit Wigners + non-resonant component with binned complex amplitude (26 bins).
- New compared to E791: Add  $I=2$   $\pi\pi$  S-wave - both, to isobar and to quasi-model independent fit. Crucial element in both fits.

Model:

$\chi^2$ -prob

isobar  
(E791 model)

$< 10^{-6}$

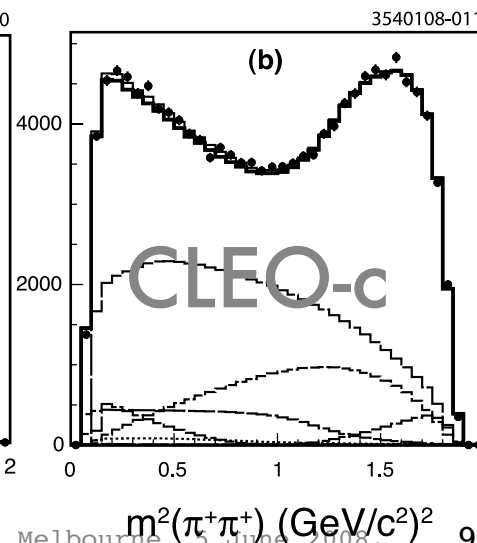
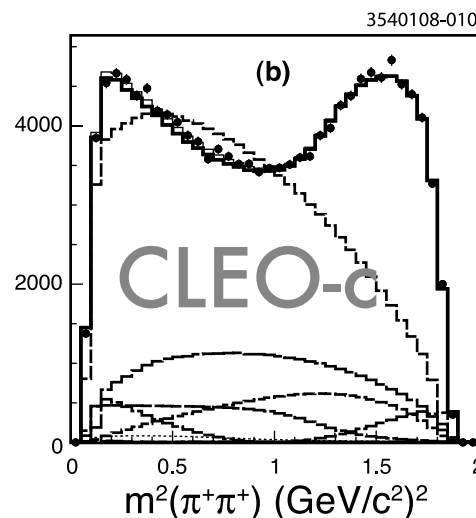
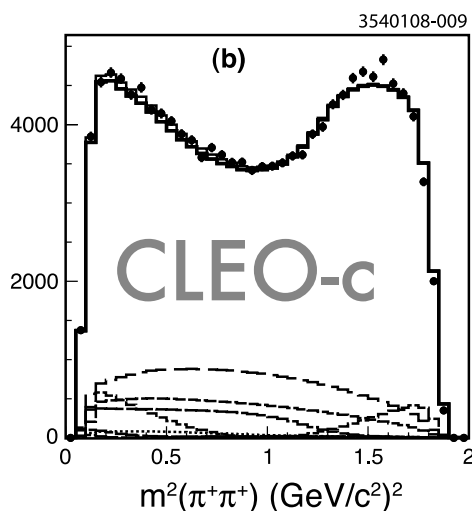
isobar  
with  $I=2$   $\pi\pi$  S-wave

13%

binned  $K\pi$  S-wave  
incl binned  $I=2$   $\pi\pi$  S-wave

32%

Fit projection  
( $\pi\pi$ )

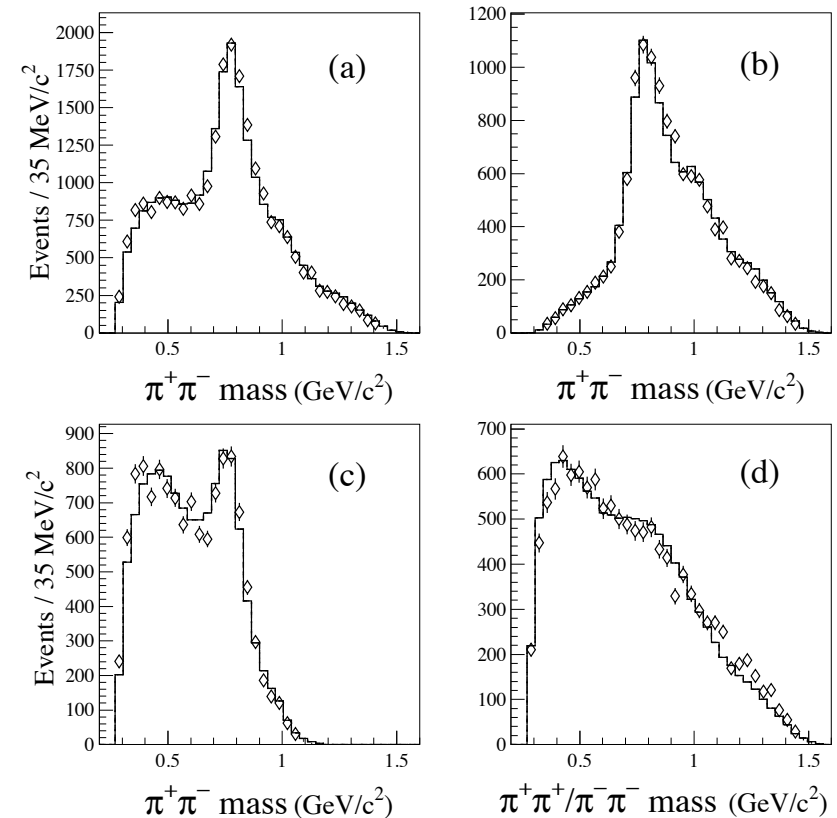


arXiv:0802.4214 (submitted to PRD)

# 4-body "Dalitz" $D \rightarrow \pi^+ \pi^+ \pi^- \pi^-$

- Similar formalism, now with 5 variables rather than 2.
- $D \rightarrow a_1(1260)\pi$  is the dominant channel, followed by  $\rho\rho$ .
- Found dominant decay of  $a_1$  is  $\sigma\pi$  (isobar analysis)
- Many more results in paper, (incl  $\rho\rho$  polarisation)
- 4-body amplitude in  $D$  also important for  $B$  physics.

FOCUS (Jan. 2007)

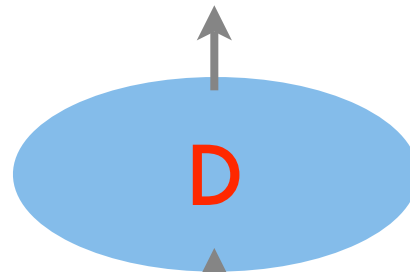


[Phys.Rev.D75:052003,2007](#)

# Dalitz Plots for Charm mixing and CP violation

$K^* \rho \alpha_1 f(980) \dots$

$\sigma(?) \kappa(?) \dots$



B

# The Charm System

- D mass eigenstates:  $D_{1,2} = p |D\rangle \pm q |\bar{D}\rangle$
- $x = \frac{\Delta m}{\bar{\Gamma}}$  ~mixing frequency
- $y = \frac{\Delta\Gamma}{2\bar{\Gamma}}$  ~lifetime difference
- CP-violation if  $|p/q| \neq 1$  or phase  $\phi \neq 0$  (measured in interference between mixing and decay, like  $-2\beta$  in B)
- SM:  $x, y \sim 10^{-3} - 10^{-2}$ , no CPV
- New Physics could affect charm very differently to K or B system (only mixing up-type quark).
- Most sensitive to NP: CP violation in charm.

# Charm mixing

- First charm mixing evidence was found with  $D \rightarrow K^+ \pi^-$ .  
Sensitive to  $(x')^2$  and  $y'$ , where

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \delta & \sin \delta \\ -\sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- $\delta$  = strong phase between the interfering CF  $D \rightarrow K^- \pi^+$  and DCS  $D \rightarrow K^+ \pi^-$ . (CLEO measured  $\cos \delta = 0.9 \pm 0.3$ ). [W. M. Sun at CHARM 2007 for CLEO, arXiv:0712.0498 \[hep-ex\]](#)
- Same principle works with  $D \rightarrow K^+ \pi^- \pi^0$  Dalitz plots and measures  $x''$ ,  $y''$ .  
See [William Lockman for BaBar at Lepton-Photon 2007, Daegu, S. Korea](#) and [backup slides](#) for details.

# Charm mixing with time-dependent $D \rightarrow K_s \pi^+ \pi^-$ Dalitz

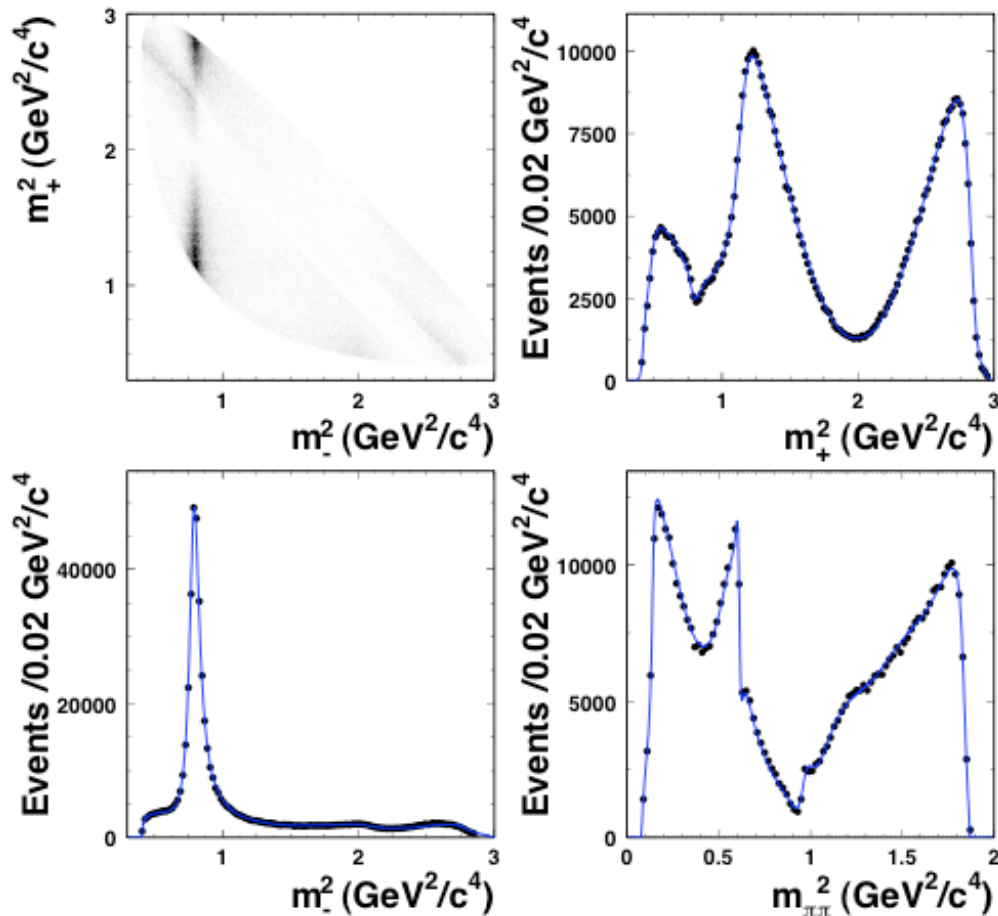
- $K_s \pi^+ \pi^-$  is self-conjugate, both CF modes like  $D \rightarrow K^{*-} \pi^+$  and DCS modes such as  $D \rightarrow K^{*+} \pi^-$  are in the same Dalitz plot  $\Rightarrow$  measure their relative phase  $\delta_i$ .
- Dalitz fit measures all phases that are needed.
- Extracts  $(x, y)$  directly; not  $(x', y')$  or  $(x'', y'')$
- Linear dependence on  $x$ , sensitivity to the sign of  $x$ .
- Pioneered by CLEO Phys. Rev.D72:012001,2005, hep-ex/0503045

# Charm mixing with time-dependent $D \rightarrow K_s \pi^+ \pi^-$ Dalitz

## $D \rightarrow K_s \pi^+ \pi^-$ at BELLE

(ca  $\frac{1}{2}M$  for 540/fb)

Phys.Rev.Lett.99:131803,2007



## isobar model

Resonance	Fit fraction
$K^*(892)^-$	0.6227
$K_0^*(1430)^-$	0.0724
$K_2^*(1430)^-$	0.0133
$K^*(1410)^-$	0.0048
$K^*(1680)^-$	0.0002
$K^*(892)^+$	0.0054
$K_0^*(1430)^+$	0.0047
$K_2^*(1430)^+$	0.0013
$K^*(1410)^+$	0.0013
$K^*(1680)^+$	0.0004
$\rho(770)$	0.2111
$\omega(782)$	0.0063
$f_0(980)$	0.0452
$f_0(1370)$	0.0162
$f_2(1270)$	0.0180
$\rho(1450)$	0.0024
$\sigma_1$	0.0914
$\sigma_2$	0.0088
NR	0.0615

# Charm mixing with time-dependent Dalitz Plots at BELLE

- HFAG FPCP 2008\*

**mix:**  $x = (0.89^{+0.26}_{-0.27})\%$     $y = (0.75^{+0.17}_{-0.18})\%$

**CPV:**  $|p/q| = 0.87^{+0.18}_{-0.15}$     $\phi = -9.1^{\circ+8.1^{\circ}}_{-7.8^{\circ}}$

- BELLE's  $D \rightarrow K_s \pi^+ \pi^-$  alone\*\*

$x = (0.81 \pm 0.30^{+0.13}_{-0.17})\%$

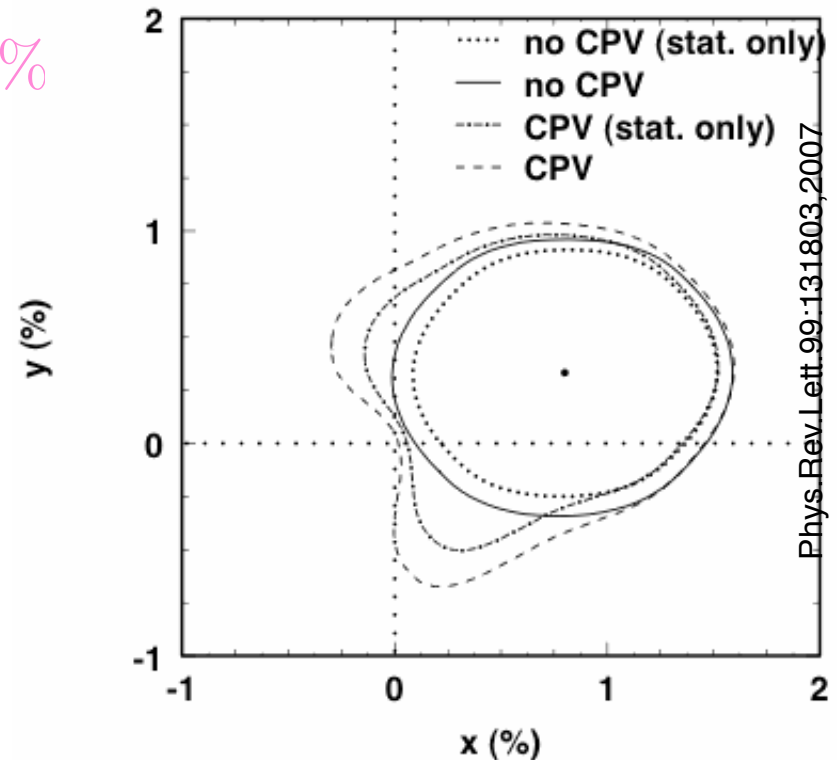
$y = (0.37 \pm 0.25^{+0.10}_{-0.15})\%$

$|p/q| = 0.86 \pm 0.30^{+0.10}_{-0.09}$

$\phi = -14^{\circ} \pm 18^{\circ} \pm 5^{\circ}$

- Best single result on  $x$ .

