# *From Colour Glass Condensate to Quark–Gluon Plasma*

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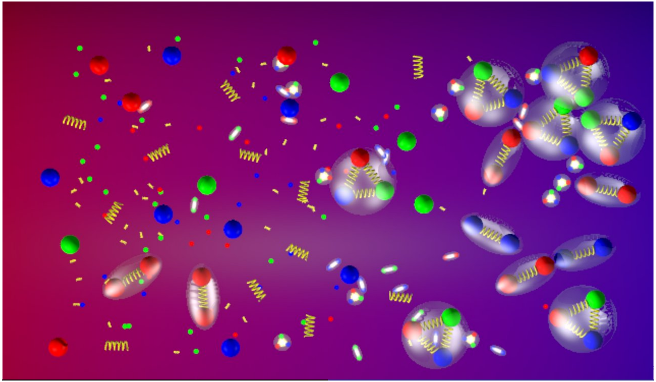
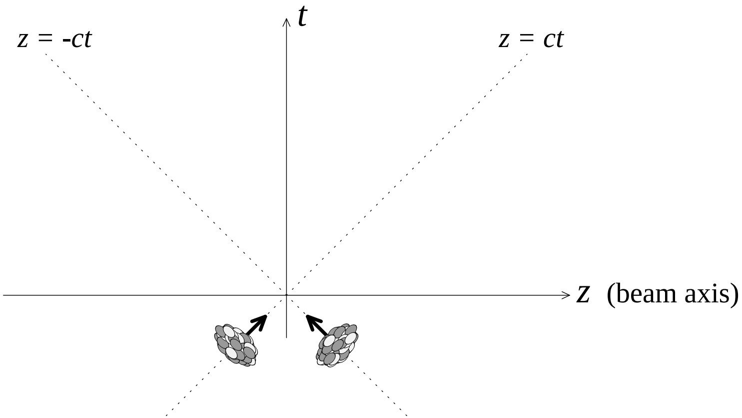
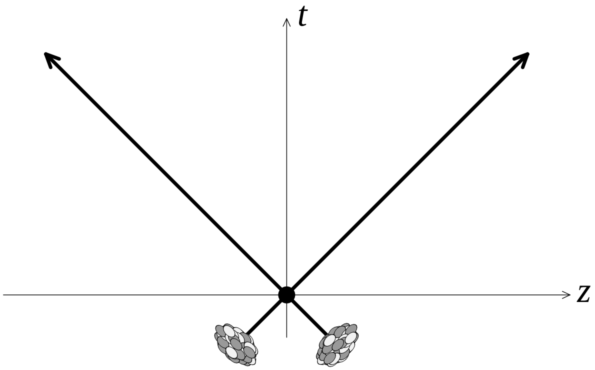


# Lecture I: Initial conditions




- $\tau < 0$ : hadronic wavefunctions prior to the collision

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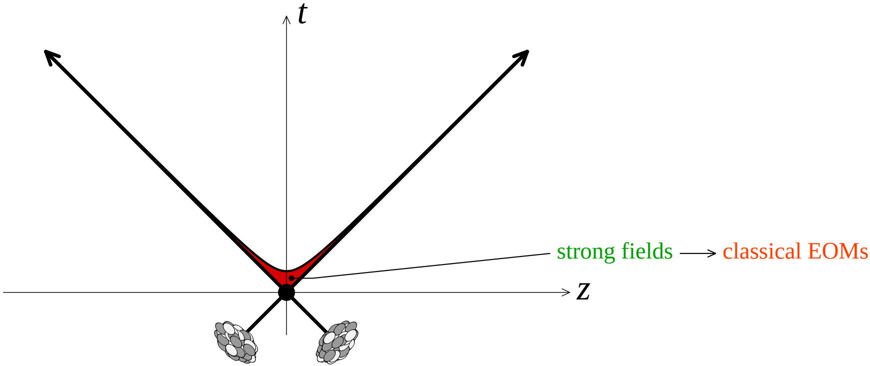
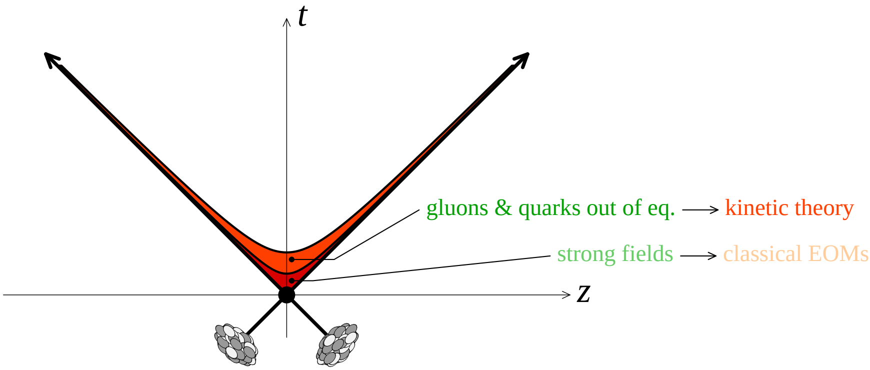
- $\tau \sim 0$ fm/c : the hard scattering

# Lecture I: Initial conditions

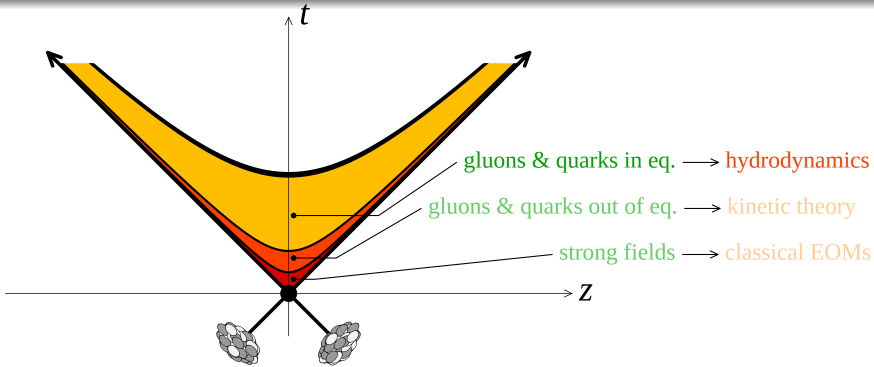



- $\tau < 0$ : hadronic wavefunctions prior to the collision
- $\tau \sim 0$ fm/c : the hard scattering
- $\tau \sim 0.2$ fm/c : strong color fields (or 'glasma')



