

Measurement of exclusive semileptonic B to D decays and the determination of exclusive $|V_{cb}|$

Phillip Urquijo
Université de Genève



HQL Melbourne June 2008

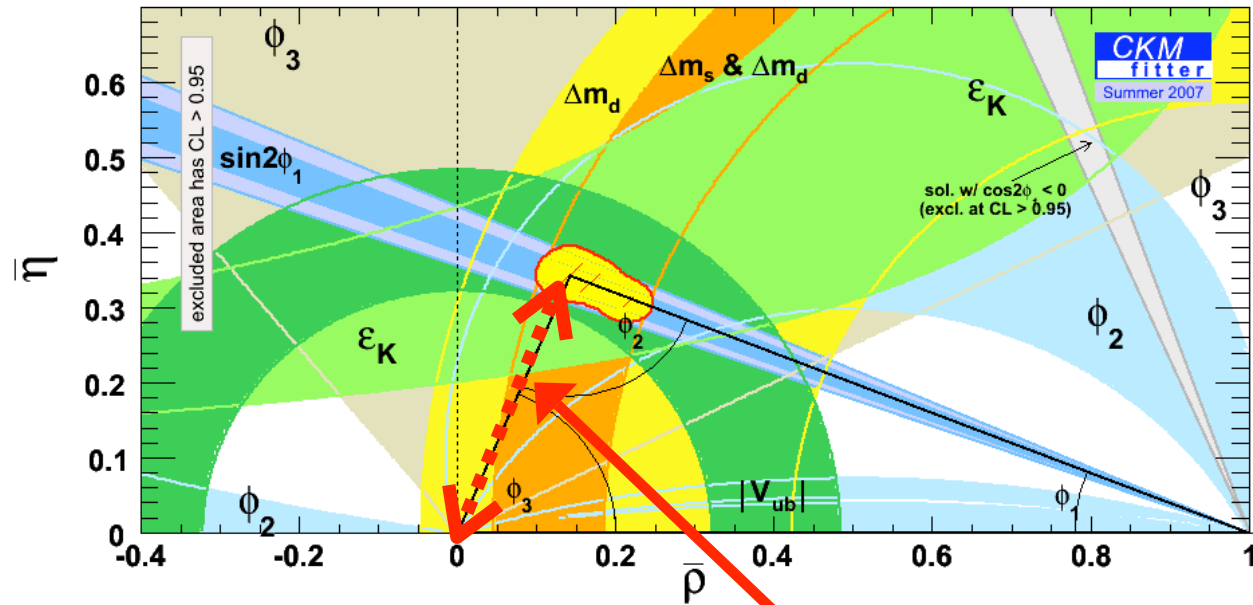
The Significance of $|V_{cb}|$ Measurements

CKM are fundamental SM parameters, size not theoretically predicted.

Constrain one side of the Unitarity Triangle

The precise determination of $|V_{ub}|/|V_{cb}|$ provides a benchmark for testing new physics in other processes.

- Current precision
 - $\sin 2\varphi_1 < 4\%$
 - $|V_{cb}| \sim 1\text{-}2\%$ inclusive
 $\sim 3\text{-}4\%$ exclusive
 - $|V_{ub}| \sim 7\%$ inclusive
 $> 10\%$ exclusive



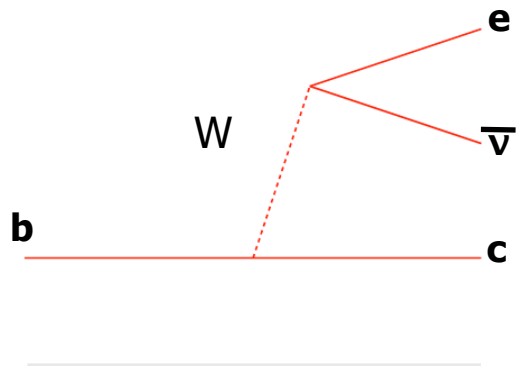
$$\propto |V_{ub}|/|V_{cb}|$$

Precise exclusive determinations used to check inclusive results, and probe our understanding of HQET

Semileptonic B decays

tree level, short distance:

$$b \rightarrow c e \nu$$

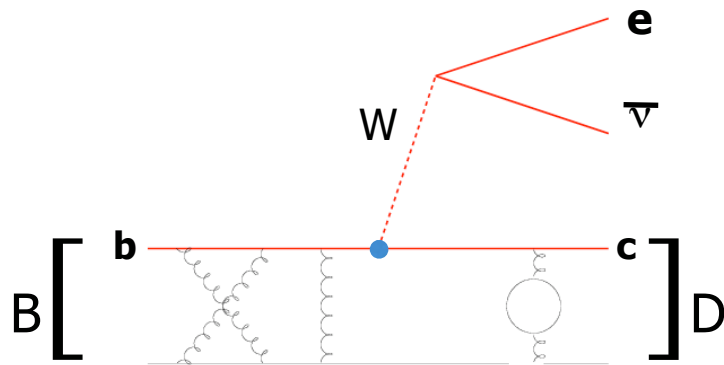


decay properties depend directly on $|V_{cb}|$ & $|V_{ub}|$ and m_b
perturbative regime (α_s^n)

Semileptonic B decays

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$$B \rightarrow D e \nu$$



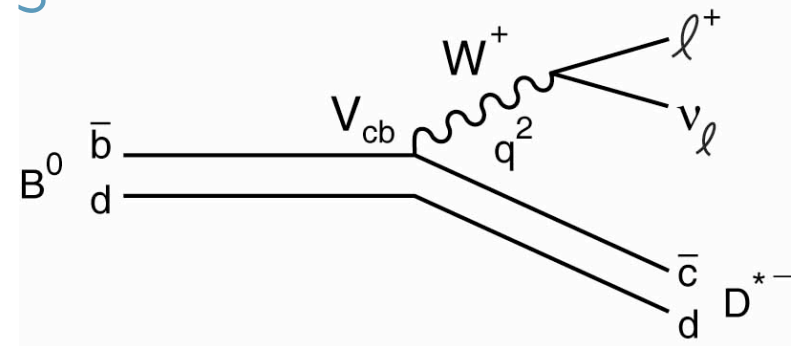
+ long distance:

decay properties depend directly on $|V_{cb}|$ & $|V_{ub}|$ and m_b
perturbative regime (α_s^n)

But quarks are bound by soft gluons: **non-perturbative**
long distance interactions of b quark with light quark

Exclusive Semileptonic Decays

One hadronic current.



Matrix element for semileptonic decays:

$$(M_{Q\bar{q} \rightarrow X_{q'\bar{q}} \rightarrow X_{q'\bar{q}} \ell^- \bar{\nu}_\ell) = -i \frac{G_F}{\sqrt{2}} V_{q'Q} L^\mu H_\mu$$

Leptonic current exactly known.

Hadronic current described by Form Factors (FF), functions of squared momentum transfer q^2 .

Exclusive rates are determined by $|V_{cb}|$ and Form Factors (FF)

- FF are calculated using non-perturbative methods. {
- Theoretically calculable at kinematic limits.
 - Lattice QCD works if D^* is at rest relative to the B.

Empirical extrapolation is necessary to extract $|V_{cb}|$ from measurements.

Measure differential rates to constrain the FF shape, then use FF normalization from the theory for $|V_{cb}|$.

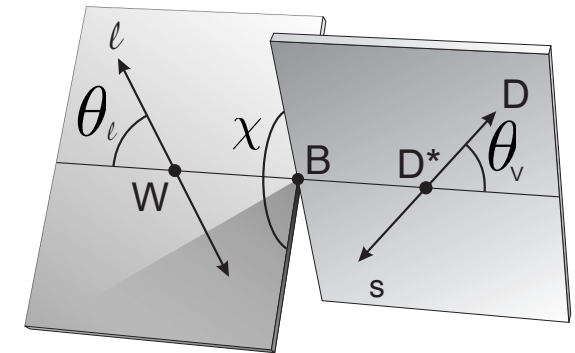
$|V_{cb}|$ and Form factors from $B \rightarrow D^* \ell \nu$

Differential decay rate

$$\frac{d\Gamma(B^0 \rightarrow D^* \ell^+ \nu_\ell)}{dw d\cos\theta_\ell \cos\theta_V d\chi} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} F(w, \theta_\ell, \theta_V, \chi) G(w)$$

Incorporates 3 non-trivial form factors
 $A_1(w)$, $A_2(w)$, and $V(w)$.

Phase space



$$w \equiv \frac{M_B^2 + M_{D^*}^2 - q^2}{2M_B M_{D^*}}$$

D^* boost in the B rest frame

HQ symmetry (b and c mass infinite) predicts a single universal FF, normalised to 1 at zero recoil.

HQET relates the 3 FF's to each other through Heavy Quark Symmetry, leaving 3 free parameters that are determined experimentally.

Amplitude ratios:

$$R_1(w) = V/A_1$$

$$R_2(w) = A_2/A_1$$

Curvature:

$$\rho^2(w) = -dF/dw|_{w=1}$$

Using parameterisation to extract 3 parameters from 4 observables in the data: $w, \theta_\nu, \theta_\ell, \chi$

Caprini et al., *Nucl. Phys.* B530 153 (1998).

Experimentally clean, a check of inclusive methods.

