B Lifetimes and Mixing

H. Evans
Indiana University, Bloomington, IN, USA – for the CDF and D0 Collaborations

The Tevatron experiments, CDF and D0, have produced a wealth of new \( B \)-physics results since the start of Run II in 2001. We’ve observed new \( B \)-hadrons, seen new effects, and increased many-fold the precision with which we know the properties of \( b \)-quark systems. In these proceedings, we will discuss two of the most fruitful areas in the Tevatron \( B \)-physics program: lifetimes and mixing. We’ll examine the experimental issues driving these analyses, present a summary of the latest results, and discuss prospects for the future.

1. Introduction

The Tevatron has been a hotbed of \( B \)-physics activity since the start of Run II in 2001. Although \( pp \) collisions at 1.96 TeV present a much more challenging environment than that seen at the B-factories, the fact that all types of \( B \)-hadrons are produced in CDF and D0 makes the Tevatron \( B \)-physics program complementary to those of BaBar, Belle, and CLEO. We will concentrate on two areas of this program: measurements of \( B \)-lifetimes and the determination of the oscillation frequencies between neutral \( B \)-mesons – \( B \)-mixing. Beside their intrinsic interest, these two topics are representative of the breadth of physics beyond the Standard Model, the CKM matrix, which describes quark mixing, is a consequence of the Higgs mechanism. Since the Standard Model uses the simplest possible method of electro-weak symmetry breaking because, within the Standard Model, the CKM matrix, which describes quark mixing, is a consequence of the Higgs mechanism. The measurement of \( B \)-hadron lifetimes and of the \( B \)-meson/baryon differences and the \( \Gamma_0 \) of \( 0 \)-hadrons are thus linear combinations of the weak eigenstates \( |q\rangle \) and \( |\bar{q}\rangle \) are governed by a Schrödinger equation with off-diagonal elements:

\[
\frac{d}{dt} \begin{pmatrix} |B^0_q(t)\rangle \\ |B^0_{\bar{q}}(t)\rangle \end{pmatrix} = \begin{pmatrix} M^{(q)} \ & i(\Gamma^{(q)}/2) \\ M^{(\bar{q})} \ & i(\Gamma^{(\bar{q})}/2) \end{pmatrix} \begin{pmatrix} |B^0_q(t)\rangle \\ |B^0_{\bar{q}}(t)\rangle \end{pmatrix} \tag{2}
\]

Eigenstates of the system with definite mass, \( |B^H_q\rangle \) and \( |B^L_q\rangle \), are thus linear combinations of the weak eigenstates. Oscillations between weak eigenstates then occur with a frequency proportional to the mass difference:

\[
\Delta m_q = M^H_q - M^L_q \sim 2|\Gamma_0^{(q)}| \tag{3}
\]

Further discussion of this, and of the \( CP \)-eigenstates can be found in [3].

Although, in principle, only \( \Delta m_d \) is necessary to extract the CKM matrix element, \( V_{td} \), in practice the theoretical uncertainty on this extraction is reduced by more than a factor of three when the ratio...
$\Delta m_d/\Delta m_s$ is used [5]:

$$\frac{\Delta m_d}{\Delta m_s} = \left( \frac{M(B_d)}{M(B_s)} \right) \left( \frac{f_{B_s}^2 B_{B_d}}{f_{B_d}^2 B_{B_s}} \right) \left| \frac{V_{td}}{V_{ts}} \right|^2 \quad (4)$$

Measurement of $\Delta m_s$ has thus been a priority of the experimental high energy physics community since the first measurements of $B$-mixing [6].

## 2. Common Experimental Issues

The busy environment surrounding a $B$-hadron produced in a $p\bar{p}$ collision at the Tevatron leads to challenges for many aspects of the CDF and D0 detectors, particularly in the areas of triggering and tracking. These challenges are apparent when considering the generic steps taken in a lifetime or mixing analysis. For any $B$-hadron decays, the first step of this process is the reconstruction of intermediate states in the chain, such as $D^- \rightarrow \phi \pi^-$ and $\phi \rightarrow K^+K^-$. Unlike at the B-factories, pion/kaon separation is not generally important here (although CDF makes use of its $dE/dx$ and time-of-flight capabilities in some analyses). Good invariant mass resolution is critical, however, as reconstructed mass is generally used to identify specific hadrons. CDF has the edge here (by nearly a factor of four) because of their large-volume tracking system. Nevertheless, both experiments are able to accumulate large samples of $B$-hadron decays with high purities.