95% CL, BELLE,  $D \rightarrow K_s \pi \pi$



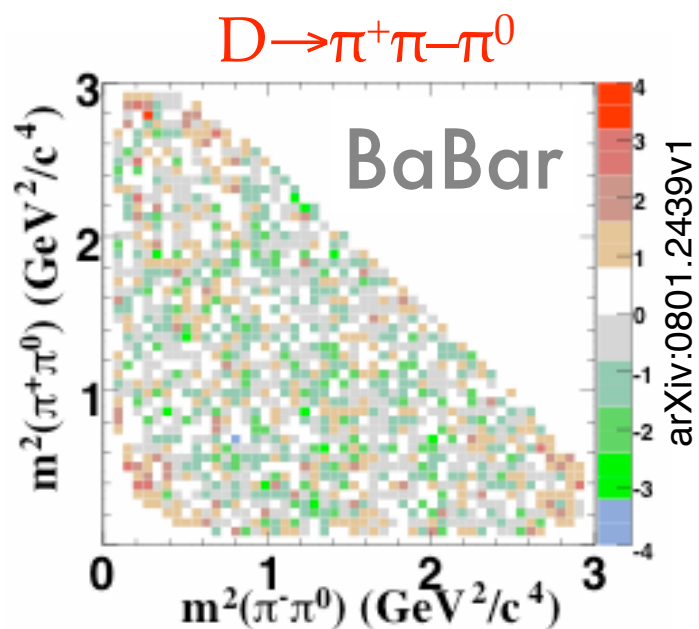
\*[http://www.slac.stanford.edu/xorg/hfag/charm/FPCP08/results\\_mix+cpv.html](http://www.slac.stanford.edu/xorg/hfag/charm/FPCP08/results_mix+cpv.html)

\*\*Phys.Rev.Lett.99:131803,2007



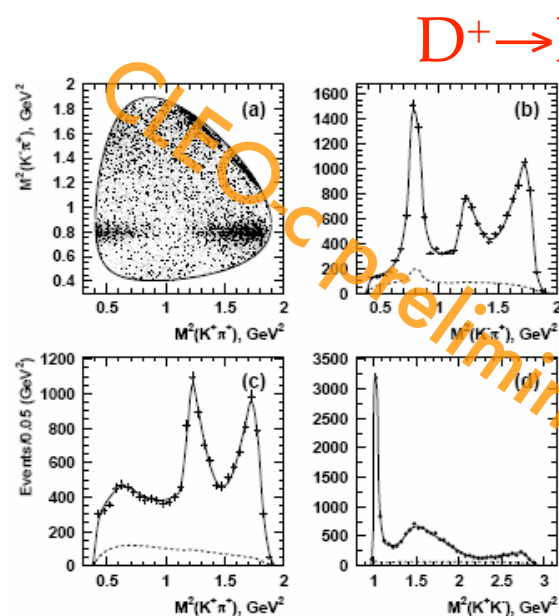
# Search for CPV in SCS decays

- New physics could affect Singly Cabibbo Suppressed decays differently from Cabibbo Favoured or Doubly Cabibbo Suppressed.
- Recent Dalitz studies by BaBar ( $D \rightarrow \pi^+ \pi^- \pi^0$ ,  $D \rightarrow K^+ K^- \pi^0$ ) and CLEO-c  $D^+ \rightarrow K^+ K^- \pi^+$ . Results: CPV in SCS smaller than few%.



BaBar result: arXiv:0802.4035, submitted to PRL

BELLE result (integrated decay rates, not Dalitz):  
arXiv:0801.2439, submitted to PRD



CLEO-c result: Presented at HADRON07 by P. Naik on behalf of CLEO-c

$$A_{CP} = \frac{N_{D^+}/\epsilon_{D^+} - N_{D^-}/\epsilon_{D^-}}{N_{D^+}/\epsilon_{D^+} + N_{D^-}/\epsilon_{D^-}}$$

Component	$A_{CP}$ (fraction) (%)
$K^*(892)^0 K^+$	$0.2 \pm 2.7^{+2.3+0.7}_{-0.4-0.4}$
$K^- \pi^+(S) K^+$	$-1 \pm 5^{+1+6}_{-2-4}$
$a_0(980) \pi^+$	$-18 \pm 23^{+4+24}_{-9-6}$
$\phi(1020) \pi^+$	$-3.7 \pm 1.9^{+0.1+0.2}_{-0.2-0.3}$
$f_2(1270) \pi^+$	$5 \pm 26^{+3+22}_{-4-46}$
$a_0(1450) \pi^+$	$-20 \pm 13^{+0+16}_{-8-9}$
$\phi(1680) \pi^+$	$-6 \pm 21^{+22+7}_{-4-3}$
$\overline{K}_2^*(1430)^0 K^+$	$59 \pm 41^{+1+8}_{-28-41}$

# Charm Dalitz for precision B physics

$K^* \rho a_1 f(980) \dots$

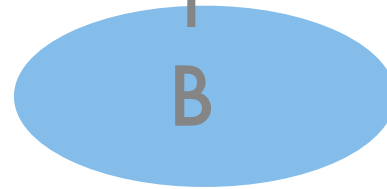
$\sigma(?) \kappa(?) \dots$



**D**

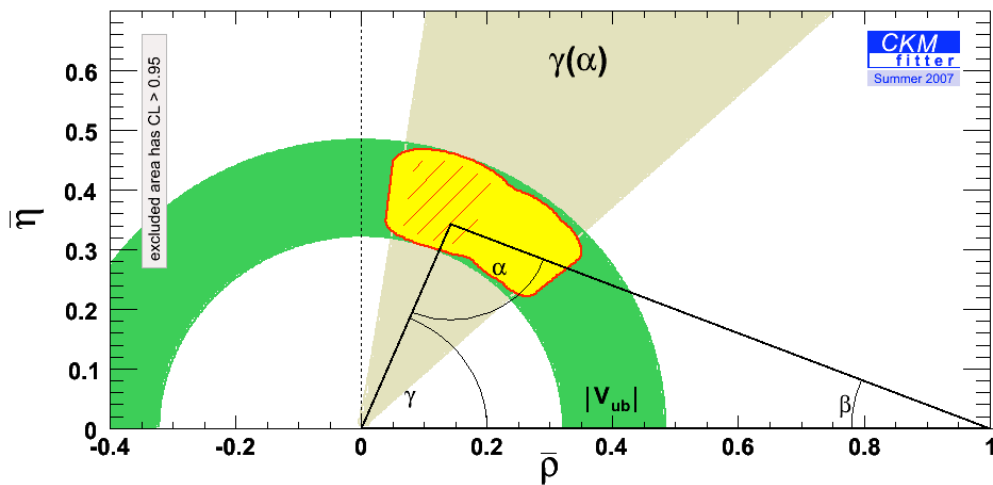


**B**

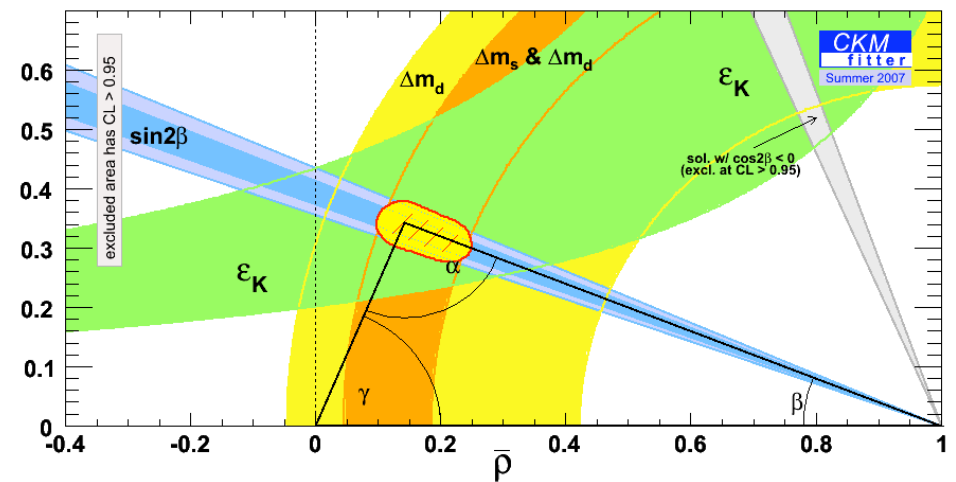


# The importance of measuring Tree-level $\gamma$

## Constraints from Trees



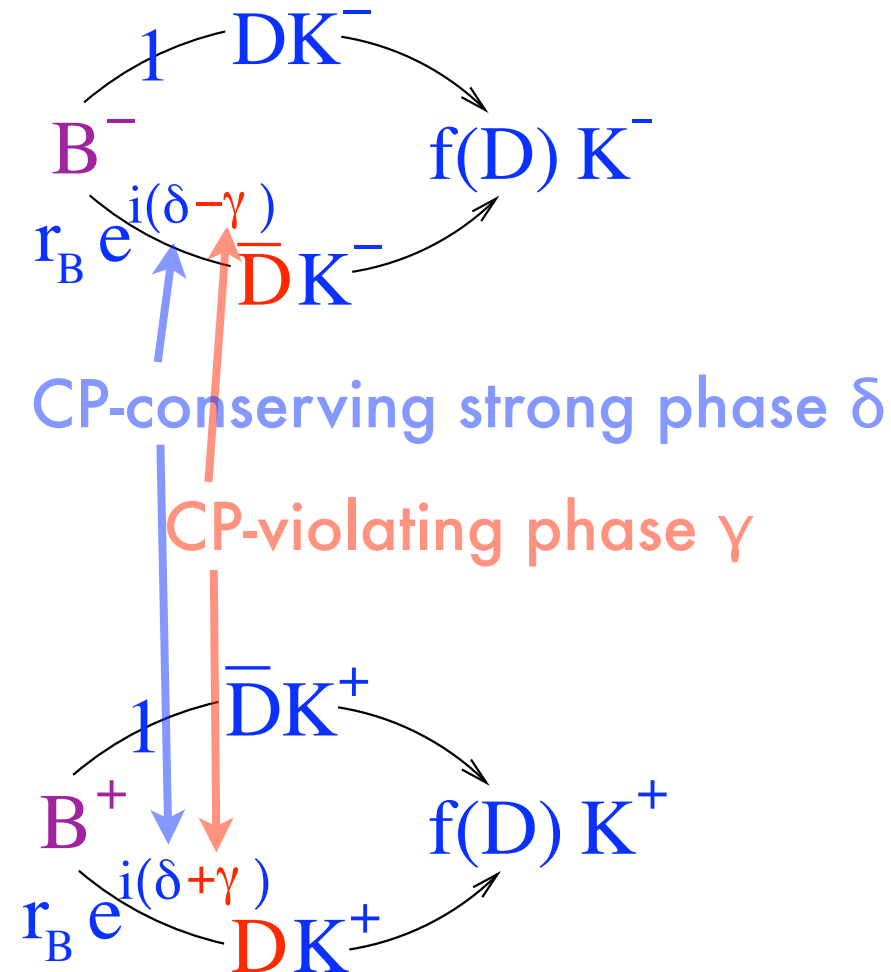
## Constraints from Loops



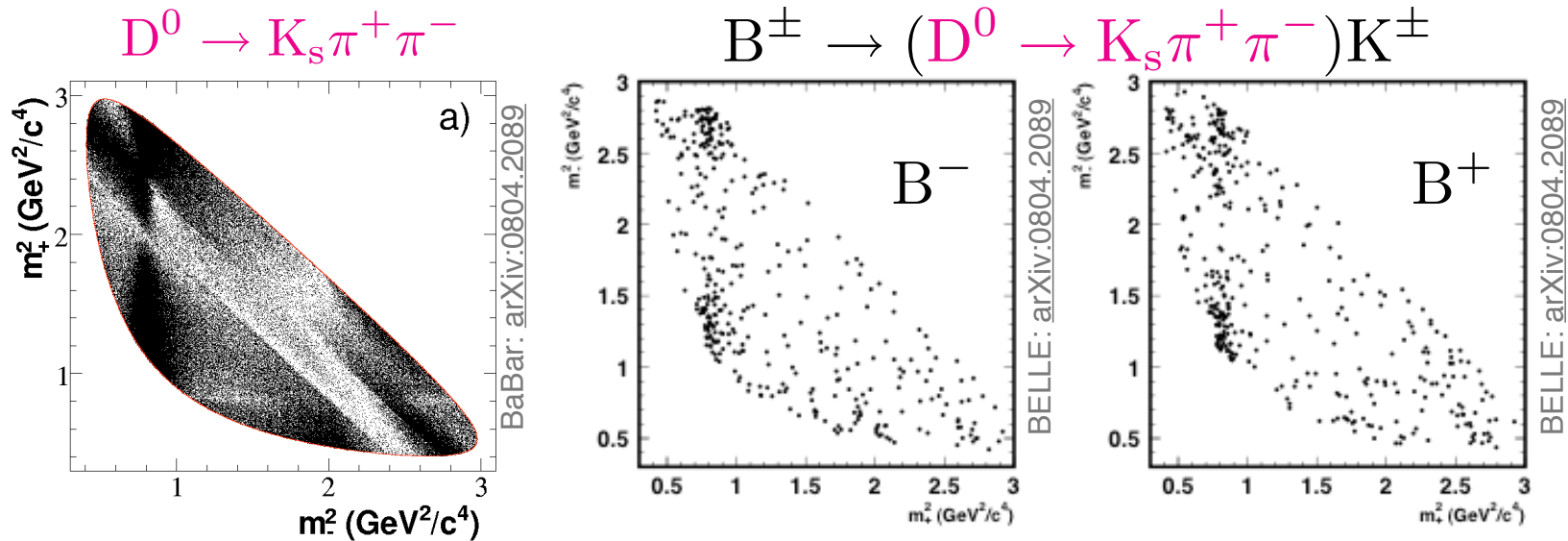
# $B^\pm \rightarrow DK^\pm$

- Tree level only - no new physics
- Crucial for NP sensitivity by providing a theoretically clean reference.
- No time measurement, no tagging.

Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW)  
 Gronau, London Phys.Lett.B253:483-488,1991 (GLW)  
 Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS)  
 Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018  
 Belle Collaboration Phys.Rev. D70 (2004) 072003



# Dalitz Plots for $\gamma$ at Belle&BaBar



BaBar\* (383M BB):  $\gamma = 76^\circ \pm 22^\circ(\text{stat}) \pm 5^\circ(\text{sys}) \pm 5^\circ(\text{model})$ ,  $r_B = 0.086 \pm 0.04$

BELLE\*\* (657M BB):  $\gamma = 76^\circ_{-13^\circ}^{+12^\circ}(\text{stat}) \pm 4^\circ(\text{sys}) \pm 9^\circ(\text{model})$ ,  $r_B = 0.16 \pm 0.04$

Using  $B^0 \rightarrow D(K_s \pi \pi) K^*$  instead of charged B (May 08): [arXiv:0805.2001](https://arxiv.org/abs/0805.2001)

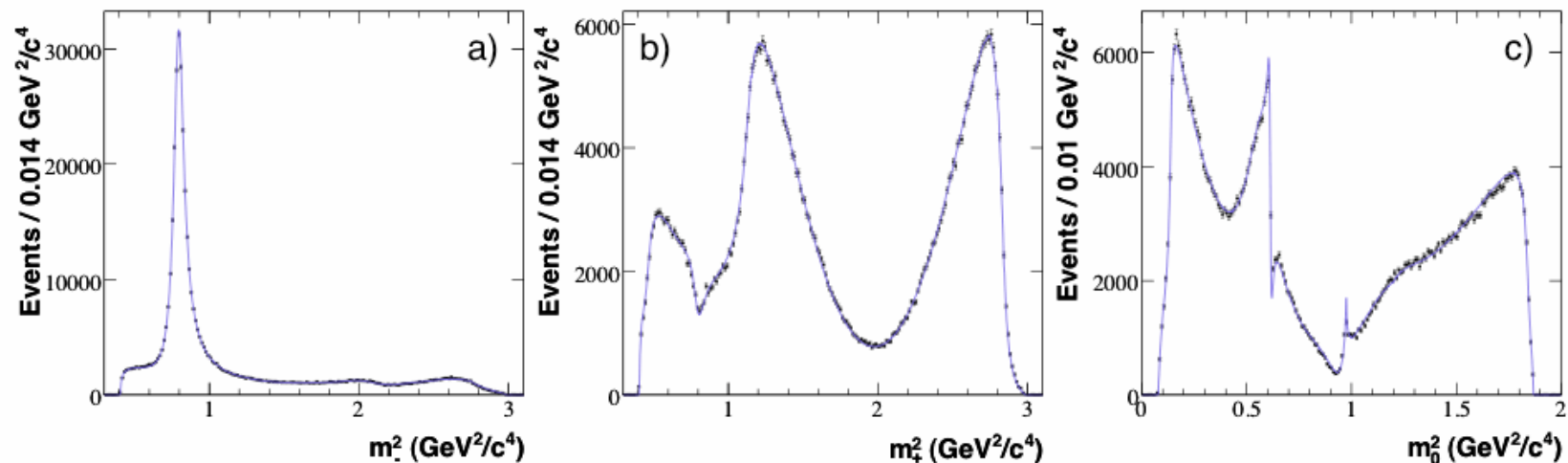
BaBar:  $162^\circ \pm 56^\circ$  or  $342^\circ \pm 56^\circ$ ,  $r_s < 0.55$  at 95%CL

\*Combined result  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D^* K^\pm$  for  $K_s \pi \pi$  and  $K_s K K$  Dalitz plot. [arXiv:0804.2089](https://arxiv.org/abs/0804.2089); April 2008

\*\*Combined result  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D^* K^\pm$  for  $K_s \pi \pi$  Dalitz plot. [arXiv:0803.3375](https://arxiv.org/abs/0803.3375), March 2008