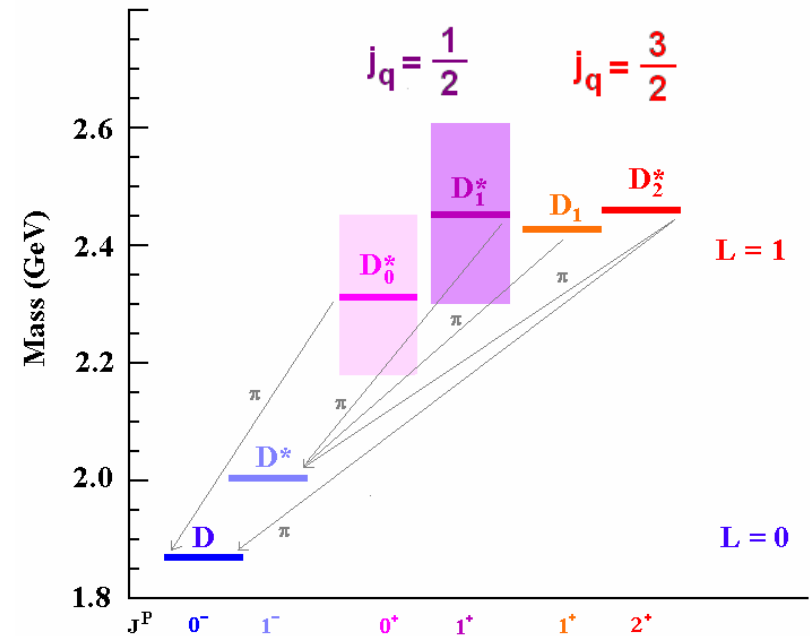
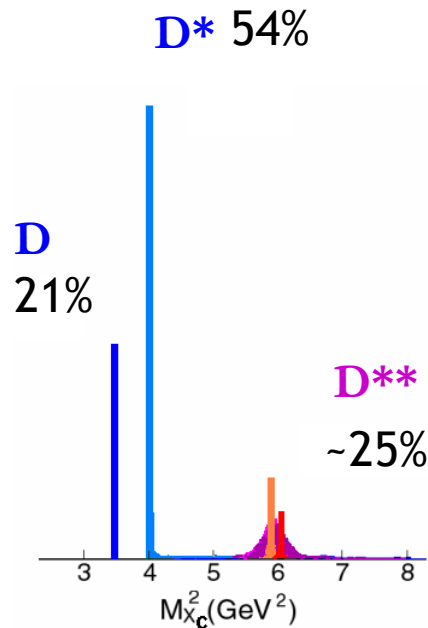
Heavy Quarks and Leptons Melbourne 2008 Phillip Urquijo

The X_c system

In addition to the well measured Δ -like $D^{(*)}$ components are the D^{**} , P-wave excitations of D-mesons.
 \Rightarrow HQET predicts **four D^{**} mesons: 2 narrow and 2 wide**, all observed in hadronic B decays.

(hep-ex/0307021, hep-ex/0611054)

$$BR(B \rightarrow X_c l \nu) \sim 10.5\%$$



BUT, Heavy Quark Symmetry also predicts $\Gamma(B \rightarrow \text{Narrow } l \nu) \gg \Gamma(B \rightarrow \text{Broad } l \nu)$

New BaBar and Belle measurements seem to defy this, eluding to a non-negligible contribution from non-resonant decay.

The components of the X_c system are not yet fully understood.

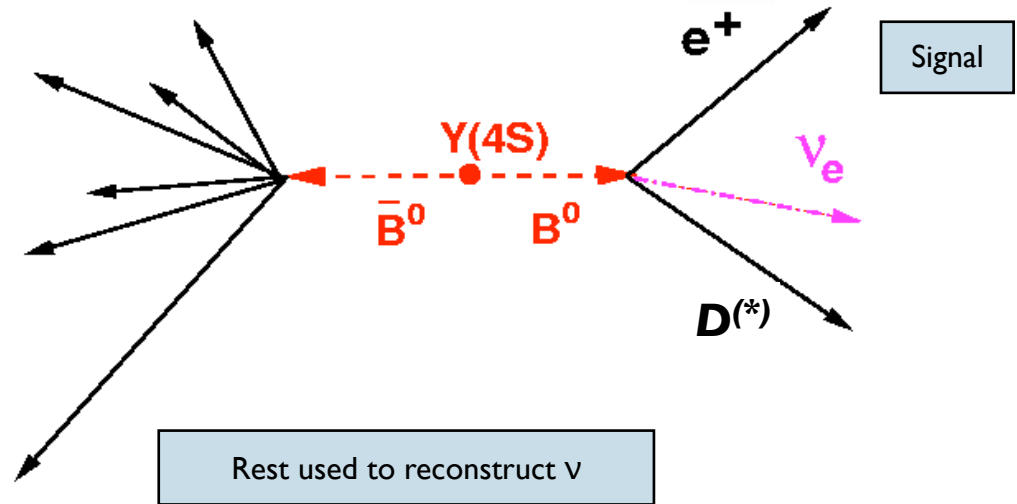
BR[%] HFAG ave.	B^0	B^+
Inclusive - Σ Exclusive	2.08 ± 0.34	1.62 ± 0.42

Measurement methods/samples



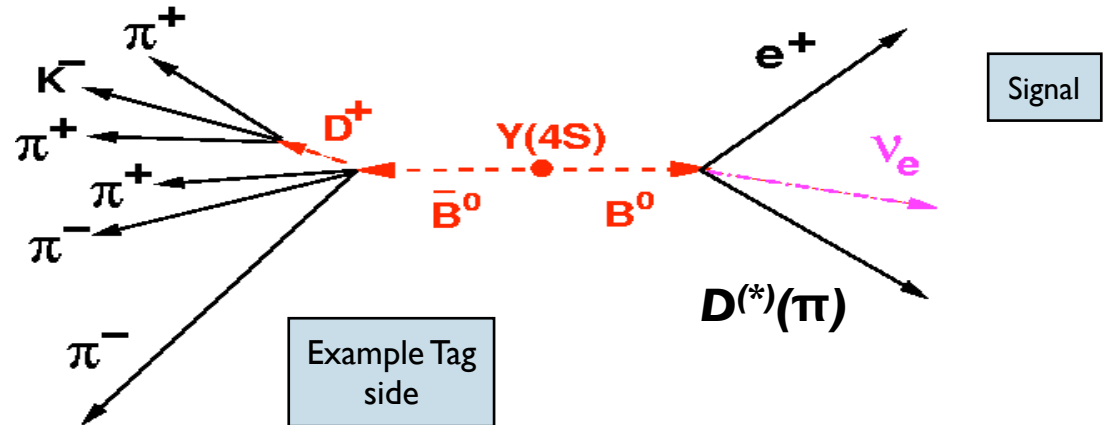
Untagged ⇒($|V_{cb}|$ and FF analyses)

- ▶ Initial 4-momentum known.
- ▶ Missing 4-momentum = \mathbf{v} .
- ▶ Reconstruct $B \rightarrow D^{(*)} | \mathbf{v}$ using m_B (beam-constrained) and $\Delta E = E_B - E_{\text{beam}}$.



Full Reconstruction Tag (“B Beam”) ⇒($BR(D^{(*)}(\pi) | \mathbf{v})$ analyses)

- ▶ One B reconstructed completely in known $b \rightarrow c$ mode.
- ▶ Many modes used.



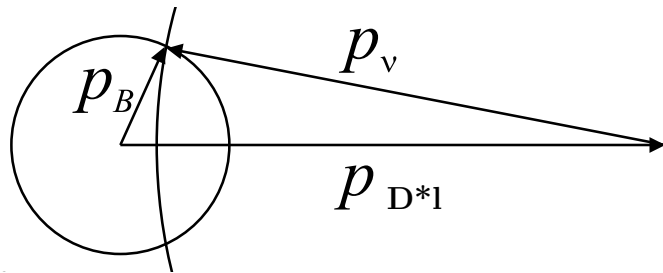
$|V_{cb}|$ and FF from $D^{(*)}$

$B^0 \rightarrow D^{*+} \ell \bar{\nu}$ selection

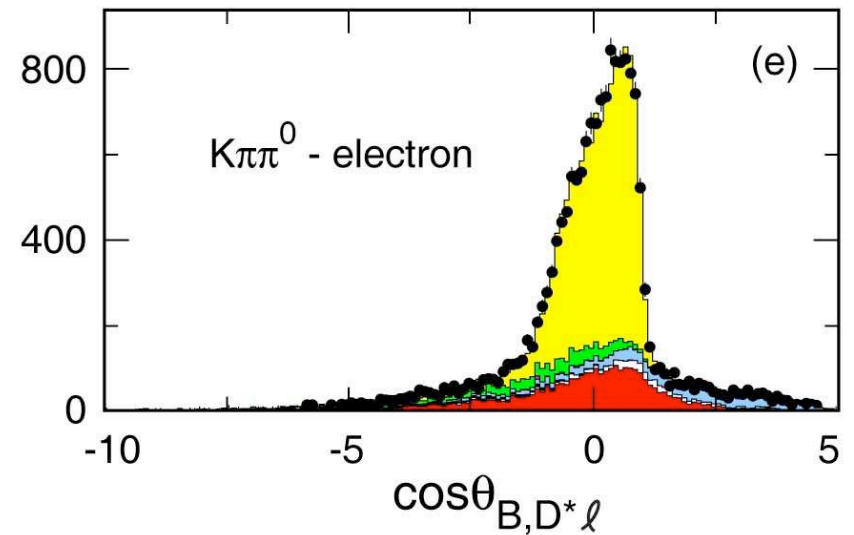
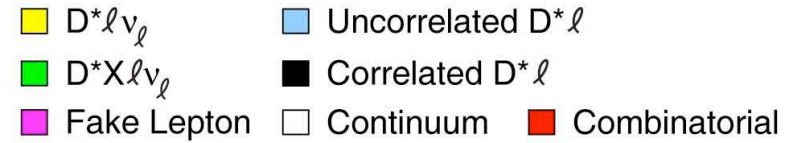


Select $B^0 \rightarrow D^{*+} \ell \bar{\nu}$ ($D^{*+} \rightarrow D^0 \pi^+$) events with $p_{T\ell}^* > 1.2$ GeV.