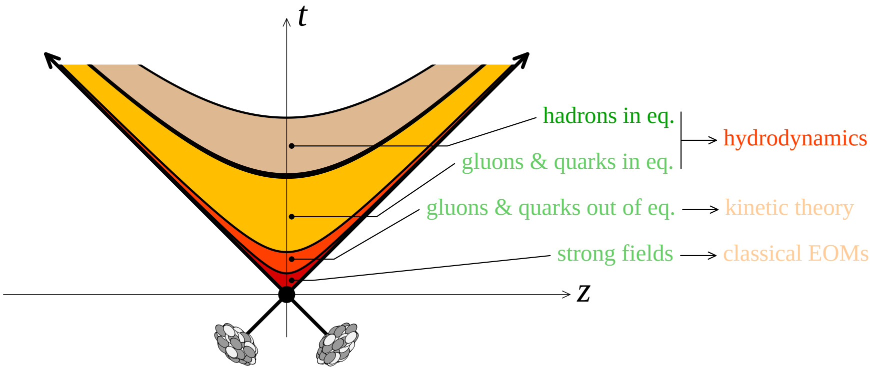

- $\tau \sim 1$ fm/c : thermalization

# Lecture II: Quark–Gluon Plasma



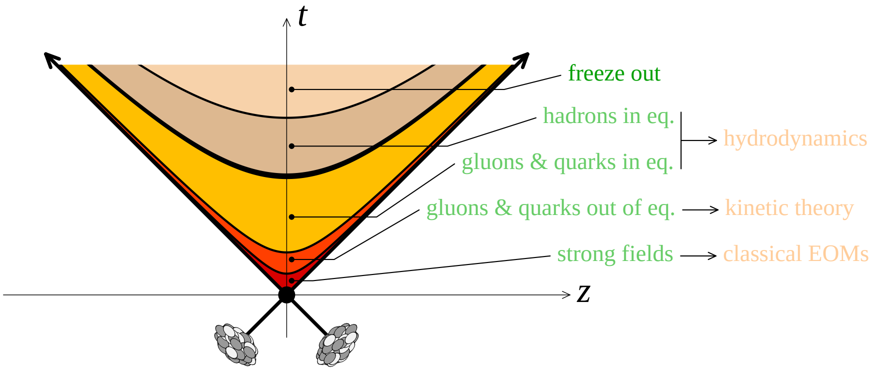

- $\tau \sim 1$ fm/c : thermalization
- $1 \lesssim \tau \lesssim 10$ fm/c : quark–gluon plasma
  - flow and hydrodynamics
  - thermodynamics: lattice QCD vs. perturbative QCD
  - collective phenomena: screening, hard thermal loops
  - jet quenching and di-jet asymmetry

# The late stages (not to be discussed here)



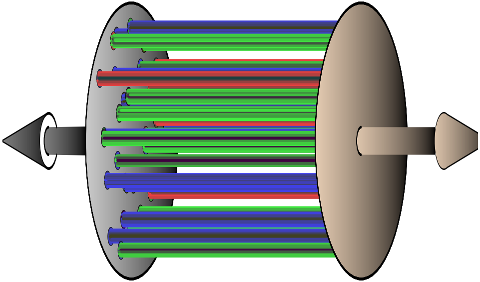

- $10 \lesssim \tau \lesssim 20$ fm/c : hot hadron gas
  - hadronisation: confinement
  - the hadron gas keeps expanding and cooling down

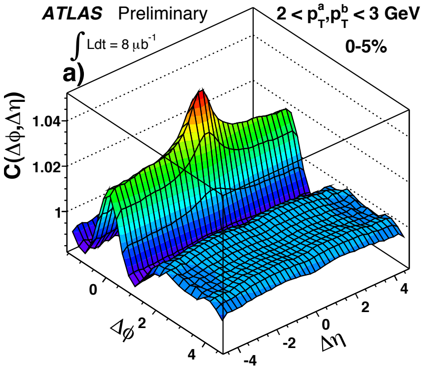
# The late stages (not to be discussed here)



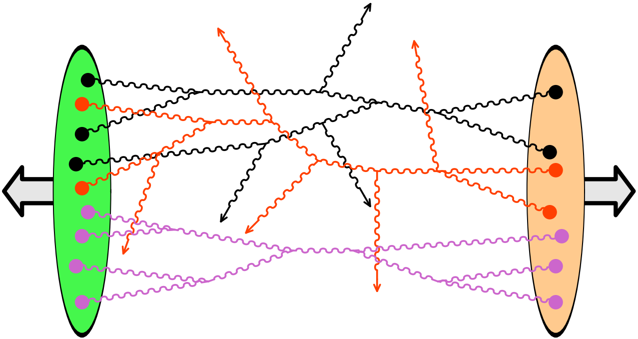

- $10 \lesssim \tau \lesssim 20$ fm/c : hot hadron gas
- $\tau > 20$ fm/c : freeze out
  - the density becomes too small to allow for interactions
  - the produced hadrons are measured by the detectors

# $AA$ collisions : Glasma & the Ridge



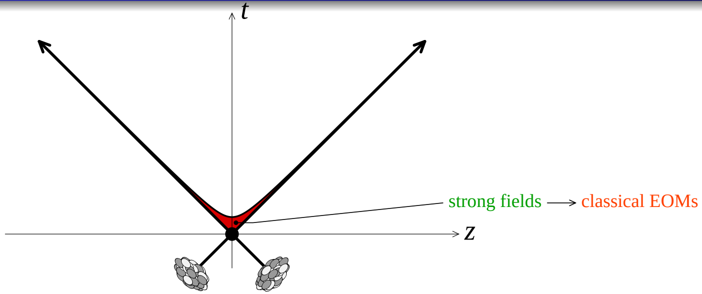


# Nucleus–nucleus collisions



- Weakly coupled ($\alpha_s \ll 1$) but dense ($n \sim 1/\alpha_s$) : highly non–linear
- Two strong color fields (CGC's) with scatter with each other
- 'Scattering' : non–linear effects in the classical Yang–Mills equation sourced by the color charges in the 2 nuclei

# Nucleus–nucleus collisions



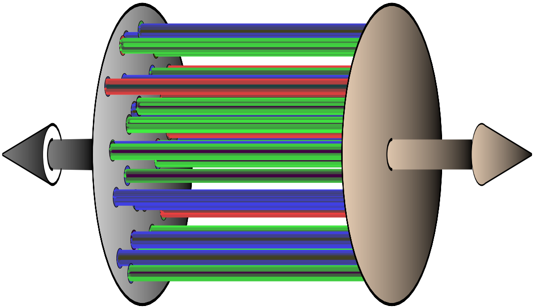

$$D_\nu F^{\nu\mu}(x) \,=\, \delta^{\mu+}\rho_R(x) \,+\, \delta^{\mu-}\rho_L(x)$$

- $\rho_{R,L}(x)$ : colour charge distributions in the 'right' and 'left' mover
- Solve the YM eqs. numerically (2D lattice) $\Longrightarrow$ the glasma field
- Average over $\rho_{R,L}(x)$ using the respective CGC weight functions
- Decompose the 2-point function in Fourier modes $\Longrightarrow$ gluon spectrum
- Initial conditions for the subsequent evolution of the fireball

# Glasma

- Right after the collision, the chromo-electric and chromo-magnetic fields are purely longitudinal
- Flux tubes which extend between the recessing nuclei

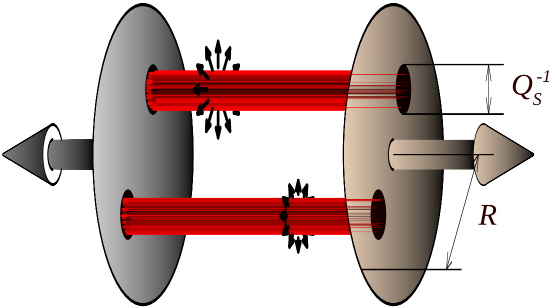
  'glasma' (from 'glass' + 'plasma') *(McLerran and Lappi, 06)*



- These anisotropic configurations are unstable (Weibel instability)

# From flux tubes to particles

- At time $\tau \sim 1/Q_s$, the glasma flux tubes break into particles (gluons)
- Gluons emitted from the same flux tube are correlated with each other




- correlation length in the transverse plane: $\Delta r_\perp \sim 1/Q_s$
- correlation length in rapidity ($Y$ or $\eta$): $\Delta\eta \sim 1/\alpha_s$
- to start with, this correlation is isotropic in $\Delta\Phi$