The final common feature in $B$-hadron reconstruction for lifetime and mixing analyses is the estimation of the proper time of the $B$-hadron’s decay. This involves reconstructing the $B$’s production and decay points using vertices found from combinations of charged tracks. Spatial resolution of the tracking systems is thus a crucial element of lifetime and mixing analyses. Both CDF and D0 have (and excellent) capabilities here, with average uncertainties on proper time reconstructed using only the charged particles from a $B$-decay of around $50 \mu$m for semi-leptonic $B$-decays and $25 \mu$m for fully hadronic decays. These resolutions are well below typical $B$-hadron lifetimes of $\sim 500 \mu$m and are also smaller than the $B_s$ oscillation period of $\sim 100 \mu$m.

Reconstruction of the momenta of $B$-hadrons is, of course, also an important element in estimating proper time. For the case of fully reconstructed, hadronic $B$-decays the uncertainty that this introduces in the estimate of proper time is negligible compared to that associated with vertexing. For semi-leptonic decays, however, the true $B$-hadron momentum cannot be measured directly because of the presence of neutrinos in the decay. To deal with this, correction factors, derived from simulation, are applied to the reconstructed (from charged tracks) proper time of each $B$-candidate based on its assumed flavor and mode.

## 3. Lifetimes

CDF and D0 play an important role in our understanding of the lifetimes of weakly decaying $B$-hadrons. Not only are these experiments the only place where higher mass $B$-mesons and all $B$-baryons can be studied, they also provide competitive lifetime results for $B^0$ and $B^+\tau$ mesons. In that area, CDF has published recent preliminary measurements using fully reconstructed $B^0$ and $B^+\tau$ decays to $D^{\mp}\tau^+$ mesons and charged pions; semi-leptonic decays involving $D$ and $D^\tau$ mesons; and decays to $J/\psi K^{(*)}$. D0 has a published result on the $B^+/B^0$ lifetime ratio using $\mu/D^{(*)}\chi$ final states. Taken together, these Tevatron results have a weight of $\sim 38\%$ in the $\tau(B^+)/\tau(B^0)$ world average [7].
The Tevatron has also been active in studies of the \( B_s\)-meson, as we have seen in several contributions to this conference [1, 3]. The \( B_s\) lifetime plays a foundational role in these studies and both CDF and D0 have measured this quantity in a variety of ways. Care must be taken when interpreting \( B_s\) lifetime results as these mesons have a non-negligible width difference between their mass eigenstates [3], which, for these purposes, are approximately equivalent to the \( CP \) eigenstates. \( B_s\) lifetime results are thus only given for those decays in which the \( CP \) content is well-known and are quoted as the average lifetime of the heavy and light mass eigenstates. CDF and D0 have several new results in this area, summarized in Table I.

Measurements of \( B\)-baryon properties are also an important component of the Tevatron \( B\)-physics program [2]. Although several new baryon states have recently been observed by CDF and D0 [2], sufficient statistics to make a lifetime determination have only been accumulated for the \( \Lambda_b\)-baryon. Both \( J/\psi\Lambda \) and \( \mu\Lambda\), \( \Lambda_\chi \) final states have been studied. Results are summarized in Table I. They indicate generally good agreement with theoretical expectations (see Fig. 1). However, there is some discrepancy between the CDF and D0 results that will need to be understood with more data.

The last lifetime measurement we will mention, that of the \( B_c\)-meson, probes different theoretical issues than those states discussed previously. Because the \( B_c\) is composed of two heavy quarks, it has more spectator-level decay possibilities than other weakly decaying \( B\)-hadrons. In fact, theory predicts that the \( B_c\) lifetime should be approximately one third of that of the other \( B\)-mesons.

On the experimental side, both CDF and D0 have now collected large samples of \( B_c\) candidates in semi-leptonic decay modes, as well as smaller sets of fully hadronic decays. Only semi-leptonic decays are currently used for the determination of the \( B_c\) lifetime, results of which are summarized in Table I.