Estimate backgrounds (comb., D^{**}) from $\Delta M = M(D^*) - M(D)$ and $\cos\theta_{BY}$.



$$\cos\theta_{BY} = \frac{2E_B E_T - m_B^2 - m_Y^2}{2|\vec{p}_B||\vec{p}_Y|} \quad Y \equiv D^{*0} e$$



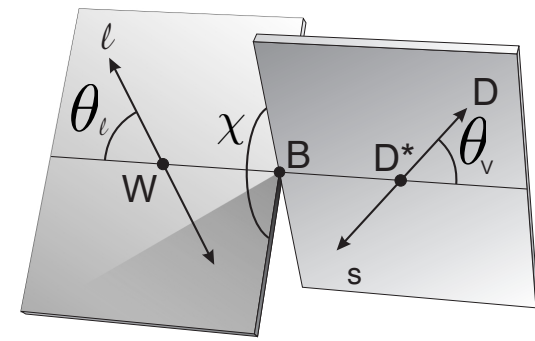
BaBar analyses:

- (1) Three D modes: $D \rightarrow K\pi$, $K\pi\pi$, $K\pi\pi\pi$, χ^2 fit to 1D projections.
- (2) One D mode: $D \rightarrow K\pi$, 4D maximum likelihood fit

$B^0 \rightarrow D^{*+} \ell \bar{\nu}$ Form factors

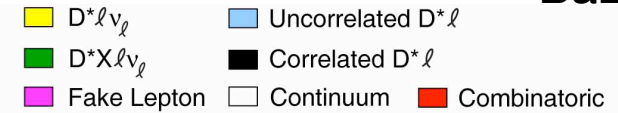
Phys.Rev. D77:032002 (2008)

Simultaneous χ^2 fit fit 1D projections in w , $\cos\theta_\ell$, $\cos\theta_\nu$ (integrated over angle χ).



BaBar

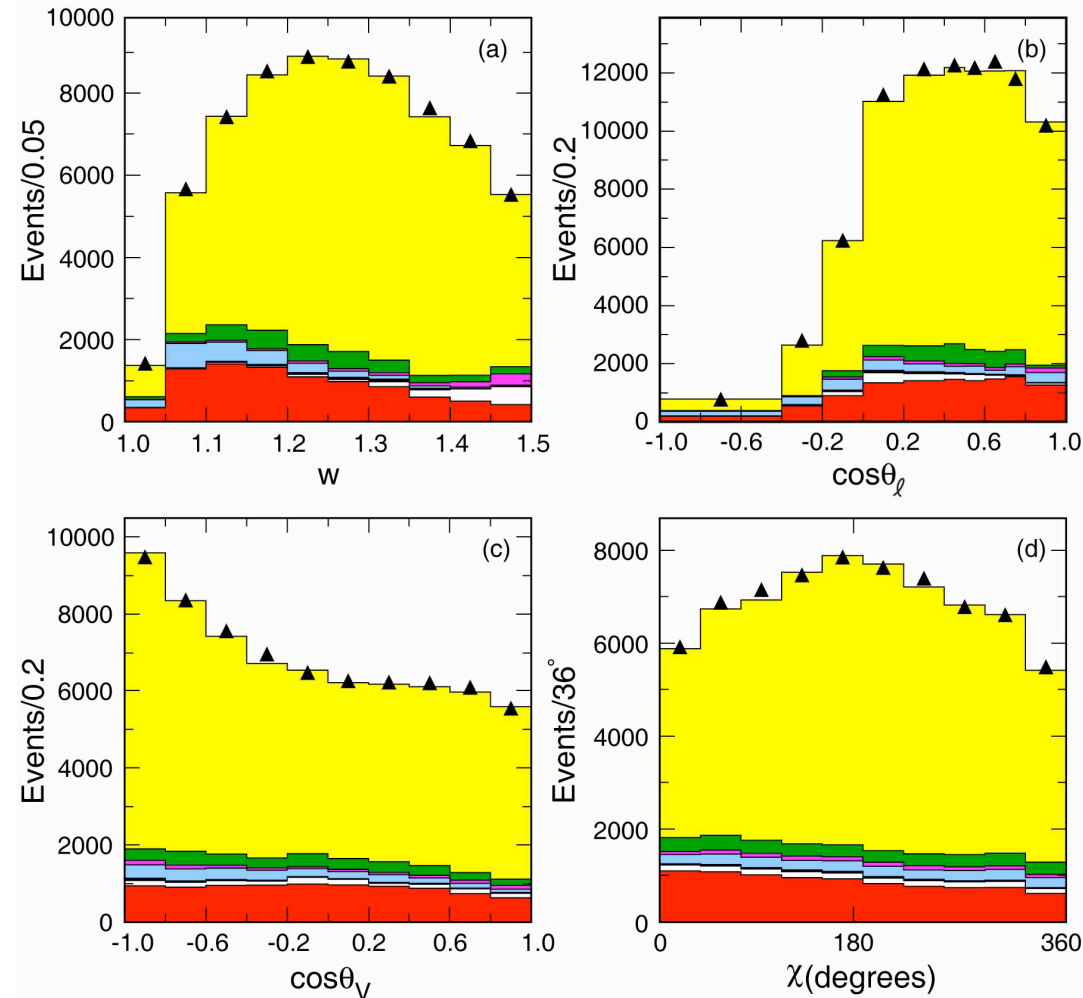
79M B/Bbar pairs



First simultaneous measurement of form factors and $|V_{cb}|$, fully accounting for all correlations.

Final results combined with Phys. Rev. D74 092004 (which uses a full 4D fit) to give:

$$\begin{aligned} \rho^2 &= 1.191 \pm 0.048 \pm 0.028 \\ R_1(1) &= 1.429 \pm 0.061 \pm 0.044 \\ R_2(1) &= 0.827 \pm 0.038 \pm 0.022 \\ F(1)|V_{cb}| &= (34.4 \pm 0.3 \pm 1.1) \times 10^{-3} \end{aligned}$$



$|V_{cb}|$ and form factors from

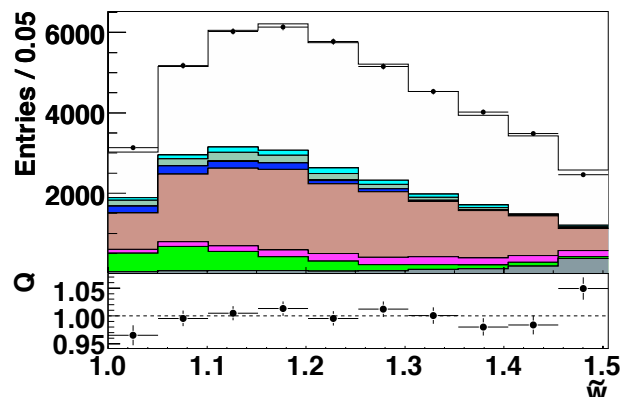
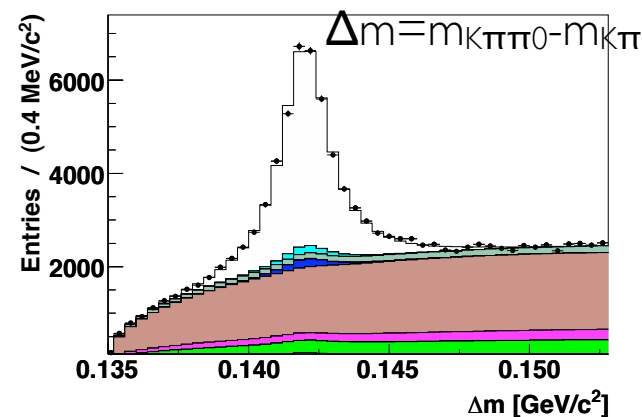
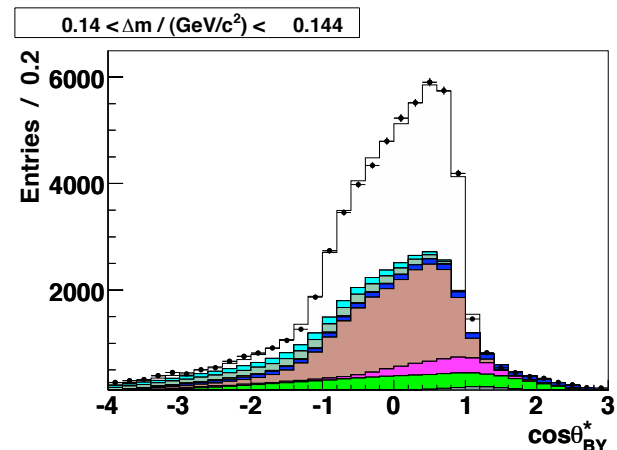


- Signal
- D^{**} (Δm -peaking)
- D^{**} (Δm -flat)
- Correlated
- Uncorrelated
- Signal-like
- $D^0 e \nu$
- Combinatorial D^{*0}
- $c\bar{c}$ events

arXiv: 0712.3493 [hep-ex]

