Neutrino Masses and Mixing (Theory)

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Outline

1. What We Have Learned About Neutrinos;
2. What We Know We Don't Know;
3. Neutrino Have Mass – So What?;
4. Where do We Go From Here? Conclusions.

neutrino: the other heavy lepton

News from ν Flavor Oscillations

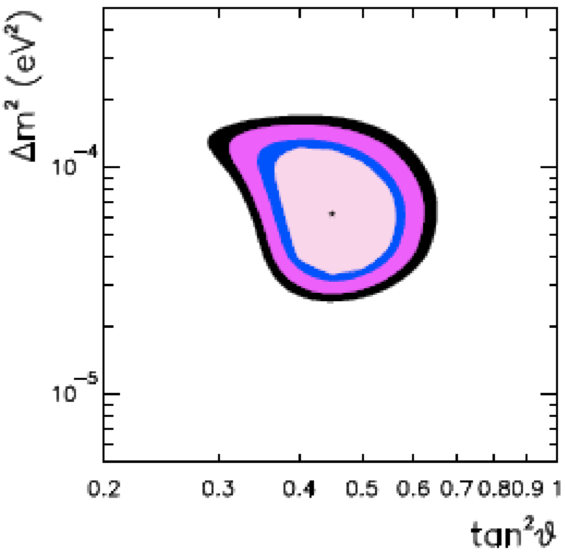
Neutrino oscillation experiments have revealed that neutrinos change flavor after propagating a finite distance. The rate of change depends on the neutrino energy E_ν and the baseline L.

- $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ — atmospheric experiments ["indisputable"];
- $\nu_e \to \nu_{\mu,\tau}$ — solar experiments ["indisputable"];
- $\bar{\nu}_e \to \bar{\nu}_{\rm other}$ — reactor neutrinos ["indisputable"];
- $\nu_\mu \to \nu_{\rm other}$ from accelerator experiments ["really strong"].

The simplest and **only satisfactory** explanation of **all** this data is that neutrinos have distinct masses, and mix.