# From flux tubes to particles

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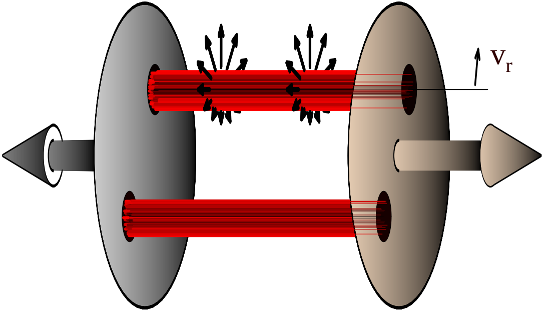
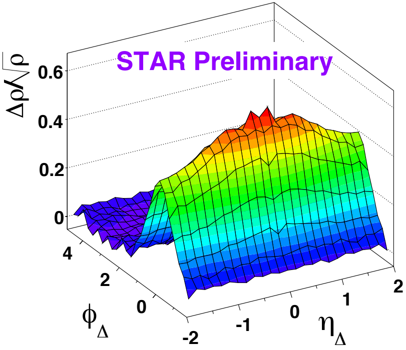
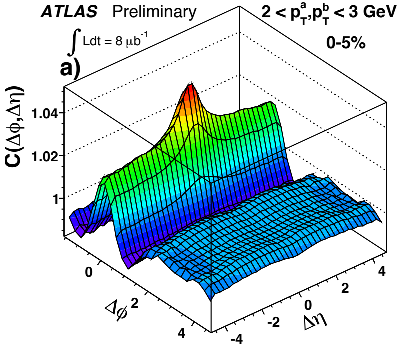



- correlation length in the transverse plane: $\Delta r_\perp \sim 1/Q_s$
- correlation length in rapidity ($Y$ or $\eta$): $\Delta\eta \sim 1/\alpha_s$
- in presence of radial flow, there is a bias leading to collimation in $\Delta\Phi$

  ▷ more particles along the radial velocity $\boldsymbol{v}_r$ than perpendicular to it

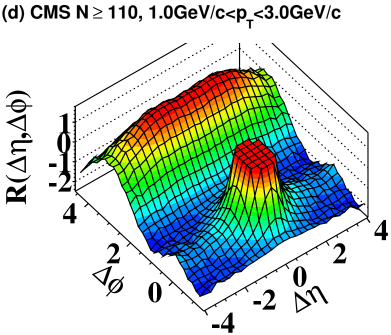
- A natural explanation for the ‘ridge’ :
  - di–hadron correlations long–ranged in $\Delta\eta$ & narrow in $\Delta\phi$
  - abundantly observed in $AA$ collisions at RHIC and the LHC

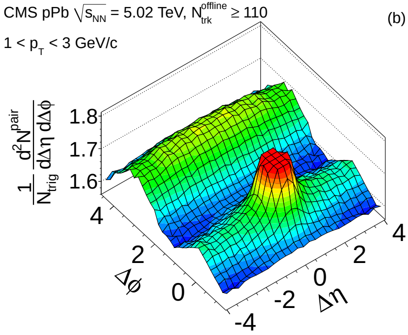


# The Ridge in $pp$ and $pA$

- LHC : quite surprisingly, a ridge is also observed in p+p and p+A events with unusually high multiplicity




- What is the origin of the azimuthal collimation ?
- Can flow develop in such small systems ($\sim 1$ fm) ?
- This might reflect the momentum correlations at early times (glasma)

# The thermalization puzzle

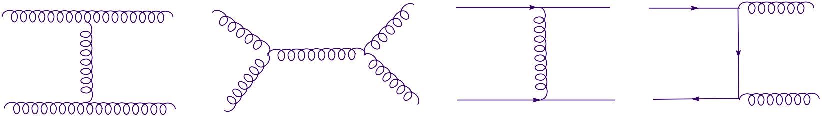
- Is there a quark–gluon plasma in the intermediate stages of a HIC ?
  - this requires local thermal equilibrium
  - to equilibrate, particles need to efficiently exchange energy and momentum
  - thermalization is not guaranteed for a system which expands and which is weakly coupled
- Just after the collision, the partonic matter is highly anisotropic
  - the glasma flux tubes have ‘negative longitudinal pressure’ : they oppose to expansion (like a string of rubber)

$$T_{\rm eq} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon/3 & 0 & 0 \\ 0 & 0 & \varepsilon/3 & 0 \\ 0 & 0 & 0 & \varepsilon/3 \end{pmatrix} \qquad T_{\rm initial} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & -\varepsilon \end{pmatrix}$$

- in equilibrium: $P_T = P_L = \varepsilon/3$ ; in the early glasma: $P_T = \varepsilon = -P_L$

# Thermalization in perturbation theory

- Particles can exchange energy and momentum through collisions.
- Weak coupling: the dominant mechanism is $2 \to 2$ elastic scattering



- Cross–section ($\sigma$) scales like $|\text{amplitude}|^2$, hence like $g^4 \sim \alpha_s^2$
- Mean free path ($\ell$) = average distance between successive collisions

$$\ell \sim \frac{1}{\text{density} \times \sigma} \sim \frac{1}{\alpha_s^2}$$

- Typical equilibration time: $\tau_{\text{eq}} \sim \ell/v \sim 1/\alpha_s^2$
- Weakly coupled systems have large equilibration times ! ☹

# The role of the strong fields

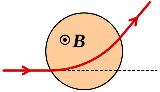
- Heisenberg's uncertainty principle requires

$$\text{mean free path } \ell \gtrsim \text{ de Broglie wavelength } \lambda \sim \frac{1}{p}$$

- In general, weakly interacting systems have $\ell \gg \lambda$
  - weakly coupled QGP, temperature $T$ : $\lambda \sim 1/T$ while $\ell \sim 1/[\alpha_s^2 T]$
- However, the situation can change for a particle interacting with a strong electric, or magnetic, field, as in the glasma
  - domain of size $Q_s^{-1}$ where the (chromo) magnetic field is $|\boldsymbol{B}| \sim Q_s^2/g$

Lorentz force : $\dfrac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = g\boldsymbol{v} \times \boldsymbol{B} \implies \dot{\theta} \sim \dfrac{gB}{p} \sim Q_s$

  - time spent in the domain $\tau \sim Q_s^{-1} \implies \Delta\theta \sim \mathcal{O}(1)$

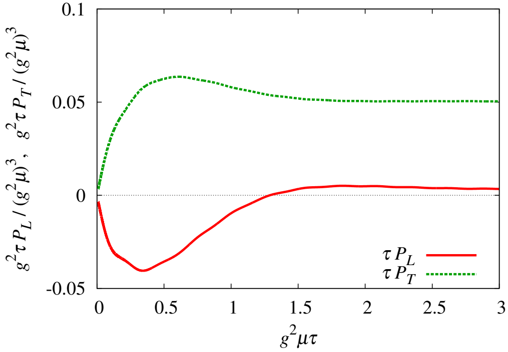



- Mean free path $\ell \sim Q_s^{-1} \sim 1/p$ : as low as permitted by Heisenberg

# Thermalization at weak coupling & strong fields

*(Epelbaum and Gelis, 2013)*

- Numerical solutions to classical Yang–Mills eq. confirm the anisotropy

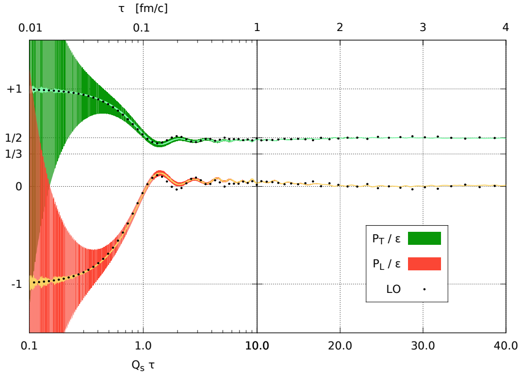



- the saturation momentum $Q_s = g^2\mu$ sets the scale
- $\tau\varepsilon = \tau\big(2P_T + P_L\big) \approx$ const. (longitudinal expansion)
- $\tau P_L$ starts by being negative, then it becomes positive, but it remains much smaller than $\tau P_T$

*(Epelbaum and Gelis, 2013)*

- However, this boost–invariant classical solution is unstable under rapidity–dependent quantum fluctuations.
- The fluctuations can be added to the initial conditions