# BaBar's improved Dalitz model

## BaBar $D \rightarrow K_s \pi \pi$ fit projections



arXiv:0804.2089

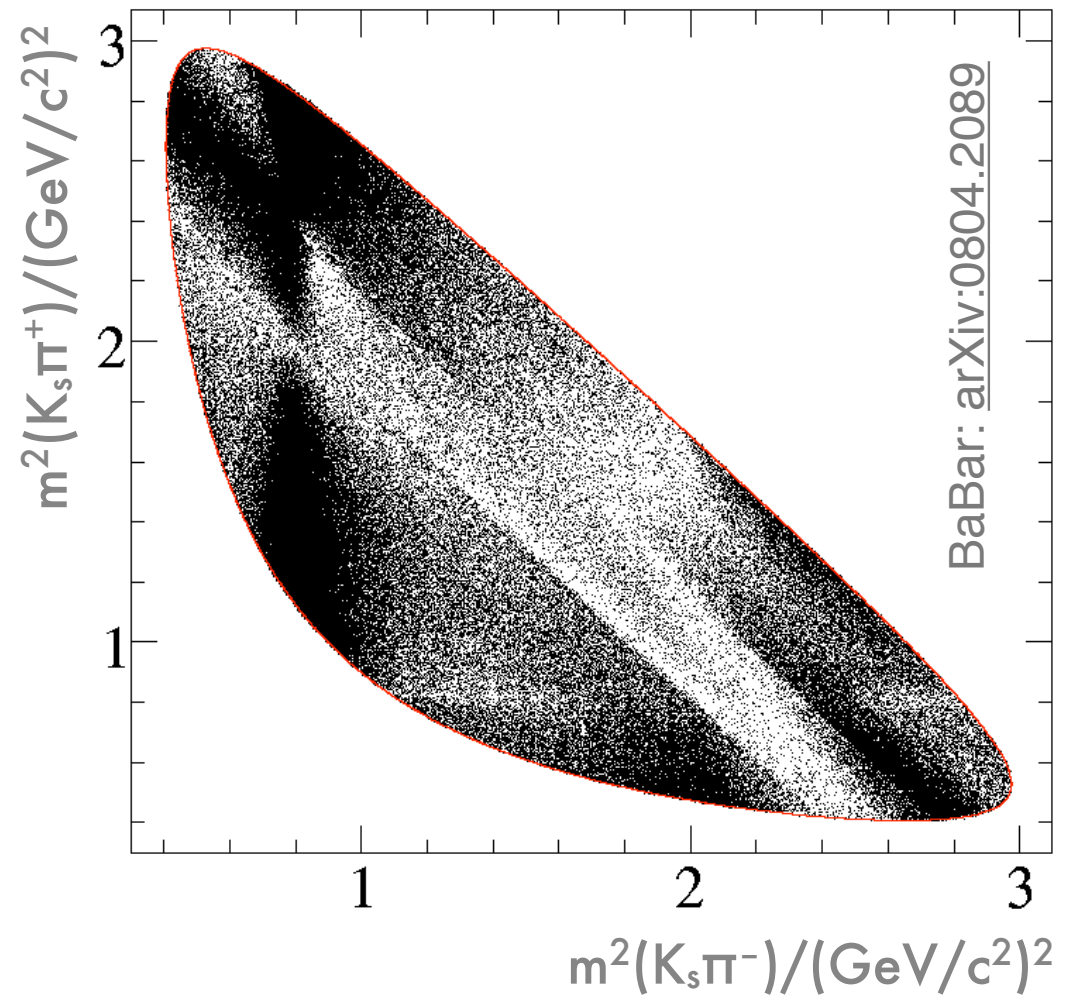
- BaBar now use K-matrix\* for  $\pi\pi$  S-wave (fit fraction  $12\% \pm 3\%$ ) and LASS-parameterisation\*\* for  $K\pi$  S-wave.
- Isobar:  $\chi^2 / \text{ndof} = 1.20$ ; K-matrix+LASS:  $\chi^2 / \text{ndof} = 1.11$ ;

\*E. P. Wigner. Phys. Rev. **70**, 15 (1946);  
I. J. R. Aitchison, Nucl. Phys. A **189**, 417 (1972)

\*\*Nucl. Phys. B **296**, 493 (1988)

# Model Dependence

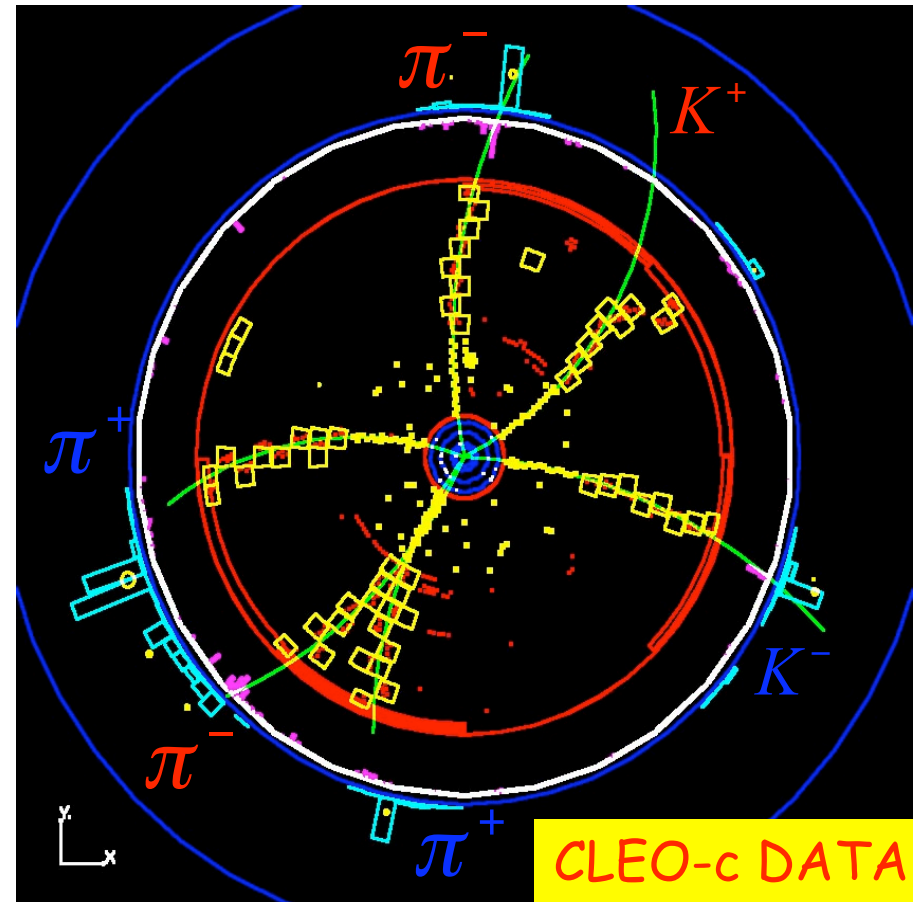
- Need D-Dalitz structure as input.
- Complex Amplitudes have 2 parameters at each point in plot.
- Measure only 1 parameter: Intensity.
- Need Model. Get model dependence. Still largest systematic in both analyses.



# CLEO-c

$$e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D}$$

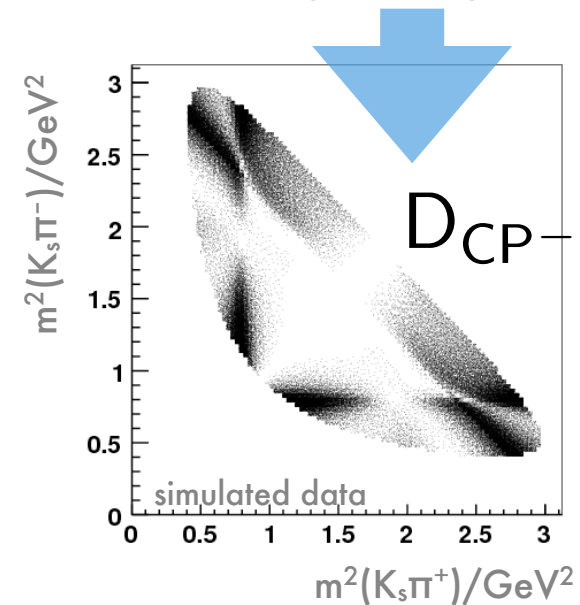
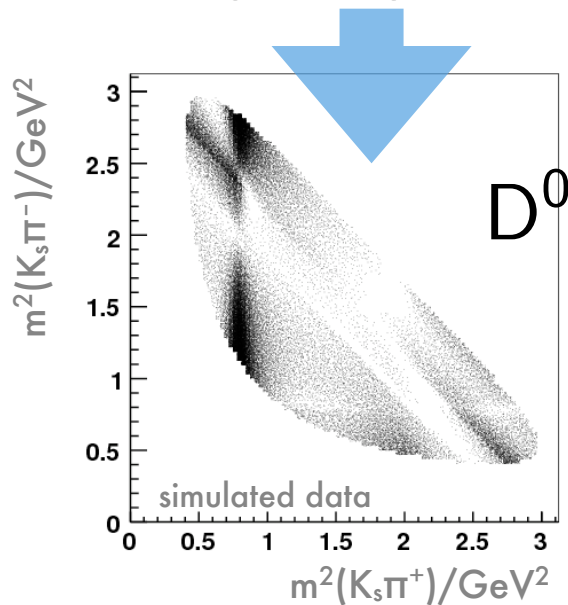
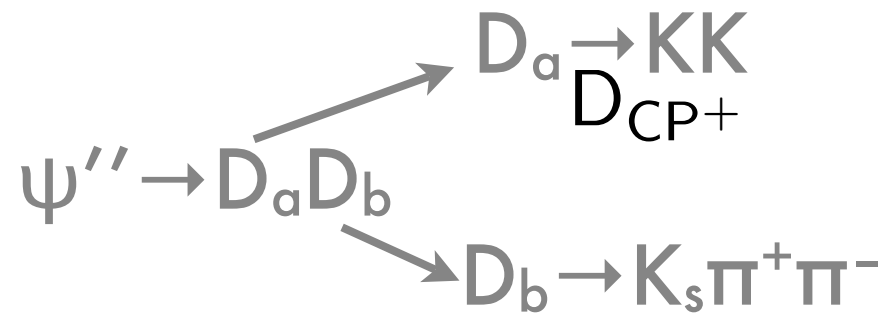
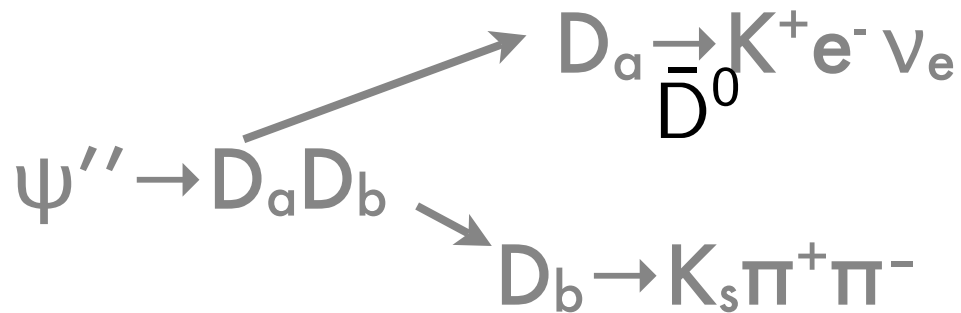
- Threshold production of correlated  $D\bar{D}$ .
- Final state must be CP-odd.
- And flavour-neutral.
- That gives us access to both amplitude and phase across the Dalitz plot.



$$\psi(3770) \rightarrow D^+(K^- \pi^+ \pi^+) D^-(K^+ \pi^- \pi^-)$$



# CP- tagged Decays



Input for a model independent (binned) analysis of



Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

Bondar, Poluektov hep-ph/0703267 (2007)

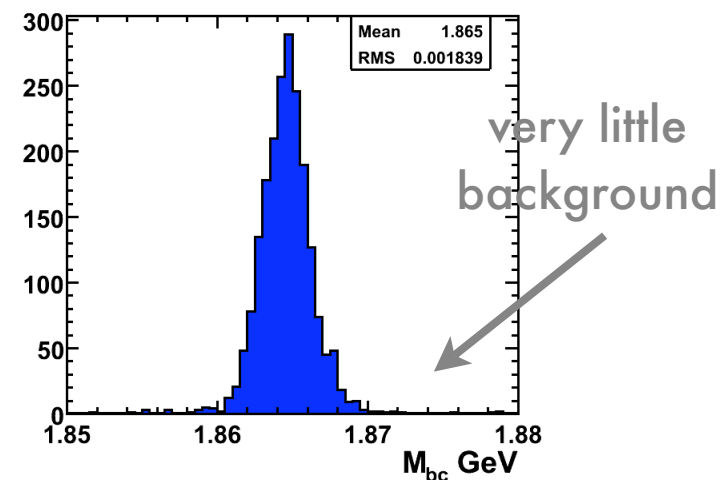
# CLEO-c's impact on $\gamma$ from $B^\pm \rightarrow D(K_s \pi \pi) K^\pm$

- Analysis not finalised. Based on 398 / pb, expect for full 818 / pb sample, using  $D_a \rightarrow CP^\pm$ ,  $D_b \rightarrow K_s \pi \pi$  and  $D_a \rightarrow K_s \pi \pi$ ,  $D_b \rightarrow K_s \pi \pi$ :

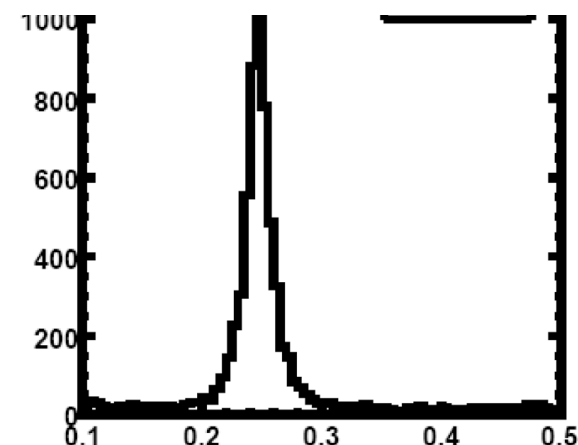
$\sigma(\gamma)$  from CLEO-c's input  $\sim 5^\circ$   
(replaces model error,  $5^\circ$ - $9^\circ$ ).

- Using  $D \rightarrow K_L \pi \pi$  could double statistics - currently investigating systematics.

Reconstructed  $D \rightarrow K_s \pi \pi$



missing mass<sup>2</sup> in  
"reconstructed"  $D \rightarrow K_L \pi \pi$

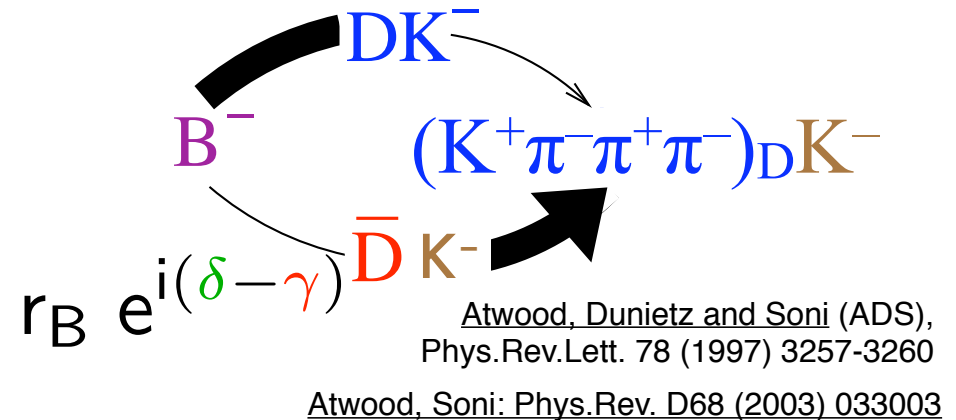
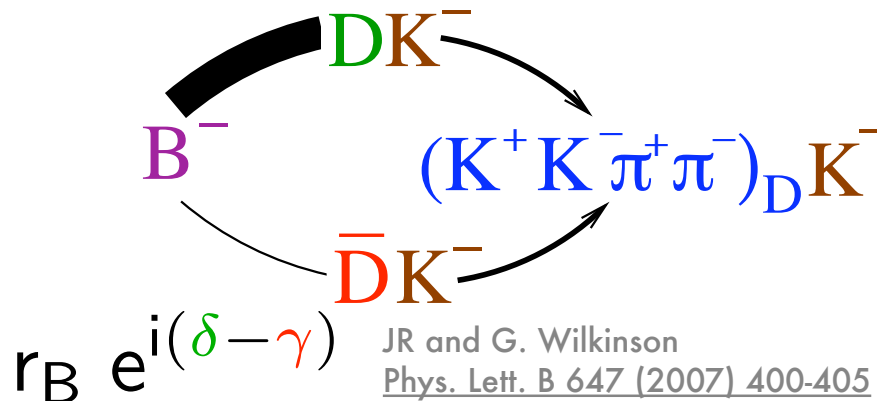


References for binned, model independent analysis of  $B^\pm \rightarrow D(K_s \pi \pi) K^\pm$ :

Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

Bondar, Poluektov hep-ph/0703267 (2007)

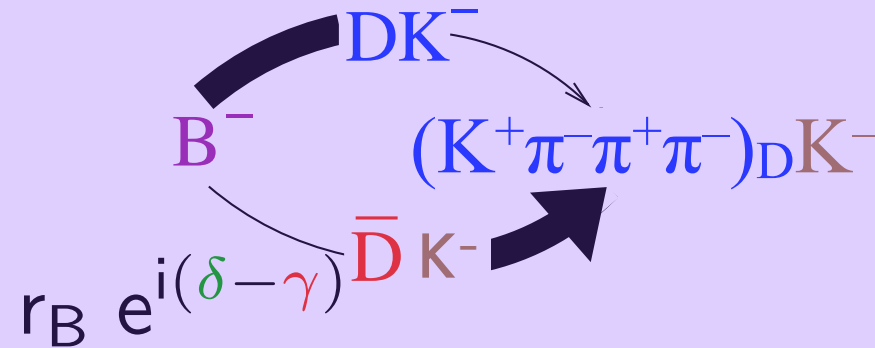
# $\gamma$ from 4-body Modes



- Same principle as for 3-body analyses.
- Four-body “Dalitz” (amplitude) analyses can be done, see for example recent FOCUS results.
- Expect similar performance from  $KK\pi\pi$  amplitude analysis as for  $K_S\pi\pi$  (see [Phys. Lett. B 647 \(2007\) 400-405](#) and public note LHCb-2007-098)
- Expect more from from  $K\pi\pi\pi$  due to “ADS effect”.