226M B/Bbar pairs

- Look at: $D^{*0} \rightarrow \pi^0 D^0$ and $D^0 \rightarrow K^- \pi^+$
- Binned maximum likelihood fit in Δm , $\cos\theta_{BY}$ and ω^2
- Main background: mis-reconstructed $B^{\pm 0} \rightarrow D^{*0\pm} e \nu$



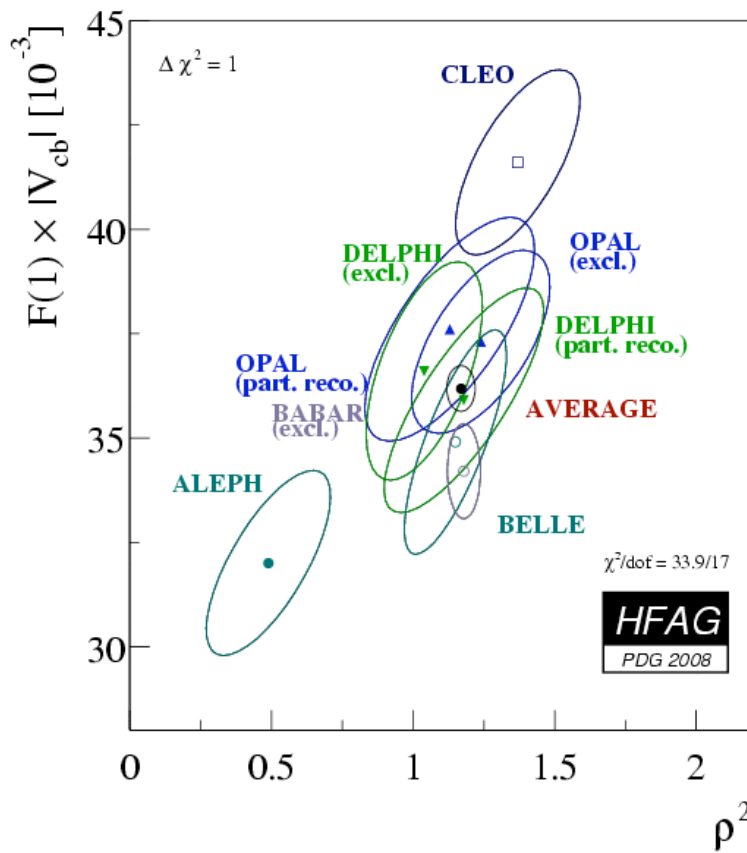
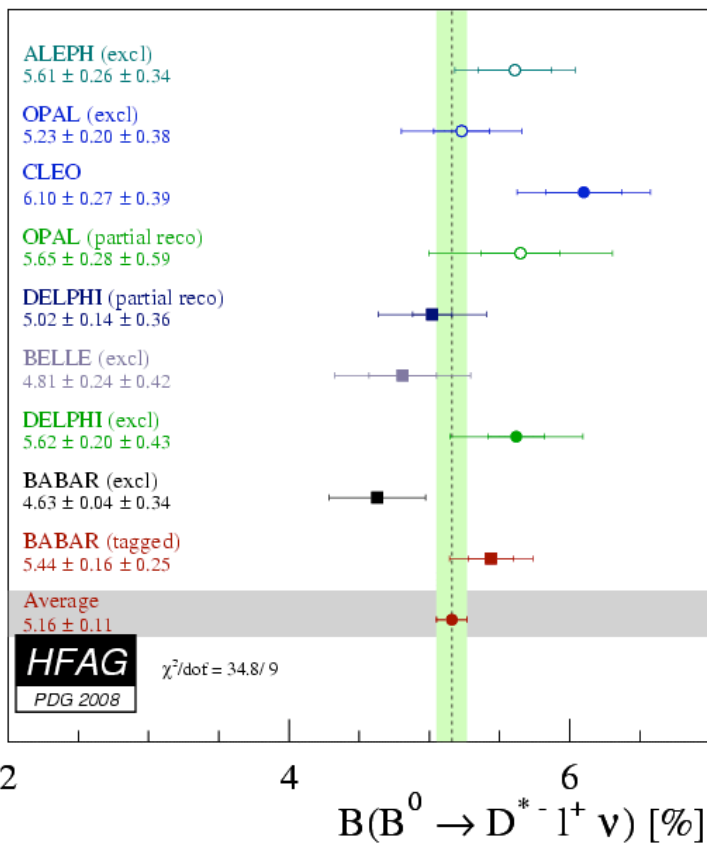
$$F(1)|V_{cb}| = (35.9 \pm 0.6 \pm 1.4) \cdot 10^{-3}$$

$$\rho^2 = 1.16 \pm 0.06 \pm 0.08$$

$$\mathcal{B}(B \rightarrow D^{*0}) = (5.56 \pm 0.08 \pm 0.41)\%$$

- Main systematic uncertainties:
 π^0 reconstruction efficiency, $BR(D^{*0} \rightarrow \pi^0 D^0)$
 $R_1(l)$ and $R_2(l)$ for ρ^2
- Complementary to $D^{*+} l \nu$ analyses

$|V_{cb}|$ World average: from $B^0 \rightarrow D^{*+} e \nu$



HFAG average:

$$F(1)|V_{cb}| = (36.18 \pm 0.55) \times 10^{-3}$$

$$\rho^2 = 1.17 \pm 0.05$$

Lattice	$F(1)^{**}$	$ V_{cb} $ exclusive	incl.* Vs excl.
Hashimoto <i>et al.</i> PRD66 104503, 2002	0.919 ± 0.033	$(39.37 \pm 0.60_{\text{exp.}} \pm 1.37_{\text{theo}}) \times 10^{-3}$	$\sim 1.6 \sigma$
Laiho <i>et al.</i> PoS LATTICE2007:358, 2006	0.930 ± 0.022	$(38.90 \pm 0.59_{\text{exp.}} \pm 0.90_{\text{theo}}) \times 10^{-3}$	$\sim 2.7 \sigma$

*Discrepancy observed w.r.t HFAG 1S scheme inclusive result $|V_{cb}| = (41.78 \pm 0.30_{\text{fit}} \pm 0.08_{\text{TB}}) \times 10^{-3}$.

Errors dominated by systematic errors in the lattice calculation of $F(1)$.

$BR(B \rightarrow D^{(*)} (\pi) / \nu)$

$B \rightarrow D^{**} / \nu$: A test of HQET

Although not directly used for $|V_{cb}|$, measurements of $B \rightarrow D^{**} / \nu$ are important for systematics in almost all semileptonic B analyses.

They also provide an essential test of HQET, relied upon for exclusive $|V_{cb}|$ determination.

- In the heavy quark limit (m_b and $m_c \rightarrow \infty$), only two independent form factors:

$\tau_{1/2}$ and $\tau_{3/2}$ for $j_l = 1/2, 3/2$

$$\Gamma(B \rightarrow D_2^* / \nu ; D_1 / \nu) \propto |\tau_{3/2}(w)|^2$$

$$\Gamma(B \rightarrow D_0^* / \nu ; D_1^* / \nu) \propto |\tau_{1/2}(w)|^2$$

Theorists consistently predict the same result:

$$\Gamma(B \rightarrow D_2^* / \nu ; D_1 / \nu) \sim 10 \times \Gamma(B \rightarrow D_0^* / \nu ; D_1^* / \nu)$$

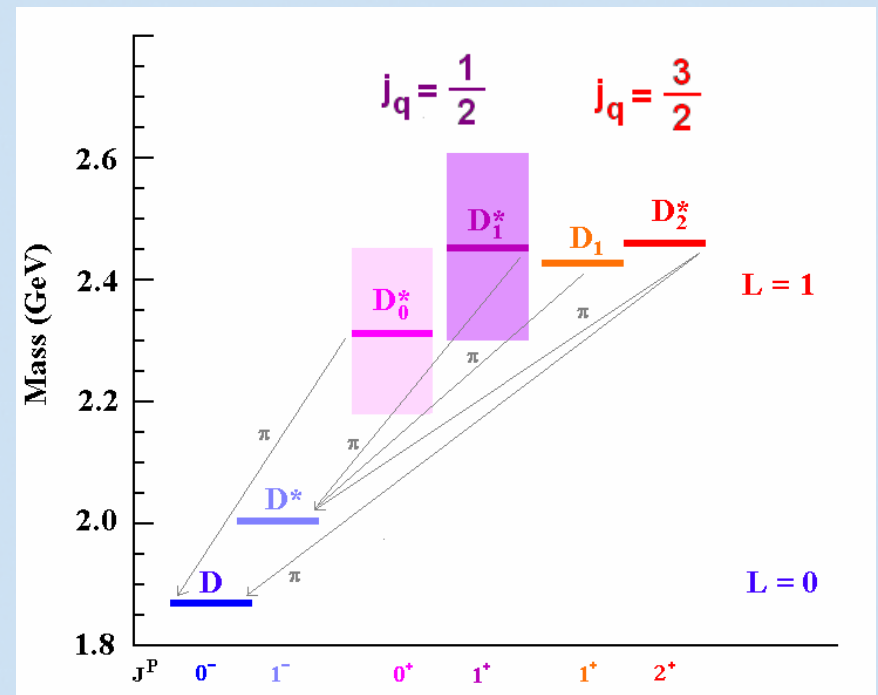
Narrow

Broad

N. Uraltsev, Phys. Lett. B501:86 (2001)

M. Neubert, Phys. Rev. D46 3914 (1992)

A. Le Yaouanc, hep-ph/0003087,



Branching Fractions of $B \rightarrow D^{(*)}\pi / \nu$



Belle

657M B/Bbar pairs

D. Liventsev et al., PRD 77, 091503 (2008)

- Reconstruct e, μ and $D^{(*)0(\pm)}$ in the recoil side of a B Beam event.