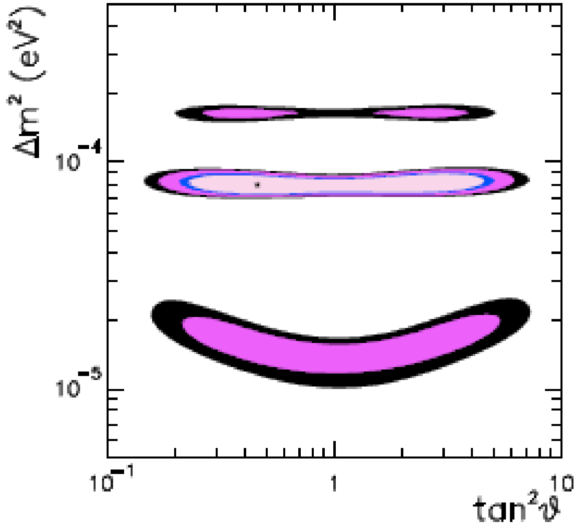
Solar

$\nu_e \to \nu_{\rm active}$

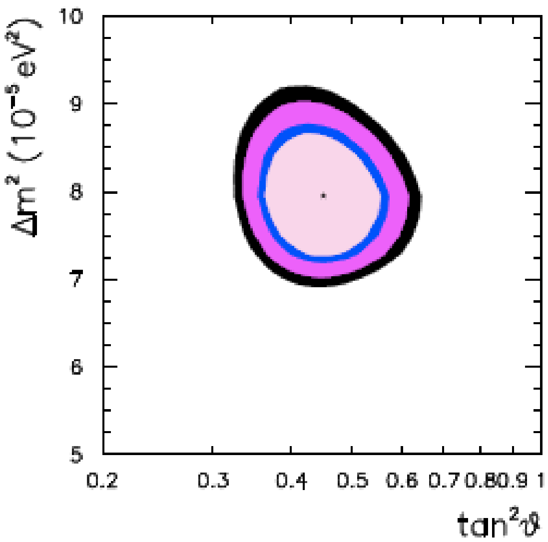


+ KamLAND

$\bar{\nu}_e \not\to \bar{\nu}_e$

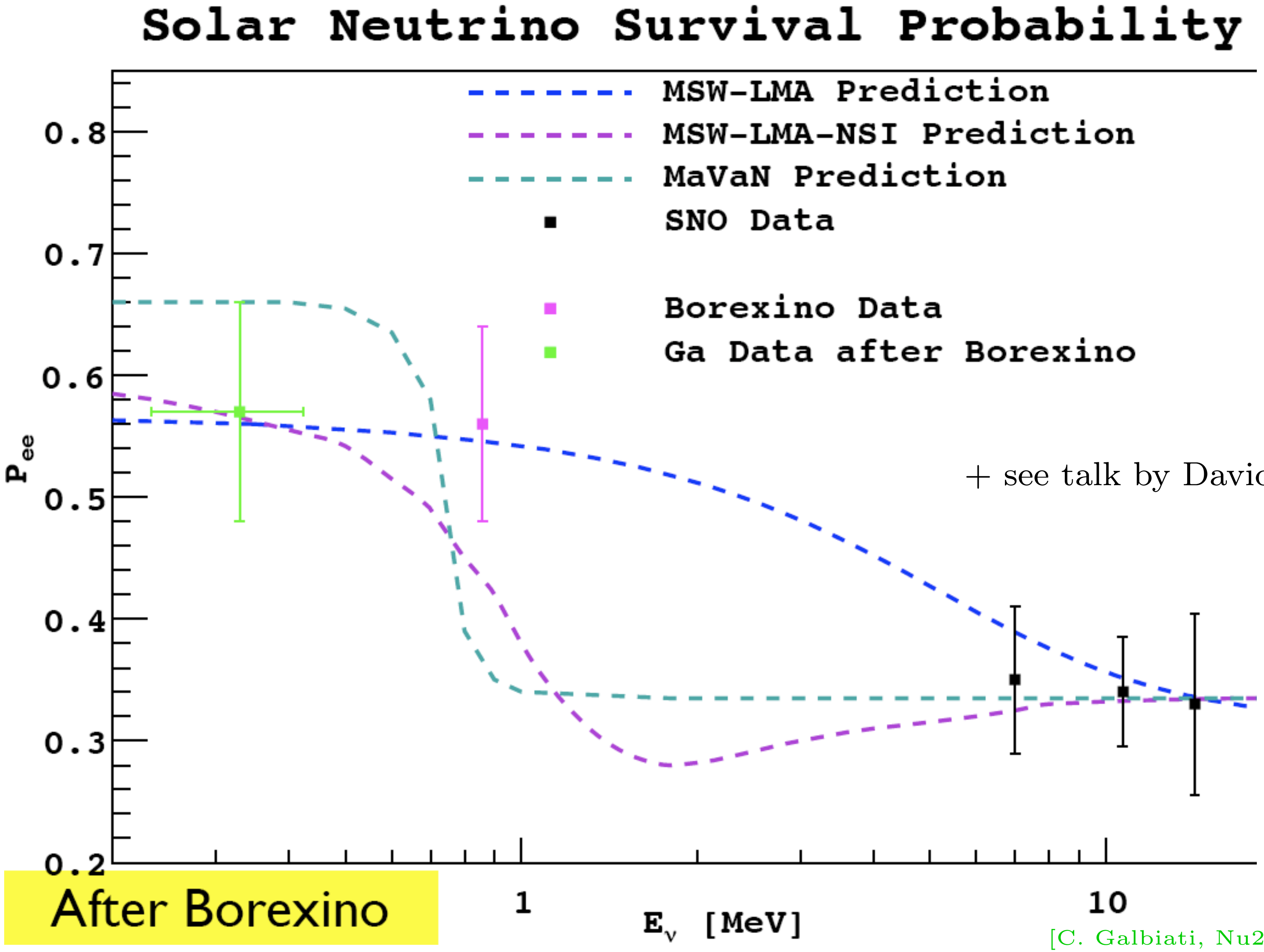


ν_e oscillation parameters compatible with $\overline{\nu}_e$: *Sensible* to assume CPT: $P_{ee} = P_{\bar{e}\bar{e}}$



$$\Delta m^2_{\odot} = (8^{+0.4}_{-0.5}) \times 10^{-5}\ \mathrm{eV}^2 \quad (1\sigma)$$
$$\tan^2\theta_{\odot} = 0.45^{+0.05}_{-0.05}$$

[Gonzalez-Garcia, PASI 2006]

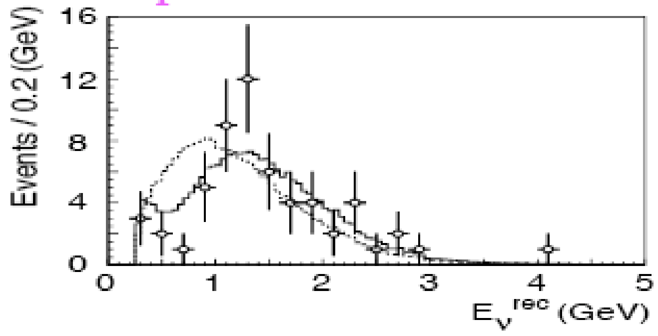


+ see talk by Davide Franco

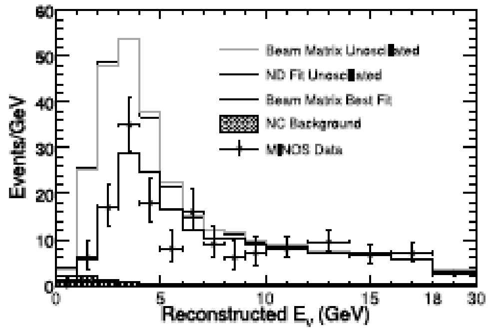
[C. Galbiati, Nu2008]

K2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km
Opera/Icarus	ν_μ at CERN	Gran Sasso	L=740 km

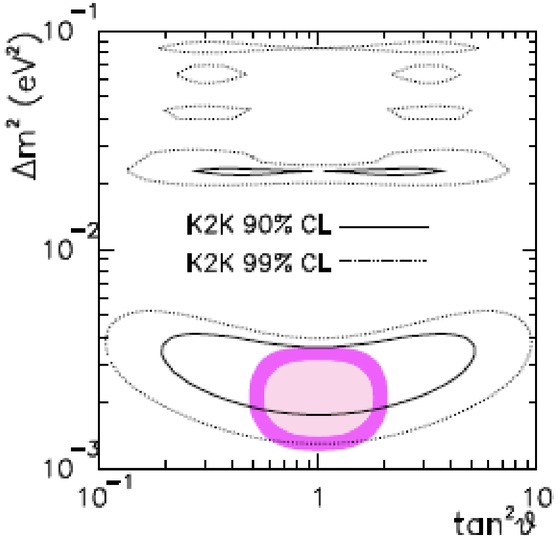
K2K 2004: spectral distortion



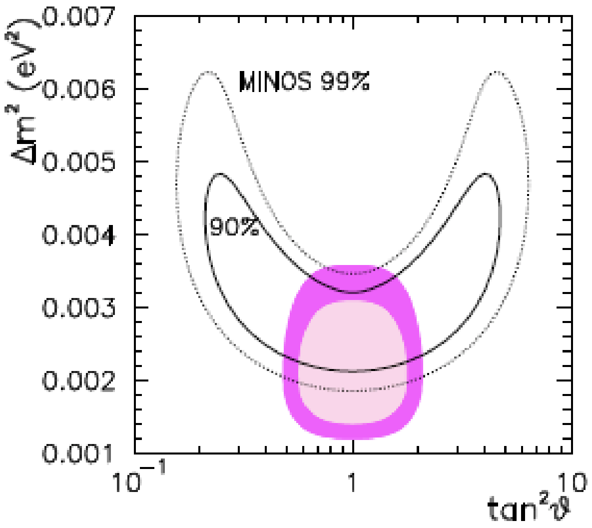
MINOS 2006: spectral distortion



Confirmation of ATM oscillations



Confirmation of ATM oscillations



[Gonzalez-Garcia, PASI 2006]

Previous fits shown assuming two-flavor mixing. Of course, there are three neutrinos...