# Quasi-two body treatment of:

Atwood, Soni: Phys.Rev. D68 (2003) 033003



- Treat  $K3\pi$  like two-body decay with and single effective strong phase  $\delta_D$ .
- New parameter: Coherence factor  $R < 1$ .

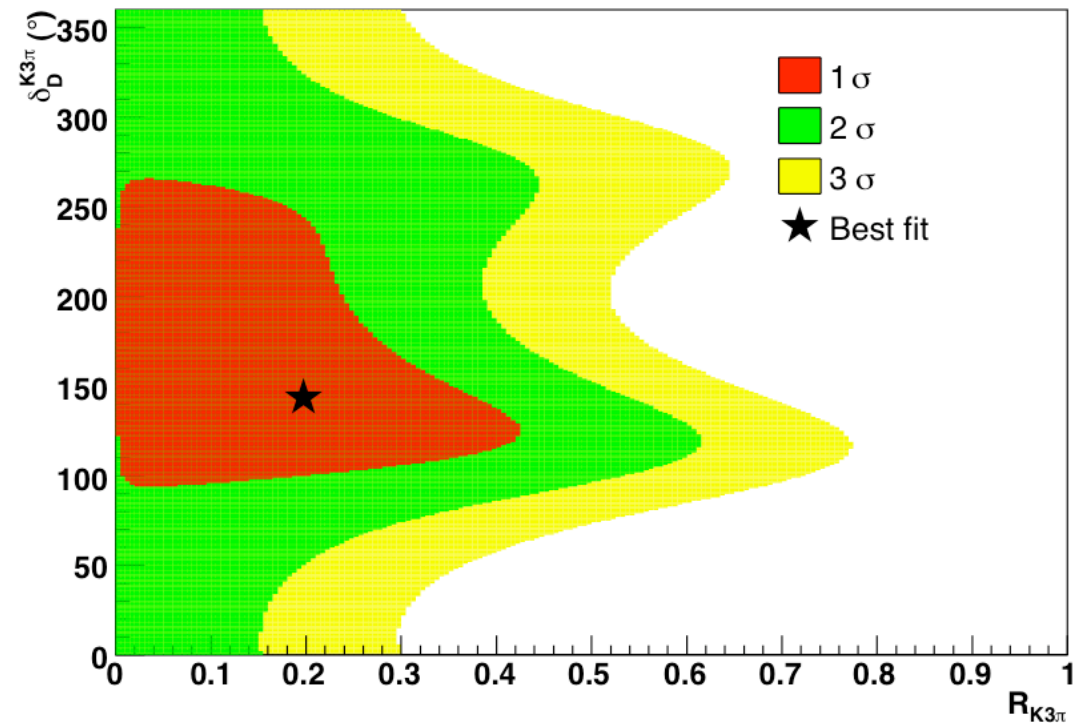
$$\Gamma(B^- \rightarrow (K^+ 3\pi)_D K^-) \propto r_B^2 + (r_D^{K3\pi})^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos(\delta_B + \delta_D^{K3\pi} - \gamma)$$

- CLEO-c's coherent  $\psi(3770) \rightarrow D_1 D_2$  events allow measurement of  $R, \delta_D$  - important input for LHCb

Double Tag Rate	Sensitive To
$K^\pm \pi^\mp \pi^+ \pi^-$ vs. $K^\pm \pi^\mp \pi^+ \pi^-$	$(R_{K3\pi})^2$
$K^\pm \pi^\mp \pi^+ \pi^-$ vs. $CP$	$R_{K3\pi} \cos(\delta^{K3\pi})$
$K^\pm \pi^\mp \pi^+ \pi^-$ vs. $K^\pm \pi^\mp$	$R_{K3\pi} \cos(\delta^{K\pi} - \delta^{K3\pi})$

# $K\pi\pi\pi$ coherence factor

- First measurement of coherence factor.
- Low value preferred. Implies that quasi-2 body analysis of  $K\pi\pi\pi$  **on its own would** not be terribly sensitive to  $\gamma$ .
- However, for **combined analysis** of  $KK$ ,  $K\pi$ ,  $\pi\pi$ ,  $K\pi\pi\pi$  this still provides powerful constraints.

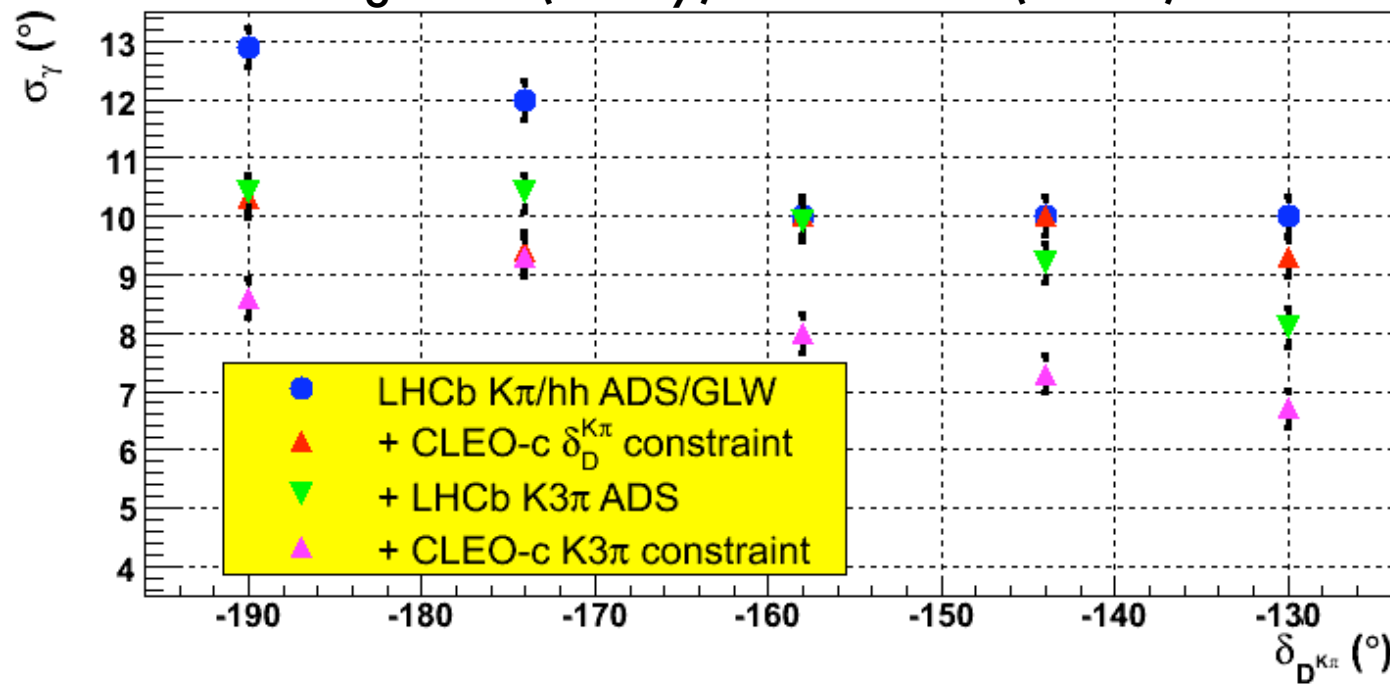


Andrew Powell on behalf of CLEO-c at the Lake Louise Winter Institute 2008. Proceedings: arXiv:0805.1722

# $K\pi\pi\pi$ coherence factor

- Significantly increases the precision of global  $\gamma$  fits in combined fit with other modes ( $KK$ ,  $\pi\pi$ ,  $K\pi$ ).

Precision on  $\gamma$  at LHCb for 2/fb (1 average year of data)  
using  $B^\pm \rightarrow D(2\text{body})K^\pm$  and  $B^\pm \rightarrow D(K\pi\pi\pi)K^\pm$



Jim Libby on behalf of CLEO-c at the  
International Workshop on e+e-  
collisions from Phi to Psi (PHIPS108).

- Other ADS-type modes such as  $K\pi\pi^0$  under study.

# Summary

- S-wave resonances not a solved problem. Progress from new high-statistics results.
- Charm mixing and especially CP violation provide unique windows to New Physics. Dalitz analyses give access to strong phases that would otherwise dilute measurements.
- $\psi(3770)$  data from CLEO-c, and in future hopefully BES III, provide essential input for  $\gamma$  measurements. Especially important for high-precision, high-statistics results at LHCb (start 2008) and future Super-Flavour Factory.

$K^* \rho a_1 f(980) \dots$   
 $\sigma(?) \kappa(?) \dots$



D

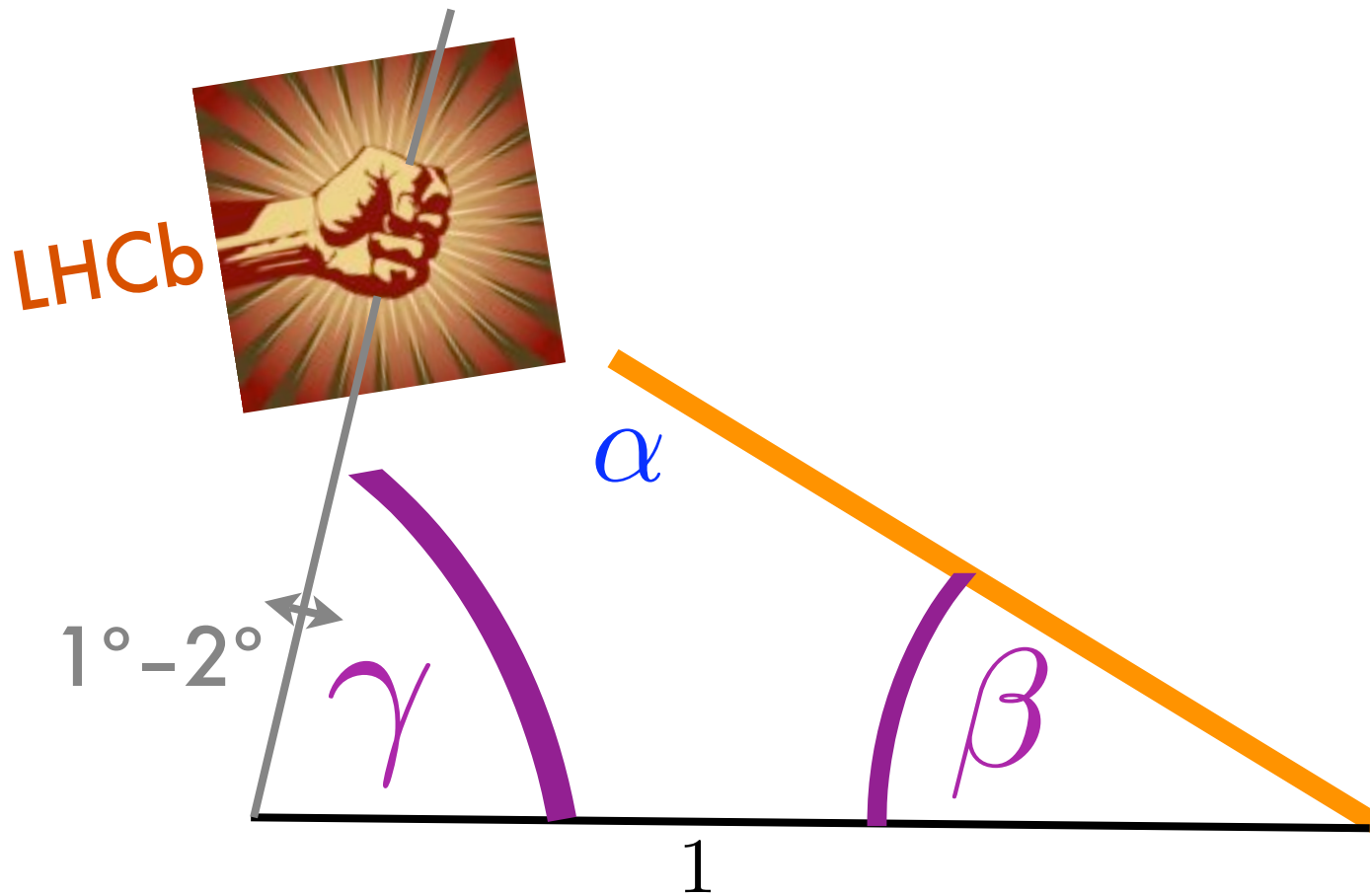


B

# Back up



# SM- $\gamma$ for New Physics



# Focus T-matrix poles in $D^+ \rightarrow 3\pi$

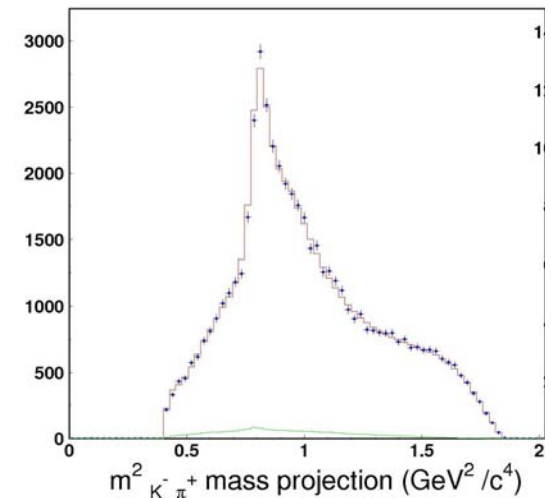
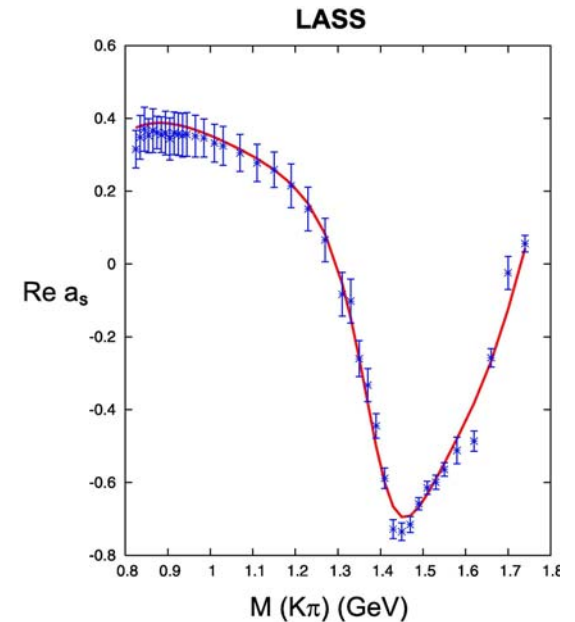
<i>T-matrix</i> pole	$(m, \Gamma/2)$ (GeV)	$D^+$ (relative) coupling constant
$f_0(980)$	(1.019, 0.038)	$1 e^{i0}$ (fixed)
$f_0(1300)$	(1.306, 0.170)	$(0.67 \pm 0.03) e^{i(-67.9 \pm 3.0)}$
$f_0(1200 - 1600)$	(1.470, 0.960)	$(1.70 \pm 0.17) e^{i(-125.5 \pm 1.7)}$
$f_0(1500)$	(1.488, 0.058)	$(0.63 \pm 0.02) e^{i(-142.2 \pm 2.2)}$
$f_0(1750)$	(1.746, 0.160)	$(0.36 \pm 0.02) e^{i(-135.0 \pm 2.9)}$

# Focus Fit Fractions in $D \rightarrow 3\pi$

$D_s^+$	
decay channel	fit fraction (%)
$(S\text{-wave}) \pi^+$	$87.04 \pm 5.60 \pm 4.17 \pm 1.34$
$f_2(1270) \pi^+$	$9.74 \pm 4.49 \pm 2.63 \pm 1.32$
$\rho^0(1450) \pi^+$	$6.56 \pm 3.43 \pm 3.31 \pm 2.90$
$D^+$	
decay channel	fit fraction (%)
$(S\text{-wave}) \pi^+$	$56.00 \pm 3.24 \pm 2.08 \pm 0.50$
$f_2(1270) \pi^+$	$11.74 \pm 1.90 \pm 0.23 \pm 0.18$
$\rho^0(770) \pi^+$	$30.82 \pm 3.14 \pm 2.29 \pm 0.17$

# $D^+ \rightarrow K^- \pi^+ \pi^+$ at FOCUS

- Focus use K-matrix approach, parameterised according to LASS elastic scattering lineshape. Separate fit to  $I=1/2$  and  $I=3/2$   $K\pi$ .
- Consistent description for  $K\pi$  elastic scattering and resonances in charm.
- S-wave fraction  $83\% \pm 1\%$ , Fit  $\chi^2$ -prob 1.2%
- Isobar fit with  $\kappa$  as x-check. Consistent with prev. experiments. Fit  $\chi^2$ -prob 6.3%



# The $\pi\pi$ S-wave in $D \rightarrow \pi\pi\pi$

- Recent results by E791 [Phys. Rev. Lett 86, 770 (2001), FOCUS (Phys. Lett. B 585, 200 (2004), CLEO-c (Phys. Rev. D 76, 012001 (2007)).
- E791 uses the isobar model, FOCUS the K-matrix\*, CLEO-c compares isobar, Schechter\*\*, and Achasov\*\*\* model.