$P_{\text{lepton}} > (0.6 \text{ GeV BaBar}, 1.0 \text{ GeV Belle}).$

- Fitting missing mass squared:

$$M_{\nu}^2 = (P_{\text{beam}} - P_{B_{\text{tag}}} - P_{B_{\text{sl}}})^2$$

- Backgrounds are subtracted using data:

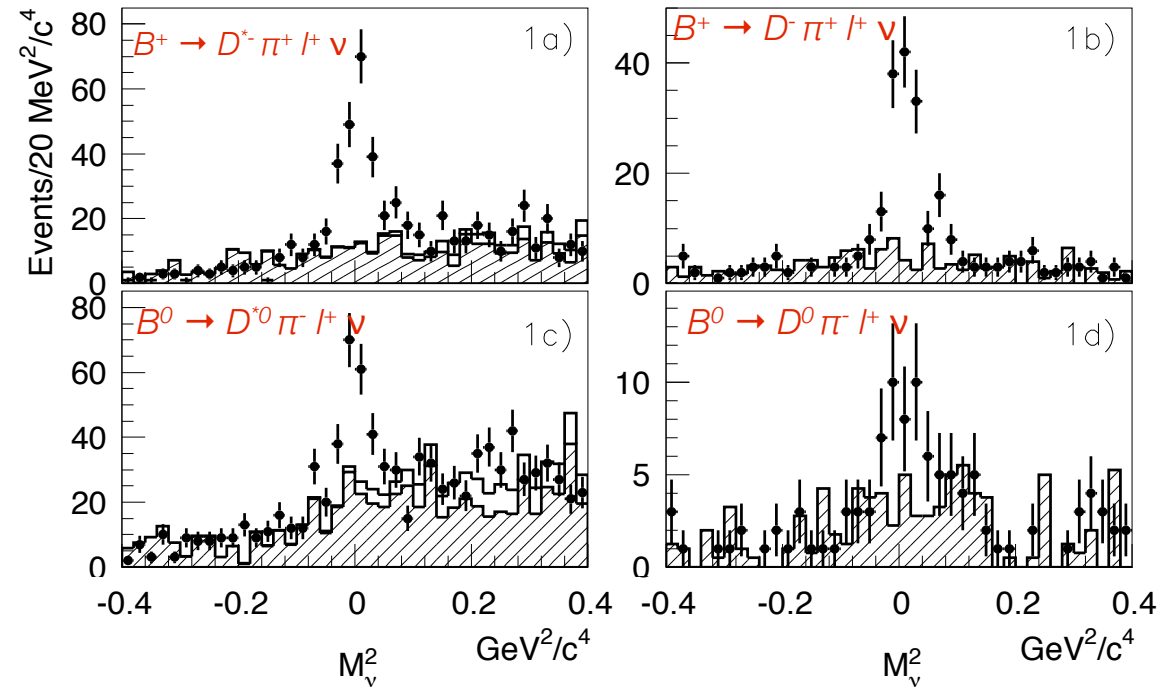
$$\Delta E \equiv E_{\text{tag}} - E_{\text{beam}} \quad M(D_{\text{sl}}) \text{ sidebands}$$

$$D^{(*)}\pi h^+ \text{ fakes}$$

- Feed-down subtracted using MC

- Branching ratios are calculated relative to the normalization modes to cancel out the Btag reconstruction efficiency

$\Delta E + M(D) - \Delta E, M(D)$ sidebands
 Fakes, Feed down



$$|M_{\nu}^2| < 0.1 \text{ GeV}^2$$

Branching Fractions of



657M B/Bbar pairs

D. Liventsev et al., PRD 77, 091503 (2008)

- Very clear peaks for narrow D1 and D2 states
- Large BR for D_0^* , but no evidence for D_1'

$$\mathcal{B}(\text{mode}) \equiv \mathcal{B}(B \rightarrow D^{**} \ell \nu) \times \mathcal{B}(D^{**} \rightarrow D^{(*)} \pi^+)$$

D π invariant mass study

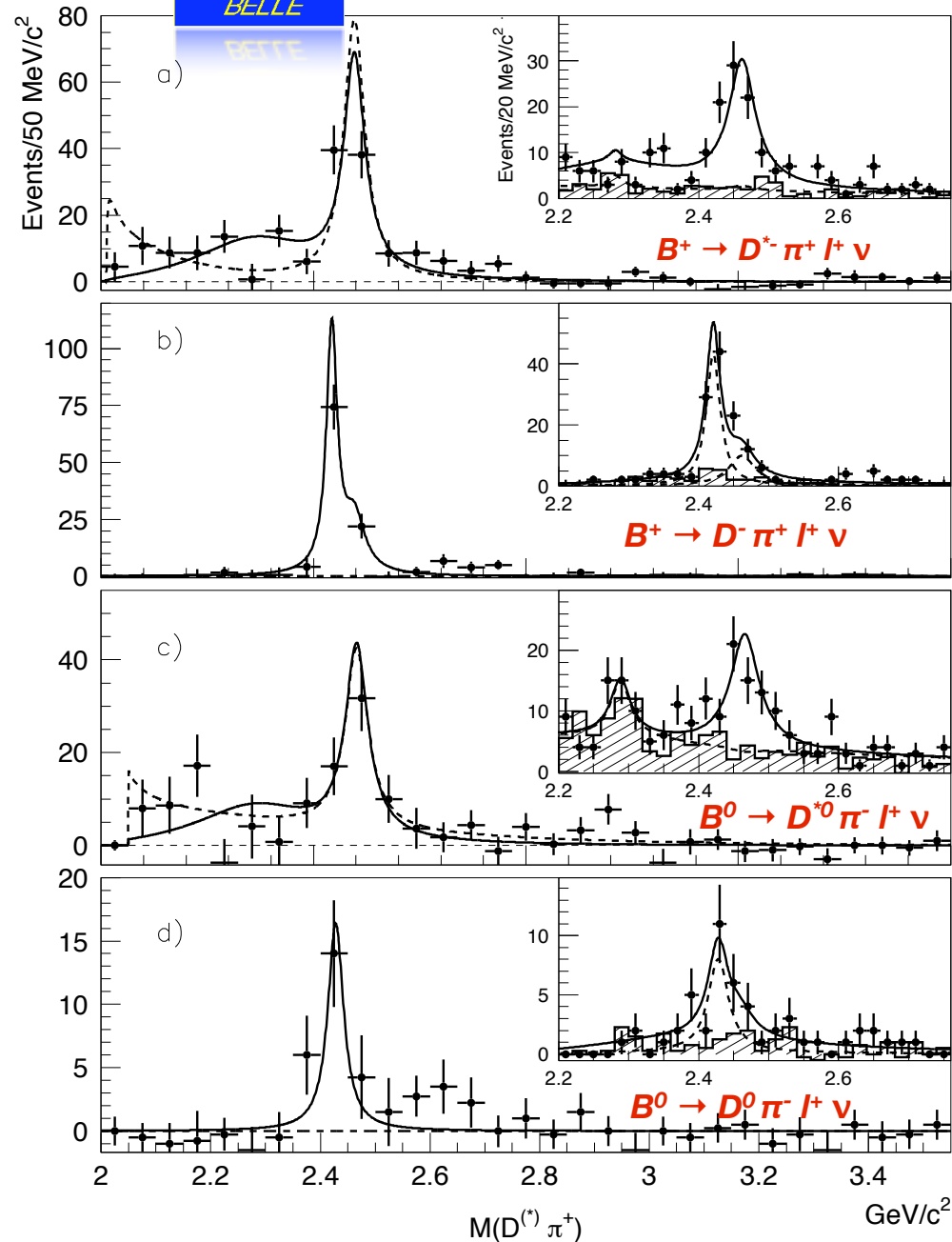
Mode	Yield	\mathcal{B} , %	Signif.
$B^+ \rightarrow \bar{D}_0^{*0} \ell^+ \nu$	102 ± 19	$0.24 \pm 0.04 \pm 0.06$	5.4
$B^+ \rightarrow \bar{D}_2^{*0} \ell^+ \nu$	94 ± 13	$0.22 \pm 0.03 \pm 0.04$	8.0
$B^0 \rightarrow D_0^{*-} \ell^+ \nu$	61 ± 22	$0.20 \pm 0.07 \pm 0.05$	2.6
$B^0 \rightarrow D_2^{*-} \ell^+ \nu$	68 ± 13	$0.22 \pm 0.04 \pm 0.04$	5.5

D $^* \pi$ invariant mass study

Mode	Yield	\mathcal{B} , %	Signif.
$B^+ \rightarrow \bar{D}_1^{*0} \ell^+ \nu$	-5 ± 11	< 0.07 @ 90% C.L.	
$B^+ \rightarrow \bar{D}_1^0 \ell^+ \nu$	81 ± 13	$0.42 \pm 0.07 \pm 0.07$	6.7
$B^+ \rightarrow \bar{D}_2^{*0} \ell^+ \nu$	35 ± 11	$0.18 \pm 0.06 \pm 0.03$	3.2
$B^0 \rightarrow D_1^{*-} \ell^+ \nu$	4 ± 8	< 0.5 @ 90% C.L.	
$B^0 \rightarrow D_1^- \ell^+ \nu$	20 ± 7	$0.54 \pm 0.19 \pm 0.09$	2.9
$B^0 \rightarrow D_2^{*-} \ell^+ \nu$	1 ± 6	< 0.3 @ 90% C.L.	



