Inclusive $|V_{cb}|$ and global fits

Christoph Schwanda, HEPHY Vienna
The CKM matrix

• Couplings of the charged current interaction in the SM

\[ -\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \, u_L \, \gamma^\mu (V_{\text{CKM}})_{ij} \, d_L \, \gamma^\nu W_j^+ + \text{h.c.} \]

\[ V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

• \( V_{\text{CKM}} \) is a unitary 3x3 matrix; it contains three real parameters and one complex phase

[Kobayashi, Maskawa, Prog. Theor. Phys. 49, 652 (1973)]
• The unitarity of $V_{\text{CKM}}$ can be probed by measuring the sides and angles of the unitarity triangle

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

• In this presentation, I’ll focus on the determination of $|V_{cb}|$ from inclusive semileptonic $B$ decays
The semileptonic width

- $\Gamma(B \to X_c l \nu)$ can be systematically calculated with the operator production expansion (OPE)

$$\Gamma_{sl}(b \to c) = \frac{G_F^2 m_b^5(\mu)}{192 \pi^3} |V_{cb}|^2 (1+A_{ew}) A_{pert}(r, \mu) \left[ z_0(r) \left( 1 - \frac{\mu_\pi^2(\mu)}{2m_b^2(\mu)} + \frac{\rho_{13}(\mu) - \rho_{31}(\mu)}{m_b(\mu)} + \rho_{33}(\mu) - \rho_{33}(\mu) \right)^2 \right]$$

- At each order in $1/m_b$, the expectation values of local operator products (heavy quark parameters) are multiplied by perturbatively calculable coefficients from [Benson et al., Nucl. Phys. B665, 367 (2003)]

$$r = m_c^2(\mu)/m_b^2(\mu)$$

- HQ parameters (non-calculable; contain soft QCD physics)

- At each order in $1/m_b$, the expectation values of local operator products (heavy quark parameters) are multiplied by perturbatively calculable coefficients
Other observables in B decays

- Moments of the lepton energy spectrum in $B \rightarrow X_{cl}\nu$

$$R_n(E_{\text{cut}}, \mu) = \int_{E_{\text{cut}}} (E_\ell - \mu)^n \frac{d\Gamma}{dE_\ell} \, dE_\ell, \quad \langle E_\ell^n \rangle_{E_{\text{cut}}} = \frac{R_n(E_{\text{cut}}, 0)}{R_0(E_{\text{cut}}, 0)}$$

- Moments of the hadronic mass spectrum in $B \rightarrow X_{cl}\nu$

$$\langle m_{X}^{2n} \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} (m_X^2)^n \frac{d\Gamma}{dm_X^2} \, dm_X^2}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dm_X^2} \, dm_X^2}$$

- Moments of the photon energy spectrum in $B \rightarrow X_s\gamma$

$$\langle E_\gamma^n \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} E_\gamma^n \frac{d\Gamma}{dE_\gamma} \, dE_\gamma}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dE_\gamma} \, dE_\gamma}$$

The OPEs of these inclusive observables contain the same HQ parameters.
Global analysis of B decays

- Dedicated predictions for each observable
  - \( \langle E^n \rangle_{E_i > E_{cut}} = f^{(n)}(E_{cut}, m_b, \text{HQ param.}) \)
  - \( \langle M^{2n}_X \rangle_{E_i > E_{cut}} = g^{(n)}(E_{cut}, m_b, \text{HQ param.}) \)
  - \( \langle E^n \rangle_{E_{\gamma} > E_{cut}} = h^{(n)}(E_{cut}, m_b, \text{HQ param.}) \)

- Determine HQ parameters by performing a minimum \( \chi^2 \) fit to all available moment measurements

- Take into account correlated experimental and theoretical errors

- External input: average B lifetime \( \tau_B = (1.585 \pm 0.006) \) ps
Available calculations

- Kinetic running mass

- 1S mass

- Non-perturbative parameters in the $1/m_b$ expansion

<table>
<thead>
<tr>
<th>Order</th>
<th>Kinetic scheme</th>
<th>1S scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O(1)$</td>
<td>$m_b, m_c$</td>
<td>$m_b$</td>
</tr>
<tr>
<td>$O(1/m_b^2)$</td>
<td>$\mu^2_\pi, \mu^2_G$</td>
<td>$\lambda_1, \lambda_2$</td>
</tr>
<tr>
<td>$O(1/m_b^3)$</td>
<td>$\rho_D, \rho_{LS}$</td>
<td>$\rho_1, \tau_{1.3}$</td>
</tr>
</tbody>
</table>
Available measurements