\* E. P. Wigner, Phys. Rev. 70 (1946) 15.

\*\* Int J. Mod. Phys. A20, 6149 (2005)

S. U. Chung et al, Annalen Phys. 4 (1995) 404.

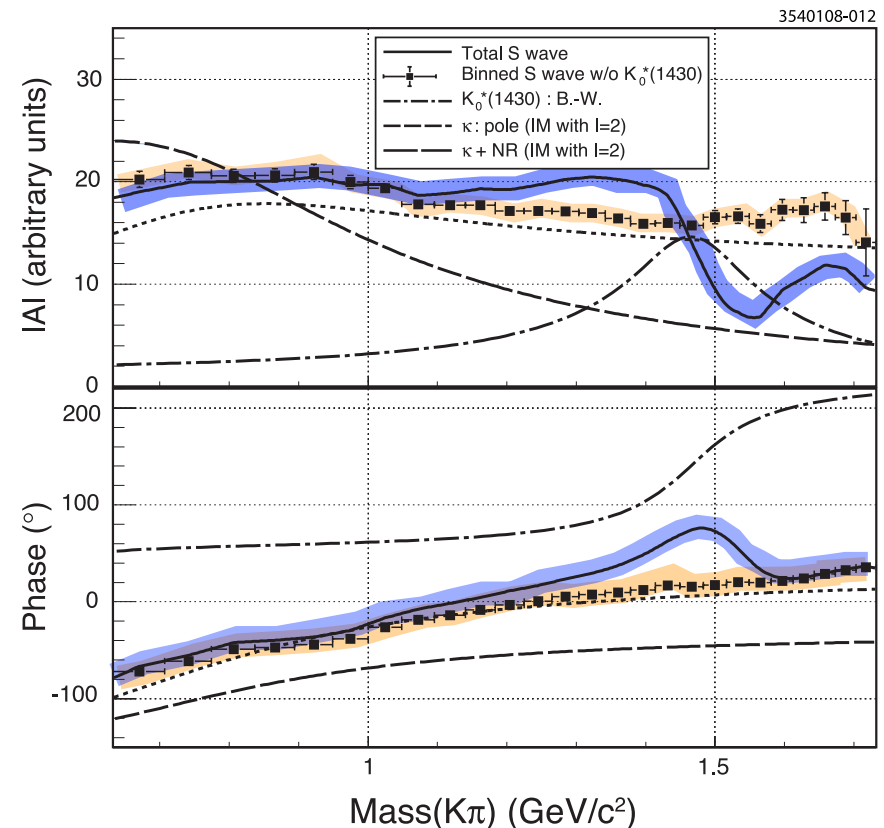
V. V. Anisovich and A. V. Sarantsev, Eur. Phys. J. A16 (2003) 229

\*\*\*private communications with the authors of the CLEO paper, and many papers - [click here](#).

# $D^+ \rightarrow K^- \pi^+ \pi^+$ Isobar and Model Independent Fit at CLEO-c

- For same parameterisation, results compatible between E791\* and CLEO-c\*\* both models.
- Crucial to achieve agreement with CLEO-c's data: Add  $I=2$   $\pi\pi$  S-wave.
- Significantly improves fit for both models. Better fit quality for binned S-wave.

— total  $K\pi$  S-wave  
—  $K\pi$  S-wave w/o  $K^*(1430)$



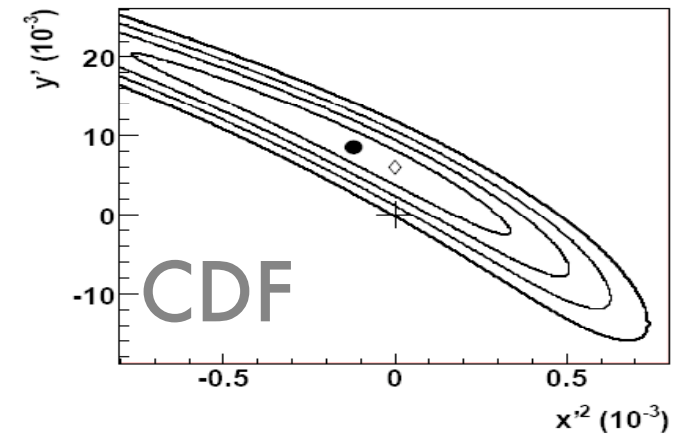
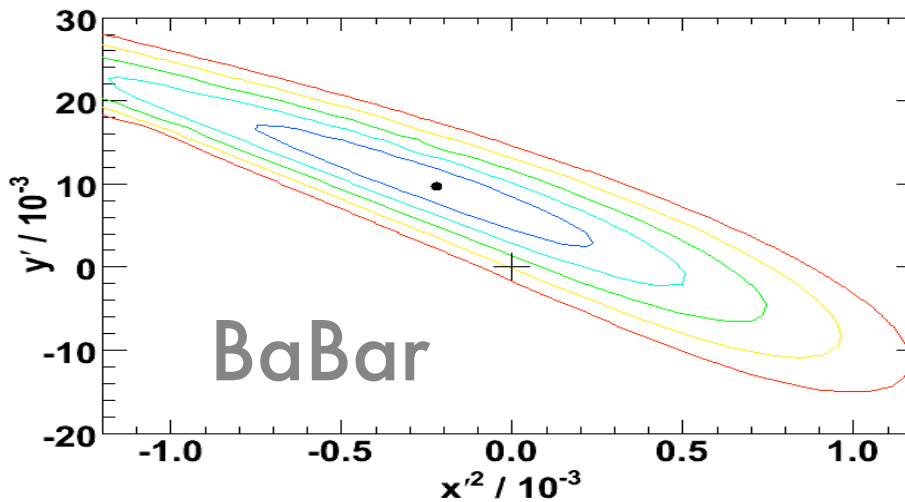
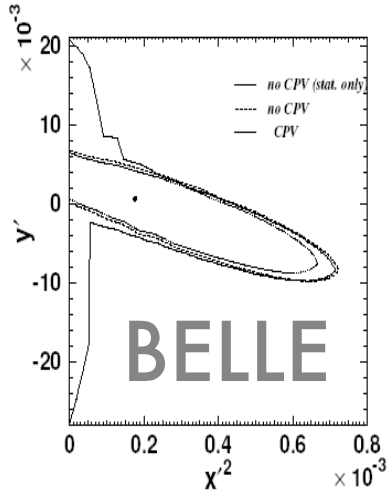
\*Phys. Rev. Lett 89, 121801 (2002); Phys. Rev. D73, 032004 (2006); \*\* [arXiv:0802.4214](https://arxiv.org/abs/0802.4214) (submitted to PRD)

# BELLE Kspipi Dalitz for mix

Resonance	Amplitude	Phase ( $^{\circ}$ )	Fit fraction
$K^*(892)^-$	$1.629 \pm 0.006$	$134.3 \pm 0.3$	0.6227
$K_0^*(1430)^-$	$2.12 \pm 0.02$	$-0.9 \pm 0.8$	0.0724
$K_2^*(1430)^-$	$0.87 \pm 0.02$	$-47.3 \pm 1.2$	0.0133
$K^*(1410)^-$	$0.65 \pm 0.03$	$111 \pm 4$	0.0048
$K^*(1680)^-$	$0.60 \pm 0.25$	$147 \pm 29$	0.0002
$K^*(892)^+$	$0.152 \pm 0.003$	$-37.5 \pm 1.3$	0.0054
$K_0^*(1430)^+$	$0.541 \pm 0.019$	$91.8 \pm 2.1$	0.0047
$K_2^*(1430)^+$	$0.276 \pm 0.013$	$-106 \pm 3$	0.0013
$K^*(1410)^+$	$0.33 \pm 0.02$	$-102 \pm 4$	0.0013
$K^*(1680)^+$	$0.73 \pm 0.16$	$103 \pm 11$	0.0004
$\rho(770)$	1 (fixed)	0 (fixed)	0.2111
$\omega(782)$	$0.0380 \pm 0.0007$	$115.1 \pm 1.1$	0.0063
$f_0(980)$	$0.380 \pm 0.004$	$-147.1 \pm 1.1$	0.0452
$f_0(1370)$	$1.46 \pm 0.05$	$98.6 \pm 1.8$	0.0162
$f_2(1270)$	$1.43 \pm 0.02$	$-13.6 \pm 1.2$	0.0180
$\rho(1450)$	$0.72 \pm 0.04$	$41 \pm 7$	0.0024
$\sigma_1$	$1.39 \pm 0.02$	$-146.6 \pm 0.9$	0.0914
$\sigma_2$	$0.267 \pm 0.013$	$-157 \pm 3$	0.0088
NR	$2.36 \pm 0.07$	$155 \pm 2$	0.0615

Fit case	Parameter	Fit result	95% C.L. interval
No	$x(\%)$	$0.80 \pm 0.29$ $^{+0.09+0.10}_{-0.07-0.14}$	(0.0, 1.6)
<i>CPV</i>	$y(\%)$	$0.33 \pm 0.24$ $^{+0.08+0.06}_{-0.12-0.08}$	(-0.34, 0.96)
<i>CPV</i>	$x(\%)$	$0.81 \pm 0.30$ $^{+0.10+0.09}_{-0.07-0.16}$	$ x  < 1.6$
	$y(\%)$	$0.37 \pm 0.25$ $^{+0.07+0.07}_{-0.13-0.08}$	$ y  < 1.04$
	$ q/p $	$0.86$ $^{+0.30+0.06}_{-0.29-0.03} \pm 0.08$	-
	$\arg(q/p)(^{\circ})$	$-14$ $^{+16+5+2}_{-18-3-4}$	-

# $D \rightarrow K\pi$ ( $x'^2, y'$ ) contours



$$x'^2 = (0.018^{+0.021}_{-0.023})\%$$

$$y' = (0.06^{+0.40}_{-0.39})\%$$

$$x'^2 = (-0.22 \pm 0.30 \pm 0.21)\%$$

$$y' = (9.7 \pm 4.4 \pm 3.1)\%$$

$$x'^2 = (-0.012 \pm 0.035)\%$$

$$y' = (0.85 \pm 0.76)\%$$

Phys.Rev.Lett.96:151801,2006

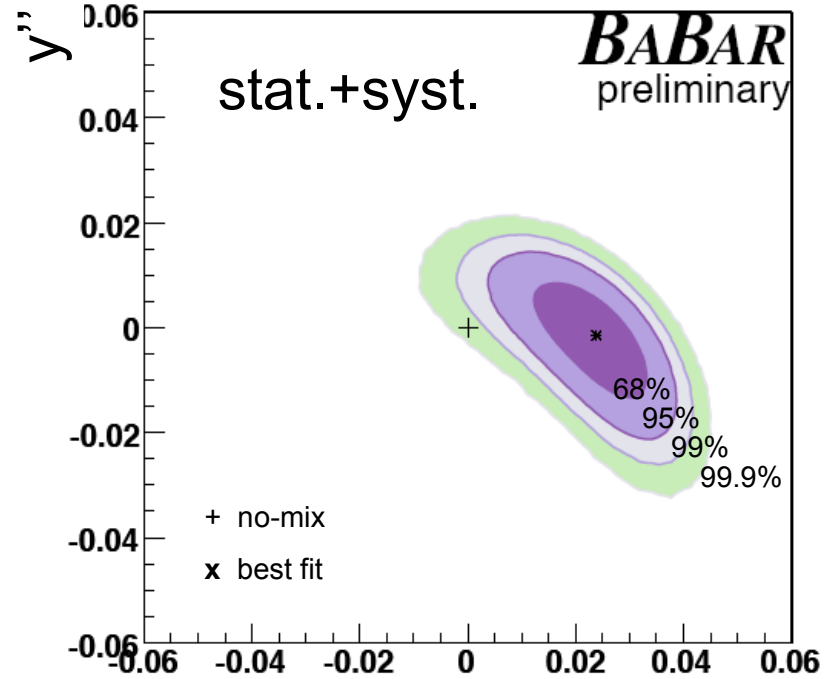
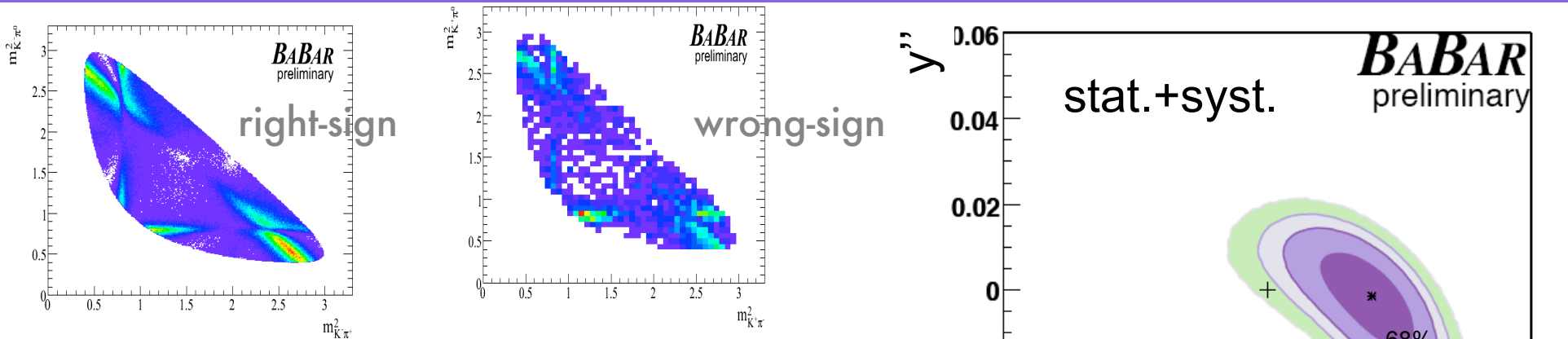
Phys.Rev.Lett.98:211802,2007

Phys.Rev.Lett.100:121802,2008

$$\Gamma(D^0(t) \rightarrow K^+\pi^-) \propto e^{-\Gamma t} \left[ R_D + \sqrt{R_D} y' (\Gamma t) + \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right]$$



# Mixing with time-dependent Dalitz analysis analysis $D/D\bar{b}ar \rightarrow K^+\pi^-\pi^0$



$$x'' = 2.39 \pm 0.61(\text{stat}) \pm 0.32(\text{syst}) \%$$

$$y'' = -0.14 \pm 0.60(\text{stat}) \pm 0.40(\text{syst}) \%$$

- Time-dependent Dalitz analysis
- Equivalent to  $K\pi$  analysis
- Measure  $(x'', y'') = \text{Rotate}(\delta_{K\pi\pi^0})$   
 $(x, y)$  where  $\delta_{K\pi\pi^0}$  = strong phase  
between CF and DCS.

