Impedance budget

First parameter estimation:

Parameter	PS2 INJECTION	PS2 EXTRACTION	HPPS-50 INJECTION	HPPS-50 EXTRACTION
Energy[GeV]	4	50	4	50
C[m]	1346.4		1174	
b[mm](pipe radius)	32.5		32.5	
σ_z[m]	1.12	0.30	4.38	4.38
$\overline{\beta_y}$[m]	32		31.61	
$\varepsilon_y[\mu m\ rad]$	1.8	0.17	2.85	0.23
η (slip factor)	-0.037	-0.0012	-5.56e-2	-8.91e-4
σ_δ	3.2e-3	1e-3	3.8e-3	3.2e-4
N_b	4.2e11		18e11	
N_p	1.1e14		2.5e14	
Q_s	18e-3	0.8e-3	6.13e-3	7.27e-5
Q_y	6.71		7.21	
I[A]	2.7		11.52	

Impedance budget

Impedance		PS2 Injection	HPPS-50 Injection	PS2 Extraction	HPPS-50 Extraction
Broad band (RW+SC) Longitudinal Impedance	$\frac{Z}{n}[\Omega]$	0.39+49i	0.74-22.80i	0.20+0.13i	0.76-0.38i
Transverse kick factor	k_y[V/pC/m]	24	10.6	47	10.6

PS2 has in account 1500 vacuum flanges

Analytic estimations following: "Impedance considerations for the design of the vacuum system of the CERN PS2 Proton Synchrotron", K.L.F Bane et al.

Instability		PS2 Injection	HPPS-50 Injection	PS2 Extraction	HPPS-50 Extraction
Microwave instability [Single bunch long.]	$\frac{N_{th}}{N_b}$	27	112	59	4.3
TMCI [Single bunch transverse]	$\frac{N_{th}}{N_b}$	10	2.2	2.5	0.3
TCBI [Multi-bunch transverse]	Turns	30	13	294	141

Impedance budget

$$Z = (1-i)\frac{l}{2\pi b}\frac{1}{\delta_s \sigma_c}$$

$$Z/n = Z/(w/w0)$$

Round pipe
Thick wall

- l pipe length
- b pipe radius
- $\sigma_c = 1.35 \cdot 10^6 \Omega^{-1} m^{-1}$ metal conductivity (assumed SS)
- Typical beam size calculated with emittance and average beta
- δ_s skin depth
  - $\delta_s = \sqrt{2c/Z_0 \sigma_c \omega}$ with $Z_0 = 377\Omega$ and $\omega = c/\sigma_z$ typical frequency of the bunch, σ_z the bunch length

$$\frac{Z}{n} \approx i\frac{Z_0}{2\gamma^2}\left(1 + 2ln\frac{b_y}{\sigma_y}\right)$$

Kick factor

$$k_y = -\langle W_y \rangle = \frac{3.63}{2^{3/2}\pi^2}\frac{cl}{b^3 \sigma_z^{1/2}}\sqrt{\frac{Z_0}{\sigma_c}}$$

"Microwave instability"
Boussard criterion

$$\frac{N_{th}}{N_b} \leq (2\pi)^{3/2}\frac{|\eta|\sigma_z E \sigma_\delta^2}{e^2 c N_b |z/n|}$$

"Transverse Mode Coupling Instability TMCI"

$$\frac{N_{th}}{N_b} \sim (0.7)\frac{4\pi E \nu_s}{e^2 N_b \overline{\beta_y} k_y}$$

"Transverse coupled bunch Instability
Assuming only RW

$$\Gamma = \frac{c}{4\gamma}\frac{m_e I}{m_p I_A}\sqrt{\frac{l}{1-[\nu_y]}}\langle \overline{\beta_y} A_y \rangle$$

- Γ the growth rate of the instability
- I average current assuming full ring filled with bunches
- $I_A = 17kA$
- $[\nu_y]$ fractional part of vertical tune
- $\overline{\beta_y}$ average beta function
- $A_y = \frac{4}{\pi^{1/2} b^3}\sqrt{\frac{1}{Z_0 \sigma_c}}$