

Inclusive $|V_{cb}|$ and global fits

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The CKM matrix

- Couplings of the charged current interaction in the SM

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \overline{u_{Li}} \gamma^\mu (V_{\text{CKM}})_{ij} d_{Lj} W_\mu^\pm + \text{h.c.}$$

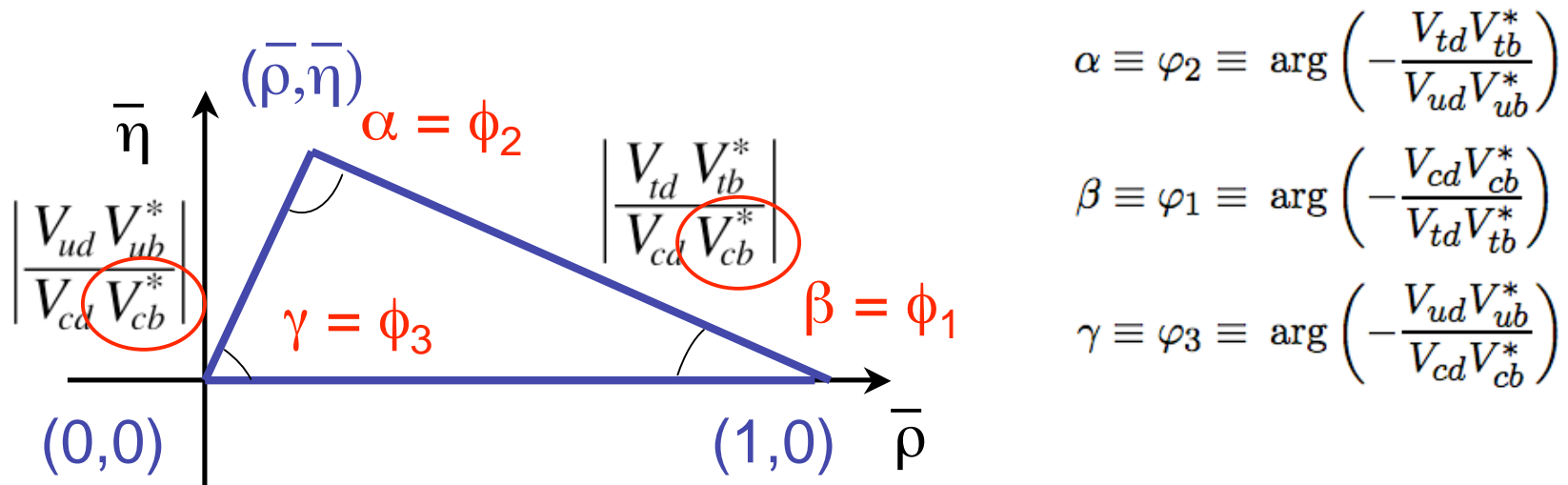
$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- V_{CKM} is a **unitary 3x3 matrix**;
it contains three real parameters and one complex phase

[Kobayashi, Maskawa, Prog. Theor. Phys. 49, 652 (1973)]

- The unitarity of V_{CKM} can be probed by measuring the sides and angles of the **unitarity triangle**

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$



- In this presentation, I'll focus on the determination of $|V_{cb}|$ from inclusive semileptonic B decays

The semileptonic width

- $\Gamma(B \rightarrow X_c \ell \nu)$ can be systematically calculated with the operator production expansion (OPE)

$$\Gamma_{\text{sl}}(b \rightarrow c) = \frac{G_F^2 m_b^5(\mu)}{192 \pi^3} |V_{cb}|^2 (1 + A_{\text{ew}}) A^{\text{pert}}(r, \mu)$$

$$\left[z_0(r) \left(1 - \frac{\mu_\pi^2(\mu) \mu_G^2(\mu) + \frac{\rho_D^3(\mu) \rho_{LS}^3(\mu)}{m_b(\mu)}}{2m_b^2(\mu)} \right) - 2(1-r)^4 \frac{\mu_G^2(\mu) \frac{\rho_D^3(\mu) \rho_{LS}^3(\mu)}{m_b(\mu)}}{m_b^2(\mu)} + d(r) \frac{\rho_D^3(\mu)}{m_b^3(\mu)} + \dots \right]$$

from [Benson et al.,
Nucl. Phys. B665,
367 (2003)]

$$r = m_c^2(\mu) / m_b^2(\mu)$$

○ ... HQ parameters (non-calculable;
contain soft QCD physics)

- At each order in $1/m_b$, the expectation values of local operator products (heavy quark parameters) are multiplied by perturbatively calculable coefficients

Other observables in B decays

- Moments of the lepton energy spectrum in $B \rightarrow X_c l \nu$

$$R_n(E_{\text{cut}}, \mu) = \int_{E_{\text{cut}}} (E_\ell - \mu)^n \frac{d\Gamma}{dE_\ell} dE_\ell, \quad \langle E_\ell^n \rangle_{E_{\text{cut}}} = \frac{R_n(E_{\text{cut}}, 0)}{R_0(E_{\text{cut}}, 0)}$$

- Moments of the hadronic mass spectrum in $B \rightarrow X_c l \nu$

$$\langle m_X^{2n} \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} (m_X^2)^n \frac{d\Gamma}{dm_X^2} dm_X^2}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dm_X^2} dm_X^2}$$

