

Inclusive $|V_{cb}|$ and global fit

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A reliable determination of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ is mandatory for precision flavor physics and for the search for CP violating phases from new, heavy particles. In this article, we review the theory of the determination of $|V_{cb}|$ from inclusive semileptonic B decays. We discuss the available measurements of the semileptonic B branching fraction and other inclusive observables in B decays relevant to the determination of $|V_{cb}|$. Finally, we perform a global fit to extract $|V_{cb}|$ and the b -quark mass m_b .

1. Introduction

The magnitude of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cb}|$ [1] can be determined from semileptonic B decays to charmed final states. At the B factory experiments Belle [2] and BaBar [3], measurements of semileptonic decays to D or D^* mesons (“exclusive measurements”) and determinations of $|V_{cb}|$ using all semileptonic final states in a given region of phase space (“inclusive measurements”) have been performed. In this article, we will review the latter approach. As experimental and theoretical systematics are different, consistency between exclusive and inclusive determinations of $|V_{cb}|$ is crucial for our understanding.

After reviewing the theory of the determination of $|V_{cb}|$ from inclusive B decays, we will discuss the measurements (mainly at the B factories) of the semileptonic branching fraction and other inclusive observables in $B \rightarrow X_c \ell \nu$ decays that allow to determine the non-perturbative parameters that appear in the calculation of the semileptonic width. Finally, we perform fits to different data sets to determine $|V_{cb}|$ and the b -quark mass m_b .

2. Calculations of the semileptonic B decay width

The semileptonic decay width $\Gamma(B \rightarrow X_c \ell \nu)$ is proportional to $|V_{cb}|^2$. Measurements of the semileptonic B branching fraction thus allow to determine $|V_{cb}|$, provided that the width can be calculated reliably. Challenges are non-perturbative QCD contributions and experimental selections applied to the data. *E.g.*, semileptonic decays can typically only be measured above a certain minimum lepton energy threshold.

These calculations are performed in the frameworks of the Heavy Quark Effective Theory (HQET) and the Operator Production Expansion (OPE) [4, 7]. The result is an expansion in inverse powers of the b -quark mass, the leading order corresponding to the result obtained assuming unconfined quarks. A problem for the practical use of these formulae is that new, so-called heavy quark (HQ) parameter appear at each

Table I Heavy quark parameters in the expressions derived in the kinetic and 1S schemes.

	Kinetic scheme	1S scheme
$\mathcal{O}(1)$	$m_b^{\text{kin}}, m_c^{\text{kin}}$	m_b^{1S}
$\mathcal{O}(1/m_b^2)$	μ_π^2, μ_G^2	λ_1, λ_2
$\mathcal{O}(1/m_b^3)$	ρ_D^3, ρ_{LS}^3	$\rho_1, \tau_1, \tau_2, \tau_3$

order in $1/m_b$. These non-perturbative quantities encode the soft QCD physics and cannot be calculated from perturbation theory.

At present, there are two independent calculations of the semileptonic width performed up to the third order in $1/m_b$ [4, 7]. Referring to the b -quark mass definition used, the two schemes are called 1S and kinetic. *E.g.*, the result in the kinetic scheme reads [4],

$$\Gamma_{\text{s.l.}} = \frac{G_F^2 m_b^5(\mu)}{192\pi^3} |V_{cb}|^2 (1 + A_{\text{ew}}) A^{\text{pert}}(r, \mu) \left[z_0(r) \left(1 - \frac{\mu_\pi^2(\mu) - \mu_G^2(\mu) + \frac{\rho_D^3(\mu) + \rho_{LS}^3(\mu)}{m_b(\mu)}}{2m_b^2(\mu)} \right) - 2(1-r)^4 \frac{\mu_G^2(\mu) - \frac{\rho_D^3(\mu) + \rho_{LS}^3(\mu)}{m_b(\mu)}}{m_b^2(\mu)} + d(r) \frac{\rho_D^3(\mu)}{m_b^3(\mu)} + \dots \right], \quad (1)$$

where G_F is the Fermi constant, μ is the renormalization scale, $r = m_c^2(\mu)/m_b^2(\mu)$, and $z_0(r)$ and $d(r)$ are phase space factors [4]. Both the electroweak and the perturbative corrections, A_{ew} and $A^{\text{pert}}(r, \mu)$, are well known [4]. The remaining quantities are the above-mentioned HQ parameters shown in Table I together with their counterparts in the 1S scheme.

