Top Mass Measurements at the Tevatron

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We present the latest measurements of the top quark mass from the Tevatron. The different top decay channels and measurement techniques used for these results are also described. The world average of the top quark mass based on some of these new results combined with previous results is $m_{\text{top}} = 172.6 \pm 1.4$ GeV.

1. Introduction

The discovery of the top quark by the DØ and CDF collaborations over a decade ago was one of the crowning achievements of Fermilab’s Tevatron collider [1]. It was a testament to the tremendous success of the standard model as a theory of particle interactions. Over the years, much effort has been devoted to the measurement and understanding of its properties. Of these, the single property that has perhaps received the most attention is the mass. Over 30 times larger than the $b$ quark mass, the sheer magnitude of the top quark mass suggests a special role in the mechanism of electroweak symmetry breaking. The current interest in a precise measurement of the top quark mass, together with that of the $W$ boson mass, is driven by the fact that they provide powerful constraints on the mass of the Higgs boson through radiative corrections, restricting the range in which to conduct searches [2]. In addition it can also help constrain possible extensions of the standard model.

We present, in this talk, the latest measurements from DØ and CDF of the top quark mass using different analysis techniques and top quark decay channels. These results are based on up to 2 fb$^{-1}$ of data collected at Fermilab’s Tevatron.

2. Decay Channels

The top quark mass measurements presented here are based on top quarks that are produced as $t\bar{t}$ pairs via the strong interaction. In the Standard Model, each top quark in the pair decays into a $W$ boson and a $b$ quark nearly 100% of the time resulting in a $W^+W^-b\bar{b}$ final state. The subsequent decays of the two $W$ bosons define the different top quark decay channels or topologies.

The first of these decay channels is the dilepton channel in which both $W$ bosons decay “leptonically” into a lepton and its associated neutrino. This is the cleanest channel with the lowest background levels dominated by Drell-Yan processes with associated jets. Additional sources include diboson production with jets, and $W^+ \geq 3$ jet production with one jet misidentified as a lepton. Despite major drawbacks which include the lowest branching fraction ($\sim 5\%$ for $e\nu$ and $\mu\nu$ modes) and the presence of two undetected neutrinos, it is an important channel because the presence of fewer jets and the sensitivity to different background sources provide a useful cross check to measurements obtained from the other channels.

The second decay channel is the hadronic channel in which both $W$ bosons decay “hadronically” into two quarks resulting in at least six jets in the event. The main advantages of this channel are the fact that it has the largest branching fraction ($\sim 44\%$) and that the jets evolving from all six quarks in the final state can, in principle, be measured. Furthermore, because the two $W$ bosons decay hadronically, the well known mass of the $W$ boson can be used to constrain the measured jet energies in order to reduce the uncertainty on the jet energy scale which is a dominant source of systematic uncertainty. The major disadvantages of this channel are the large amount of background from multijet events and the combinatoric problem when assigning jets to parton identities.

The final decay channel is the lepton+jets ($\ell+jets$) channel where one $W$ boson decays hadronically and the other leptonically. Drawing from the strengths of the other two channels, it combines a decent branching fraction of $\sim 29\%$ and a manageable level of background from $W^+j$ets and multijet events. It also benefits from an in-situ jet energy calibration due to the presence of a hadronically decaying $W$ boson. This channel has yielded the most precise measurements of the top quark mass.

3. A Challenging Measurement

Top quark mass measurements are challenging because, except for the leptons from the $W$ boson decay in two of the channels, the daughters of the top and $W$ boson are not identified directly. What is measured in the detector are the high transverse momentum jets evolving from the quarks and a large amount of missing energy signifying the presence of neutrinos. In general, one does not know how to associate the jets with the quarks and must try all possible combinations. The use of $b$-tagging helps greatly in reducing the combinatorics. Although detached vertices are associated with the $b$-jets, there are none associated with the short-lived top quark itself that can be used...
to cleanly identify daughters in the same way they are used, for instance, in mass measurements involving long lived particles like hyperons. In the following section, we discuss the different analysis techniques that make precise measurements of the top quark mass possible despite such challenges.