$\alpha_s = 8\,10^{-4}\ (g = 0.1)$



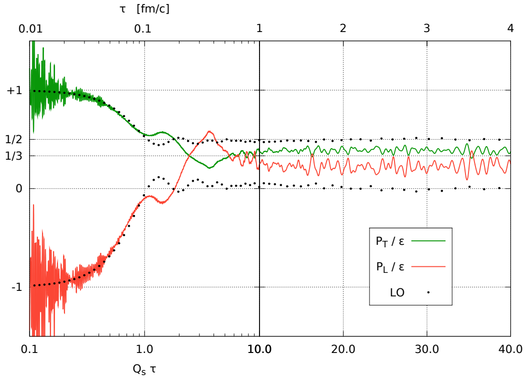

- for very small $g = 0.1$, the solution shows anisotropy, as at LO

# Thermalization at weak coupling & strong fields

*(Epelbaum and Gelis, 2013)*

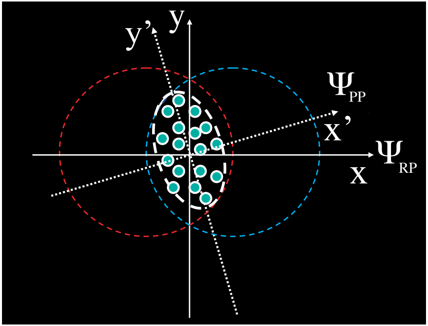
- However, this boost–invariant classical solution is unstable under rapidity–dependent quantum fluctuations.
- The fluctuations can be added to the initial conditions

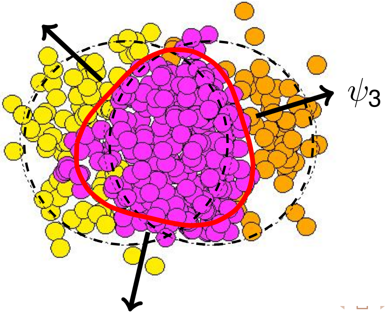
$\alpha_s = 2\,10^{-2}$ $(g = 0.5)$



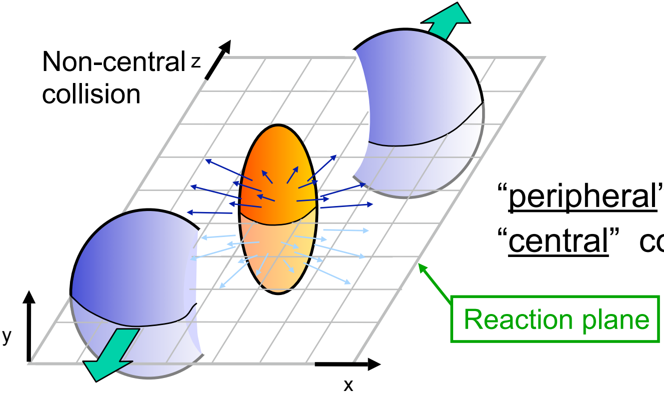

- for $g \gtrsim 0.5$, it approaches isotropy: $P_L/P_T \simeq 0.7$ ☺

# Flow and Thermalization



# The geometry of a HIC

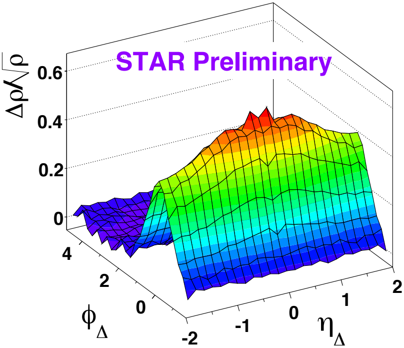



"peripheral" collision ($b \sim b_{max}$)

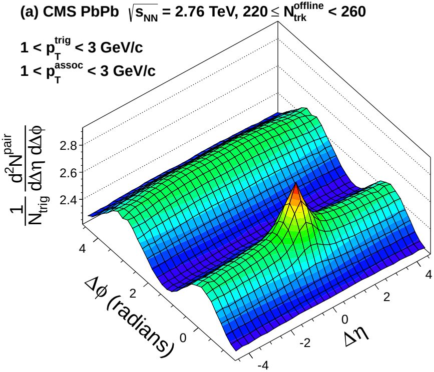
"central" collision ($b \sim 0$)

Number of participants ($N_{part}$): number of incoming nucleons (participants) in the overlap region

- Di–hadron correlations long–ranged in $\Delta\eta$ & narrow in $\Delta\phi$




central Au–Au

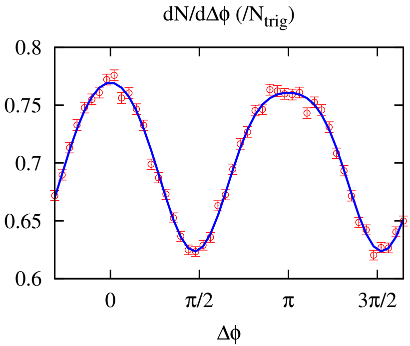


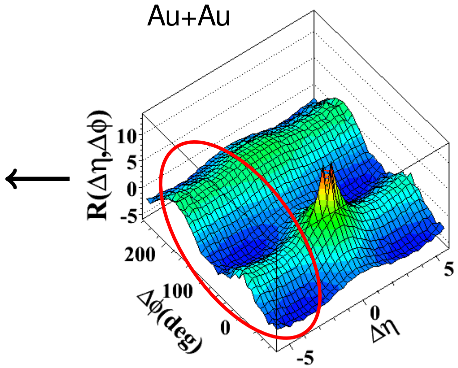

non-central Pb–Pb

- peak at $(\Delta\phi,\,\Delta\eta) = (0, 0)$ : pairs of hadrons from a same jet
- the 'ridge' : $\Delta\phi \approx 0$ and $|\Delta\eta| > 4$
- non–central collisions : a long–range correlation on the 'away' side: $\Delta\phi \approx \pi$ and $|\Delta\eta| > 4$

# From ridge to flow

- What is the origin of the double peak structure ($\Delta\phi = 0$ and $\pi$) ?

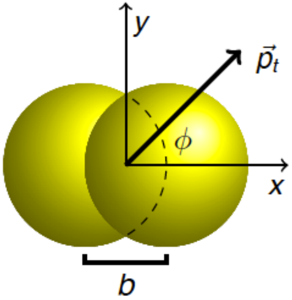



$$\mathcal{R} \equiv \frac{\langle N_1\, N_2\rangle - \langle N_1\rangle\,\langle N_2\rangle}{\langle N_1\rangle\,\langle N_2\rangle} \propto v_2^2 \cos\left(2\Delta\phi\right)$$

- This is elliptic flow !

# Elliptic flow $v_2$



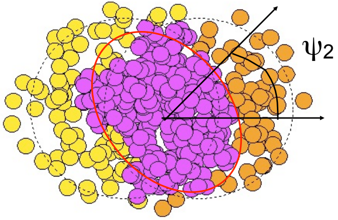

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi$$

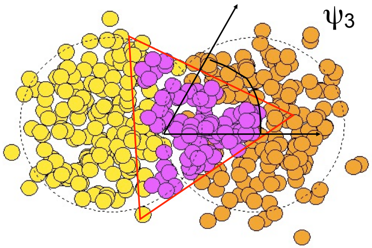
$v_2$ : the 'coefficient of the elliptic flow'

- Non–central $AA$ collision: impact parameter $b_\perp > 0$
- The interaction region is (roughly) elliptic
- Pressure gradient is larger along the smaller axis $(x)$
- Fluid velocity is proportional to the pressure gradient
- Particle emerge predominantly parallel to the fluid velocity

$\Longrightarrow$ the particle distribution is not axially symmetric !