Although these HQ parameters cannot be determined theoretically, they can be obtained from experiment: Other inclusive observables in B decays, such as the moments of the lepton energy and the X_c mass distributions in $B \rightarrow X_c \ell \nu$ decays, and the moments of the photon energy spectrum in $B \rightarrow X_s \gamma$ decays, have a similar expansion in inverse powers of m_b with

the *same* HQ parameters. By measuring these quantities in the experiment, one can determine the parameters in Table I and substitute them into Eq. 1 to obtain $|V_{cb}|$ with a precision of 1 – 2%. This type of analysis is often referred to as *global fit*.

On the theory side, this approach requires calculations of the $B \rightarrow X_c \ell \nu$ and $B \rightarrow X_s \gamma$ moments with the same precision in $1/m_b$ as in Eq. 1. In the kinetic scheme, calculations of the lepton energy and hadronic mass spectra up to $\mathcal{O}(1/m_b^3)$ for different truncations in the lepton energy are available [5]. Also, the moments of the photon energy spectrum in $B \rightarrow X_s \gamma$ decays have been calculated [6]. Note that the photon energy spectrum cannot be described by the OPE alone but some modeling of additional non-perturbative contributions is necessary. In Ref. [6] these are referred to as bias corrections. Calculations up to the same order in $1/m_b$ are also available in the 1S scheme [7].

3. Measurements of $B \rightarrow X_c \ell \nu$ decay distributions

Measurements of the semileptonic B branching fraction and inclusive observables in $B \rightarrow X_c \ell \nu$ decays have been obtained by the BaBar [8–10], Belle [11, 12], CDF [13], CLEO [14] and DELPHI [15] experiments. The photon energy spectrum in $B \rightarrow X_s \gamma$ decays has been studied by BaBar [16, 17], Belle [18, 19] and CLEO [20]. In this section, we will briefly review the new or updated measurements of $B \rightarrow X_c \ell \nu$ decays.

BaBar has updated their previous measurement of the hadronic mass moments $\langle M_X^{2n} \rangle$ [9] and obtained preliminary results based on a dataset of 210 fb^{-1} taken at the $\Upsilon(4S)$ resonance [10]. In the updated analysis, the hadronic decay of one B meson in $\Upsilon(4S) \rightarrow B\bar{B}$ is fully reconstructed (B_{tag}) and the semileptonic decay of the second B is inferred from the presence of an identified lepton (e or μ) within the remaining particles in the event (B_{sig}). This so-called full reconstruction tag allows to significantly reduce combinatorial backgrounds and select semileptonic decays with a purity of about 80%. Particles used neither for the reconstruction of B_{tag} nor for the charged lepton are considered to belong to the X_c system, and the hadronic mass spectrum M_X is calculated using some kinematic constraints (Fig. 1).

From this spectrum, BaBar calculates the hadronic mass moments $\langle M_X^n \rangle$, $n = 1, \dots, 6$ for minimum lepton momenta in the center-of-mass (c.m.) frame ranging from 0.8 to 1.9 GeV/ c . These moments are however distorted by acceptance and finite resolution effects and an event-by-event correction is derived from Monte Carlo (MC) simulated events. These corrections are linear functions with coefficients depending on the lepton momentum, the multiplicity of the