William Lockman for the BaBar Collaboration  
Lepton-Photon 2007, Daegu, S. Korea

Compatible with no mixing at  $<0.8\%$  CL

# Charm mixing with time-dependent Dalitz Plots at BELLE

- HFAG 2008\* (no CPV):

$$x=(0.91\pm 0.26)\%$$

$$y=(0.73\pm 0.18)\%$$

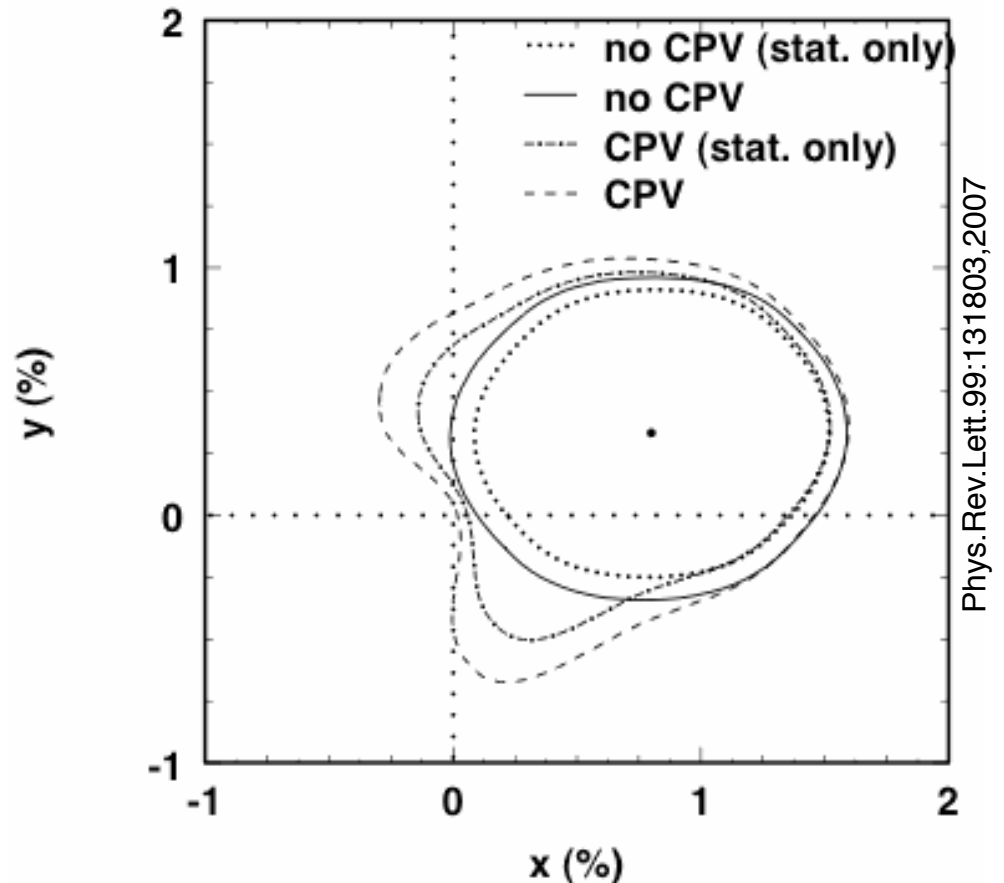
- BELLE's  $D\rightarrow K_s\pi^+\pi^-$  alone (assuming no CPV)\*

$$x=(0.80\pm 0.29\pm 0.16)\%$$

$$y=(0.33\pm 0.24\pm 0.14)\%$$

- Best result on  $x$ .

95% CL, BELLE,  $D\rightarrow K_s\pi\pi$

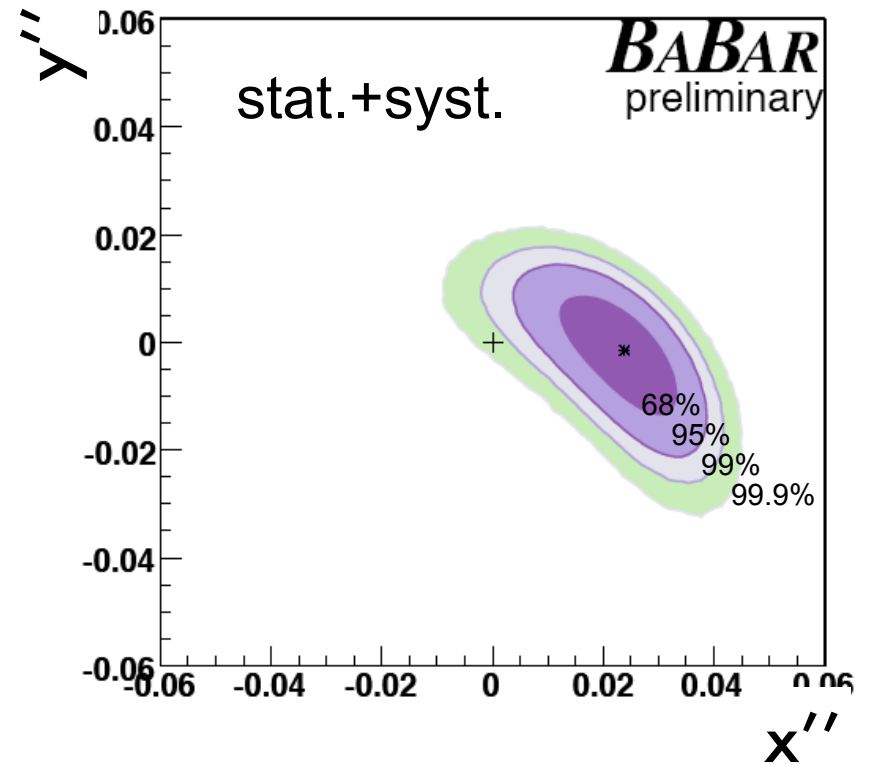


\*\*Phys.Rev.Lett.99:131803,2007

\*[http://www.slac.stanford.edu/xorg/hfag/charm/FPCP08/results\\_mix+cpv.html](http://www.slac.stanford.edu/xorg/hfag/charm/FPCP08/results_mix+cpv.html)

# Mixing with $D/D\bar{b}ar \rightarrow K^+\pi^-\pi^0$

- Requires Dalitz fit since each point in Dalitz space has different  $\delta(m_{12}, m_{23})$ .
- Can fit for relative phases within Dalitz plot, but an overall phase difference remains between the plots, hence measure  $(x'', y'') = \text{Rotate}(\delta_{K\pi\pi^0})(x, y)$



$$x'' = 2.39 \pm 0.61(\text{stat}) \pm 0.32(\text{sys}) \%$$

$$y'' = -0.14 \pm 0.60(\text{stat}) \pm 0.40(\text{sys}) \%$$

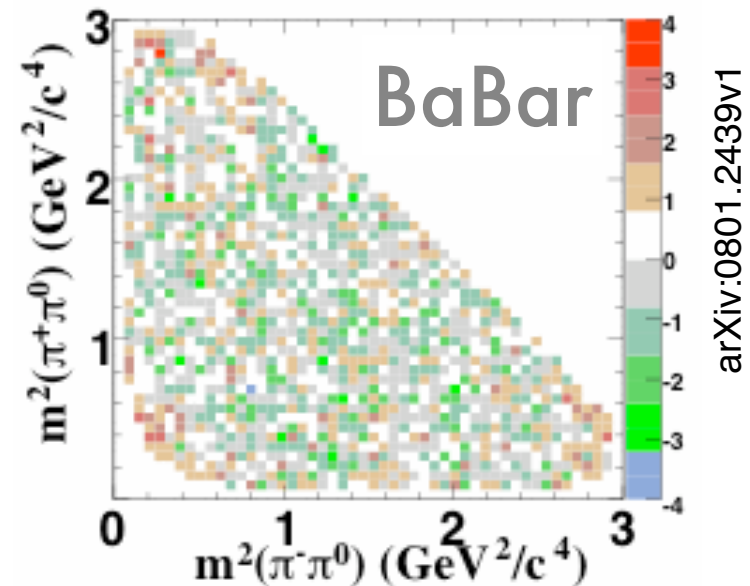
# CPV in $D \rightarrow \pi^+ \pi^- \pi^0$ , $D \rightarrow K^+ K^- \pi^0$ ?

- Four approaches (BaBar): Look for differences between  $D$ ,  $\bar{D}$ ...
  - across Dalitz space (plot  $\rightarrow$ )
  - in fit to Dalitz model
  - angular distributions
  - total decay rates (also BELLE)
- No evidence for CPV. Limits CPV in SCS charm to  $< \text{few } \%$  level.

BaBar result: arXiv:0801.2439, submitted to PRL

BELLE result: arXiv:0801.2439, submitted to PRD

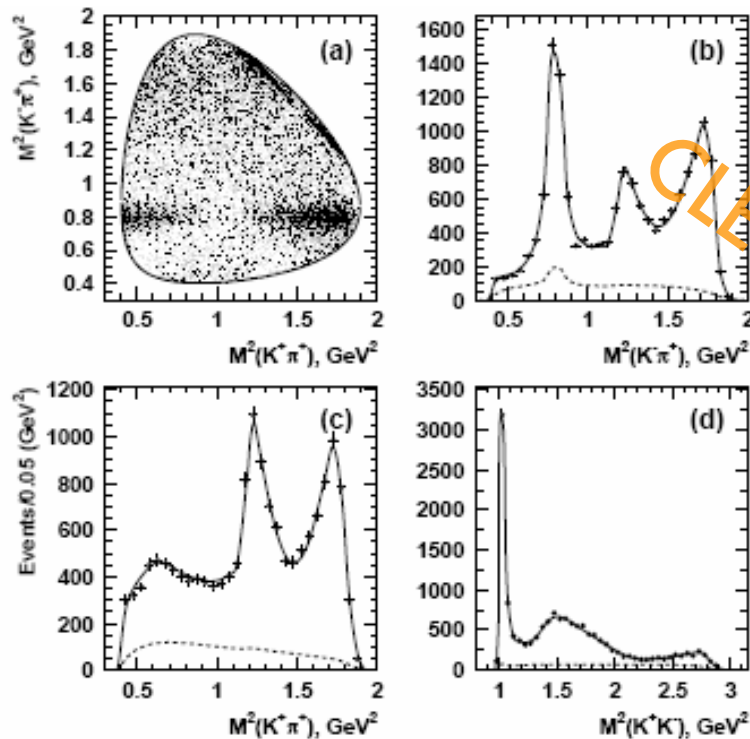
Normalised residuals  
(Difference/error) between  
 $D \rightarrow \pi^+ \pi^- \pi^0$  and its CP conjugate



$\chi^2$  test: consistent with no  
CPV at 33% CL  
(17% for  $KK\pi^0$ )

# CPV in SCS decay $D^+ \rightarrow K^+ K^- \pi^+$ ?

## The Fit



$K\pi$  S-wave described by  
LASS\* model for  $K\pi \rightarrow K\pi$   
elastic scattering

\*Nucl. Phys. B **296**, 493 (1899)

## The CP asymmetry

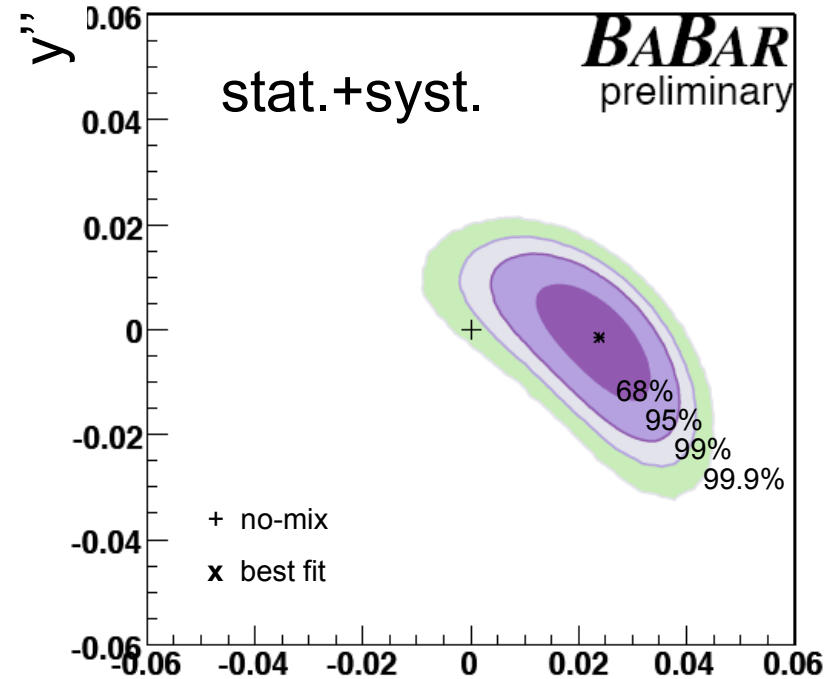
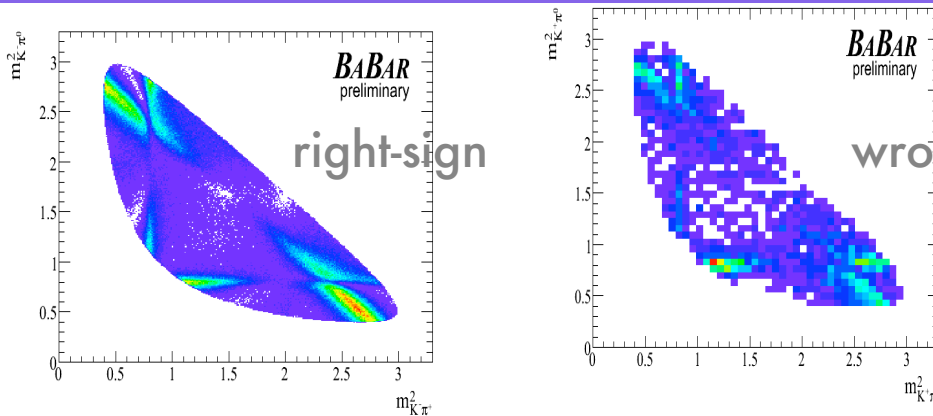
$$A_{CP} = \frac{N_{D^+}/\epsilon_{D^+} - N_{D^-}/\epsilon_{D^-}}{N_{D^+}/\epsilon_{D^+} + N_{D^-}/\epsilon_{D^-}}$$

Component	$A_{CP}$ (fraction) (%)
$K^*(892)^0 K^+$	$0.2 \pm 2.7^{+2.3+0.7}_{-0.4-0.4}$
$K^-\pi^+(S)K^+$	$-1 \pm 5^{+1+6}_{-2-4}$
$\rho_0(980)\pi^+$	$-18 \pm 23^{+4+24}_{-9-6}$
$\phi(1020)\pi^+$	$-3.7 \pm 1.9^{+0.1+0.2}_{-0.2-0.3}$
$f_2(1270)\pi^+$	$5 \pm 26^{+3+22}_{-4-46}$
$a_0(1450)\pi^+$	$-20 \pm 13^{+0+16}_{-8-9}$
$\phi(1680)\pi^+$	$-6 \pm 21^{+22+7}_{-4-3}$
$\bar{K}_2^*(1430)^0 K^+$	$59 \pm 41^{+1+8}_{-28-41}$

No evidence for CPV

precision in some channels: few %

# Mixing with time-dependent Dalitz analysis analysis $D/D\bar{b}ar \rightarrow K^+\pi^-\pi^0$

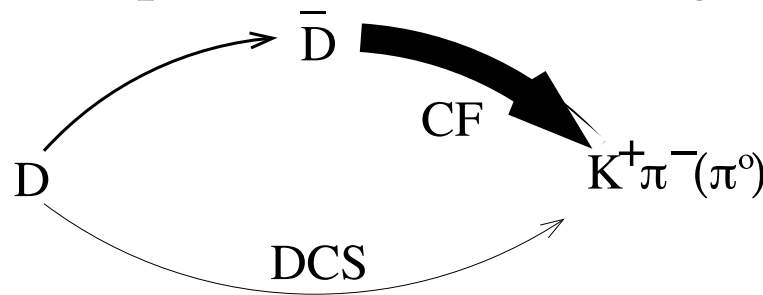


$$x'' = 2.39 \pm 0.61(\text{stat}) \pm 0.32(\text{syst}) \%$$

$$y'' = -0.14 \pm 0.60(\text{stat}) \pm 0.40(\text{syst}) \%$$

Compatible with no mixing at  $<0.8\%$  CL

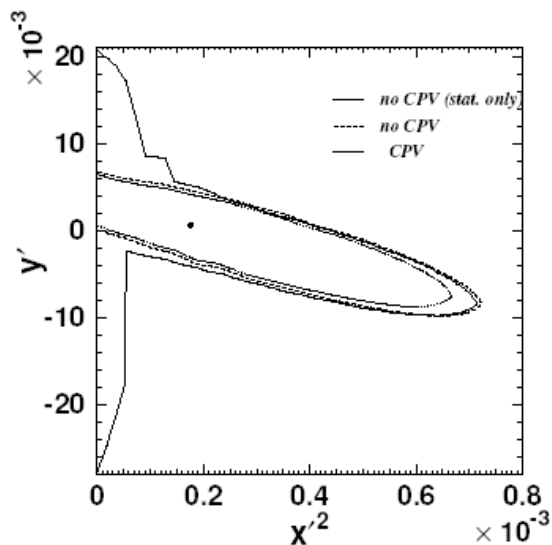
- Time-dependent Dalitz analysis



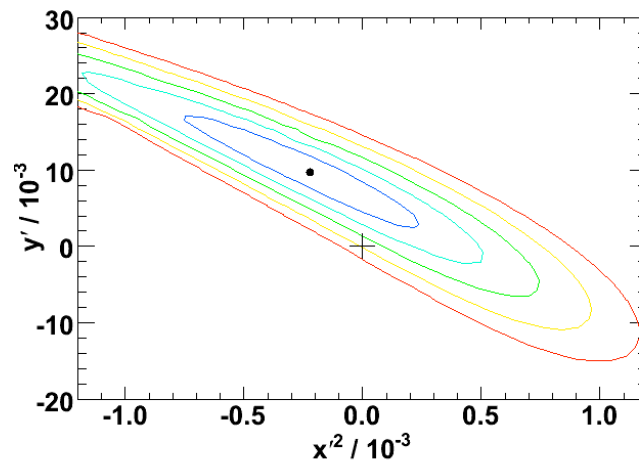
- Measure  $(x'', y'') = \text{Rotate}(\delta_{K\pi\pi^0})$   
 $(x, y)$  where  $\delta_{K\pi\pi^0}$  = strong phase  
between CF and DCS.