- Moments of the photon energy spectrum in $B \rightarrow X_s \gamma$

$$\langle E_\gamma^n \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} E_\gamma^n \frac{d\Gamma}{dE_\gamma} dE_\gamma}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dE_\gamma} dE_\gamma}$$

The OPEs of these inclusive observables contain the same HQ parameters

Global analysis of B decays

- Dedicated predictions for each observable
 - $\langle E_l^n \rangle_{E_l > E_{\text{cut}}} = f^{(n)}(E_{\text{cut}}, m_b, \text{HQ param.})$
 - $\langle M_X^{2n} \rangle_{E_l > E_{\text{cut}}} = g^{(n)}(E_{\text{cut}}, m_b, \text{HQ param.})$
 - $\langle E_\gamma^n \rangle_{E_\gamma > E_{\text{cut}}} = h^{(n)}(E_{\text{cut}}, m_b, \text{HQ param.})$
- Determine HQ parameters by performing a **minimum χ^2 fit to all available moment measurements**
- Take into account correlated experimental and theoretical errors
- External input: average B lifetime $\tau_B = (1.585 \pm 0.006)$ ps

Available calculations

- Kinetic running mass
 - [P.Gambino, N.Uraltsev, Eur.Phys.J. C34, 181 (2004)]
 - [D.Beson, I.Bigi, N.Uraltsev, Nucl.Phys. B710, 371 (2005)]
- 1S mass
 - [C.Bauer, Z.Ligeti, M.Luke, A.Manohar, M.Trott, Phys.Rev. D70, 094017 (2004)]
- Non-perturbative parameters in the $1/m_b$ expansion

both calculations up to $O(1/m_b^3)$

	Kinetic scheme	1S scheme
$O(1)$	m_b, m_c	m_b
$O(1/m_b^2)$	μ_π^2, μ_G^2	λ_1, λ_2
$O(1/m_b^3)$	ρ_D, ρ_{LS}	ρ_1, τ_{1-3}

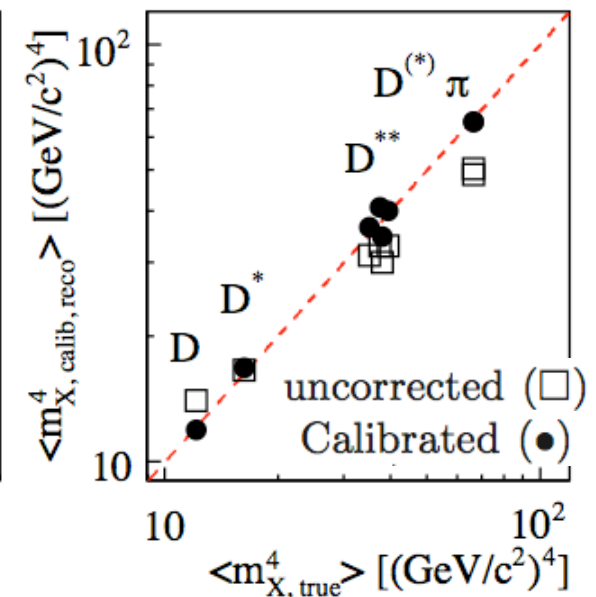
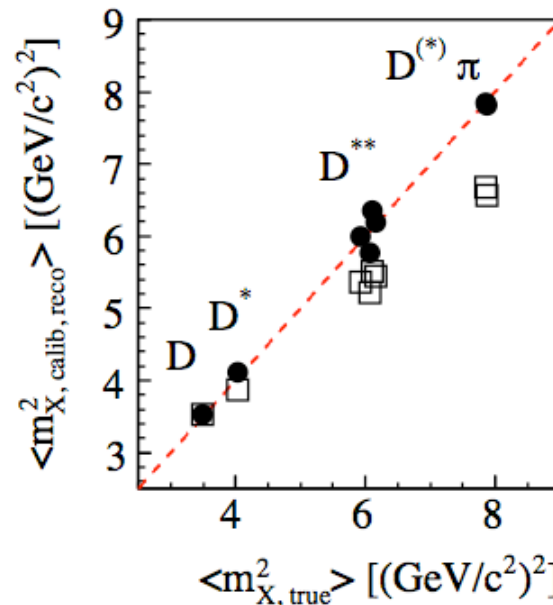
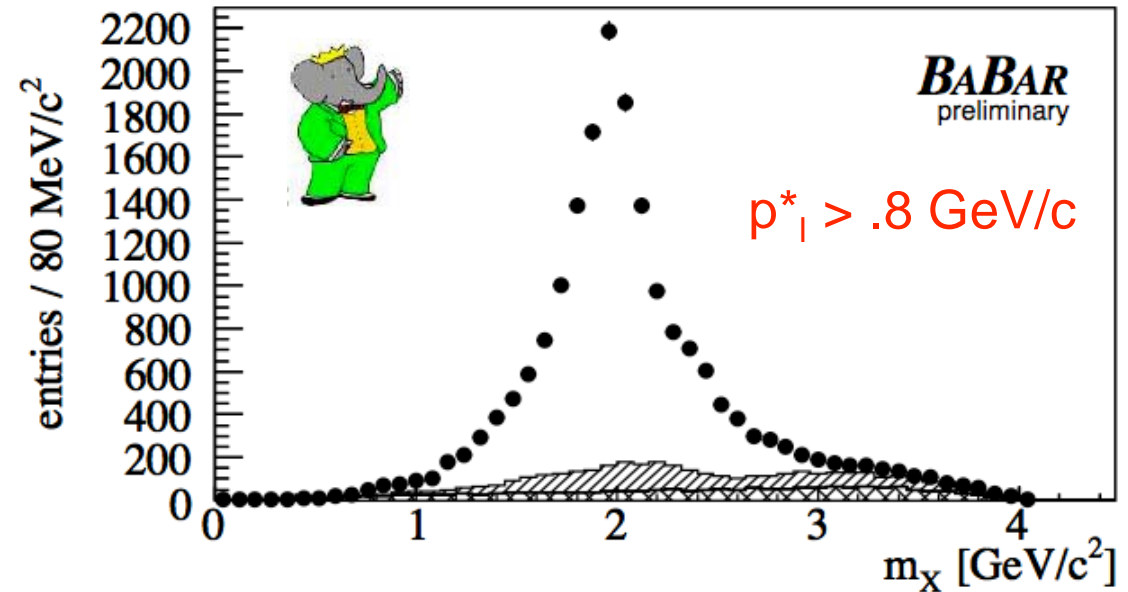
Available measurements