Phenomenological Understanding of Neutrino Masses & Mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{e\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

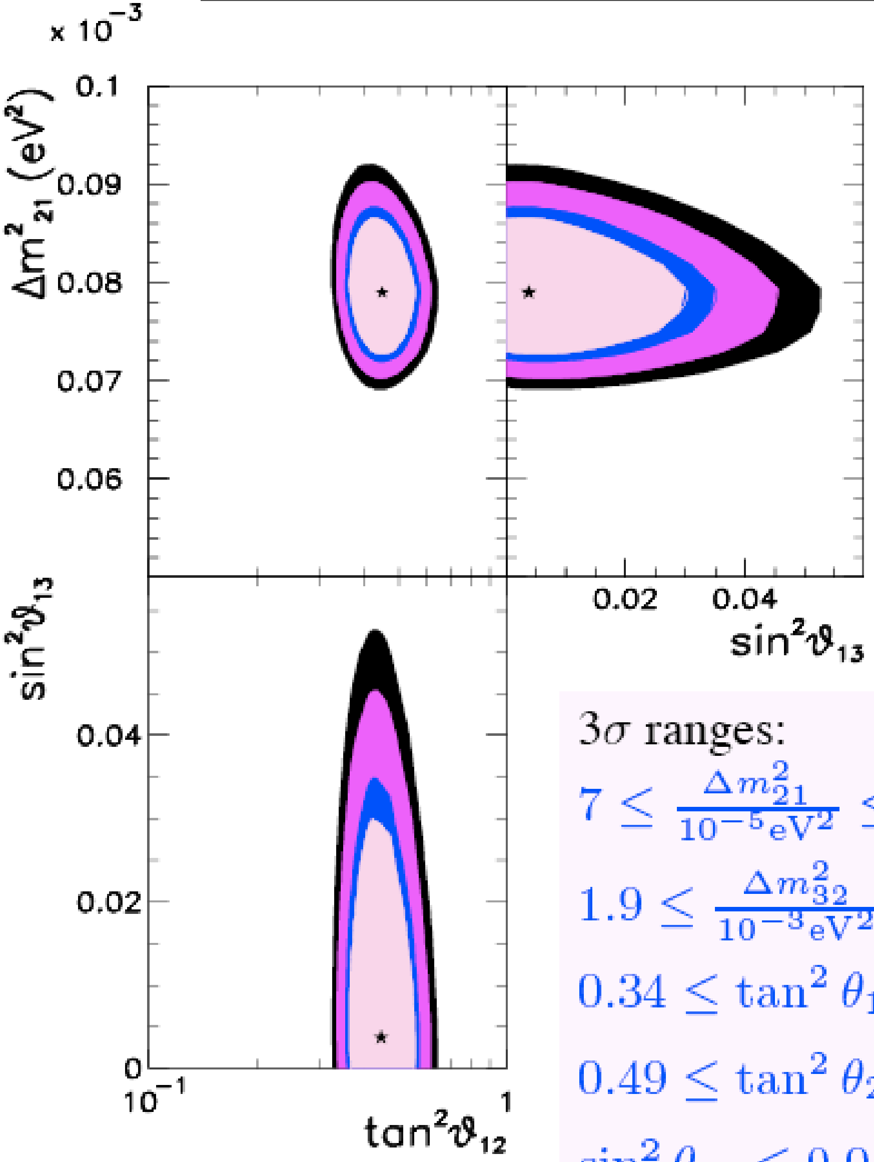
Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3?):

- $m_1^2 < m_2^2$ — $\Delta m_{13}^2 < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ — $\Delta m_{13}^2 > 0$ – Normal Mass Hierarchy

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu3}|^2}{|U_{\tau3}|^2}; \quad U_{e3} \equiv \sin\theta_{13} e^{-i\delta}$$

Putting It All Together:

[absent: new MINOS results]



3σ ranges:

$7 \leq \frac{\Delta m^2_{21}}{10^{-5}\mathrm{eV}^2} \leq 9.1$

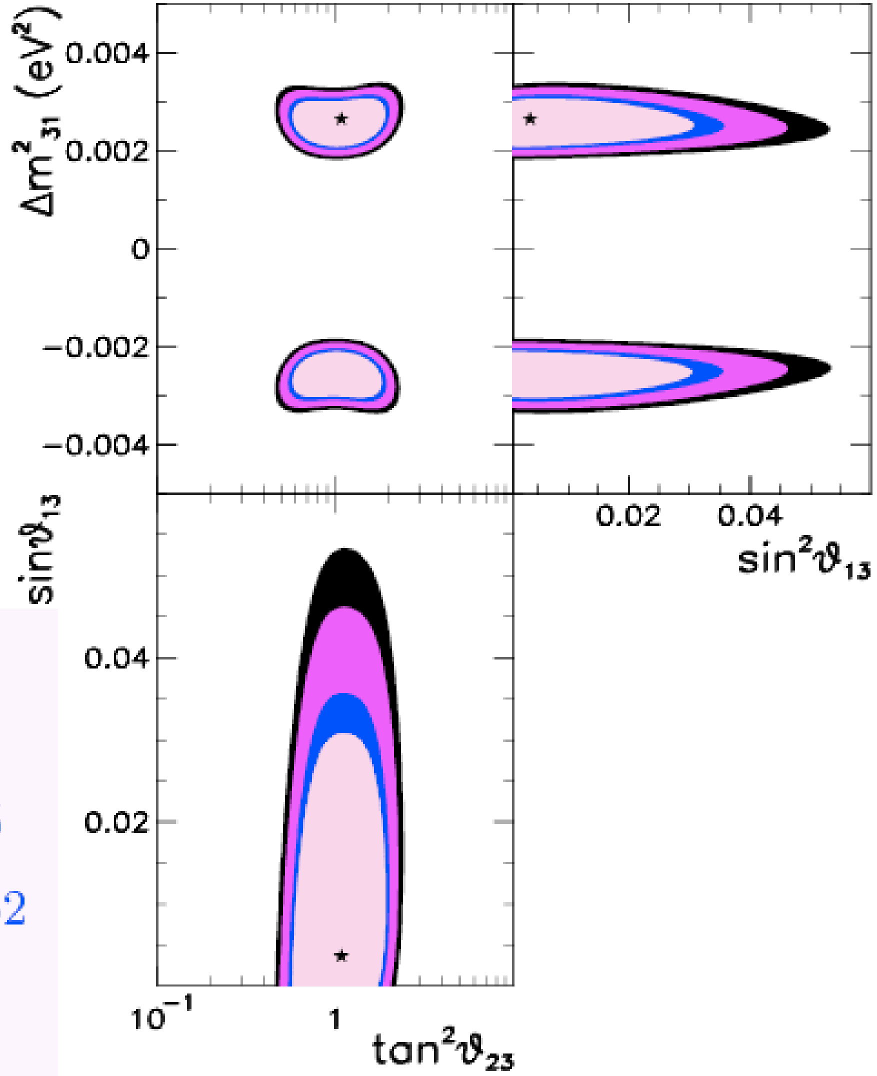
$1.9 \leq \frac{\Delta m^2_{32}}{10^{-3}\mathrm{eV}^2} \leq 3.25$