# Granularity and fluctuations




- Nucleons are randomly distributed inside a nucleus.
- In some events, the shape of the interaction can be quite different from an ellipse !
- Then one speaks about triangular flow (or even higher harmonics)

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2v_2 \cos 2(\phi - \Psi_2) + 2v_3 \cos 3(\phi - \Psi_3) + \ldots$$

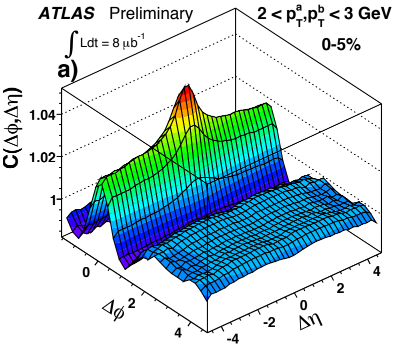
- The small disks need not be nucleons: they can be also color flux tubes

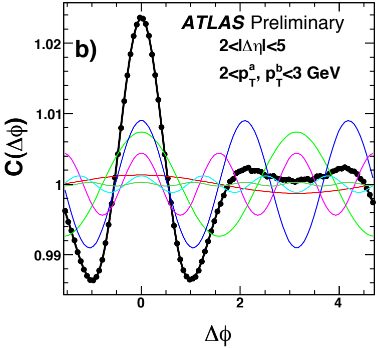
# $v_n$ from 2–particle correlations

- The reference phases $\Psi_n$ randomly vary from event to event

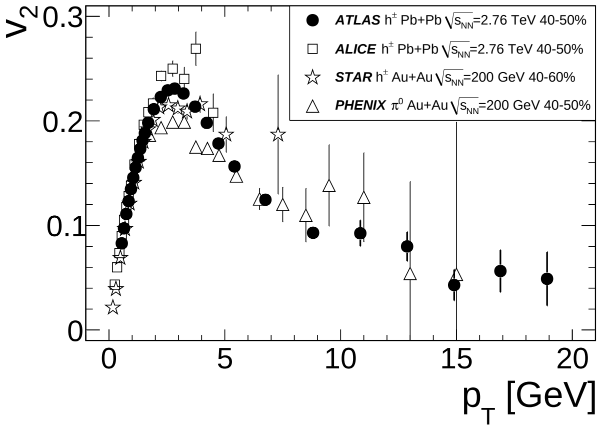
$$\left\langle \frac{\mathrm{d}N_{pairs}}{\mathrm{d}\Delta\phi} \right\rangle \propto 1 + 2\sum_{n=1}^{\infty} \langle v_n^2 \rangle \cos(n\Delta\phi)$$

- They drop out in the di–hadron correlations !



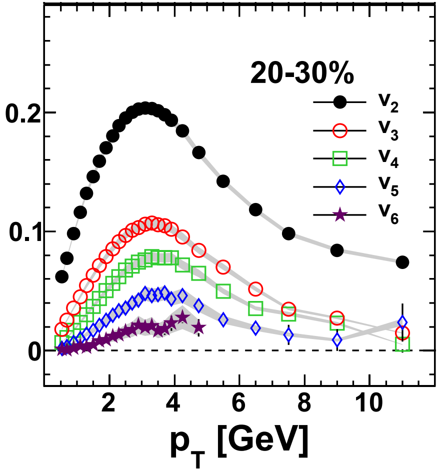



# Momentum dependence for $v_2$




- $v_2$ first rises up to $3 \div 4$ GeV, then decreases again
  - the flow is collective motion which contributes to $p_\perp$
  - relatively hard/fast particles cannot be driven by the flow
- No significant increase in $v_2$ from RHIC to LHC

# $p_\perp$ dependence for $v_n$, $n = 2 - 6$ (ATLAS)




- Similar $p_\perp$ dependence for all $n$: rise up to 3-4 GeV, then fall

# Hydrodynamics in a nut shell

- Thermodynamics: a system in global thermal equilibrium
  - pressure ($P$), temperature ($T$), chemical potential ($\mu$) are independent of time ...
  - and uniform throughout the volume $V$ of the system
- Hydrodynamics is about local thermal equilibrium
  - $P$, $T$ and $\mu$ can vary with space and time ...
  - ... but they vary so slowly that one can still assume thermal equilibrium to hold locally, in the neighborhood of any point
  - the velocity $\mathbf{v}$ can be different for different fluid elements
  - equations of motion for $P(x)$, $T(x)$, $\mu(x)$, $\mathbf{v}(x)$
- This holds when the scales for space–time variations ('system size $L$') are much larger than the mean free path $\ell$
- Based on gradient expansion in powers of $\ell/L$

# Hydro equations = the conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J_B^\mu = 0$$

- $T^{\mu\nu}$ (energy–momentum tensor) and $J_B^\mu$ (baryonic current) :
  - fluid 4–velocity: $u^\mu(x) = \gamma(1, \mathbf{v}),\ \gamma = 1/\sqrt{1-v^2}$
  - energy density $\varepsilon(x)$ & pressure $P(x)$
  - additional parameters ('viscosities') for a non–ideal fluid
- 'Ideal fluid' $\equiv$ local thermal equilibrium

$$T_{\rm id} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$ in the local rest frame at $x$ : $u^\mu = (1, 0)$

- After a boost to the laboratory frame, this becomes:

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

# Viscous hydrodynamics

- Ideal hydro assumes that there is no dissipation (no friction)
- You may think this means the coupling is weak...but you'd be wrong !
- ... it actually means that the coupling is infinite !
  - weak coupling $\Longrightarrow$ large mean free path $\Longrightarrow$ strong brownian motion $\Longrightarrow$ energy and momentum can be transmitted in directions other than that of the collective flow $\Longrightarrow$ dissipation
  - strong coupling $\Longrightarrow$ small mean free path $\Longrightarrow$ little energy or momentum transfer except in the direction of flow $\Longrightarrow$ little dissipation

# Viscous hydrodynamics

- Ideal hydro assumes that there is no dissipation (no friction)
- You may think this means the coupling is weak...but you'd be wrong !
- ... it actually means that the coupling is infinite !
- Real fluids have no infinite coupling, so they have dissipation.
- This is described by transport coefficients known as viscosities

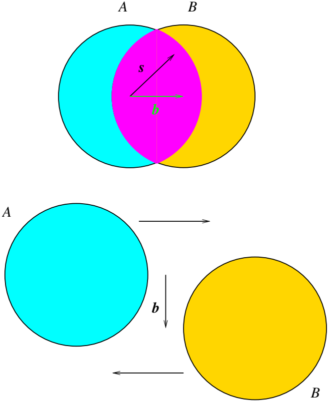
$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu} \oplus (\eta, \zeta) \otimes \partial u \oplus \cdots$$

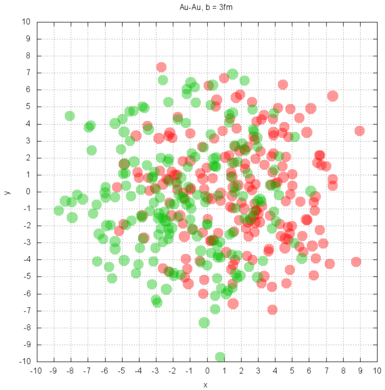
N.B. Viscous effects enter $T^{\mu\nu}$ as gradient corrections

- For the hydro problem to be well defined, one needs to specify:
  - the equation of state which relates $\varepsilon$ to $P$
  - the initial conditions (at $\tau = \tau_0$) for $\varepsilon$ and $\mathbf{v}$
  - the viscosities $\eta$, $\zeta$

- From models assuming smooth (or average) initial conditions ...

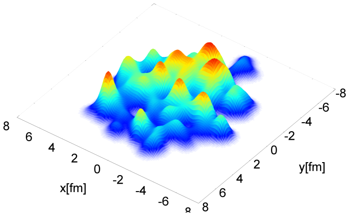
Optical Glauber



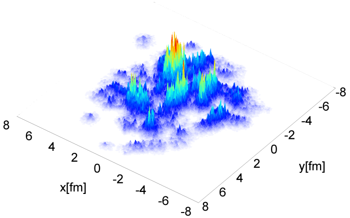




- ... to more sophisticated descriptions including fluctuations

- From a random superposition of nucleons within the nuclear disks ...