4. Analysis Techniques

One of the most commonly used analysis techniques is the template method. This method makes use of a variable that is sensitive to the top quark mass. An obvious choice of this variable used by many analyses is the reconstructed top quark mass. Monte Carlo (MC) samples of \( \bar{t}t \) events generated with different values of the top quark mass are then used to produce distributions of the selected variable called templates. Since each of these templates is associated with a particular value of the true top quark mass used to generate the events, one can extract the top quark mass from the data by comparing each of the templates to the corresponding data distribution. To do this, one can either fit the templates directly to the data distribution or parametrize the templates as probability density functions from which likelihoods can be constructed that are used to extract the mass.

The analysis technique that has yielded the most precise top quark mass measurements to date is the Matrix Element (ME) method which was pioneered by DØ in Run I of the Tevatron using the \( \ell + \text{jets} \) channel [3]. In this method, the probability to observe a given event is calculated by taking into account the different physics processes that could contribute to the observed event. The total probability is simply the properly normalized sum of the differential cross sections for each contributing process calculated from the MEs characterizing the process which gives the technique its name. The probabilities are calculated for each event as a function of the parameters one wishes to extract such as the top quark mass and jet energy scale factor. A joint likelihood is constructed from the probability distribution of each event from which the best estimate of each parameter and its uncertainty can be determined. The power of this technique is based on the maximal use of the available kinematic information in order to fully specify an event. While providing discrimination between different events it also deals with the combinatoric problem within each event mentioned above by properly weighting each jet permutation based on its probability. Furthermore, detector resolution effects are also accounted for with transfer functions which give the probability density for a measured value of a given quantity as a function of the true parton value.

Both techniques described above rely on fully simulated MC \( \bar{t}t \) events for calibrating the procedure in order to determine the true value of the top quark mass from the extracted value. The MC generators currently used for this purpose are all based on leading order (LO) matrix elements with higher orders simulated by parton showers leading to a concept of the top quark mass that is not well defined. To address this, a third technique extracts the top quark mass by comparing the theoretically predicted and experimentally measured values of the \( \bar{t}t \) production cross section. Theory and measurement likelihoods which are defined as a function of the top quark mass and \( \bar{t}t \) production cross section are multiplied together to construct a joint likelihood. Integrating over the \( \bar{t}t \) production cross section then gives a one dimensional likelihood from which the top quark mass is extracted.

Finally a fourth technique addresses the fact that most top quark mass measurements are dominated by jet energy scale uncertainties. Assuming top quarks in \( pp \) collisions are produced nearly at rest, it can be shown that the Lorentz boost imparted to the \( b \) quark is a function of the top quark mass. Because of this, the velocity of the \( b \) quark and hence the \( b \) hadron is strongly correlated with the top quark mass. Therefore, the average momentum of the \( b \) hadron can be used to determine the top quark mass. In practice, this technique uses the highly correlated average transverse decay length of the \( b \) hadron instead of its average momentum. The dependence of the decay length on the top quark mass is parameterized with MC events and this parameterization is used to extract the top quark mass from data. This technique depends mainly on tracking to determine the decay length precisely and is largely insensitive to the jet energy scale uncertainties that dominate other techniques.