- Belle E_γ , 605/fb [[arXiv:0804.1580](#)] preliminary
- BaBar E_l , M^2_X , 210/fb [[arXiv:0707.2670](#)] preliminary
- Belle E_l , 140/fb [[PRD 75, 032001 \(2007\)](#)]
- Belle M^2_X , 140/fb [[PRD 75, 032005 \(2007\)](#)]
- DELPHI E_l , M^2_X , 3.4M Z [[EPJ C45, 35 \(2006\)](#)]
- BaBar, E_γ , 82/fb [[PRL 97, 171803 \(2006\)](#)]
- BaBar, E_γ , 82/fb [[PRD 72, 052004 \(2005\)](#)]
- CDF, M^2_X , 180/pb [[PRD 71, 051103 \(2005\)](#)]
- Belle, E_γ , 140/fb [[PRL 93, 061803 \(2004\)](#)]
- CLEO, M^2_X , 9/fb [[PRD 70, 032002 \(2004\)](#)]
- BaBar, E_l , 47/fb [[PRD 69, 111104 \(2004\)](#)]
- BaBar, M^2_X , 89M BB [[PRD 69, 111103 \(2004\)](#)]
- CLEO, E_γ , 9/fb [[PRL 87, 251807 \(2001\)](#)]

BaBar M_X^2 moments

[arXiv:0707.2670] preliminary

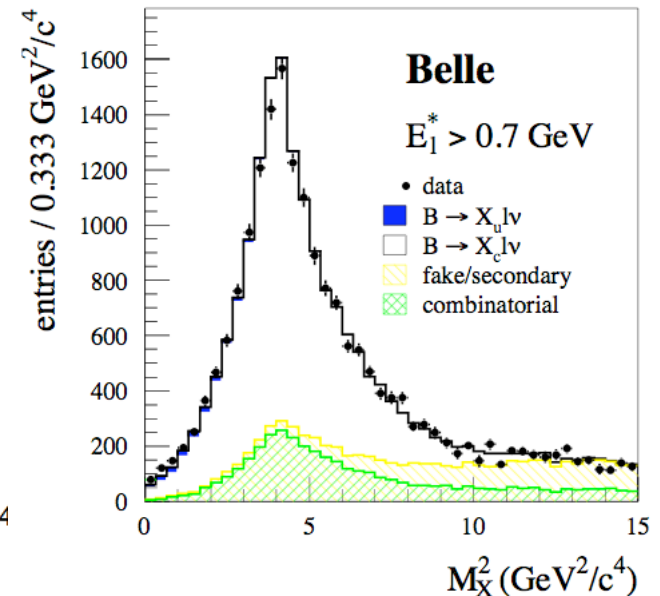
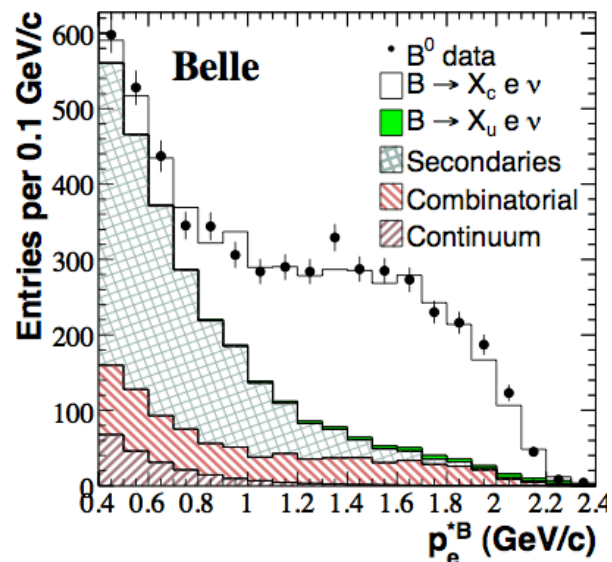
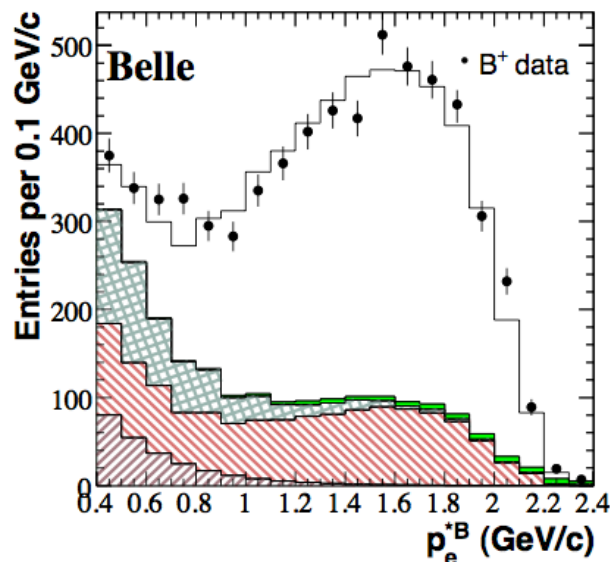
- 210/fb of Y(4S) data
- Hadronic decay of one B meson fully reconstructed
- Semileptonic decay of other B selected by requiring identified lepton (e/ μ)
- Reconstructed moments corrected event-by-event for detector effects
- $\langle M_X^k \rangle$ measured for $k=1, \dots, 6$ and p_{\perp}^* from 0.8 to 1.9 GeV/c