- Monte-Carlo Glauber (nucleons)
- size of flucts: $R_p \sim 1$ fm

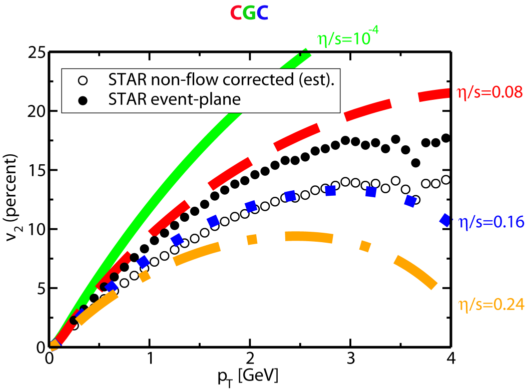



- Glasma (color flux tubes)
- size of flucts: $1/Q_s \sim 0.2$ fm

- ... to a fully dynamical Glasma simulation (classical Yang–Mills equations with randomly distributed color charges)

# Hydro simulations for $v_2$ *(from Luzum and Romatschke, 08)*

- The data are well described by nearly ideal hydrodynamics
  - local thermal equilibrium
  - a rather short equilibration time $\tau_0 \lesssim 1$ fm/c
  - a small viscosity/entropy ratio $\eta/s < 0.2$




- Both properties are puzzling ... at least at weak coupling !

# Viscosity over entropy density ratio

- $\eta \sim \ell \times \varepsilon$ ($\ell$ : mean free path; $\varepsilon$ : energy density). Thus,

$$\frac{\eta}{s} \sim \ell\,\frac{\varepsilon}{s} \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}} \gtrsim \hbar$$

*(since $\varepsilon/s \sim$ energy per particle $\sim 1/\lambda$)*

- Ideal fluids cannot exist in nature (Heisenberg) !
- Weakly coupled QGP *(Arnold, Moore, Yaffe, 2003)* :

$$\frac{\eta}{s} \sim \frac{\hbar}{\alpha_s^2 \ln(1/\alpha_s)} \gg \hbar$$

- Conjectured limit at strong coupling *(Kovtun, Son, Starinets, 05)*

$$\frac{\eta}{s} \to \frac{\hbar}{4\pi} \quad \text{when} \quad \lambda \equiv g^2 N_c \to \infty \quad \text{(AdS/CFT)}$$

- The RHIC value is at most a few times $1/4\pi \simeq 0.08$

# RHIC serves up the perfect liquid !

## RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

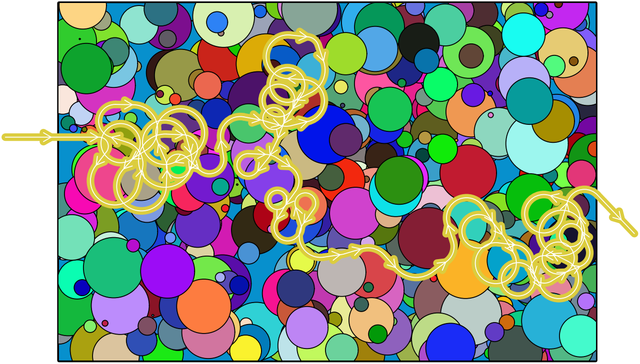
Monday, April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

- 'Strongly–coupled quark–gluon plasma' or 'perfect fluid'

# Small $\eta/s$ at weak coupling but strong fields

- Remember : in the presence of a strong (chromo) magnetic field, the mean free path can be as small as $\lambda$ : $\ell \sim Q_s^{-1} \sim 1/p$
- Glasma : classical Yang–Mills with noisy initial conditions
  $\Longrightarrow$ random superposition of color flux tubes (strong fields)

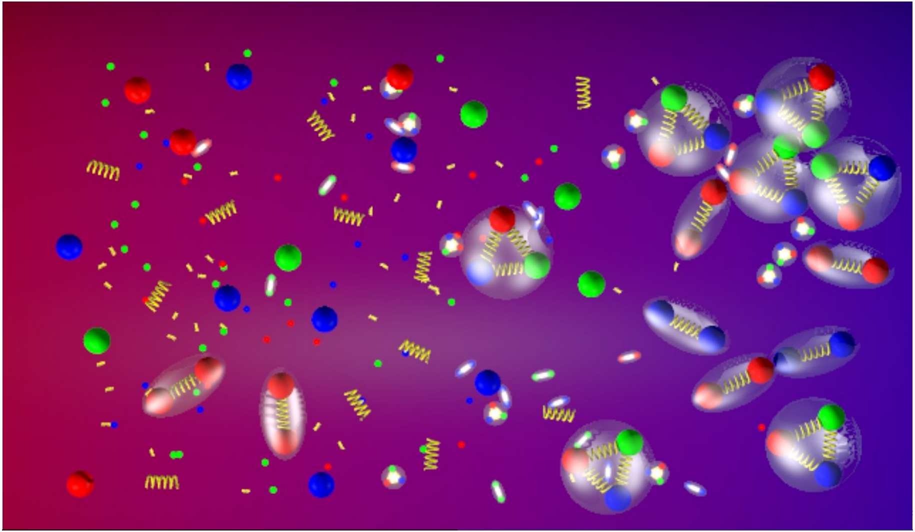


$$\frac{\eta}{s} \sim \frac{\ell}{\lambda} \sim \mathcal{O}(1)$$

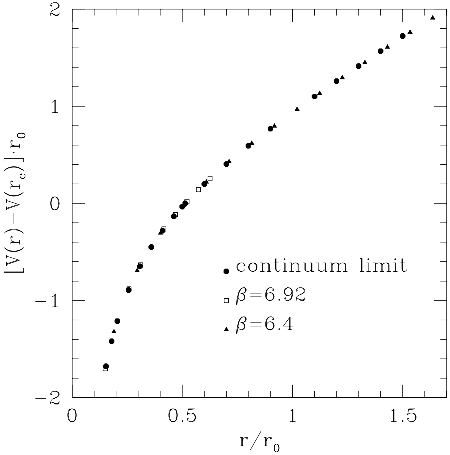
(in units of $\hbar$)

- Consistent with the numerical results by Epelbaum and Gelis (2013)

# Quark–Gluon Plasma



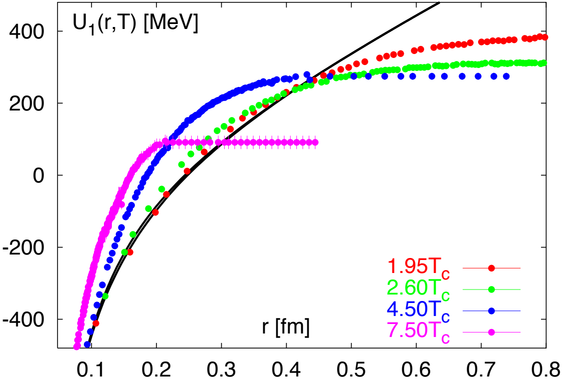
# Confinement




- The quark–antiquark potential increases linearly with the distance.
- Quarks (and gluons) are confined into colorless hadrons

# Quark–antiquark potential at finite $T$

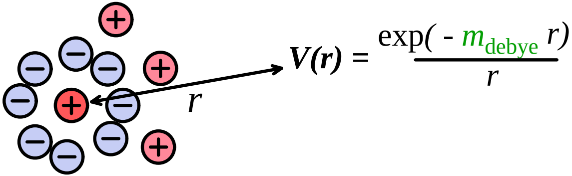
- With increasing the temperature $T$, the potential flattens out at shorter and shorter distances




- This leads to a 'phase transition' at some 'critical temperature' $T_c$ : from Hadron Gas to a Quark–Gluon Plasma (QGP)

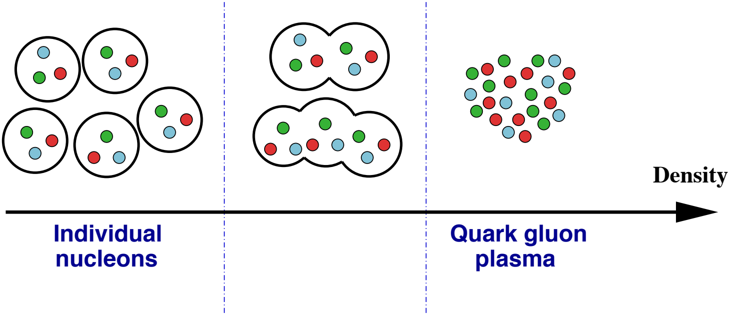
# Debye screening

- QGP : a system of quarks and gluons which got free of confinement
- How is that possible ???