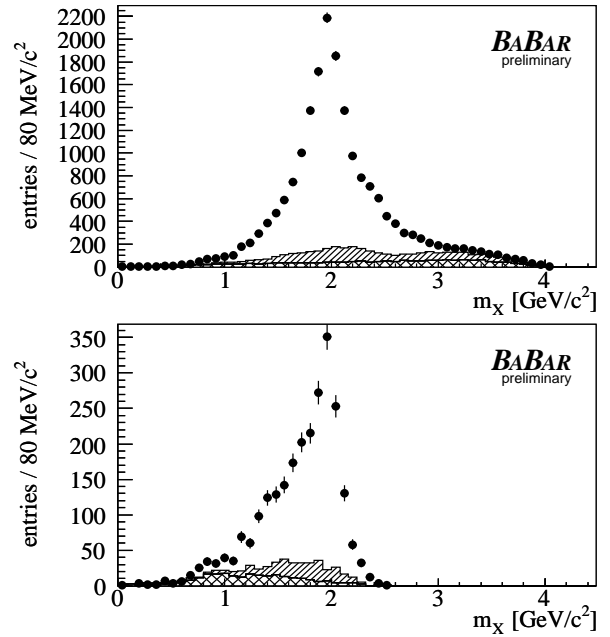


Figure 1: Hadronic mass spectra in BaBar’s recent analysis [10] for minimum c.m. lepton momenta of 0.8 GeV/ c (top) and 1.9 GeV/ c (bottom). The residual backgrounds are shown by the histograms.

X_c system and $E_{\text{miss}} - c|\vec{p}_{\text{miss}}|$, where E_{miss} and \vec{p}_{miss} are the missing energy and 3-momentum in the event. Note that this analysis measures also mixed mass and c.m. energy moments $\langle N_X^{2n} \rangle$, $n = 1, 2, 3$, with $N_X = M_X^2 c^4 - 2\tilde{\Lambda}E_X + \tilde{\Lambda}^2$ and $\tilde{\Lambda} = 0.65 \text{ GeV}$ in addition to ordinary hadronic mass moments. These mixed moments are expected to better constrain some HQ parameters though they are not used in current global fit analyses yet.

Belle has recently measured the c.m. electron energy [11] and the hadronic mass [12] spectra in $B \rightarrow X_c \ell \nu$ decays, based on 140 fb^{-1} of $\Upsilon(4S)$ data. The experimental procedure is very similar to the BaBar analysis, *i.e.*, one B meson in the event is fully reconstructed in a hadronic channel (Fig. 2). The main difference to the analysis discussed above is that detector effects in the spectra are removed by unfolding using the Singular Value Decomposition (SVD) algorithm [21] with a detector response matrix found from MC simulation. The moments are then calculated from the unfolded spectra. In Ref. [11], Belle measures the partial semileptonic branching fraction and the c.m. electron energy moments $\langle E_e^n \rangle$, $n = 1, \dots, 4$, for minimum c.m. electron energies ranging from 0.4 to 2.0 GeV. The hadronic mass analysis [12] measures the first and second moments of M_X^2 for minimum c.m. lepton energies between 0.7 and 1.9 GeV.

An interesting analysis of inclusive $B \rightarrow X_c \ell \nu$ decays comes also from the DELPHI experiment [15]: In this study, the b -frame lepton energy $\langle E_l^n \rangle$, $n = 1, 2, 3$,

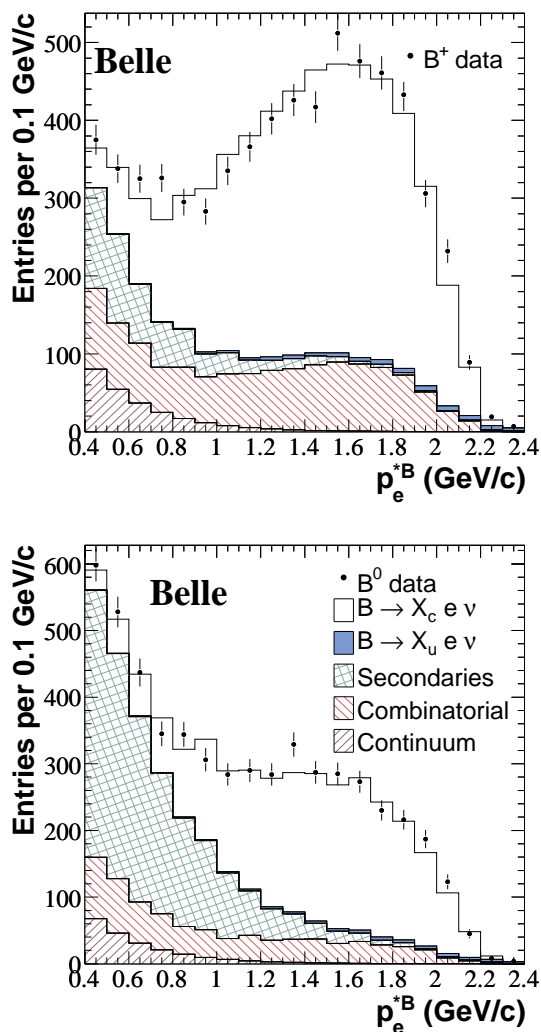


Figure 2: Electron energy spectrum in the c.m. frame for charged (top) and neutral (bottom) tags, as measured in Ref. [11].