Belle



- dominant syst due to Breco $\mathcal{B}(D^{(*)0(\pm)})$ and track reconstruction.

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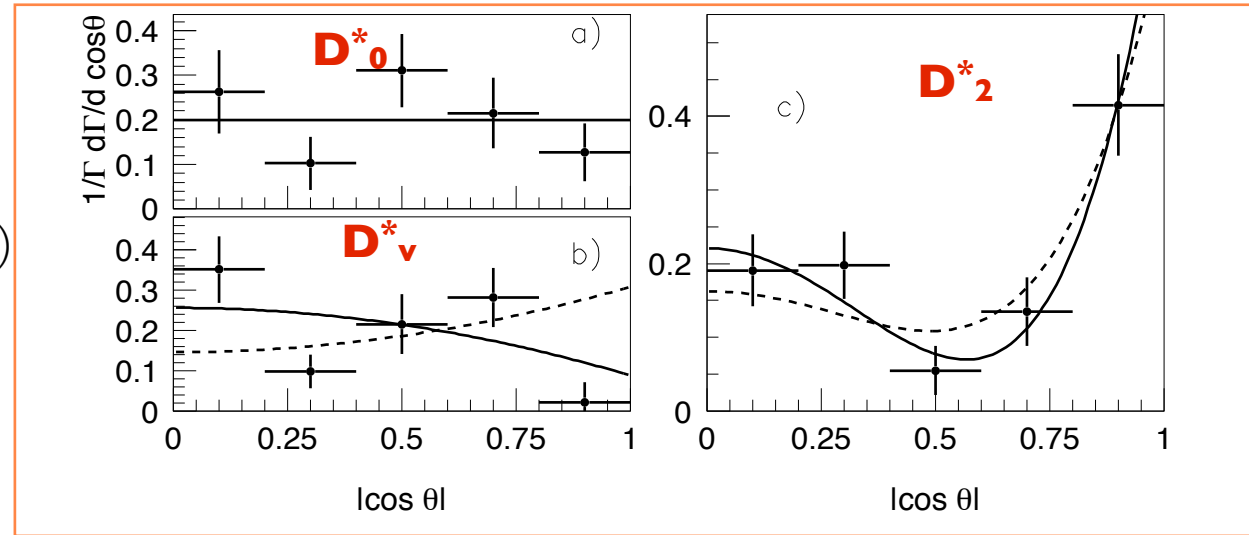
Helicity distributions of $(B \rightarrow D_0^*, D_2^* \pi / \nu)$



Belle

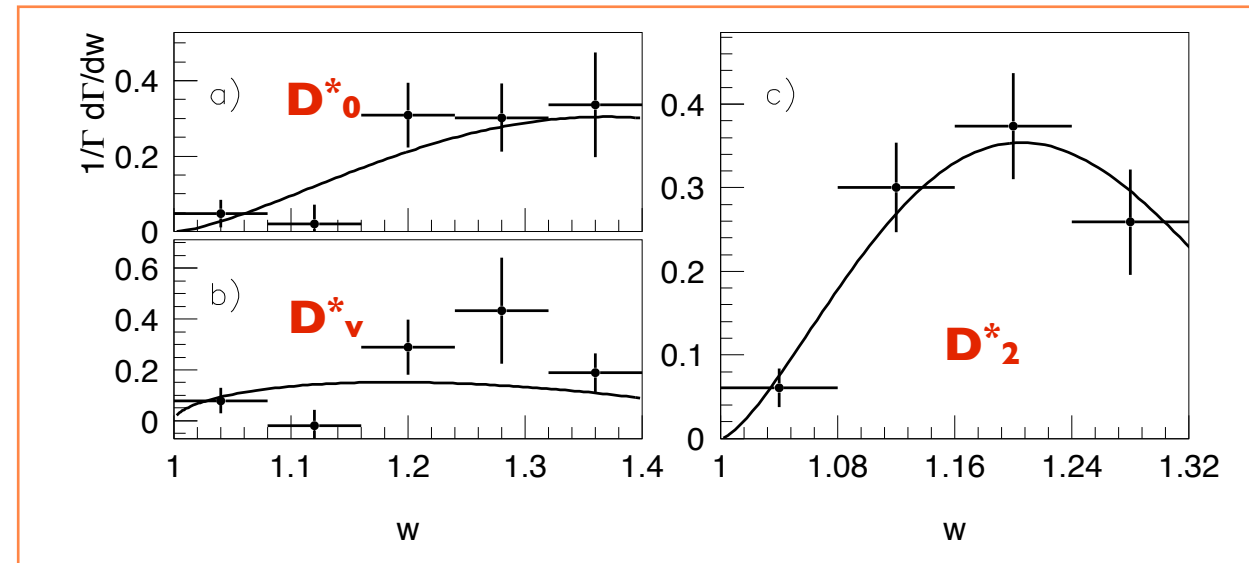
D. Liventsev et al., PRD 77, 091503 (2008)

- $D\pi$ invariant mass fit in bins of helicity.
- θ angle between
 - π in D^{**} rest frame and D^{**} boost vector



- Helicity distributions for D^{*0} and D^{*2} fit with

- $J=0$ ($\chi^2/\text{ndf} = 6.0/4$)
- $J=2$ ($\chi^2/\text{ndf} = 2.0/3$)



Branching Fractions of $B \rightarrow D^{(*)}(\pi) l \bar{\nu}$

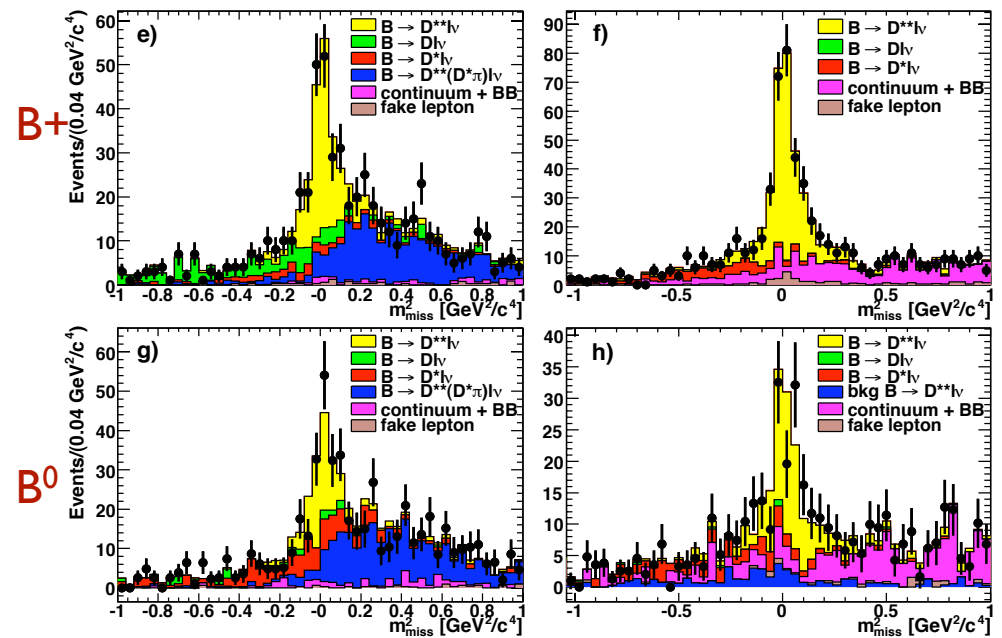
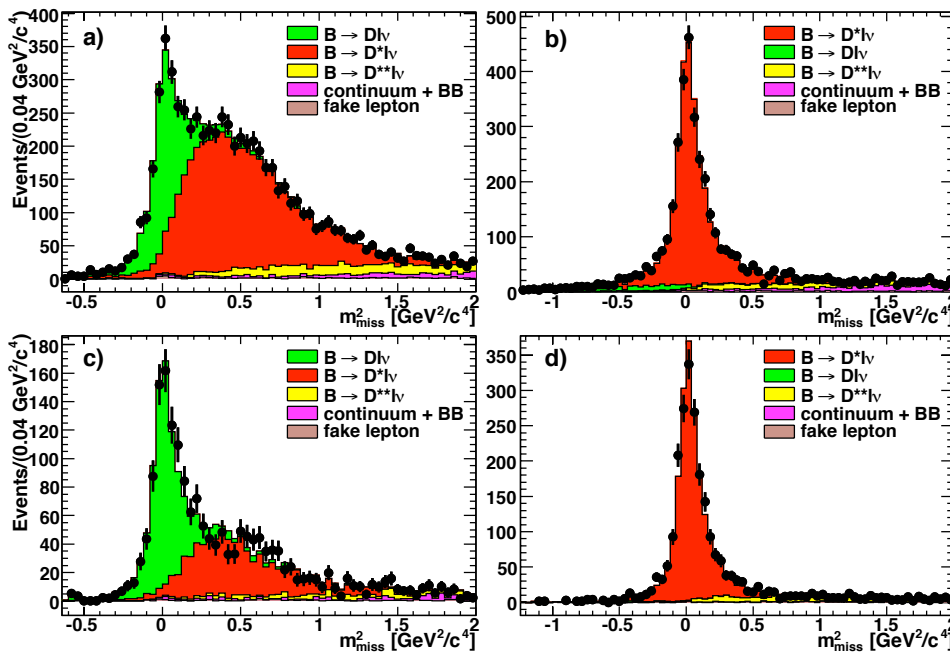