- Belle $E_\gamma$, 605/ fb [arXiv:0804.1580] preliminary
- Belle $E_l$, 140/ fb [PRD 75, 032001 (2007)]
- Belle $M^2_X$, 140/ fb [PRD 75, 032005 (2007)]
- DELPHI $E_l$, $M^2_X$, 3.4M Z [EPJ C45, 35 (2006)]
- BaBar, $E_\gamma$, 82/ fb [PRL 97, 171803 (2006)]
- BaBar, $E_\gamma$, 82/ fb [PRD 72, 052004 (2005)]
- CDF, $M^2_X$, 180/ pb [PRD 71, 051103 (2005)]
- Belle, $E_\gamma$, 140/ fb [PRL 93, 061803 (2004)]
- CLEO, $M^2_X$, 9/ fb [PRD 70, 032002 (2004)]
- BaBar, $E_l$, 47/ fb [PRD 69, 111104 (2004)]
- BaBar, $M^2_X$, 89M BB [PRD 69, 111103 (2004)]
- CLEO, $E_\gamma$, 9/ fb [PRL 87, 251807 (2001)]
BaBar $M_X^2$ moments

- 210/fb of Y(4S) data
- Hadronic decay of one B meson fully reconstructed
- Semileptonic decay of other B selected by requiring identified lepton (e/μ)
- Reconstructed moments corrected event-by-event for detector effects
- $<M_X^k>$ measured for $k=1,\ldots,6$ and $p^*_\text{cut}$ from 0.8 to 1.9 GeV/c

[p^*_1 > .8 GeV/c]
Belle $E_i$ and $M^2_X$ moments

- 140/mb of Y(4S) data
- Measurement also done with fully reconstructed events
- The finite detector resolution is unfolded with SVD algorithm [NIM A372, 469 (1996)]
- $<E^n_{e}>$ measured for $n=0,\ldots,4$ and $E_{cut}=0.4-2.0$ GeV
- $<M^{2n}_X>$ measured for $n=1,2$ and $E_{cut}=0.7-1.9$ GeV

[PRD 75, 032001 (2007)]
[PRD 75, 032005 (2007)]
The hadronic moments are derived from the fitted $D^{**}$ mass spectrum; assumptions on the $D^{**}$ decay are made.

$\langle E_1^n \rangle$, $n=1,\ldots,3$ and $\langle M_{2n}^2 \rangle$, $n=1,\ldots,5$ measured at $E_{\text{cut}} = 0$ as in $Z$ events the $b$-quark is produced with a boost.
$|V_{cb}|$ and $m_b$ from the fit to the Belle moment data


Similar analysis recently done on the BaBar moment data [arXiv:0707.2670] preliminary
### Belle measurements used

| Electron moments $<E^n_\gamma>$ | n=0: $E_{\text{cut}}=0.6, 1.0, 1.4$ GeV  
| n=1: $E_{\text{cut}}=0.6, 0.8, 1.0, 1.2, 1.4$ GeV  
| n=2: $E_{\text{cut}}=0.6, 1.0, 1.4$ GeV  
| n=3: $E_{\text{cut}}=0.8, 1.0, 1.2$ GeV |
| Hadron moments $<M^{2n}_X>$ | n=1: $E_{\text{cut}}=0.7, 1.1, 1.3, 1.5$ GeV  
| n=2: $E_{\text{cut}}=0.7, 0.9, 1.3$ GeV |
| Photon moments $<E^n_\gamma>$ | n=1: $E_{\text{cut}}=1.8, 2.0$ GeV  
| n=2: $E_{\text{cut}}=1.8, 2.0$ GeV |

- Exclude measurements
  - with no (reliable) theory prediction
  - with excessive correlations
Fit result in the 1S scheme

\[ \chi^2/\text{ndf.} = 7.3 / (25-7) \]
Fit result in the kinetic scheme

$\chi^2/\text{ndf.} = 4.7 / (25-7)$
$|V_{cb}|$ and $m_b$

Kinetic scheme ($X_c l\nu + X_s \gamma$ data)