$0.34 \leq \tan^2\theta_{12} \leq 0.62$

$0.49 \leq \tan^2\theta_{23} \leq 2.2$

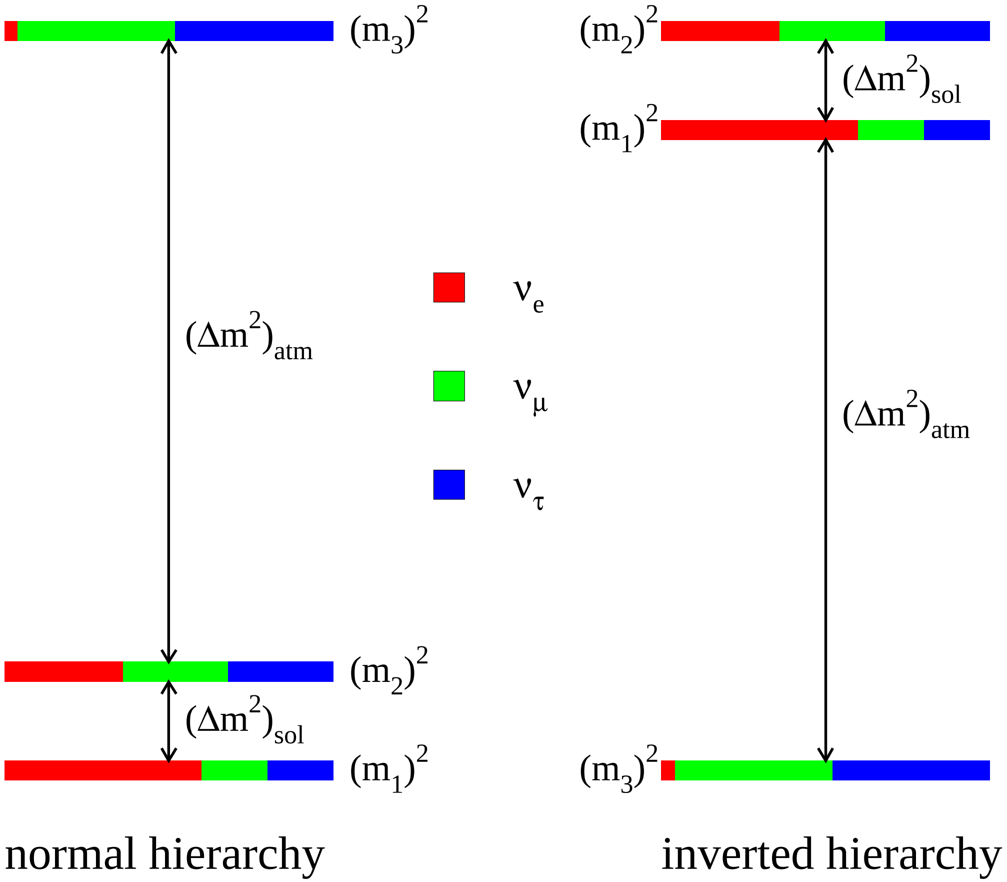
$\sin^2\theta_{13} \leq 0.045$

$-\pi \leq \delta \leq \pi$



[Gonzalez-Garcia, PASI 2006]

What We Know We Don't Know (1): Missing Oscillation Parameters



- What is the ν_e component of ν_3? ($\theta_{13} \neq 0$?)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?)
- Is ν_3 mostly ν_μ or ν_τ? ($\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, or $\theta_{23} = \pi/4$?)
- What is the neutrino mass hierarchy? ($\Delta m^2_{13} > 0$?)

$\Rightarrow$ All of the above can "only" be addressed with neutrino oscillation experiments.

Ultimate goal not just to measure parameters $\rightarrow$ test formalism (over-constrain parameters?)

The "Holy Graill" of Neutrino Oscillations – CP Violation

In the old Standard Model, there is only one[a] source of CP-invariance violation:

$\Rightarrow$ The complex phase in V_{CKM}, the quark mixing matrix.

Indeed, as far as we have been able to test, all CP-invariance violating phenomena agree with the CKM paradigm:

- ϵ_K;
- ϵ'_K;
- $\sin 2\beta$;
- etc.

Neutrino masses and lepton mixing provide strong reason to believe that other sources of CP-invariance violation exist.

[a] modulo the QCD θ-parameter, which will be "willed away" as usual.

CP-invariance Violation in Neutrino Oscillations

The most promising approach to studying CP-violation in the leptonic sector seems to be to compare $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

$$A_{\mu e} = U_{e2}^* U_{\mu 2} \left(e^{i\Delta_{12}} - 1\right) + U_{e3}^* U_{\mu 3} \left(e^{i\Delta_{13}} - 1\right)$$

where $\Delta_{1i} = \frac{\Delta m^2_{1i} L}{2E}$, $i = 2, 3$.

The amplitude for the CP-conjugate process is

$$\bar{A}_{\mu e} = U_{e2} U_{\mu 2}^* \left(e^{i\Delta_{12}} - 1\right) + U_{e3} U_{\mu 3}^* \left(e^{i\Delta_{13}} - 1\right).$$

[remember: according to unitarty, $U_{e1} U_{\mu 1}^* = -U_{e2} U_{\mu 2}^* - U_{e3} U_{\mu 3}^*$]

In general, $|A|^2 \neq |\bar{A}|^2$ (CP-invariance violated) as long as:

- Nontrivial "Weak" Phases: $\arg(U_{ei}^* U_{\mu i}) \to \delta \neq 0, \pi$;
- Nontrivial "Strong" Phases: Δ_{12}, $\Delta_{13} \to L \neq 0$;
- Because of Unitarity, we need all $|U_{\alpha i}| \neq 0 \to$ three generations.

All of these can be satisfied, with a little luck: given that two of the three mixing angles are known to be large, **we need $|U_{e3}| \neq 0$.**