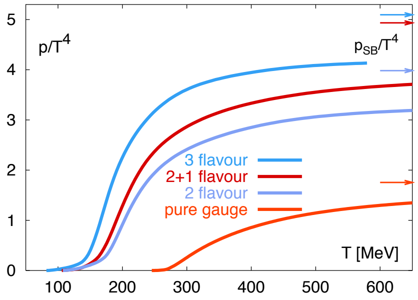

- In a dense medium, color charges are screened by their neighbors
- The interaction potential decreases exponentially beyond the Debye radius $R_{\text{Debye}} = 1/m_{\text{Debye}}$
- Hadrons whose sizes are larger than $R_{Debye}$ cannot bind anymore

# Deconfinement phase transition




- When the nucleon density increases, they merge, enabling quarks and gluons to hop freely from a nucleon to its neighbors
- For sufficiently high density (or temperature), the Debye radius $R_{Debye}$ becomes much smaller than the typical hadron radius $R_h \sim 1$ fm
- The hadrons melt into quarks and gluons

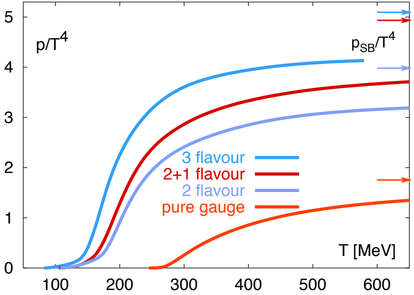
- Lattice calculations of the pressure in QCD at finite $T$




- Rapid increase of the pressure
  - at $T \simeq 270$ MeV with gluons only ('pure gauge')
  - at $T \simeq 150$ to $180$ MeV with light quarks

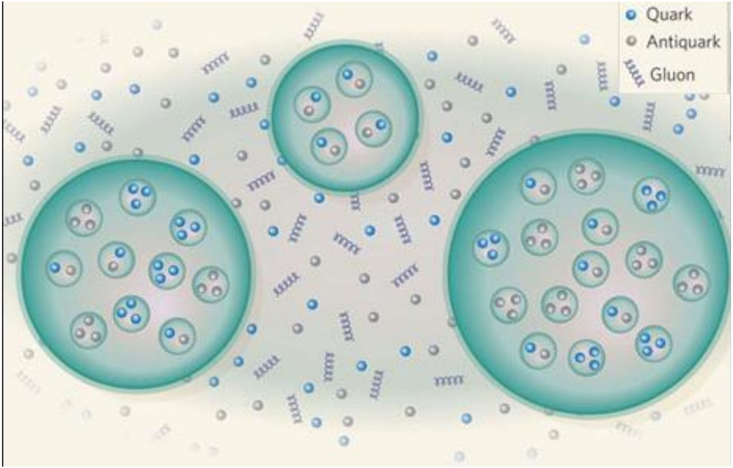
# Quark–Gluon Plasma

- Lattice calculations of the pressure in QCD at finite $T$



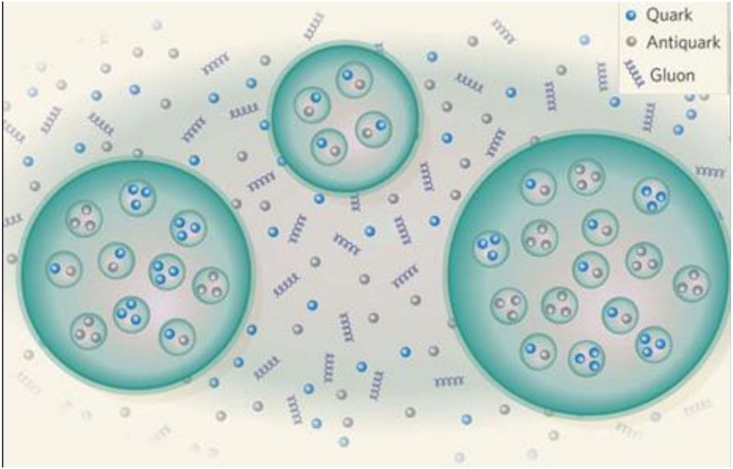

- The expected rise in the number of active degrees of freedom due to the liberation of quarks and gluons
  - at $T < T_c$ : 3 light mesons $(\pi^0, \pi^{\pm})$
  - at $T > T_c$ : 52 d.o.f. (gluons: $8 \times 2 = 16$; quarks: $3 \times 3 \times 2 \times 2 = 36$)

# Possible first–order scenario with critical bubbles




- If the transition was first–order, it would go through a mixed phase containing a mixture of nucleons and plasma

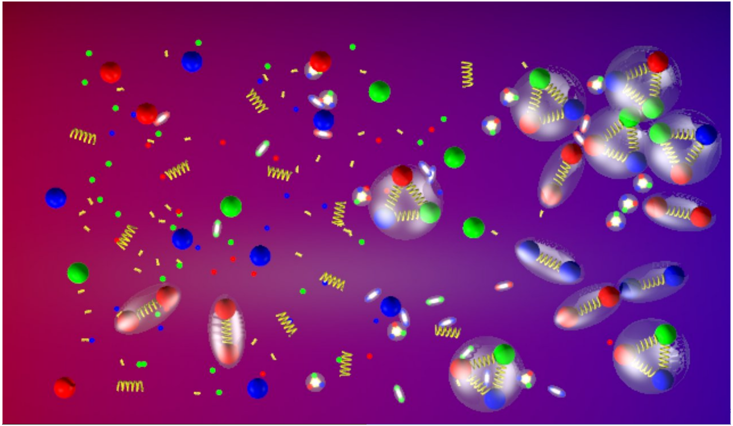
# Possible first–order scenario with critical bubbles




- This would be the case if the 3 'active' quarks ($u$, $s$, $d$) were either massless or infinitely massive ('pure gauge')

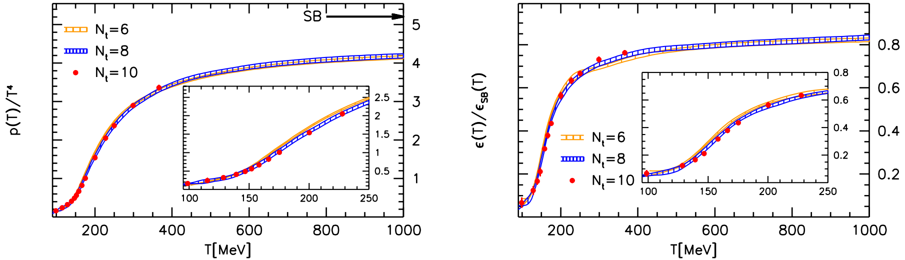
# A cross–over

- This is not the case for the physical quark masses (2 light + 1 massive)



- The actual scenario is a 'cross–over' (no discontinuity)
  *the Wuppertal–Budapest lattice group, Nature, 443 (2006) 675)*

# QCD thermodynamics: lattice




- With increasing temperature, the coupling $g(T)$ decreases, so the exact result approaches towards the Stefan–Boltzmann limit

$$P_{SB} = \frac{\pi^2}{90}\left\{2(N_c^2-1) + \frac{7}{2}N_c N_f\right\}T^4$$

- For $T \gtrsim 2.5T_c$, $P(T) - P_{SB}(T)$ is about 20%

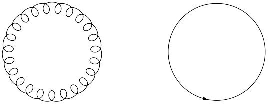
  ... is this small or large ?