5. Results from the Tevatron

The first result is from DØ in the \( \ell + \text{jets} \) channel [4]. It uses the ME technique and is based on 2.1 fb\(^{-1}\) of data. The well known \( W \) mass is used as constraint to perform an in-situ jet energy calibration through the introduction of a \( JES \) parameter which is a global factor multiplying the energies of all the jets. The top quark mass (\( m_{\text{top}} \)) is then extracted by maximizing the joint likelihood simultaneously in terms of the top quark mass and the \( JES \) parameter. Figure 1 (left) shows a one dimensional projection of the likelihood onto the top quark mass axis for the second 1 fb\(^{-1}\) of data yielding \( m_{\text{top}} = 173.0 \pm 1.9(\text{stat}+\text{JES}) \pm 1.0(\text{syst}) \) GeV. Figure 1 (right) shows the corresponding distribution of expected uncertainties from MC ensemble tests with the arrow indicating the measurement uncertainty. The result for the first 1 fb\(^{-1}\) of data is \( m_{\text{top}} = 170.5 \pm 2.5(\text{stat}+\text{JES}) \pm 1.4(\text{syst}) \) GeV. Combining both results gives \( m_{\text{top}} = 172.2 \pm 1.1(\text{stat}) \pm 1.6(\text{syst}) \) GeV for the entire data set. The systematic uncertainty is dominated by the uncertainty on the \( b \)
jet energy scale and the uncertainty in the modeling of extra jets from radiation.

The second result is from CDF in the $\ell+$jets channel and is based on 1.9 fb$^{-1}$ of data [5]. It also uses the ME technique and performs an in-situ jet energy calibration in a similar way. Unlike the DØ result, it uses a neural network to discriminate between signal and background events and, in addition, also takes the angular resolution of the jets into account. Figure 2 (left) shows the two dimensional likelihood in top quark mass and jet energy scale parameter. Figure 2 (right) shows the expected uncertainty distribution from MC ensemble tests with the measurement uncertainty indicated by the arrow. The measured result is $n_{\text{top}} = 171.4 \pm 1.5(\text{stat}+\text{JES}) \pm 1.0(\text{syst})$ GeV. The dominant sources of systematic uncertainty are the modeling of initial and final state radiation and the residual uncertainty in the jet energy scale representing uncertainties that cannot be addressed by a global scale factor.

The third result is from DØ in the dilepton channel and is based on 1 fb$^{-1}$ of data [6]. It is a combination of two results, both of which are based on the template method and which differ mainly in the manner weights are assigned to the neutrino solutions. The first is based on the neutrino weighting technique which assigns a weight to each neutrino solution based on the compatibility between the calculated transverse momentum of the neutrinos and the observed missing transverse energy. The second is based on the matrix weighting technique which assigns weights based on the probability for the lepton to have the observed energy in the top quark rest frame for a given value of the top quark mass. Negative log likelihoods as a function of the top quark mass for both results are shown Figure 3. Combining both results gives a measurement of $m_{\text{top}} = 173.7 \pm 5.4(\text{stat}) \pm 3.4(\text{syst})$ GeV. The dominant sources of uncertainty are the uncertainties in the light and $b$ jet energy scales.

The fourth result is also in the dilepton channel [7]. It is from CDF and is based on 1.9 fb$^{-1}$ of data. Like the first two lepton+jets results, it is also based on the ME technique. In addition, it also uses a neural network based selection that is optimized for precision. Figure 4 (left) shows the negative log likelihood as a function of the top quark mass and Figure 4 (right) shows the distribution of expected uncertainties from MC ensemble tests with the arrow indicating the measurement uncertainty. The measured result is $m_{\text{top}} = 171.2 \pm 2.7(\text{stat}) \pm 2.9(\text{syst})$ GeV. The dominant sources of uncertainty are the uncertainties in the jet energy scale and in the MC generator.

The fifth result is from CDF in the all jets channel and is based on 1.9 fb$^{-1}$ of data [8]. It is based on the template method with background shapes that are determined from data. Like the first two $\ell+$jets results, it also takes advantage of the $W$ mass constraint to perform an in-situ jet energy calibration. The two dimensional likelihood in the jet energy scale parameter and top quark mass for data is shown in Figure 5 (left). The expected uncertainty distribution from MC ensemble tests is shown in Figure 5 (right) with the measurement uncertainty indicated by the vertical line. The measured result is $m_{\text{top}} = 177.0 \pm 3.7(\text{stat}+\text{JES}) \pm 1.6(\text{syst})$ GeV. The systematic uncertainty is dominated by the uncertainty in the background shapes and the residual uncertainty in the jet energy scale not accounted for by the jet energy scale parameter.