Belle E_1 and M_X^2 moments

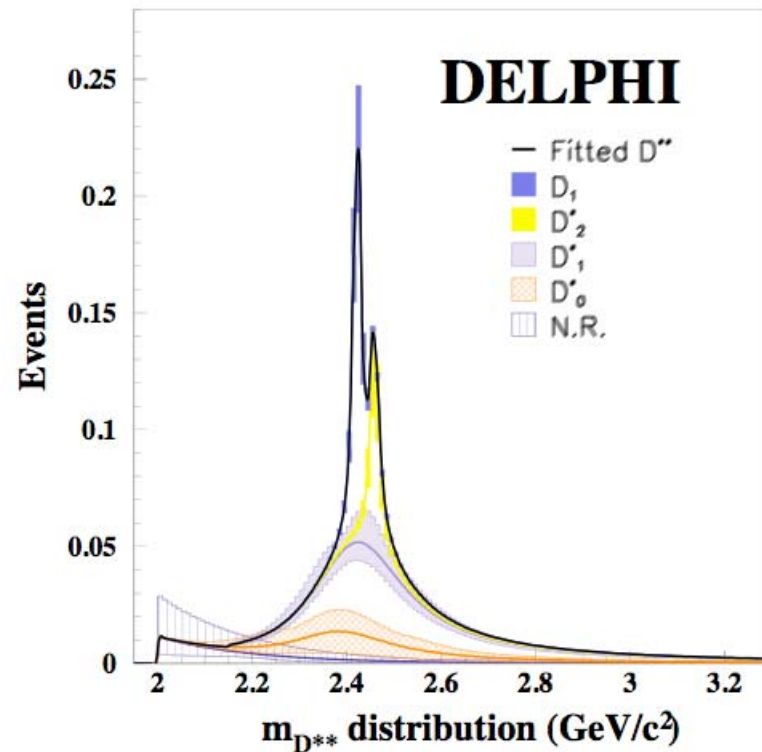
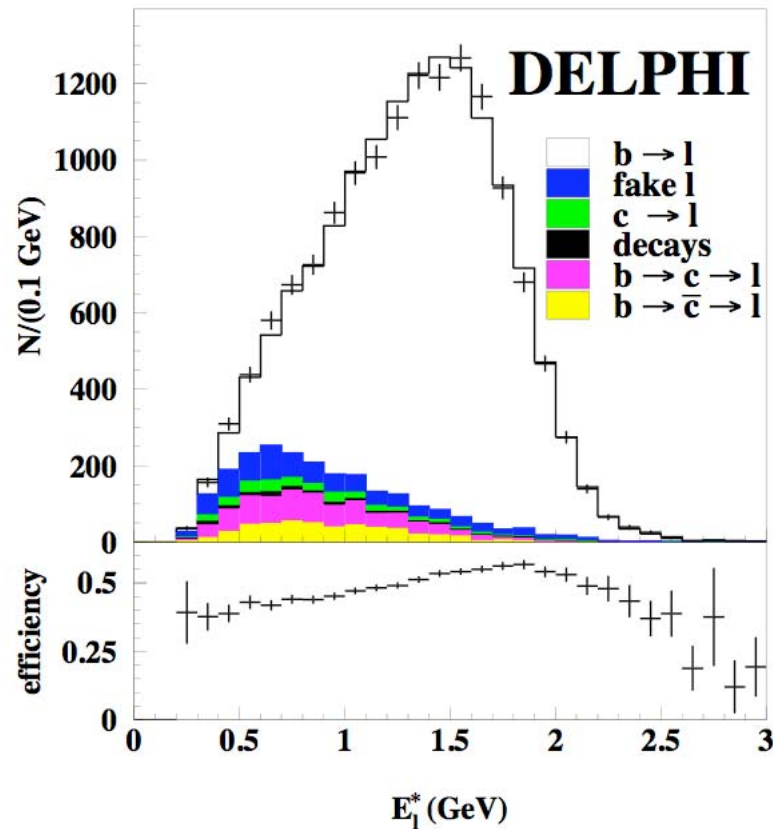
[PRD 75, 032001 (2007)]
[PRD 75, 032005 (2007)]