$|V_{cb}| = (41.52 \pm 0.69_{\text{fit}} \pm 0.08_{\text{th}} \pm 0.58_{\text{th}}) \times 10^{-3}$

$m_b^{\text{kin}} = 4.543 \pm 0.075$ GeV

$m_c^{\text{kin}} = 1.055 \pm 0.118$ GeV

Results for $m_b$ compatible after scheme translation

$1S$ scheme ($X_c l\nu + X_s \gamma$ data)

$|V_{cb}| = (41.56 \pm 0.68_{\text{fit}} \pm 0.08_{\text{th}}) \times 10^{-3}$

$m_b^{1S} = 4.723 \pm 0.055$ GeV
$|V_{cb}|$ and $m_b$ from the fit to all available moment measurements

preliminary
# Measurements used

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>BaBar</td>
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<tr>
<td>Belle</td>
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<tr>
<td>CDF</td>
</tr>
<tr>
<td>CLEO</td>
</tr>
<tr>
<td>DELPHI</td>
</tr>
</tbody>
</table>

| | \( <E_n^i> \): n=0,1,2,3 [PRD 69, 111104 (2004)] |
| | \( <M_{2n}^X> \): n=1,2 [PRD 69, 111103 (2004)] |
| | \( <E_n^\gamma> \): n=1,2 [PRL 97, 171803 (2006)] and [PRD 72, 052004 (2005)] |
| | \( <E_n> \): n=0,1,2,3 [PRD 75, 032001 (2007)] |
| | \( <M_{2n}^X> \): n=1,2 [PRD 75, 032005 (2007)] |
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| | \( <M_{2n}^X> \): n=1,2 [PRD 70, 032002 (2004)] |
| | \( <E_n^\gamma> \): n=1 [PRL 87, 251807 (2001)] |
| | \( <E_n^i> \): n=1,2,3 |
| | \( <M_{2n}^X> \): n=1,2 [EPJ C45, 35 (2006)] |

- 70 measurements in total
$|V_{cb}|$ and $m_b$

Kinetic scheme ($X_c|\nu+X_s\gamma$ data)

$|V_{cb}| = (41.55 \pm 0.43_{\text{fit}} \pm 0.08_{\tau B} \pm 0.58_{\text{th}}) \times 10^{-3}$

$m_b^{\text{kin}} = 4.613 \pm 0.033 \text{ GeV}$

$m_c^{\text{kin}} = 1.178 \pm 0.049 \text{ GeV}$

$\chi^2/\text{ndf.} = 30.6 / (70-7)$

1S scheme ($X_c|\nu+X_s\gamma$ data)

$|V_{cb}| = (41.74 \pm 0.29_{\text{fit}} \pm 0.08_{\tau B}) \times 10^{-3}$

$m_b^{1S} = 4.708 \pm 0.024 \text{ GeV}$

$\chi^2/\text{ndf.} = 26.1 / (70-7)$

Δ$\chi^2 = 1$ contours
Summary and conclusion

- Calculations based on heavy quark effective theory and operator product expansion can reproduce inclusive observables in B decays to a high degree of precision.

- Results by experiment (kinetic scheme)

|                | $|V_{cb}|$ (10^{-3}) | $m_b$ (GeV) |
|----------------|---------------------|------------|
| BaBar [arXiv:0707.2670] | $41.88\pm0.56_{\text{fit}}\pm0.08_{\tau B}\pm0.59_{\text{th}}$ | $4.552 \pm 0.055$ |
| Belle [arXiv:0803.2158]   | $41.52\pm0.69_{\text{fit}}\pm0.08_{\tau B}\pm0.58_{\text{th}}$ | $4.543 \pm 0.075$ |

- Fits to all available measurements (preliminary)

|                | $|V_{cb}|$ (10^{-3}) |
|----------------|---------------------|
| Kinetic scheme | $41.55\pm0.43_{\text{fit}}\pm0.08_{\tau B}\pm0.58_{\text{th}}$ |
| 1S scheme      | $41.74\pm0.29_{\text{fit}}\pm0.08_{\tau B}$ |