### Table I

<table>
<thead>
<tr>
<th>( B_c \rightarrow J/\psi\phi )</th>
<th>Exp</th>
<th>Lumi [fb]</th>
<th>Signal</th>
<th>( \tau ) [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF</td>
<td>1.7</td>
<td>2500</td>
<td>1.52±0.04±0.02 [8]</td>
<td></td>
</tr>
<tr>
<td>D0</td>
<td>2.8</td>
<td>1976</td>
<td>1.52±0.05±0.01 [9]</td>
<td></td>
</tr>
<tr>
<td>( B_c \rightarrow \pi D_s )</td>
<td>CDF</td>
<td>1.3</td>
<td>3340</td>
<td>1.517±0.041±0.025 (prel)</td>
</tr>
<tr>
<td>( B_c \rightarrow D_s X )</td>
<td>CDF</td>
<td>0.4</td>
<td>1155</td>
<td>1.381±0.053±0.048 (prel)</td>
</tr>
<tr>
<td>D0</td>
<td>0.4</td>
<td>5176</td>
<td>1.398±0.044±0.028 (prel)</td>
<td></td>
</tr>
<tr>
<td>( A_b \rightarrow J/\psi\Lambda )</td>
<td>CDF</td>
<td>1.0</td>
<td>557</td>
<td>1.58±0.08±0.01 (prel)</td>
</tr>
<tr>
<td>D0</td>
<td>1.2</td>
<td>171</td>
<td>1.22±0.11±0.04 (prel)</td>
<td></td>
</tr>
<tr>
<td>( A_b \rightarrow \mu\Lambda X )</td>
<td>D0</td>
<td>1.3</td>
<td>3727</td>
<td>1.29±0.12±0.09 (prel)</td>
</tr>
<tr>
<td>( B_s \rightarrow J/\psi\ell )</td>
<td>CDF</td>
<td>1.0</td>
<td>916</td>
<td>0.47±0.06±0.018 (prel)</td>
</tr>
<tr>
<td>D0</td>
<td>1.3</td>
<td>881</td>
<td>0.44±0.038±0.032 (prel)</td>
<td></td>
</tr>
</tbody>
</table>

As can be seen in Fig. 1, Run II Tevatron measurements of \( B\)-hadron lifetimes have dramatically increased the precision with which we can probe QCD. In this figure we see a comparison of world average results from 2000 [14] and 2002 [15] (before Run II results) and the current world averages from HFAG [7]. Also included are theoretical predictions of lifetime ratios [16] and \( B_s\) lifetimes [17]. The latest experimental measurements represent improvements of factors of three over pre-Run II results. They tend to be in good agreement with theoretical expectations. In some cases, experimental precision is smaller than uncertainties in the calculations allowing constraints to be put on models of \( B\)-decays.

![Figure 1](image.png)

Figure 1: A comparison of world average lifetime ratios, and \( B_s\) lifetimes in 2000, 2002, and 2008 with current theoretical predictions.

### 4. Mixing

Measurement of the oscillation frequency between \( B_s^0 \) and \( B_s^0 \) mesons (or equivalently, the mass difference, \( \Delta m_s \), between the heavy and light mass eigenstates) was one of the goals of Run II at the Tevatron. This goal was achieved in the spring of 2006 when D0 saw first hints of a mixing signal [18], followed quickly by a \( >3\sigma \) significance measurement by CDF [19]. These early results have now been updated with a new published measurement from CDF [20] and a set of preliminary results from D0. Basic features of the analyses are given in Table II, and are described in more detail in the following.

The analyses producing the results mentioned above are similar to lifetime analyses in that they examine the proper time evolution of candidate \( B_s\) decays. An
added feature is the use of tagging to determine the flavor ($B^0$ or $\bar{B}^0$) of the meson at production and decay. Tagging the decay flavor is straightforward using charges of the decay products. Production flavor tagging, however, is harder. Two classes of techniques are used here: opposite side tagging (OST) and same side tagging (SST).

Because $b$-quarks are produced in quark-antiquark pairs at the Tevatron, the OST technique uses information about the "other" (non-$B_i$) $B$-hadron in the event to determine its production flavor. The $B_i$ candidate is then assumed to have been produced with the opposite flavor. SST, on the other hand, uses information gleaned from fragmentation and other particles associated with the $B_i$-meson itself to determine its flavor at production. The figure of merit associated with these techniques is called the tagging power, $\varepsilon D^2$.

It is composed of the efficiency, $\varepsilon$, for an event to be tagged as either oscillated or non-oscillated and the dilution, $D$, a quantity related to the purity of the tagging method: $D = 2\eta - 1$, where $\eta$ is the fraction of tags where the oscillated or non-oscillated state is correctly identified. Tagging powers for the various modes used in CDF and D0 $B_i$ mixing analyses are give in Table II.

Understanding tagging is obviously a critical component of mixing analyses into which the experiments have put a lot of effort. Briefly speaking, OST is calibrated by measuring the well-known $B_d$ oscillation frequency. The $B_d$ mass difference, $\Delta m_d$, has been measured extremely accurately by BaBar and Belle (see [7] for a compilation of these results). CDF and D0 [22] cannot compete with these measurements, but do use $\Delta m_d$ analyses to simultaneously determine OST calibration parameters. The results obtained for $\Delta m_d$ are fully consistent with the world average, giving us confidence in the OST technique.

Same side tagging performance, unlike that of the OST, is dependent upon the particular $B$-meson flavor being considered. The method to verify its calibration therefore consists of ensuring that the SST gives similar results in data and MC control samples, such as $B^+ \rightarrow J/\psi K^+$. The MC is then assumed to give a correct description of the SST in $B_i$-events.