- 140/fb of Y(4S) data
- Measurement also done with fully reconstructed events
- The finite detector resolution is unfolded with SVD algorithm [NIM A372, 469 (1996)]
- $\langle E_e^n \rangle$ measured for $n=0, \dots, 4$ and $E_{\text{cut}}=0.4-2.0$ GeV
- $\langle M_X^{2n} \rangle$ measured for $n=1, 2$ and $E_{\text{cut}}=0.7-1.9$ GeV



DELPHI E_1 and M^2_x moments

[EPJ C45, 35 (2006)]



- $\langle E_1^n \rangle$, $n=1, \dots, 3$ and $\langle M_x^{2n} \rangle$, $n=1, \dots, 5$ measured at $E_{\text{cut}} = 0$ as in Z events the b -quark is produced with a boost
- The hadronic moments are derived from the fitted D^{**} mass spectrum; assumptions on the D^{**} decay are made

$|V_{cb}|$ and m_b from the fit to the Belle moment data



[[arXiv:0803.2158](#)] submitted to Phys.Rev.D

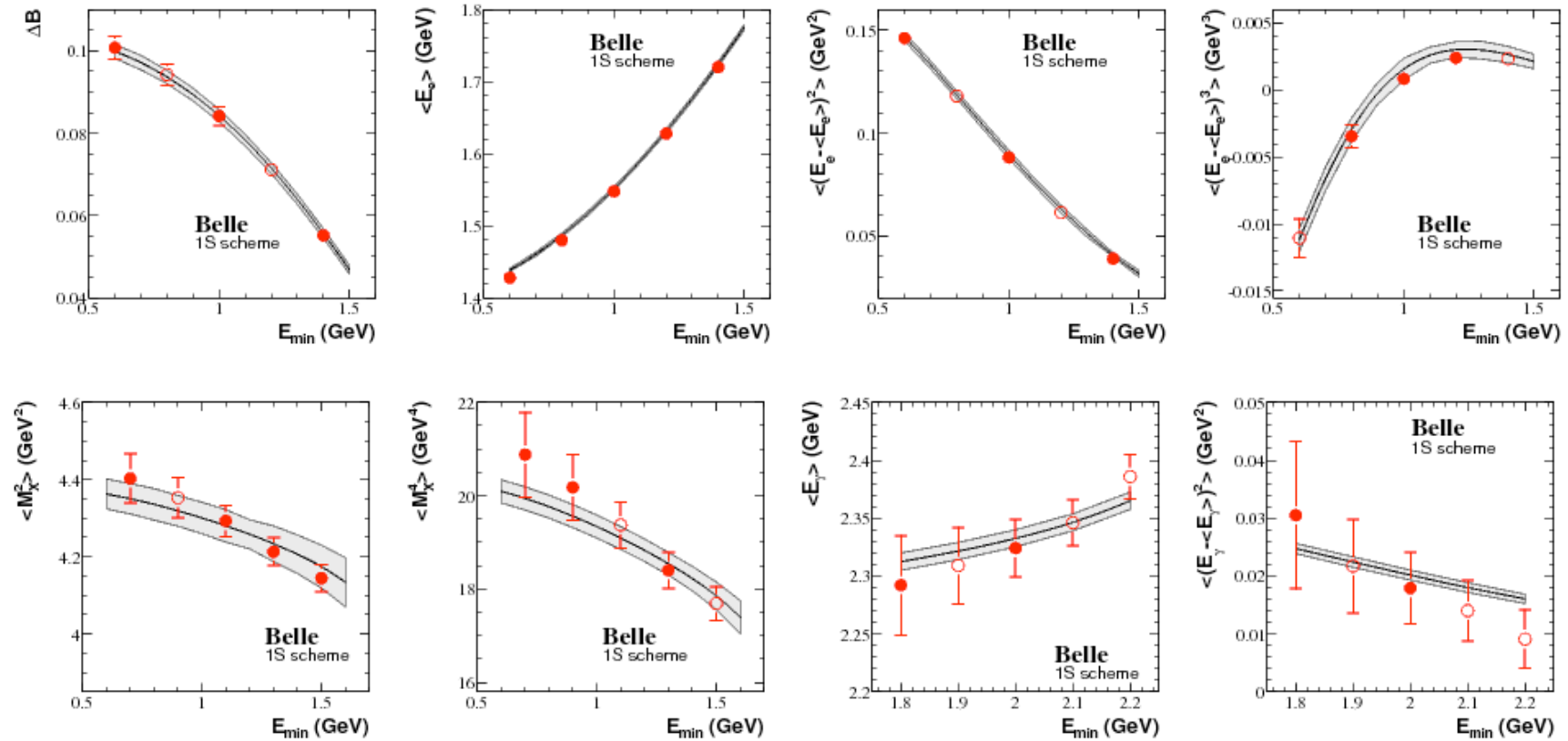
Similar analysis recently done on the BaBar
moment data [[arXiv:0707.2670](#)] preliminary