The sixth result uses the indirect approach of determining the top quark mass by comparing the theoretical and measured $t\bar{t}$ production cross sections [9]. It is from DØ and is based on 1 fb$^{-1}$ of data. Figure 6 (left) shows the theoretical and measured cross sections as a function of the top quark mass including the 68% C.L. contour. The top quark mass is found to be $m_{\text{top}} = 170 \pm 7$ GeV at 68% C.L. in agreement with the world average from direct measurements indicated.
by the cross hatched bar in Fig. 6.

The last result uses the decay length technique to determine the top quark mass. It is from CDF and is based on 1.9 fb$^{-1}$ of data [10]. It is a combination of two results. The first is based on the average transverse decay length as described in the previous section and the second is based on the average transverse momentum of the lepton using exactly the same technique as that for the decay length. Figure 6 (right) shows the dependence of the average transverse decay length on the top quark mass including one sigma confidence intervals determined from MC events. Also shown are the one sigma statistical uncertainties from data represented by the horizontal bar. The combination of the two results gives $m_{\text{top}} = 175.3 \pm 6.2(\text{stat}) \pm 3.0(\text{syst})$ GeV.

The world average of the top quark mass from March 2008, $m_{\text{top}} = 172.6 \pm 1.4$ GeV, based on the best Run 1 and Run 2 measurements from the Tevatron, is shown in Fig. 7 [11]. Most of the results presented above are included in this world average. The exceptions are the CDF ME $\ell$+jets (“CDF-II lepton+jets”) and $b$ decay length (“CDF-II $b$ decay length”) measurements which are from a previous version and the DØ measurement based on the $t\bar{t}$ cross section which is not included.

6. Conclusion

Although the large mass of the top quark is interesting in itself, a precise measurement of this quantity is important because of the constraint it imposes on the Higgs mass and on possible extensions of the standard model. Measuring the top quark mass is a daunting task but, fortunately, various sophisticated techniques have been developed that make the precise measurements presented here possible. For consistency, these measurements are performed in the different top decay channels which are sensitive to different systematic effects and background sources. In addition to the ME and template based measurements, also included are two measurements based on interesting approaches such as the DØ cross section technique and the CDF $b$ decay length technique. These results still have large uncertainties but seem promising and complement the measurements based on the ME and template methods. As the total uncertainty on the world average top quark mass quickly approaches 1 GeV, it is now more important than ever to continue refining

Figure 2: CDF result in the $\ell$+jets channel using the ME method. 2D likelihood in top quark mass and jet energy scale parameter for data (left). Expected uncertainty distribution from MC ensemble tests (right).

Figure 3: DØ results in the dilepton channel. -Log likelihoods for data as a function of top quark mass using the neutrino weighting method (left) and matrix weighting method (right).
measurement techniques. This will require deepening our understanding of the systematic uncertainties and hopefully developing new ways to control the dominant ones.

References

3 V. Abazov et al. (D0 Collaboration), Nature 429, 638 (2004).
4 The DØ Collaboration, DØ Note 5610-CONF (2008).
6 The DØ Collaboration, DØ Note 5460 (2007).
7 The CDF Collaboration, CDF note 9130 (2007).
8 The CDF Collaboration, CDF note 9165 (2008).
10 The CDF Collaboration, CDF note 9414 (2008).
Figure 6: Comparison of measured and theoretically predicted cross sections versus top quark mass from DØ (right). Expected average transverse decay length including $1\sigma$ confidence intervals as a function of top quark mass from CDF (right).

Figure 7: World average top quark mass from Tevatron results.