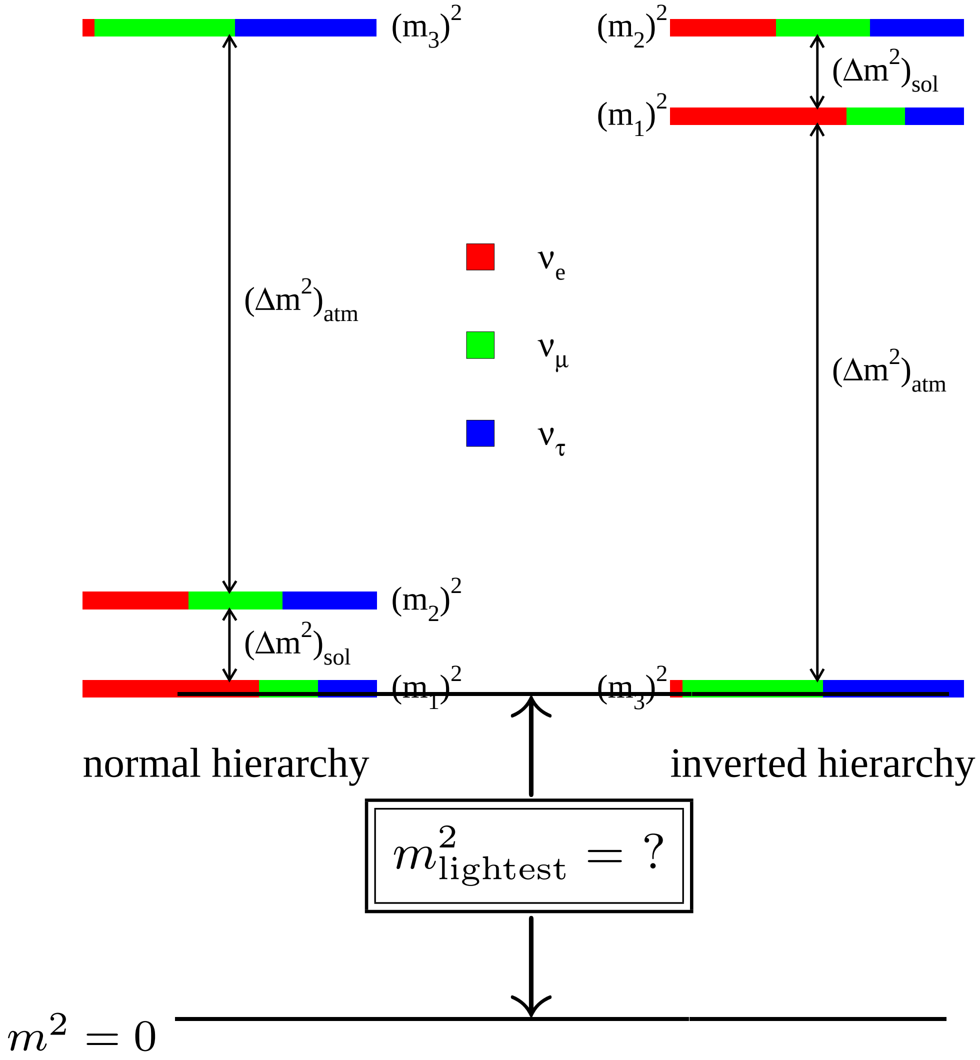
The goal of next-generation neutrino experiments is to determine the magnitude of $|U_{e3}|$. We need to know this in order to understand how to study CP-invariance violation in neutrino oscillations!

In the real world, life is much more complicated. The lack of knowledge concerning the mass hierarchy, θ_{13}, and θ_{23}, for example, leads to several degeneracies and ambiguities.

Note that, in order to see CP-invariance violation, we **need** the "subleading" terms (and need to make sure that the leading atmospheric terms do not average out)!

In order to ultimately measure a new source of CP-invariance violation, we will need to combine different measurements:
– oscillation of muon neutrinos and antineutrinos,
– oscillations at accelerator and reactor experiments,
– experiments with different baselines (or broad energy spectrum),
– etc.

What We Know We Don't Know (2): How Light is the Lightest Neutrino?



So far, we've only been able to measure neutrino mass-squared differences.

The lightest neutrino mass is only poorly constrained: $m^2_{\rm lightest} < 1$ eV^2

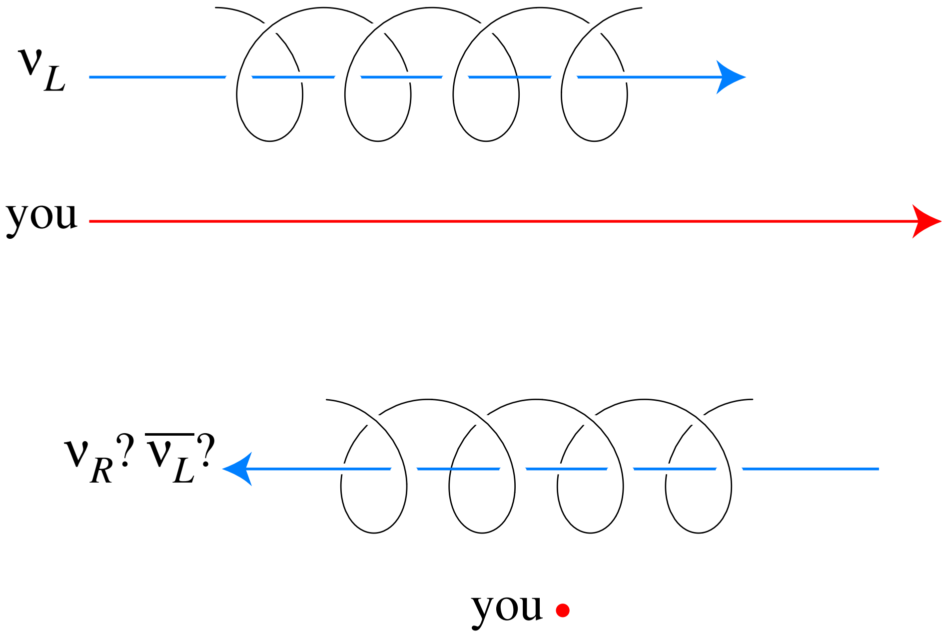
qualitatively different scenarios allowed:

- $m^2_{\rm lightest} \equiv 0$;
- $m^2_{\rm lightest} \ll \Delta m^2_{12,13}$;
- $m^2_{\rm lightest} \gg \Delta m^2_{12,13}$.

Need information outside of neutrino oscillations.

[talk by Markus Steidl]

What We Know We Don't Know (3) – Are Neutrinos Majorana Fermions?



A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$(e^-_L \leftarrow \text{CPT} \rightarrow e^+_R)$$
$$\updownarrow \text{Lorentz}$$
$$(e^-_R \leftarrow \text{CPT} \rightarrow e^+_L)$$

A massive neutral fermion (s=1/2) is described by 4 or 2 degrees of freedom:

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz} \qquad \text{"DIRAC"}$$
$$(\nu_R \leftarrow \text{CPT} \rightarrow \bar{\nu}_L)$$

"MAJORANA"

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz}$$
$$(\bar{\nu}_R \leftarrow \text{CPT} \rightarrow \nu_L)$$

How many degrees of freedom are required to describe massive neutrinos?