341M B/Bbar pairs

B. Aubert *et al.*, PRL 100, 151802 (2008)

Mode	$\mathcal{B}(B^-)$ [%]	$\mathcal{B}(\bar{B}^0)$ [%]
$Dl^- \bar{\nu}_\ell$	$2.33 \pm 0.09 \pm 0.09$	$2.21 \pm 0.11 \pm 0.12$
$D^*l^- \bar{\nu}_\ell$	$5.83 \pm 0.15 \pm 0.30$	$5.49 \pm 0.16 \pm 0.25$
$D\pi^\pm l^- \bar{\nu}_\ell$	$0.42 \pm 0.06 \pm 0.03$	$0.43 \pm 0.08 \pm 0.03$
$D^*\pi^\pm l^- \bar{\nu}_\ell$	$0.59 \pm 0.05 \pm 0.04$	$0.48 \pm 0.08 \pm 0.04$
$\mathcal{B}(B^- \rightarrow D^{(*)}\pi l^- \bar{\nu}_\ell) = (1.52 \pm 0.12_{stat.} \pm 0.10_{syst.})\%$		
$\mathcal{B}(\bar{B}^0 \rightarrow D^{(*)}\pi l^- \bar{\nu}_\ell) = (1.37 \pm 0.17_{stat.} \pm 0.10_{syst.})\%$		

Results consistent with isospin symmetry

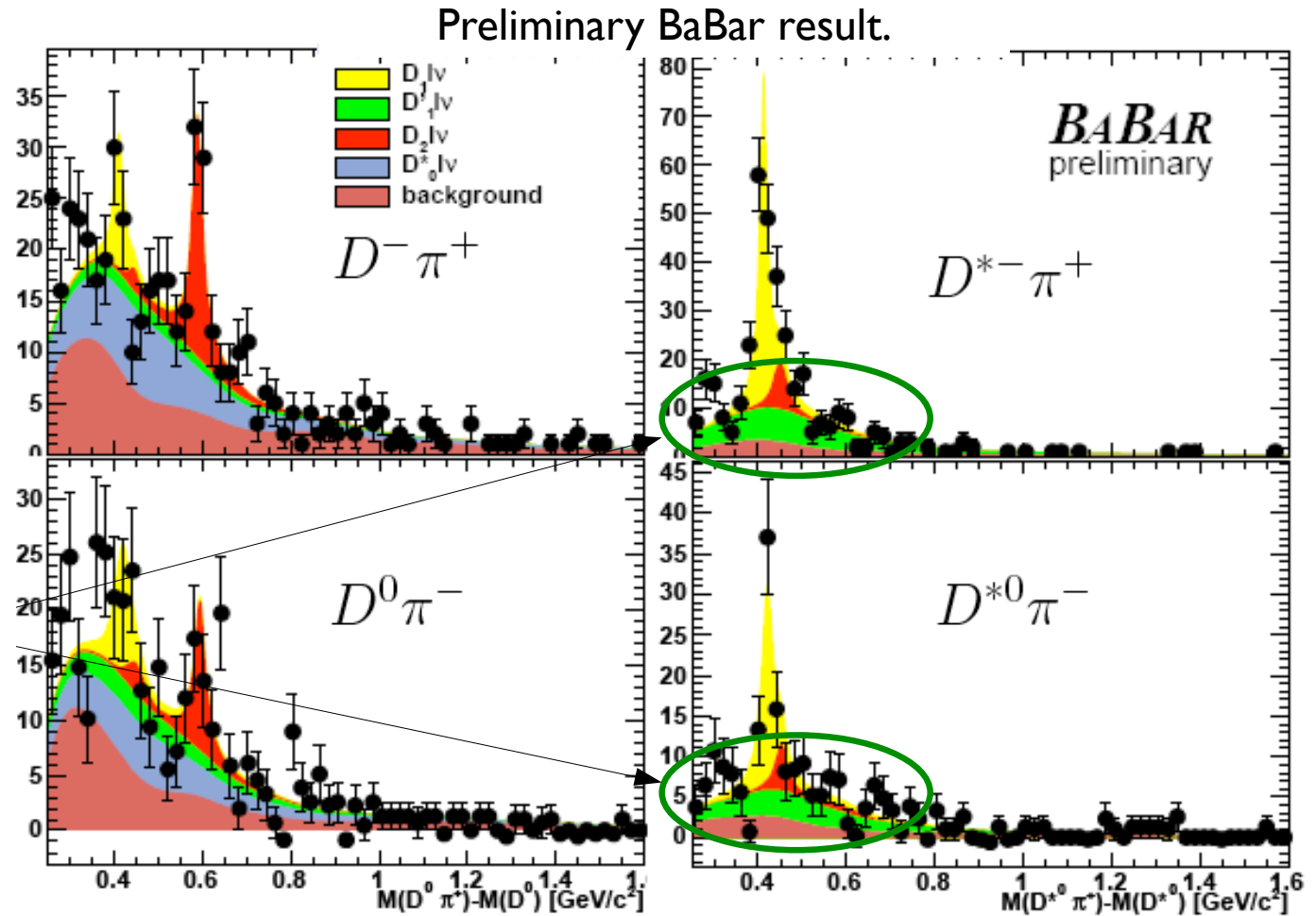


Fit to $B \rightarrow D^{**} | \nu$ components

BaBar



- Simultaneous fit to all 4 channels, including crossfeeds.
- BG normalised from fits to B_{tag} mass.
- Large BR for wide states - in disagreement with HQET.



$B \rightarrow D^{**} \ell \bar{\nu}$ Branching fraction results

Decay Mode	$\mathcal{B}(\bar{B} \rightarrow D^{**} \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D^{**} \rightarrow D^{(*)} \pi) \%$ (BELLE)	BABAR Branching Fraction
$D\pi$ invariant mass fit		
$B^- \rightarrow D_0^{*0} \ell^- \bar{\nu}_\ell$	$0.24 \pm 0.04 \pm 0.06$	$0.28 \pm 0.05 \pm 0.04$
$B^- \rightarrow D_2^{*0} \ell^- \bar{\nu}_\ell$	$0.22 \pm 0.03 \pm 0.04$	$0.16 \pm 0.03 \pm 0.01$
$\bar{B}^0 \rightarrow D_0^{*+} \ell^- \bar{\nu}_\ell$	$0.20 \pm 0.07 \pm 0.05$	$0.47 \pm 0.09 \pm 0.07$
$\bar{B}^0 \rightarrow D_2^{*+} \ell^- \bar{\nu}_\ell$	$0.22 \pm 0.04 \pm 0.04$	$0.08 \pm 0.04 \pm 0.02$
$D^* \pi$ invariant mass fit		
$B^- \rightarrow D_1^0 \ell^- \bar{\nu}_\ell$	< 0.07 @ 90CL	$0.27 \pm 0.05 \pm 0.05$
$B^- \rightarrow D_1^0 \ell^- \bar{\nu}_\ell$	$0.42 \pm 0.07 \pm 0.07$	$0.29 \pm 0.03 \pm 0.03$
$B^- \rightarrow D_2^{*0} \ell^- \bar{\nu}_\ell$	$0.18 \pm 0.06 \pm 0.03$	$0.07 \pm 0.01 \pm 0.01$
$\bar{B}^0 \rightarrow D_1^+ \ell^- \bar{\nu}_\ell$	< 0.5 @ 90CL	$0.37 \pm 0.07 \pm 0.05$
$B^0 \rightarrow D_1^+ \ell^- \bar{\nu}_\ell$	$0.54 \pm 0.19 \pm 0.09$	$0.25 \pm 0.05 \pm 0.03$
$\bar{B}^0 \rightarrow D_2^{*+} \ell^- \bar{\nu}_\ell$	< 0.3 @ 90CL	$0.04 \pm 0.02 \pm 0.01$

Belle: non-resonant $D^{(*)} \pi \ell \bar{\nu}$ consistent with zero.

BaBar: non-resonant fixed to zero in fit.

D_1' BR > 6 sigma significance in BaBar; not found in Belle.