With tagging well in hand, fits are performed to determine the value of $\Delta m_s$. Scans of $\ln \mathcal{L}$ vs. assumed $\Delta m_s$ are shown in Fig. 2 for all analysis modes in CDF [20] and for the preliminary, combined D0 result. Both experiments see minima in $-\ln \mathcal{L}$ around 18 ps$^{-1}$, measuring the following values for $\Delta m_s$:

$$17.77 \pm 0.10 \pm 0.07 \text{ps}^{-1} \text{CDF} \ [20]$$
$$18.53 \pm 0.93 \pm 0.30 \text{ps}^{-1} \text{D0 (prelim)} \ (5)$$

Scans of $-\ln \mathcal{L}$ vs $\Delta m_s$ for the two experiments are shown in Fig. 2. The significance of the CDF result is 5.4$\sigma$ (background fluctuation probability of $8 \times 10^{-8}$), while that of the D0 measurement is 2.9$\sigma$ (with systematic effects included).

Using the two measurements above and Eq. 4, we find an average value of $|V_{td}/V_{ts}|$ of:

$$|V_{td}/V_{ts}| = 0.2060 \pm 0.0012(\text{exp})^{+0.0081}_{-0.0060}(\text{theor}) \ (6)$$

where the “exp” error includes all statistical and systematic errors on the measurements of $\Delta m_s$, while the “theor” error comes from the uncertainty on the ratio of decay constants and bag parameters from lattice calculations [5]:

$$\xi \equiv \frac{f_{B_s}\sqrt{f_{B_s}}}{f_{B_d}\sqrt{f_{B_d}}} = 1.210^{+0.047}_{-0.035} \ (7)$$

It is interesting to note that the theoretical error on $\xi$ completely dominates the uncertainty of the CKM element ratio (by a factor of nearly 7) and also that the relative error on the world average value of $\Delta m_s$ (0.3%) is now smaller than that on $\Delta m_d$ (0.5%) [7].

These new measurements of $\Delta m_s$ have a large impact on tests of the consistency of the CKM picture of quark mixing. This is particularly evident when comparing experimental reconstruction of the “unitarity triangle” (formed from one element of the CKM unitarity condition: $V_{ud}V_{td}^* + V_{cd}V_{td}^* + V_{td}V_{td}^* = 0$). Figure 3 shows a comparison of the state of our knowledge of this triangle in 2003 and 2007 [23] (see also [24]). The new measurement of $\Delta m_s$ significantly increases the accuracy with which we know the apex of the unitarity triangle and favors those models of new physics that have a Standard Model, CKM-like flavor structure.

5. Conclusions and Future Prospects

We’ve made remarkable progress in the field of $B$-physics since the start of Run II at the Tevatron. To cite just a few examples discussed here: our knowledge of $B$-hadron lifetimes has improved by a factor of two or more depending on the hadron; the accuracy
to which we measure the $B_d$ mixing frequency has increased by a factor of more than three, thanks to hard work at the B-factories; and we have finally observed $B_s$ oscillations, which are now measured with an uncertainty of only 0.3%. All these measurements, and many others, point to a picture of flavor that is consistent with the Standard Model CKM description. However, some cracks may be appearing in the mirror due to recent measurements of $CP$ violation in $B_s$-mesons [3].

Certainly then, the $B_s$ is a system to watch – particularly in the $CP$-sector. Advances on the oscillation frequency side will need to come from improved calculations though, since that is where the main source of uncertainty now lies. For lifetime measurements, the increasingly large data sets being collected at the Tevatron (more than 4 fb$^{-1}$ have now been delivered to each experiment) hold out the prospect of comprehensive tests of QCD models using a wide range of $B$-hadrons. The experimental focus here will shift to baryons (and the $B_c$), including the newly observed $\Xi_b$ and $\Sigma_b$ states.

With many past successes and a bright future ahead, $B$-physics will remain a vital part of the Tevatron program while we await the next big step – LHCb.
Acknowledgments

I am pleased to acknowledge the assistance of my collaborators on D0 and CDF in the preparation of this presentation. I would also like to thank the HQL08 organizers for a very interesting and enjoyable conference!

References

1 I. Bertram, these proceedings.
2 M. Paulini, these proceedings.
3 J. Boudreau, these proceedings.
7 E. Barberio, et al. (HFAG), arXiv:0808.1297 [hep-ex].
9 V.M. Abazov, et al. (D0), arXiv:0802.2255 [hep-ex].
13 V.M. Abazov, et al. (D0), arXiv:0805.2614 [hep-ex].