Why Don't We Know the Answer?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are proportional to the Majorana nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

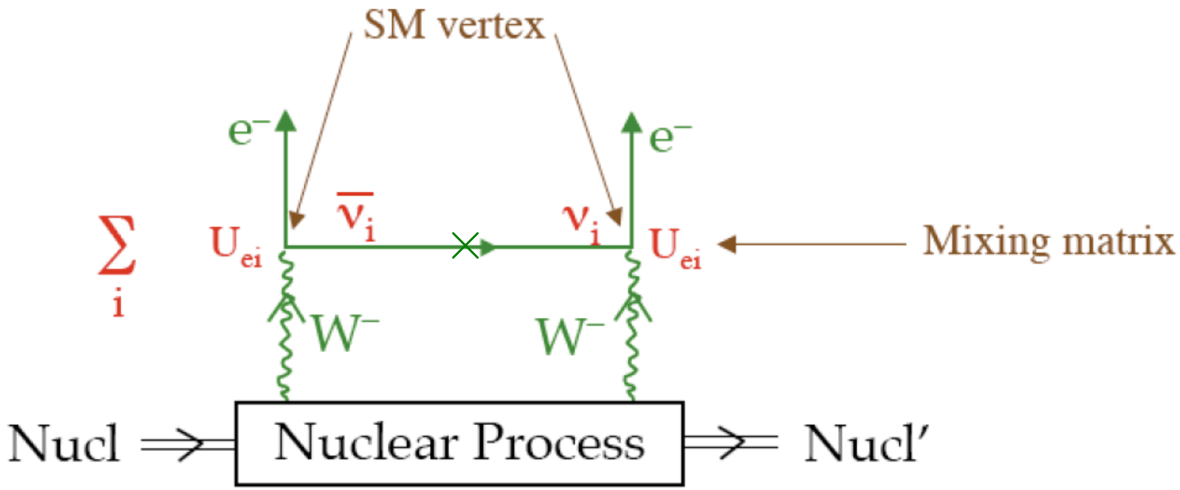
The "smoking gun" signature is the observation of LEPTON NUMBER violation. This is easy to understand: Majorana neutrinos are their own antiparticles and, therefore, cannot carry "any" quantum numbers — including lepton number.

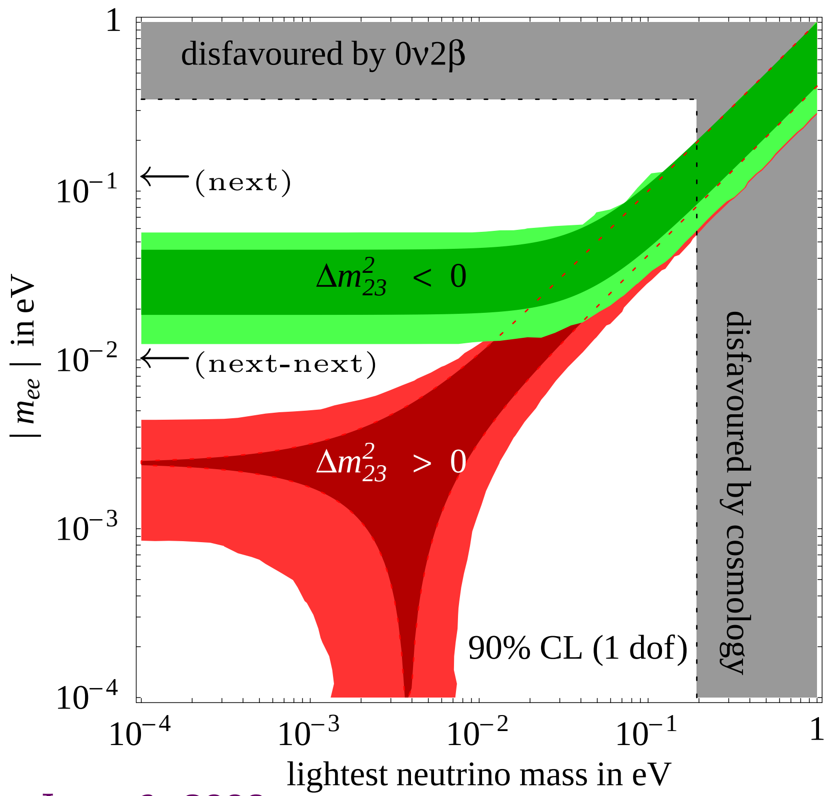
Search for the Violation of Lepton Number (or $B-L$)

Best Bet: search for

Neutrinoless Double-Beta

Decay: $Z \to (Z+2)e^- e^-$

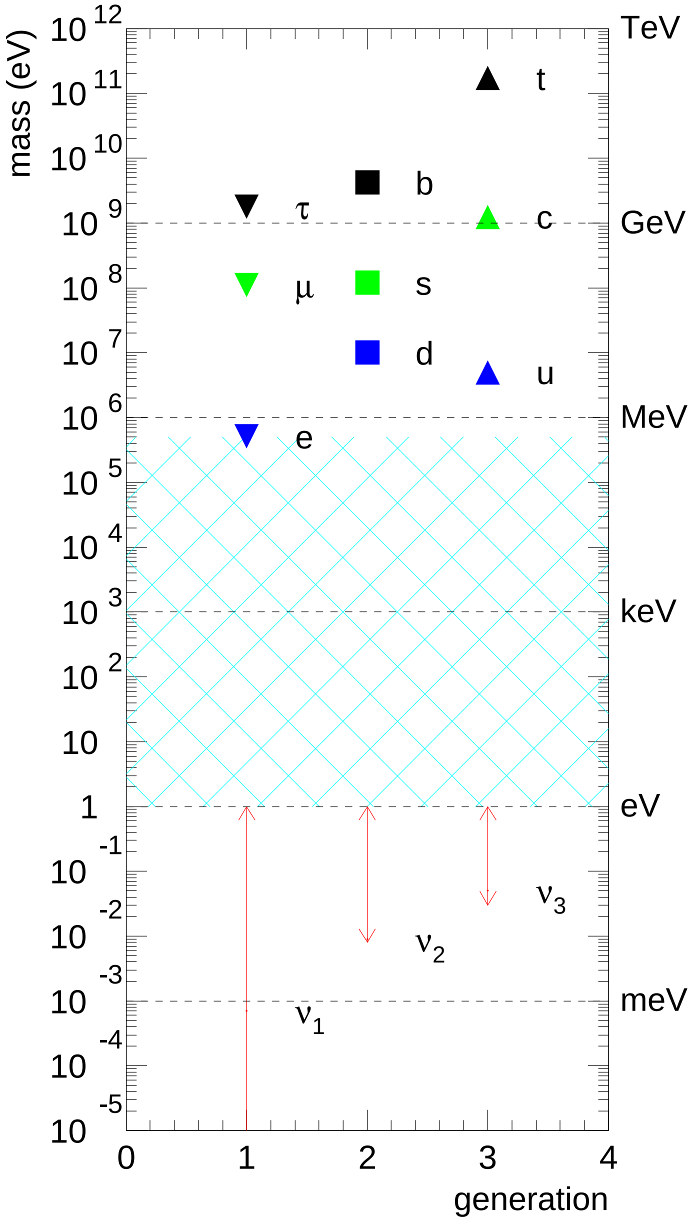




Helicity Suppressed Amplitude $\propto \frac{m_{ee}}{E}$

Observable: $m_{ee} \equiv \sum_i U_{ei}^2 m_i$

Are other probes competitive?