and the hadronic mass M_X^{2n} , $n = 1, \dots, 5$, moments are measured without applying any selection on the lepton energy in the b -frame. This is possible as DELPHI studies b -decays in $Z \rightarrow b\bar{b}$ events. b -hadrons are thus produced with significant kinetic energy in the laboratory frame, so that charged leptons at rest in the b -frame can be seen in the detector.

4. Global fit and determination of $|V_{cb}|$ inclusive

The different global fit analyses differ by the theory expressions and the data sets they are based onto: As discussed in Sect. 2, at present there are two sets of theoretical expressions available for this analysis de-

Table II Experimental inputs used in the Belle $|V_{cb}|$ inclusive analysis [18].

Moments	Measurements used
Lepton energy $\langle E_\ell^n \rangle$ [11]	$n = 0$: $E_{\min} = 0.6, 1.0, 1.4$ GeV
	$n = 1$: $E_{\min} = 0.6, 0.8, 1.0, 1.2, 1.4$ GeV
	$n = 2$: $E_{\min} = 0.6, 1.0, 1.4$ GeV
	$n = 3$: $E_{\min} = 0.8, 1.0, 1.2$ GeV
Hadronic mass $\langle M_X^{2n} \rangle$ [12]	$n = 1$: $E_{\min} = 0.7, 1.1, 1.3, 1.5$ GeV
	$n = 2$: $E_{\min} = 0.7, 0.9, 1.3$ GeV
Photon energy $\langle E_\gamma^n \rangle$ [18]	$n = 1$: $E_{\min} = 1.8, 2.0$ GeV
	$n = 2$: $E_{\min} = 1.8, 2.0$ GeV

Table III Results of the Belle $|V_{cb}|$ analysis [18]. The results for m_b are compatible after scheme translation.

	Kinetic scheme	1S scheme
$ V_{cb} $ (10^{-3})	$41.58 \pm 0.69(\text{fit})$ $\pm 0.08(\tau_B) \pm 0.58(\text{th})$	$41.56 \pm 0.68(\text{fit})$ $\pm 0.8(\tau_B)$
m_b (GeV)	4.543 ± 0.075	4.723 ± 0.055
$\chi^2/\text{ndf.}$	7.3/18	4.7/18

rived in the kinetic [4–6] and 1S schemes [7]. As for the data sets, DELPHI [15], BaBar [10] and Belle [18] have determined $|V_{cb}|$ inclusive from their own data. We will discuss the recently published Belle analysis in more detail. Finally, to achieve the ultimate statistical precision, we will combine all available data to measure $|V_{cb}|$ and the b -quark mass m_b .

In Ref. [18], Belle uses 25 measurements of the lepton energy and hadronic mass moments in $B \rightarrow X_c \ell \nu$ and of the photon energy moments in $B \rightarrow X_s \gamma$ (Table II). Though more moment measurements are available, moments without matching theoretical prediction or highly correlated measurements are excluded. A χ^2 -fit of these measurements is done to both the kinetic and 1S scheme expressions. The only external input in the analysis is the average B lifetime $\tau_B = (1.585 \pm 0.006)$ ps [22].

There are 7 free parameters in both fits. Only $|V_{cb}|$ can be compared directly, while a scheme translation must be performed for the HQ parameters, including the b -quark mass m_b . The main challenge is to properly account for correlations: the covariance matrix of the χ^2 -fit is the sum of experimental and theoretical correlations. While the experimental correlation coefficients have been determined in the respective analyses [11, 12, 18], theoretical correlations are estimated following the prescriptions of the theoretical authors [5–7]. The results of the Belle analysis are given in Table III.

Table IV Experimental inputs used for a global fit analysis to all available moment data.