Belle measurements used

Electron moments $\langle E^n_l \rangle$	$n=0$: $E_{\text{cut}}=0.6, 1.0, 1.4$ GeV $n=1$: $E_{\text{cut}}=0.6, 0.8, 1.0, 1.2, 1.4$ GeV $n=2$: $E_{\text{cut}}=0.6, 1.0, 1.4$ GeV $n=3$: $E_{\text{cut}}=0.8, 1.0, 1.2$ GeV
Hadron moments $\langle M^{2n}_X \rangle$	$n=1$: $E_{\text{cut}}=0.7, 1.1, 1.3, 1.5$ GeV $n=2$: $E_{\text{cut}}=0.7, 0.9, 1.3$ GeV
Photon moments $\langle E^n_\gamma \rangle$	$n=1$: $E_{\text{cut}}=1.8, 2.0$ GeV $n=2$: $E_{\text{cut}}=1.8, 2.0$ GeV

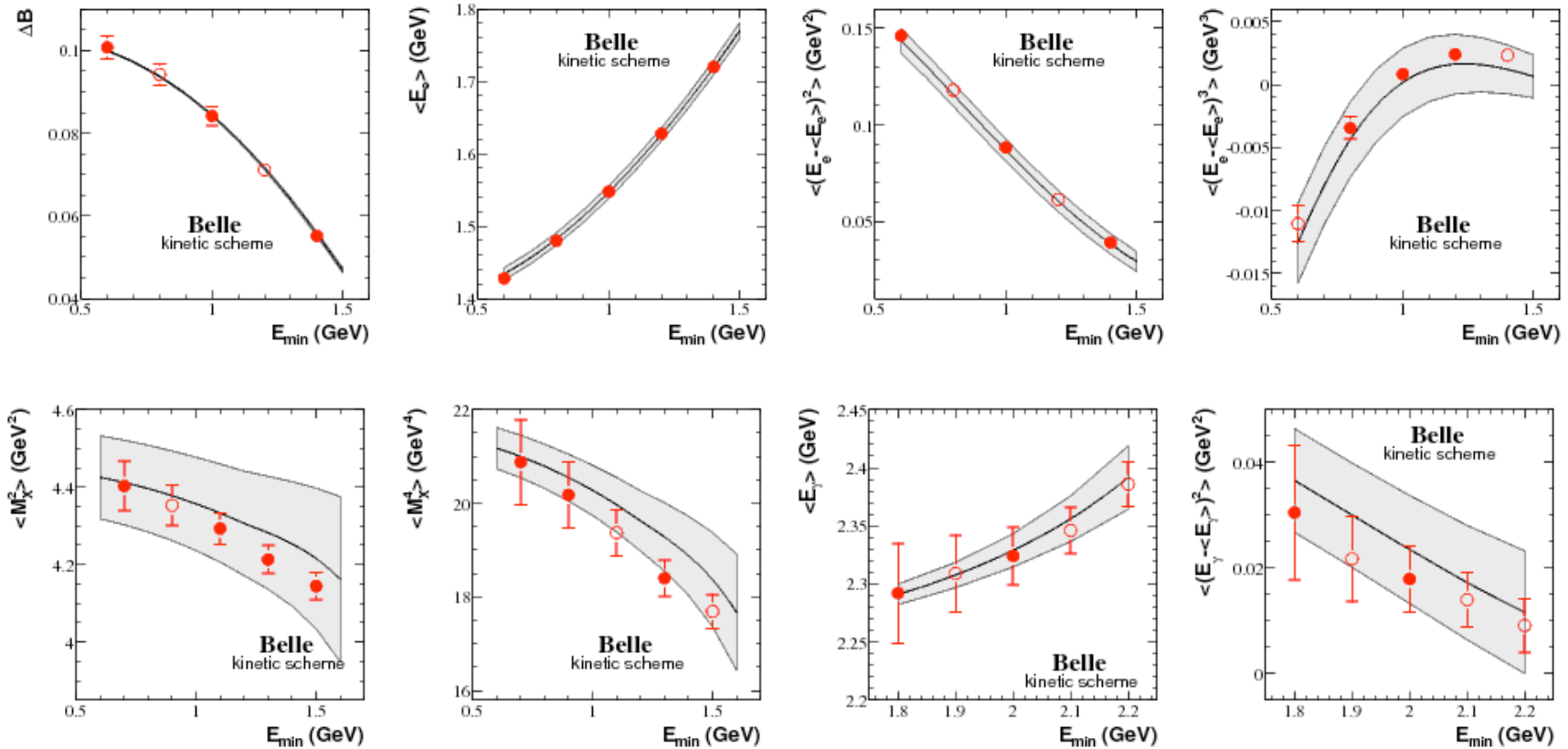
- Exclude measurements
 - with no (reliable) theory prediction
 - with excessive correlations