# $D \rightarrow K\pi$ $y'$ , $x'^2$ contours

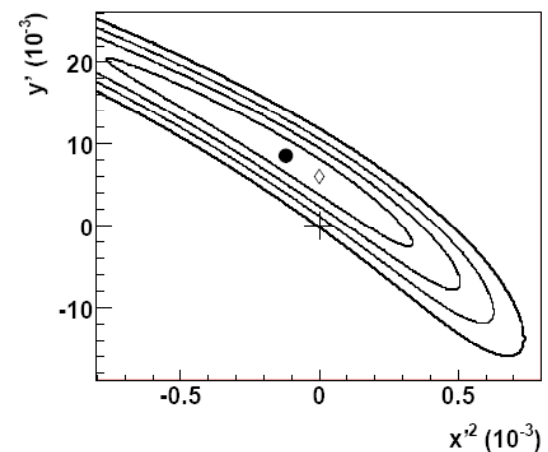
BELLE



BaBar



CDF



$$\Gamma(D^0(t) \rightarrow K^+ \pi^-) \propto e^{-\Gamma t} \left[ R_D + \sqrt{R_D} y' (\Gamma t) + \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right]$$

# CPV in $D \rightarrow \pi^+ \pi^- \pi^0$ , $D \rightarrow K^+ K^- \pi^0$

- Dalitz fit allows to differentiate the contributions of different resonances, CP and flavour eigenstates, etc
- Tables (unreadable, but included for completeness) differences between  $D$  and  $\bar{D}$  amplitudes at BaBar. Results:  $\sim$ percent, none significant.
- So, no evidence for CPV.

State	$f_r$ (%)	$\Delta a_r$ (%)	$\Delta \phi_r$ ( $^\circ$ )	$\Delta f_r$ (%)
$K^*(892)^+$	45	$2 \pm 3 \pm 2$	$10 \pm 12 \pm 3$	$0.8 \pm 1.1 \pm 0.4$
$K^*(1410)^+$	4	$101 \pm 65 \pm 37$	$1 \pm 21 \pm 6$	$1.7 \pm 1.8 \pm 0.6$
$K^+ \pi^0(S)$	16	$-130 \pm 64 \pm 51$	$-9 \pm 10 \pm 6$	$-2.3 \pm 4.7 \pm 1.0$
$\phi(1020)$	19	$-1 \pm 2 \pm 1$	$-10 \pm 20 \pm 5$	$-0.4 \pm 0.8 \pm 0.2$
$f_0(980)$	7	$14 \pm 16 \pm 6$	$-12 \pm 25 \pm 8$	$0.4 \pm 2.6 \pm 0.2$
$[a_0(980)^0]$	[6]	$[19 \pm 16 \pm 6]$	$[-7 \pm 16 \pm 8]$	$[0.6 \pm 1.9 \pm 0.2]$
$f_2'(1525)$	0.1	$-38 \pm 74 \pm 8$	$6 \pm 36 \pm 12$	$0.0 \pm 0.1 \pm 0.3$
$K^*(892)^-$	16	$1 \pm 3 \pm 1$	$-7 \pm 4 \pm 2$	$1.7 \pm 1.3 \pm 0.4$
$K^*(1410)^-$	5	$133 \pm 93 \pm 68$	$-23 \pm 13 \pm 9$	$1.7 \pm 2.8 \pm 0.7$
$K^- \pi^0(S)$	3	$8 \pm 68 \pm 36$	$32 \pm 39 \pm 14$	$0.4 \pm 2.4 \pm 0.5$

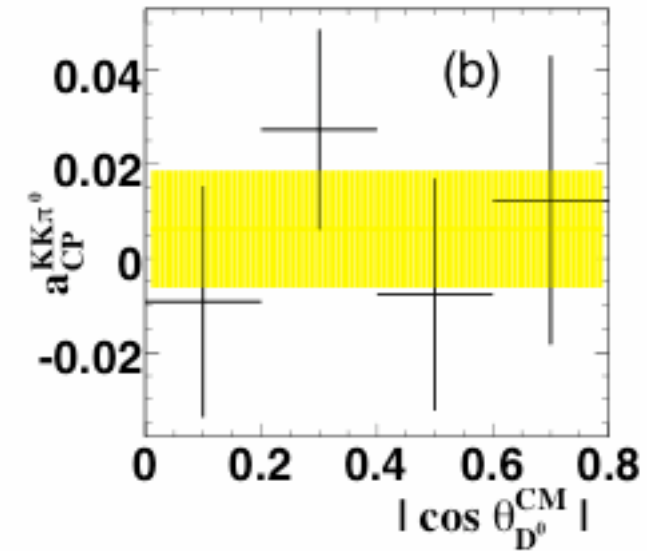
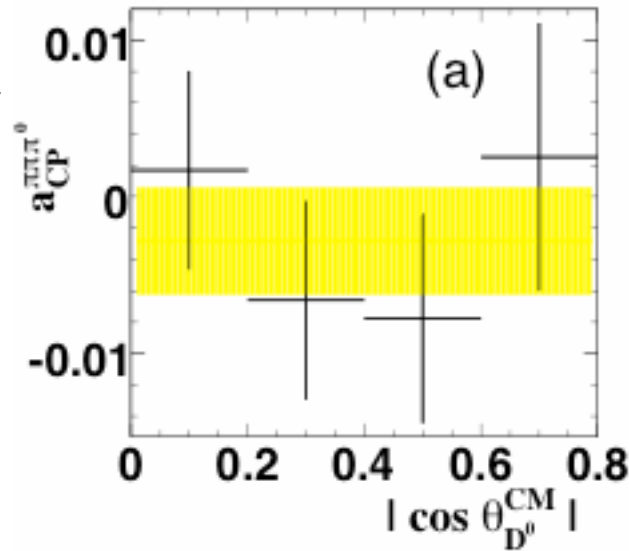
State	$f_r$ (%)	$\Delta a_r$ (%)	$\Delta \phi_r$ ( $^\circ$ )	$\Delta f_r$ (%)
$\rho^+(770)$	68	$-3.2 \pm 1.7 \pm 0.8$	$-0.8 \pm 1.0 \pm 1.0$	$-1.6 \pm 1.1 \pm 0.4$
$\rho^0(770)$	26	$2.1 \pm 0.9 \pm 0.5$	$0.8 \pm 1.0 \pm 0.4$	$1.6 \pm 1.4 \pm 0.6$
$\rho^-(770)$	35	$2.0 \pm 1.1 \pm 0.8$	$-0.6 \pm 0.9 \pm 0.4$	$0.7 \pm 1.1 \pm 0.5$
$\rho^+(1450)$	0.1	$2 \pm 11 \pm 8$	$-30 \pm 25 \pm 9$	$0.0 \pm 0.1 \pm 0.1$
$\rho^0(1450)$	0.3	$13 \pm 8 \pm 6$	$-1 \pm 14 \pm 3$	$0.1 \pm 0.2 \pm 0.1$
$\rho^-(1450)$	1.8	$-3 \pm 6 \pm 5$	$8 \pm 7 \pm 3$	$-0.2 \pm 0.3 \pm 0.1$
$\rho^+(1700)$	4	$19 \pm 27 \pm 9$	$9 \pm 7 \pm 3$	$0.4 \pm 1.0 \pm 0.4$
$\rho^0(1700)$	5	$-31 \pm 20 \pm 12$	$-7 \pm 6 \pm 2$	$-1.3 \pm 0.8 \pm 0.3$
$\rho^-(1700)$	3	$-3 \pm 14 \pm 11$	$-3 \pm 8 \pm 3$	$-0.5 \pm 0.6 \pm 0.3$
$f_0(980)$	0.2	$0.0 \pm 0.1 \pm 0.2$	$-3 \pm 7 \pm 4$	$0.0 \pm 0.1 \pm 0.1$
$f_0(1370)$	0.4	$-0.3 \pm 1.3 \pm 1.2$	$7 \pm 14 \pm 5$	$-0.2 \pm 0.1 \pm 0.1$
$f_0(1500)$	0.4	$0.4 \pm 1.1 \pm 0.7$	$-1 \pm 12 \pm 1$	$0.0 \pm 0.1 \pm 0.1$
$f_0(1710)$	0.3	$-3 \pm 3 \pm 2$	$-25 \pm 13 \pm 11$	$0.0 \pm 0.1 \pm 0.1$
$f_2(1270)$	1.3	$8 \pm 4 \pm 5$	$2 \pm 5 \pm 2$	$0.1 \pm 0.1 \pm 0.1$
$\sigma(400)$	0.8	$-0.3 \pm 0.7 \pm 2.0$	$-4 \pm 7 \pm 3$	$-0.1 \pm 0.1 \pm 0.1$
Nonres	0.8	$12 \pm 7 \pm 8$	$11 \pm 9 \pm 4$	$0.2 \pm 0.3 \pm 0.2$

Results from BaBar: arXiv:0801.2439, submitted to PRL



# CPV in $D \rightarrow KK\pi$ , $D \rightarrow 3\pi$

BaBar's  $A_{CP}$  result by angular distribution  $\rightarrow$



$A_{CP}$  from integrated decay rates:

- $A_{CP}(3\pi)$  BaBar =  $(-0.31 \pm 0.41 \text{ (stat)} \pm 0.17 \text{ (syst)})\%$

$$A_{CP}(3\pi) \text{ BELLE} = (-0.43 \pm 1.30 \text{ (stat+syst)})\% \\ (0.43 \pm 0.41 \text{ (stat)} \pm 1.01 \text{ (track)} \pm 0.70 \text{ (other syst)})\%$$

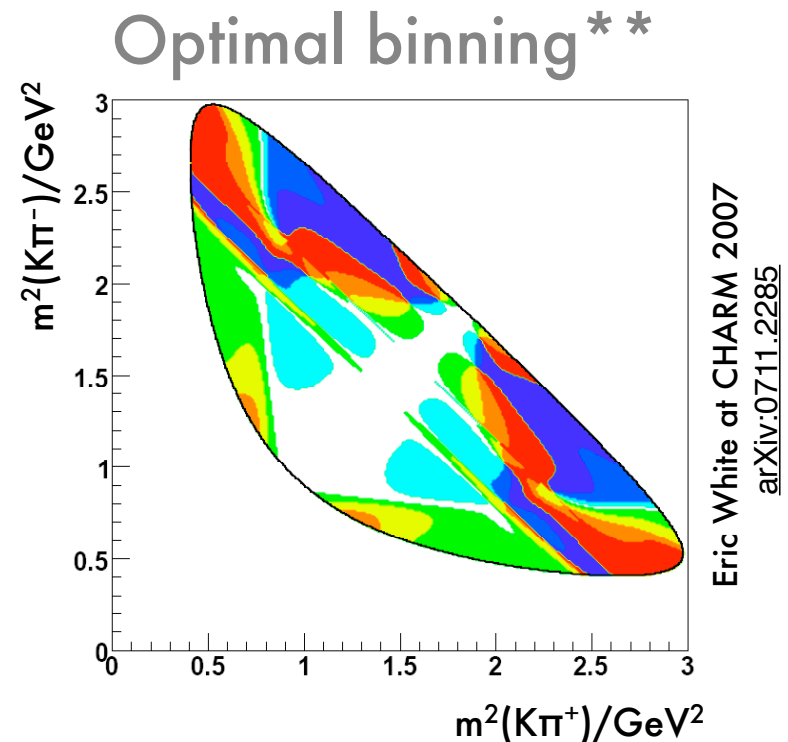
- $A_{CP}(KK\pi)$  BaBar =  $(1.00 \pm 1.67 \text{ (stat)} \pm 0.25 \text{ (syst)})\%$

BaBar result: arXiv:0801.2439, submitted to PRL

BELLE result: arXiv:0801.2439, submitted to PRD

# Model-independent Fit

- Binned analysis allows model independent fit\* for  $\gamma$  at B-factories, LHCb.
- CLEO-c's quantum-correlated data provide crucial input \*\*:
  - CP-tagged Dalitz plots  
 $D_a \rightarrow CP_{\pm}, D_b \rightarrow K_S \pi \pi$
  - Simultaneous Dalitz analysis  
of  $D_a \rightarrow K_S \pi \pi, D_b \rightarrow K_S \pi \pi$



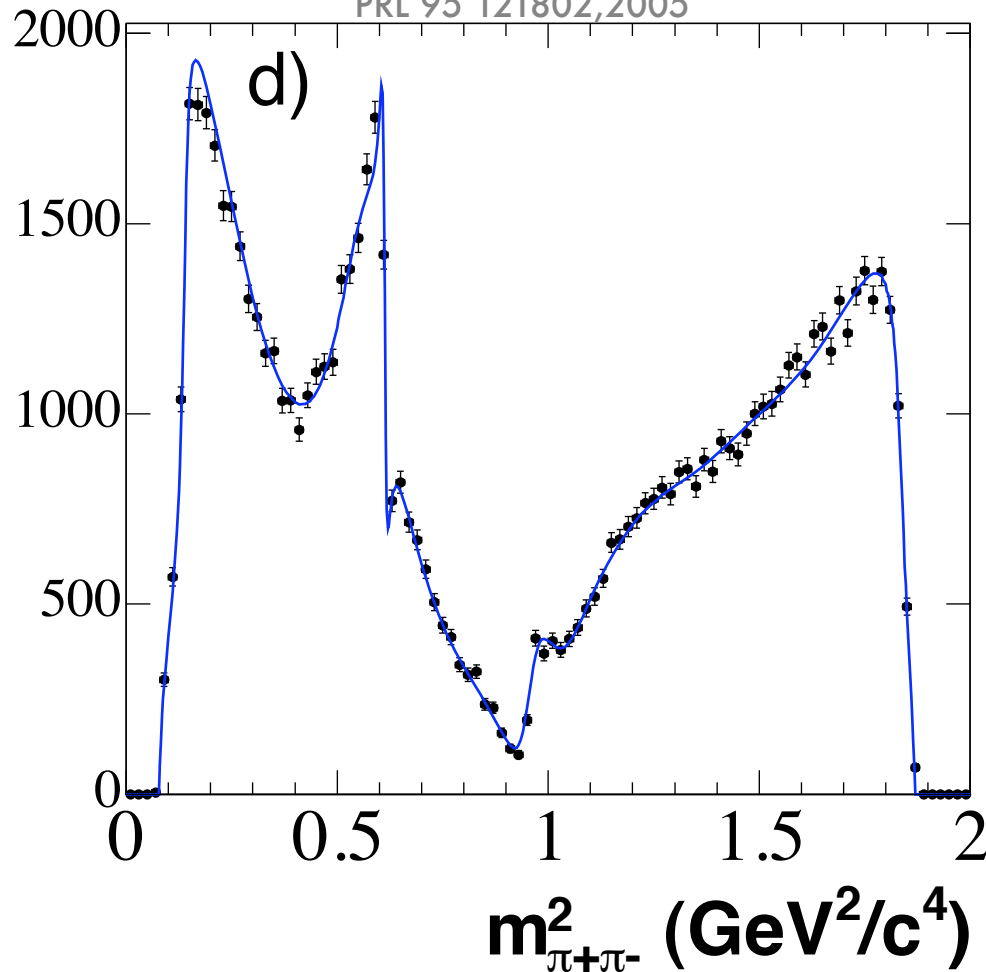
\*Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