Experiment	Measurements used
BaBar	$\langle E_\ell^n \rangle, n = 0, 1, 2, 3$ [8] $\langle M_X^{2n} \rangle, n = 1, 2$ [9] $\langle E_\gamma^n \rangle, n = 1, 2$ [16, 17]
Belle	$\langle E_\ell^n \rangle, n = 0, 1, 2, 3$ [11] $\langle M_X^{2n} \rangle, n = 1, 2$ [12] $\langle E_\gamma^n \rangle, n = 1, 2$ [19]
CDF	$\langle M_X^{2n} \rangle, n = 1, 2$ [13]
CLEO	$\langle M_X^{2n} \rangle, n = 1, 2$ [14] $\langle E_\gamma^n \rangle, n = 1$ [20]
DELPHI	$\langle E_\ell^n \rangle, n = 1, 2, 3$ [15] $\langle M_X^{2n} \rangle, n = 1, 2$ [15]

Table V Preliminary results of the analysis combining all available moment data (Table IV).

	Kinetic scheme	1S scheme
$ V_{cb} $ (10^{-3})	41.55 ± 0.43 (<i>fit</i>) ± 0.08 (τ_B) ± 0.58 (<i>th</i>)	41.74 ± 0.29 (<i>fit</i>) ± 0.8 (τ_B)
m_b (GeV)	4.613 ± 0.033	4.708 ± 0.024
$\chi^2/\text{ndf.}$	30.6/63	26.1/63

Finally, we attempt to combine all available moment measurements to optimize the statistical precision in $|V_{cb}|$ and m_b . Using 70 measurements (Table IV) from different experiments, we follow the Belle approach to derive numbers in the kinetic and 1S schemes. The preliminary results are shown in Table V and Fig. 3.

5. Summary

We have reviewed the theory and surveyed the experimental data for the determination of the CKM matrix element $|V_{cb}|$ from inclusive decays $B \rightarrow X_c \ell \nu$. The results for $|V_{cb}|$ using the data of the Belle experiment alone are given in Table III. Also, an attempt is made to combine all available experimental data: The preliminary results in terms of $|V_{cb}|$ and the b -quark mass m_b are shown in Table V.

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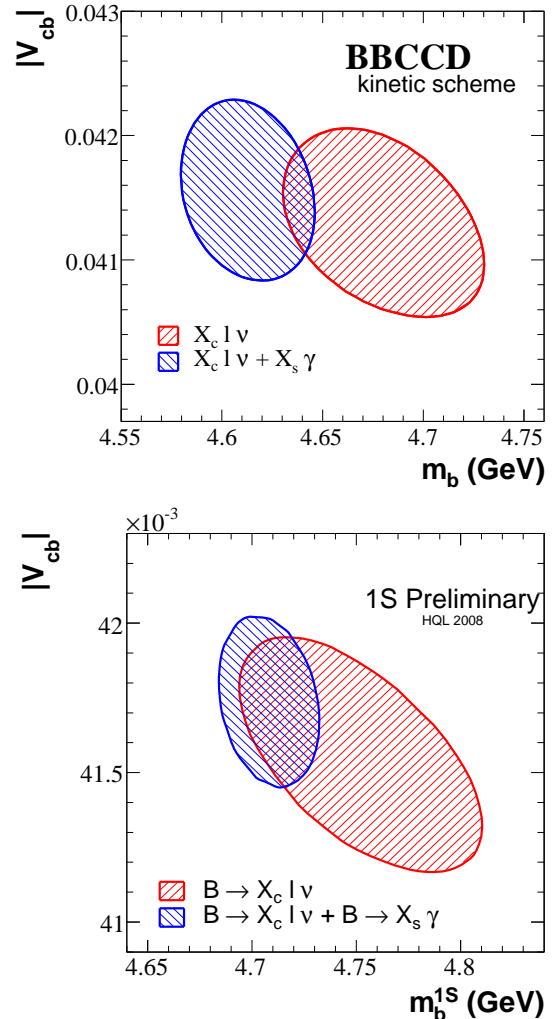


Figure 3: Preliminary results of the fits in the kinetic (top) and 1S (bottom) schemes combining all available moment data (Table IV). The $\Delta\chi^2 = 1$ contours are shown for the fit with and without $B \rightarrow X_s \gamma$ data.

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