Fit result in the 1S scheme



$$\chi^2/\text{ndf.} = 7.3 / (25-7)$$

Fit result in the kinetic scheme



$$\chi^2/\text{ndf.} = 4.7 / (25-7)$$

$|V_{cb}|$ and m_b

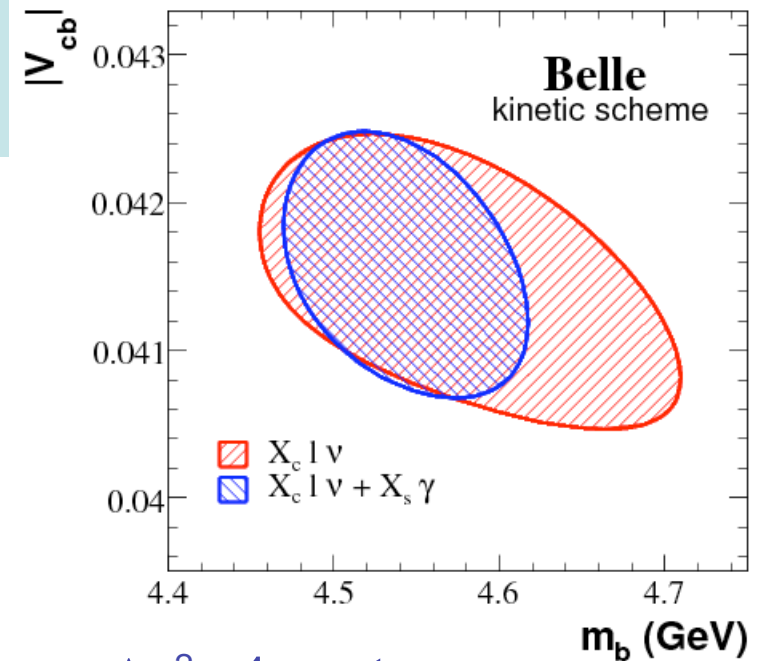
Kinetic scheme ($X_c l \nu + X_s \gamma$ data)

$$|V_{cb}| = (41.52 \pm 0.69_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.58_{\text{th}}) \times 10^{-3}$$
$$m_b^{\text{kin}} = 4.543 \pm 0.075 \text{ GeV}$$
$$m_c^{\text{kin}} = 1.055 \pm 0.118 \text{ GeV}$$

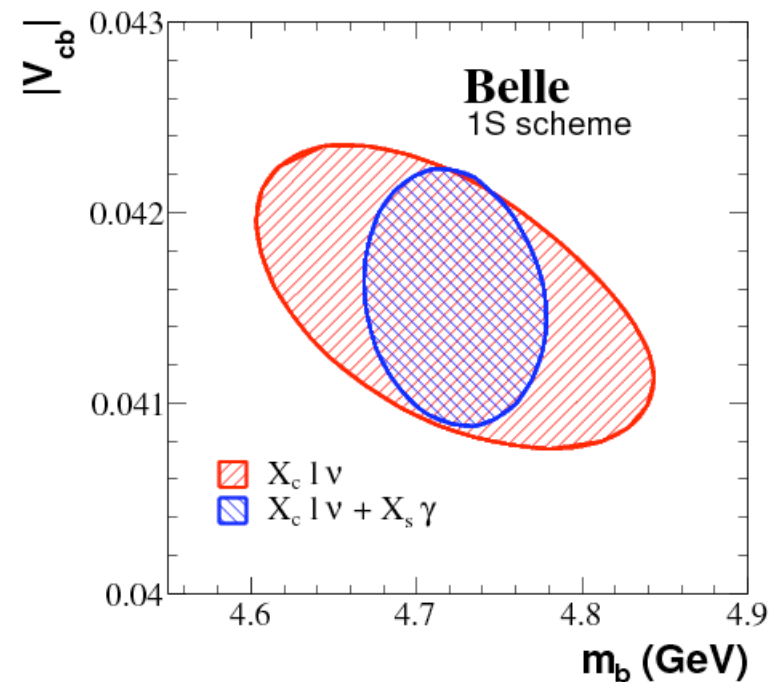
Results for m_b compatible after
scheme translation

1S scheme ($X_c l \nu + X_s \gamma$ data)

$$|V_{cb}| = (41.56 \pm 0.68_{\text{fit}} \pm 0.08_{\tau_B}) \times 10^{-3}$$
$$m_b^{1S} = 4.723 \pm 0.055 \text{ GeV}$$



$\Delta\chi^2 = 1$ contours



$|V_{cb}|$ and m_b from the fit to
all available moment measurements



preliminary

Measurements used

BaBar	$\langle E^n_l \rangle$: $n=0,1,2,3$ [PRD 69, 111104 (2004)] $\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 69, 111103 (2004)] $\langle E^n_\gamma \rangle$: $n=1,2$ [PRL 97, 171803 (2006)] and [PRD 72, 052004 (2005)]
Belle	$\langle E^n_l \rangle$: $n=0,1,2,3$ [PRD 75, 032001 (2007)] $\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 75, 032005 (2007)] $\langle E^n_\gamma \rangle$: $n=1,2$ [arXiv:0804.1580] preliminary
CDF	$\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 71, 051103 (2005)]
CLEO	$\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 70, 032002 (2004)] $\langle E^n_\gamma \rangle$: $n=1$ [PRL 87, 251807 (2001)]
DELPHI	$\langle E^n_l \rangle$: $n=1,2,3$ $\langle M^{2n}_X \rangle$: $n=1,2$ [EPJ C45, 35 (2006)]