\*\*Bondar, Poluektov hep-ph/0703267v1 (2007)

# Comparing the $\pi\pi$ projections

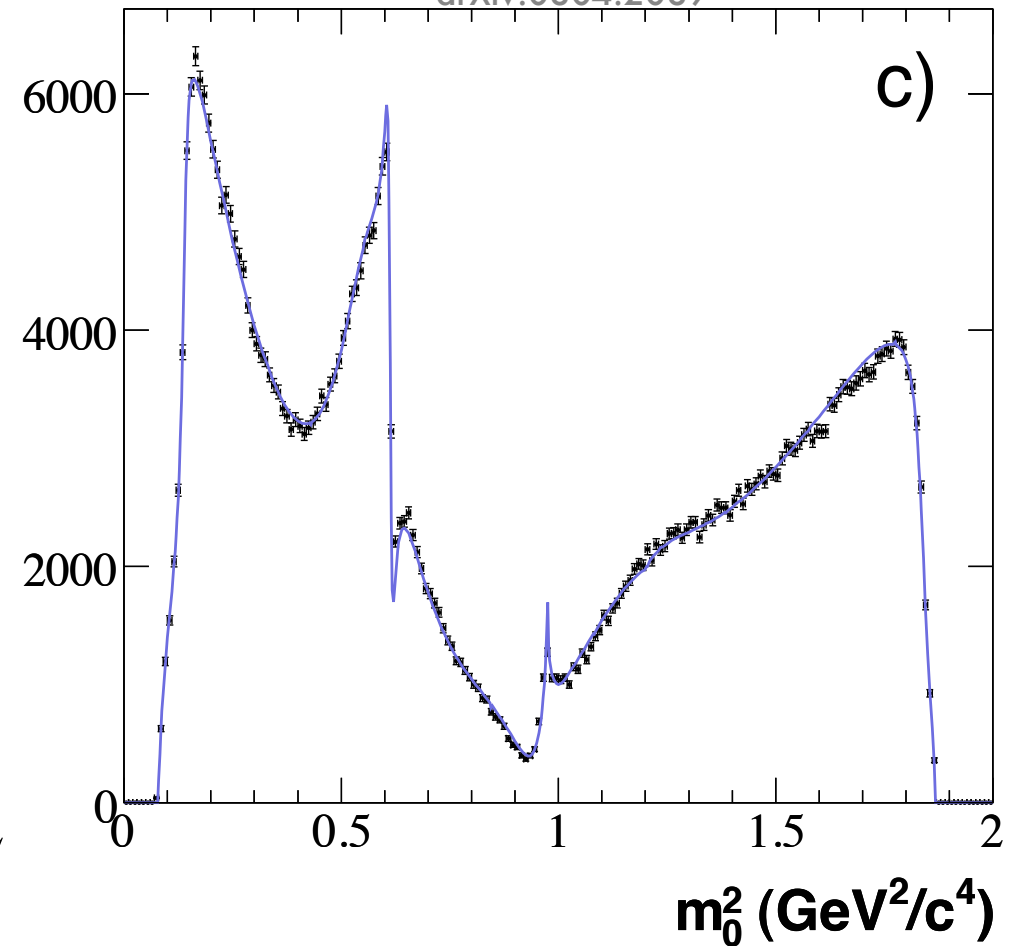
227M BB, isobar model

PRL 95 121802,2005

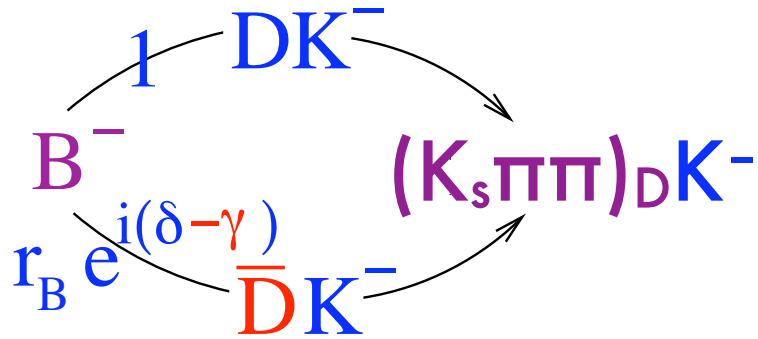


383M BB, K-matrix model

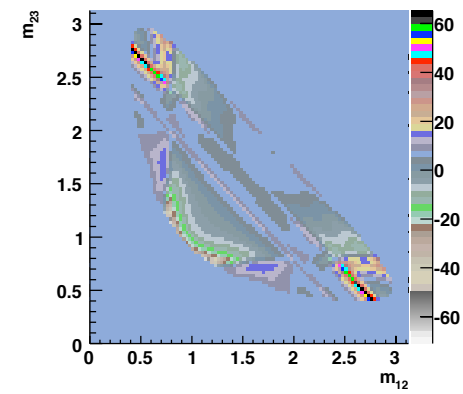
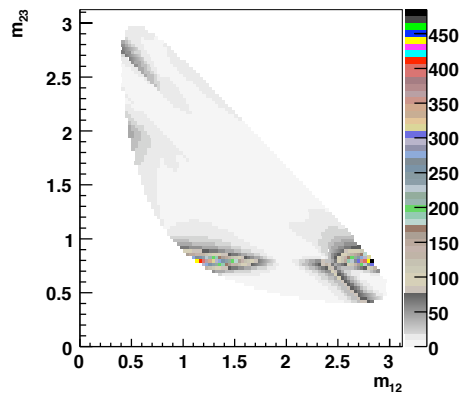
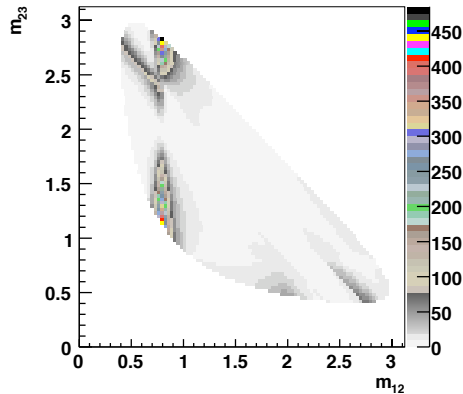
arXiv:0804.2089



# D → K<sub>s</sub>ππ Dalitz plot



$$|A_B|^2 = |A_D|^2 + r_B^2 |A_D|^2 + 2r_B \text{Re}(A_D \bar{A}_D^* e^{i(\delta-\gamma)})$$



Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018  
 Belle Collaboration Phys.Rev. D70 (2004) 072003

$$\sigma(\gamma) \propto 1/r_B$$

# Same trick works with $B \rightarrow DK^*$

- $K^*$  tags flavour of B at decay.
- Same  $K_S\pi\pi$  analysis, but different  $r$ ,  $\delta$ .
- BaBar analysis (May 2008): [arXiv:0805.2001](https://arxiv.org/abs/0805.2001)

$$\begin{aligned}\gamma &= (162 \pm 56)^\circ \text{ or } (342 \pm 56)^\circ; \\ \delta_S &= (62 \pm 57)^\circ \text{ or } (242 \pm 57)^\circ; \\ r_S &< 0.55 \text{ at } 95\% \text{ probability.}\end{aligned}$$

# Model independent $\gamma$ fit

Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

- Binned decay rate:

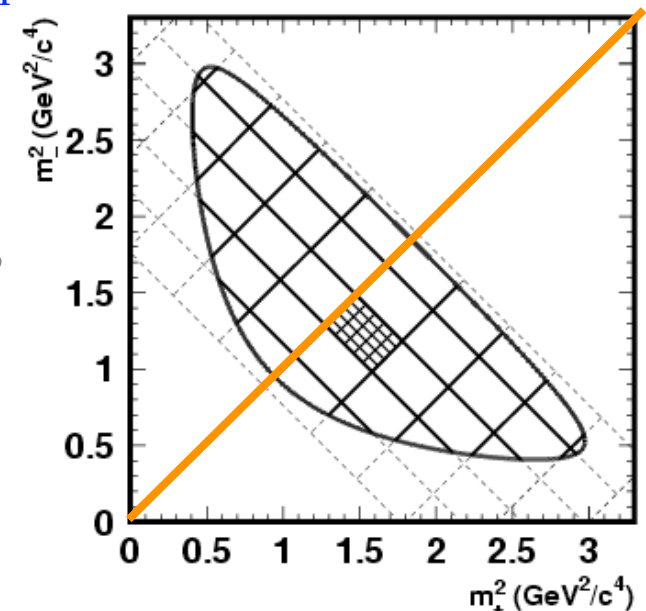
$$\Gamma(B^\pm \rightarrow D(K_s \pi^+ \pi^-)K^\pm)_i =$$

$$\mathcal{T}_i + r_B^2 \mathcal{T}_{-i} + 2r_B \sqrt{\mathcal{T}_i \mathcal{T}_{-i}} \{c_i \cos(\delta - \gamma) + s_i \sin(\delta - \gamma)\}$$

$\cos(\delta_D(s_{12}, s_{34}))$  and  $\sin(\delta_D(s_{12}, s_{34}))$  averaged over bin  $i$

$\mathcal{T}_i$  known from  $D^*$  decays

- Binning such that  $c_i = c_{-i}$ ,  $s_i = -s_{-i}$
- Can in principle fit  $c_i$ ,  $s_i$ ,  $\delta$  and  $\gamma$ .
- However, not feasible with with statistics at BaBar, BELLE, or even LHCb.
- Need external input from CLEO.



# CLEO-c's impact on $\gamma$

- CPV-sensitive factor in  $B^\pm \rightarrow D(K_s \pi \pi) K^\pm$  Dalitz analysis:

$$c_i \cos(\delta - \gamma) + s_i \sin(\delta - \gamma)$$

$\swarrow$   $\nwarrow$   
 $\cos(\delta_D(s_{12}, s_{34}))$  and  $\sin(\delta_D(s_{12}, s_{34}))$  averaged over bin  $i$

- CLEO-c can
  - Measure  $c_i$  with CP-tagged  $D \rightarrow K_{S,L} \pi \pi$  Dalitz plot
  - Measure both  $c_i, s_i$  with simultaneous Dalitz plot analysis  $D_1 \rightarrow K_s \pi \pi, D_2 \rightarrow K_s \pi \pi$

# Optimal binning

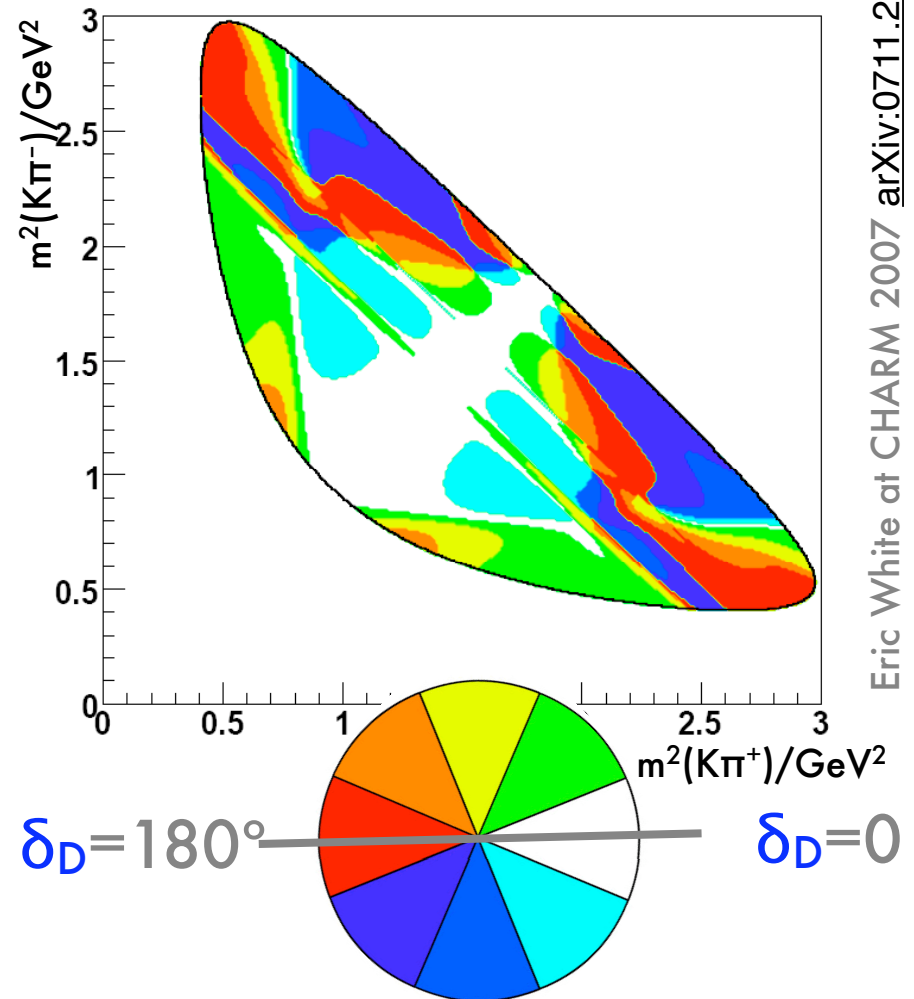
Bondar, Poluektov hep-ph/0703267v1 (2007)

$$c_i \cos(\delta - \gamma) + s_i \sin(\delta - \gamma)$$

$\swarrow$   $\searrow$   
 $\cos(\delta_D(s_{12}, s_{34}))$  and  $\sin(\delta_D(s_{12}, s_{34}))$   
 averaged over bin  $i$

- Best if strong phase  $\delta_D$  is as constant as possible over each bin, since then  $s_i^2 = 1 - c_i^2$
- Plot shows CLEO-c's 8 bins (based on BELLE model)

## CLEO-c's binning

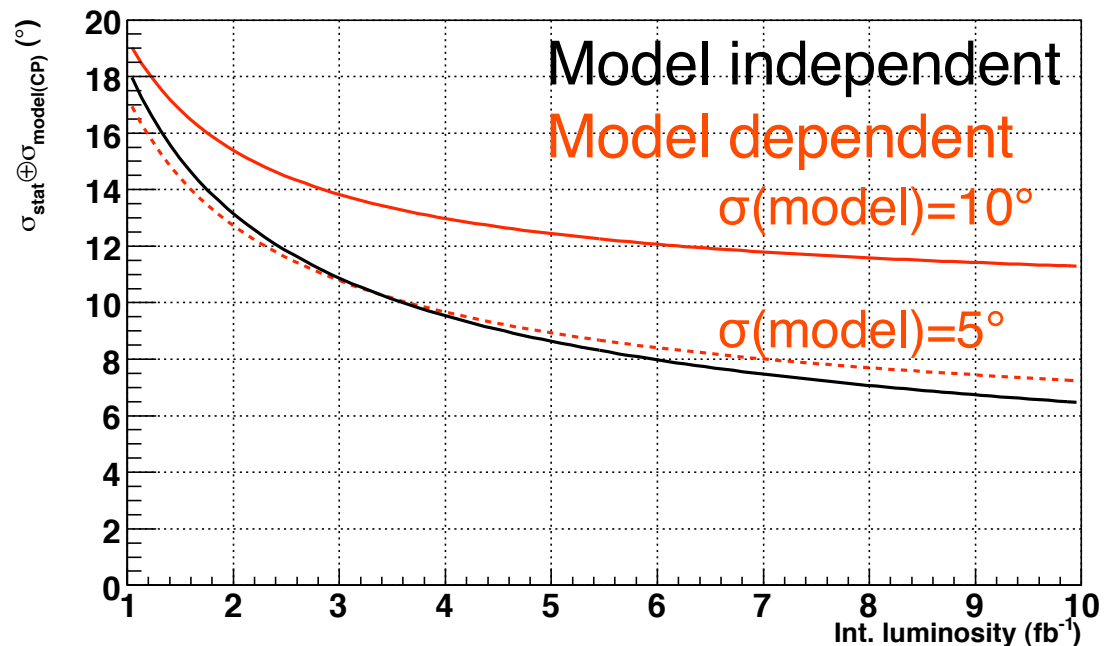




# LHCb-reach (with CLEO's input)

- Result will benefit BaBar and BELLE's measurement;
- But above all, future measurements of  $\gamma$  with high stats, such as LHCb.

LHCb's  $\gamma$  reach from  $B^\pm \rightarrow D(K_s \pi \pi) K^\pm$ ,  
assuming  $r_B=0.1$  (note:  $\sigma_\gamma \propto 1/r_B$ )



LHCb expects  $2\text{fb}^{-1}$  per year

# Achasov references

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