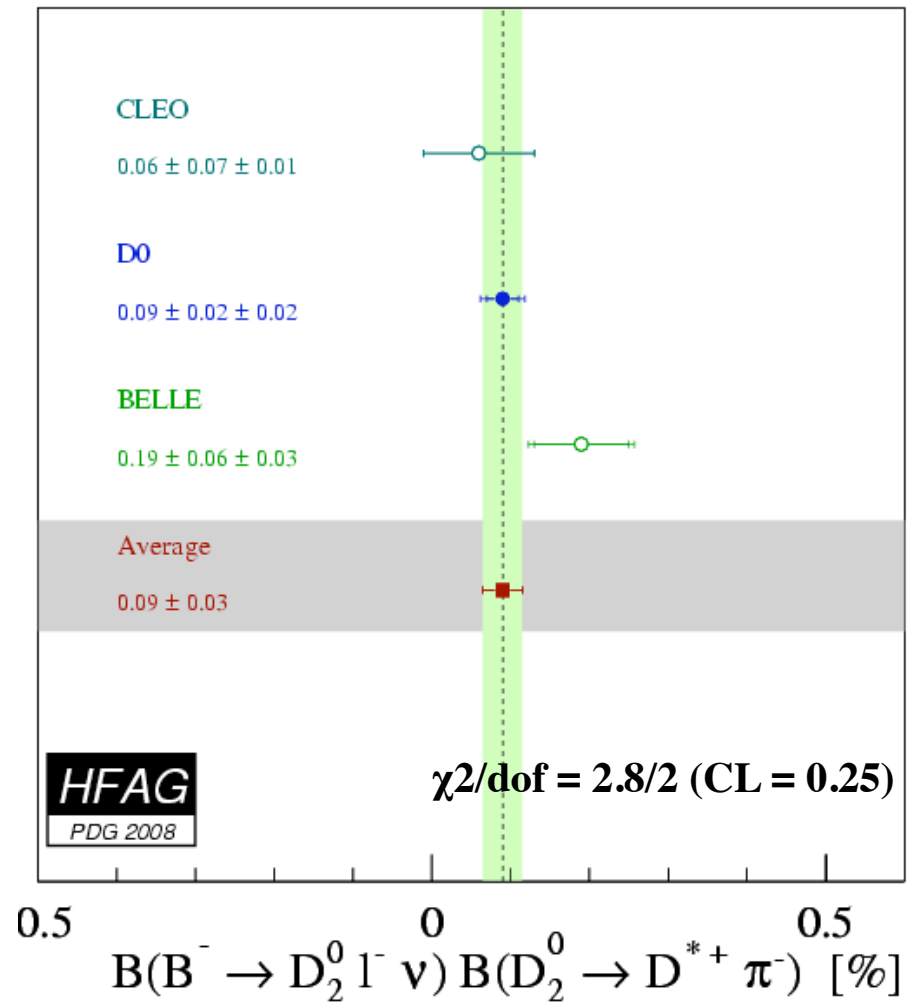
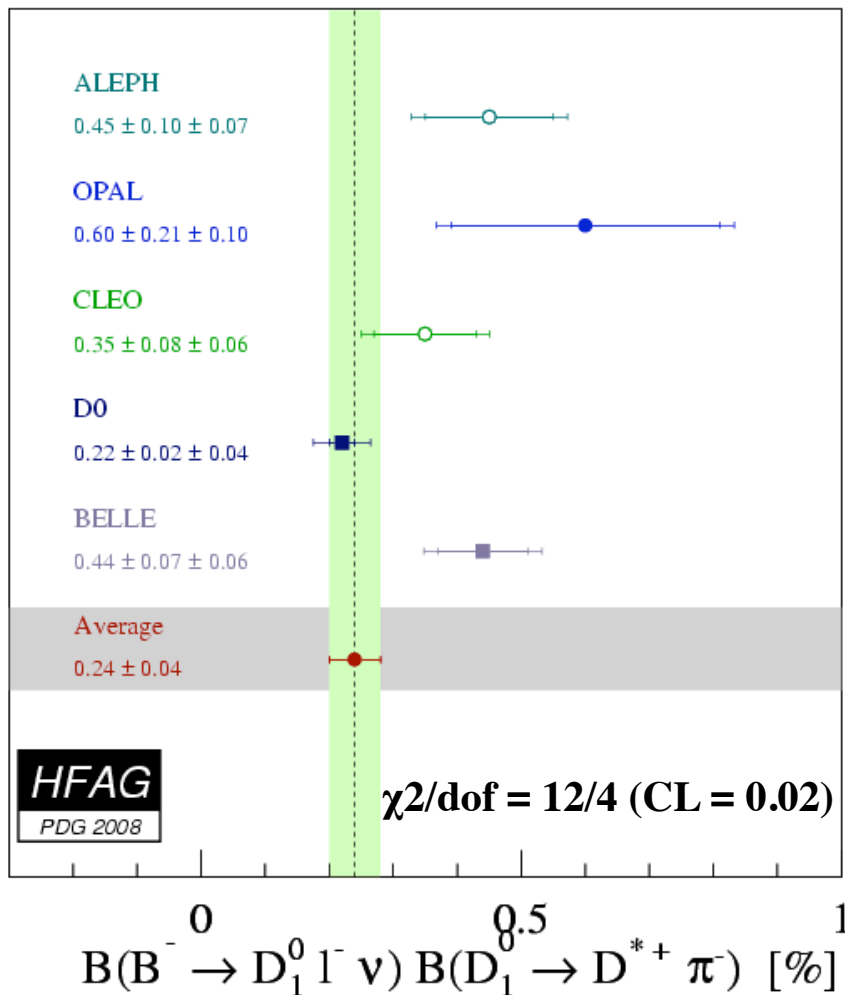
World averages of D^{**} / ν measurements

- Many measurements of narrow states, with the common assumptions:

$$\mathcal{B}(b \rightarrow B) = 37.8 - 39.7\%$$

$$\mathcal{B}(D_1 \rightarrow D^* \pi^+) = 66.7\%$$

$$\mathcal{B}(D_2^* \rightarrow D^* \pi^+) = 20\%$$



Inconsistency with HQET

2 big questions...

Experiment:

There's a big hole in the measurement of the exclusive X_c components. What's missing?

Theory:

Why is there such a discrepancy between HQET prediction and measurement for the narrow and wide state BR?

BR[%] HFAG ave.	B^0	B^+
D^*	5.16 ± 0.11	6.07 ± 0.29
D	2.17 ± 0.12	2.27 ± 0.11
$D\pi$	0.43 ± 0.06	0.42 ± 0.05
$D^*\pi$	0.49 ± 0.08	0.61 ± 0.05
Σ Exclusive (measured)	8.25 ± 0.19	9.37 ± 0.32
inclusive	10.33 ± 0.28	10.99 ± 0.28
Inclusive - Σ Exclusive	2.08 ± 0.34	1.62 ± 0.42

Phys. Lett. B609, 298 (2005)

Quenched lattice computations of using static heavy quarks (infinitely massive b and c) give:

$$\tau_{1/2}(1) = 0.41(5) < \tau_{3/2}(1) = 0.57(10)$$

BPS symmetry (N. Uraltsev, hep-ph/0312001)

Use an approximate non-perturbative symmetry for heavy mesons, predicting $|\tau_{1/2}(w)|^2 = 0$.

Jugeau et al., hep-ph/0407176

HQET rules apply at $w=1$ (zero recoil) and infinite mass. Extensions have been performed to first order in $1/m_c$ and all w .

N. Isgur, hep-ph/9811377

Isgur predicted a non-negligible non-resonant contribution 10 years ago!

Conclusions

$D^{(*)}$ and exclusive $|V_{cb}|$

$$B^- \rightarrow D^{0*} | \nu$$

New measurement with simultaneous extraction of form factor parameters.

$$|V_{cb}| = (39.37 \pm 0.60 \pm 1.37) 10^{-3} \sim 1.5 \sigma \text{ difference to inclusive results.}$$

\Rightarrow uncertainty from $F(1)$ is dominant, requiring more input from Lattice!

$$\text{BR} (B^- \rightarrow D^{0*} | \nu)$$

no different to expectation from $\text{BR} (B^0 \rightarrow D^{+*} | \nu)$

D^{**} Narrow, $B \rightarrow D_1/D_2 | \nu$ decays observed and properties studied.

Large BR for *wide* $B \rightarrow D_0^* | \nu$ observed by BaBar **and** Belle, contradicting HQET predictions.

BaBar has found the D_1' state, not yet seen by Belle.

But, if $j_f=1/2$ states dominate there is a problem with HQET!

15-20 % of the semileptonic decay rate is not allocated.

Experiment

Heavier states or non-resonant contributions?

Need further analysis of broad states and what saturates the inclusive sum.

Theory

Missing $1/m_c$ corrections in theoretical predictions.

Need unquenched lattice calculations.

If we do not control $B \rightarrow D^{**} | \nu$ can we fully trust the HQET based extraction of exclusive $|V_{cb}|$?

backup

BaBar $B \rightarrow D^{**} \ell \bar{\nu}$ systematics.

	Systematic uncertainty on $\Gamma(\bar{B} \rightarrow D^{**}(D^{(*)}\pi)\ell^{-}\bar{\nu}_\ell)/\Gamma(\bar{B} \rightarrow X\ell^{-}\bar{\nu}_\ell)$			
	$B \rightarrow D_1\ell^{-}\bar{\nu}_\ell$	$B \rightarrow D'_1\ell^{-}\bar{\nu}_\ell$	$B \rightarrow D_2^*\ell^{-}\bar{\nu}_\ell$	$B \rightarrow D_0^*\ell^{-}\bar{\nu}_\ell$
Tracking efficiency	1.5	1.9	1.2	1.6
Neutral reconstruction	2.6	1.8	1.2	0.9
lepton ID	1.2	1.2	1.2	1.5
Soft particle efficiency	1.2	1.2	0.4	-
Monte Carlo corrections				
$B^0 - B^-$ cross-feed	0.3	0.3	0.2	0.2
$D^{(*)}$ Form factors	0.8	0.8	0.5	0.4
D^{**} Form factors	1.0	2.5	1.2	2.0
$D^{(*)}$ branching fractions	3.9	3.9	3.8	3.8
$\bar{B} \rightarrow X\ell^{-}\bar{\nu}_\ell$ branching fraction	1.7	1.7	1.7	1.7
B_{tag} selection	4.8	4.8	4.7	4.6
Fit technique				
$\bar{B} \rightarrow X\ell^{-}\bar{\nu}_\ell$ yield	0.7	0.7	0.7	0.7
$\bar{B} \rightarrow D^{**}\ell^{-}\bar{\nu}_\ell$ yield	5.6	10.6	7.4	8.5
Total systematic error	9.3	13.0	10.0	11.0
Statistical error	9.9	12.2	15.2	13.1