- 70 measurements in total

$|V_{cb}|$ and m_b

Kinetic scheme ($X_c l \nu + X_s \gamma$ data)

$$|V_{cb}| = (41.55 \pm 0.43_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.58_{\text{th}}) \times 10^{-3}$$

$$m_b^{\text{kin}} = 4.613 \pm 0.033 \text{ GeV}$$

$$m_c^{\text{kin}} = 1.178 \pm 0.049 \text{ GeV}$$

$$\chi^2/\text{ndf.} = 30.6 / (70-7)$$

preliminary

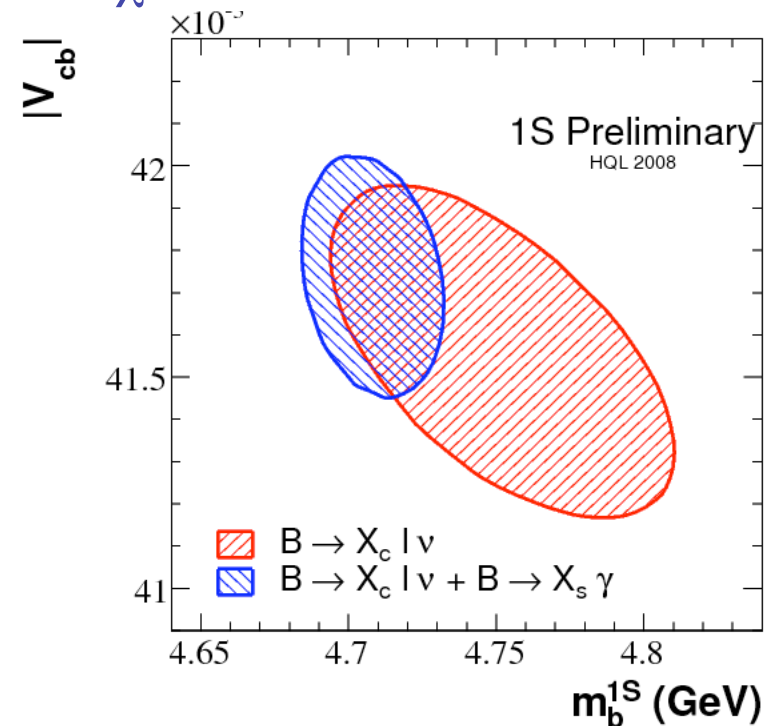
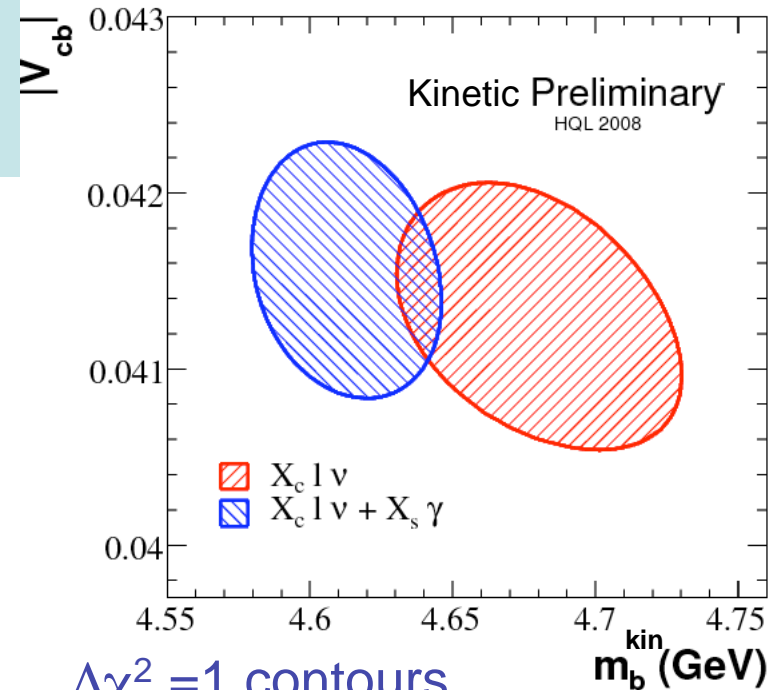


1S scheme ($X_c l \nu + X_s \gamma$ data)

$$|V_{cb}| = (41.74 \pm 0.29_{\text{fit}} \pm 0.08_{\tau_B}) \times 10^{-3}$$

$$m_b^{1S} = 4.708 \pm 0.024 \text{ GeV}$$

$$\chi^2/\text{ndf.} = 26.1 / (70-7)$$



Summary and conclusion

- Calculations based on heavy quark effective theory and operator product expansion can reproduce inclusive observables in B decays to a high degree of precision
- Results by experiment (kinetic scheme)

	$ V_{cb} (10^{-3})$	m_b (GeV)
BaBar [arXiv:0707.2670]	$41.88 \pm 0.56_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.59_{\text{th}}$	4.552 ± 0.055
Belle [arXiv:0803.2158]	$41.52 \pm 0.69_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.58_{\text{th}}$	4.543 ± 0.075

- Fits to all available measurements (preliminary)

	$ V_{cb} (10^{-3})$
Kinetic scheme	$41.55 \pm 0.43_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.58_{\text{th}}$
1S scheme	$41.74 \pm 0.29_{\text{fit}} \pm 0.08_{\tau_B}$