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

So What?

Who Cares About Neutrino Masses: Only* "Palpable" Evidence of Physics Beyond the Standard Model

The SM we all learned in school predicts that neutrinos are strictly massless. Massive neutrinos imply that the the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

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* There is only a handful of questions our model for fundamental physics cannot explain properly. These are, in order of "palpability" (my opinion):

- What is the physics behind electroweak symmetry breaking? (Higgs *or* not in SM).
- What is the dark matter? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM – is this "particle physics?").

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

$$\Updownarrow$$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input, and it looks like it may be coming in the near/intermediate future!

Options include:

- modify SM Higgs sector (e.g. Higgs triplet) and/or
- modify SM particle content (e.g. $SU(2)_L$ Triplet or Singlet) and/or
- modify SM gauge structure and/or
- supersymmetrize the SM and add R-parity violation and/or
- augment the number of space-time dimensions and/or
- etc

Important: different options $\rightarrow$ different phenomenological consequences

Understanding Fermion Mixing

The other puzzling phenomenon uncovered by the neutrino data is the fact that Neutrino Mixing is Strange. What does this mean?

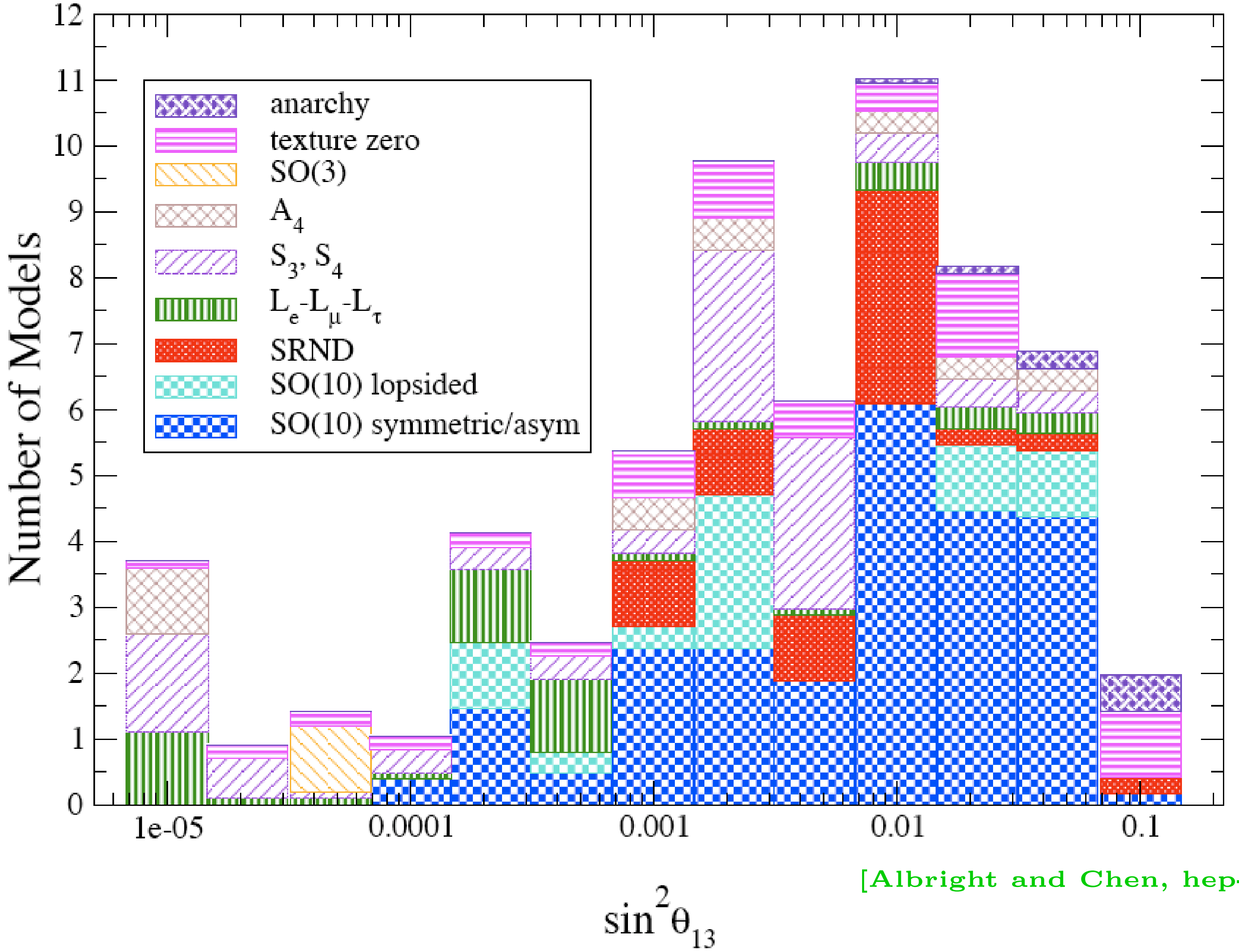
It means that lepton mixing is very different from quark mixing:

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

$[|(V_{MNS})_{e3}| < 0.2]$

WHY?

They certainly look VERY different, but which one would you label as "strange"?



[Albright and Chen, hep-ph/0608137]

pessimist – "We can't compute what $|U_{e3}|$ is – must measure it!"

(same goes for the mass hierarchy, δ)

Comments On Current Flavor Model-Building Scene:

- VERY active research area. Opportunity to make *bona fide* prediction regarding parameters that haven't been measured yet but will be measured for sure in the near future → θ_{13}, δ, mass hierarchy, etc;
- For flavor symmetries, more important than determining the values of the parameters is the prospect of establishing non-trivial relationships among several interesting unkowns;

  e.g.,

$$\sin^2\theta_{13} \sim \Delta m^2_{12}/|\Delta m^2_{13}| \text{ if hierarchy is normal,}$$
$$\sin^2\theta_{13} \sim (\Delta m^2_{12}/|\Delta m^2_{13}|)^2 \text{ if hierachy is inverted}$$

  is common "prediction" of many flavor models (often also related to $\cos 2\theta_{23}$).

How Do We Learn More?

In order to learn more, we need more information. Any new data and/or idea is welcome, including

- searches for charged lepton flavor violation ($\mu \to e\gamma$, etc);
- searches for lepton number violation (neutrinoless double beta decay, etc);
- precision measurements of the neutrino oscillation parameters;
- searches for fermion electric/magnetic dipole moments (electron edm, muon $g-2$, etc);
- searches for new physics at the TeV scale – we need to understand the physics at the TeV scale before we can really understand the physics behind neutrino masses (is there low-energy SUSY?, etc).

CONCLUSIONS

The venerable Standard Model has finally sprung a leak – neutrinos are not massless!

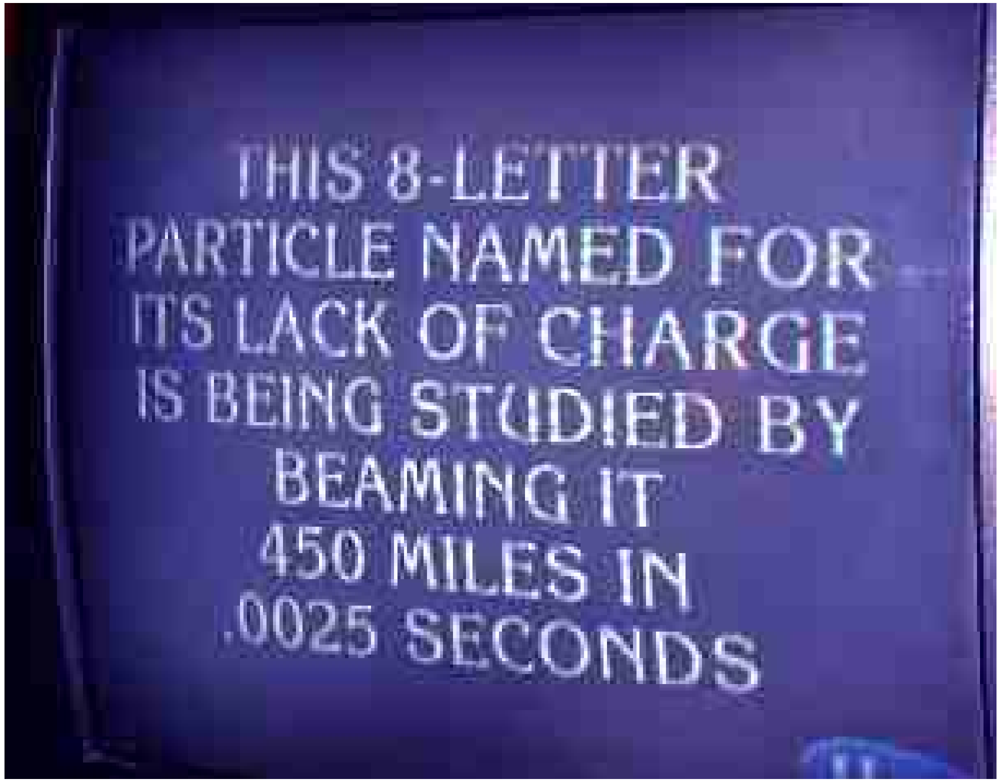
1. we have a very successful parametrization of the neutrino sector, and we have identified what we know we don't know → Well defined experimental program.

2. neutrino masses are very small – we don't know why, but we think it means something important.

3. lepton mixing is very different from quark mixing – we don't know why, but we think it means something important.

4. We need more experimental input – and more seems to be on the way (this is a truly data driven field right now). We only started to figure out what is going on.

5. There is plenty of room for surprises, as neutrinos are very narrow but deep probes of all sorts of physical phenomena. Remember that neutrino oscillations are "quantum interference devices" – potentially very sensitive to whatever else may be out there (e.g., neutrino masses are often interpreted as evidence for new physics at the 10^{14} GeV!).



(GameShow Neutrinos)

BACK-UPs

What I Mean By the Standard Model

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group ($SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y}$);
- Particle Content (fermions: Q, u, d, L, e, scalars: H).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done! (after several decades of hard experimental work…)

If you follow these rules, neutrinos have no mass. Something has to give.

Candidate νSM

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\mathrm{SM}} \supset -\lambda_{ij}\frac{L^i H L^j H}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB } \mathcal{L}_{\nu\mathrm{SM}} \supset \frac{m_{ij}}{2}\nu^i\nu^j; \quad m_{ij} = \lambda_{ij}\frac{v^2}{\Lambda}.$$

- Neutrino masses are small: $\Lambda \gg v \to m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- νSM effective theory – not valid for energies above *at most* Λ/λ.
- What is Λ? First naive guess is that M is the Planck scale – does not work. Data require $\Lambda < 10^{15}$ GeV (anything to do with the GUT scale?).

What else is this “good for”? Depends on the ultraviolet completion!

Another νSM

Why don't we just enhance the fermion sector of the theory?

One may argue that it is trivial and simpler to just add

$$\mathcal{L}_{\rm Yukawa} = -y_{i\alpha} L^i H N^\alpha + H.c.,$$

and neutrinos get a mass like all other fermions: $m_{i\alpha} = y_{i\alpha} v$

- Data requires $y < 10^{-12}$. Why so small?
- Neutrinos are Dirac fermions. $B - L$ exactly conserved.
- νSM is a renormalizable theory.

This proposal, however, violates the rules of the SM (as I defined them)! The operator $\frac{M_N}{2} NN$, allowed by all gauge symmetries, is absent. In order to explain this, we are forced to add a symmetry to the νSM. The simplest candidate is a global $U(1)_{B-L}$.

$U(1)_{B-L}$ is upgraded from accidental to fundamental (global) symmetry.

Old Standard Model, Encore

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group $(SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y})$;
- Particle Content (fermions: Q, u, d, L, e, scalars: H).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done.

This model has *accidental global symmetries*. In particular, the anomaly free global symmetry is preserved: $U(1)_{B-L}$.

New Standard Model, Dirac Neutrinos

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group $(SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y})$;
- Particle Content (fermions: Q, u, d, L, e, N, scalars: H);
- Global Symmetry $U(1)_{B-L}$.

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done.

Naively not too different, but nonetheless qualitatively different $\rightarrow$ enhanced symmetry sector!

On very small Yukawa couplings

We would like to believe that Yukawa couplings should naturally be of order one.

Nature, on the other hand, seems to have a funny way of showing this. Of all known fermions, only one (1) has a "natural" Yukawa coupling – the top quark!

Regardless there are several very different ways of obtaining "naturally" very small Yukawa couplings. They require more new physics.

"Natural" solutions include flavor symmetries, extra-dimensions of different "warping," ...