- Can one understand this difference in perturbation theory ?

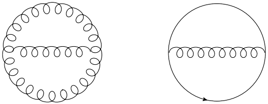
# Perturbation theory for the pressure

$$P = \frac{T}{V} \ln \mathcal{Z}\,, \qquad \mathcal{Z} \equiv \sum_n \mathrm{e}^{-\beta E_n} = \mathrm{Tr}\; \mathrm{e}^{-\beta H} \qquad \text{(partition function)}$$

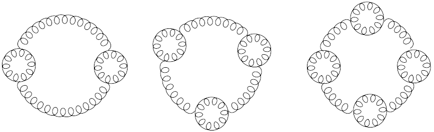
- Zero order $(g \to 0)$ : one-loop graphs



- Order $g^2 \sim \alpha_s$ : two-loop graphs



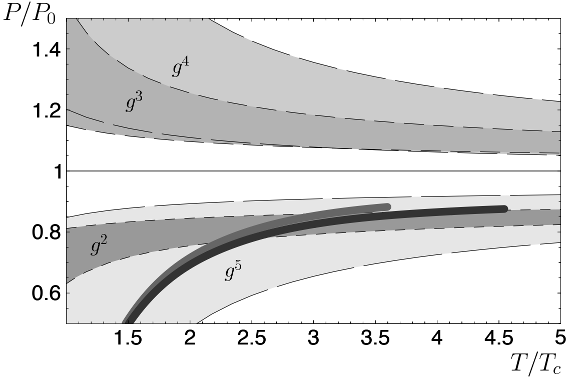
- Order $g^3 \sim \alpha_s^{3/2}$ : ring diagrams



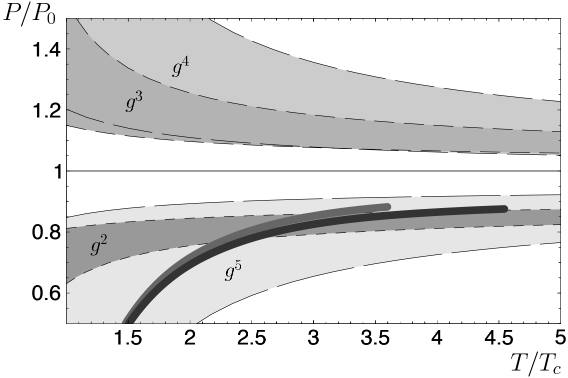
...

  - Infinitely many diagrams formally starting at $\mathcal{O}(g^4)$ but which contribute already at $\mathcal{O}(g^3)$ : 'plasmon effect' (see below)

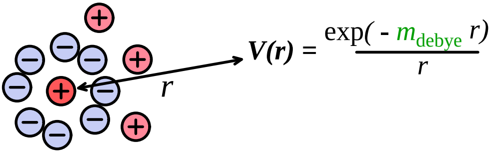
# QCD thermodynamics: perturbation theory



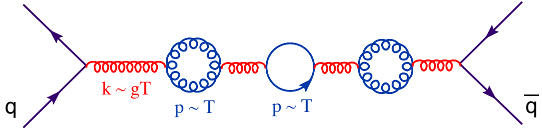

- By itself, the $\mathcal{O}(g^2)$ seems to do a pretty good job. However...
- Successive perturbative approximations — $\mathcal{O}(g^2)$, $\mathcal{O}(g^3)$, $\mathcal{O}(g^4)$, $\mathcal{O}(g^5)$ — jump up and down, without any sign of convergence.
- Larger and larger renormalization scale uncertainties




- Is this a non-perturbative effect inherent to QCD ?
  An indication of strong coupling ?
- A similar problem appears for any field theory at finite temperature, including weakly coupled QED, or scalar $\phi^4$ theory !
- At finite $T$, perturbation theory gets complicated by medium effects




- Thermal effect associated with dressing the propagator: $m_{\rm Debye} \sim gT$

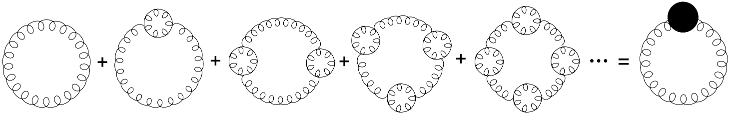



- The electric gluon acquires a mass which is 'non–perturbative' at 'soft' momenta $k \sim gT$ :

$$G_{00}(k) = \underbrace{\frac{1}{k^2 + m_{\rm D}^2}}_{\text{fine !}} = \underbrace{\frac{1}{k^2}\left[1 - \frac{m_{\rm D}^2}{k^2} + \left(\frac{m_{\rm D}^2}{k^2}\right)^2 \cdots\right]}_{\text{not fine !}}$$

# Ring diagrams

- The sum of the ring diagrams reconstructs the dressed propagator :



- The Bose-Einstein thermal distribution is divergent as $k \to 0$

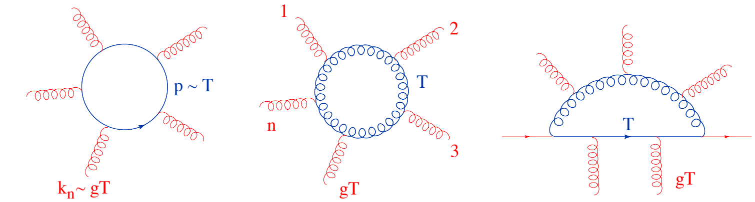
$$f_B(k) = \frac{1}{e^{\beta k} - 1} \simeq \frac{T}{k} \sim \frac{1}{g} \qquad \text{when} \quad k \sim gT$$

▷ large occupation numbers for the soft thermal gluons

- This divergence is cut off by Debye screening at $k \sim gT$, but this results in an enhancement $\sim 1/g$
- The resummation of $m_D \Longrightarrow$ odd powers in $g$ in perturbation theory
- An expansion in powers of $g$ and not $\alpha_s \Longrightarrow$ lack of convergence !

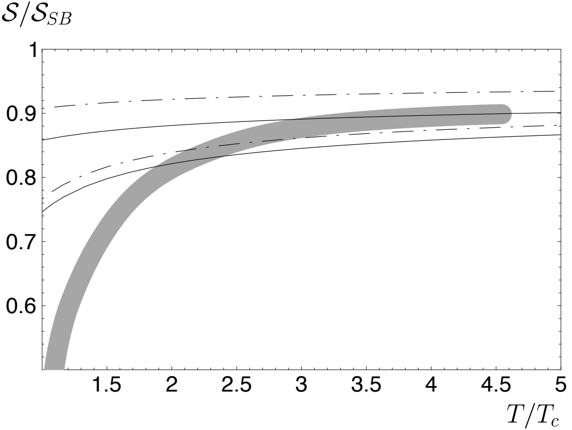
$$\alpha_s = g^2/4\pi = 0.2 \div 0.3 \implies g \simeq 1.5 \div 2$$

- In a gauge theory, gauge symmetry requires (via Ward identities) the generalization of the Debye mass to generic $n$–point amplitudes:
  'Hard Thermal Loops' *(Braaten and Pisarski, 1990; Blaizot, E. I., 1992)*




- HTL's : one loop diagrams with internal momenta $p \sim \mathcal{O}(T)$ ('hard') and external momenta $k_i \sim \mathcal{O}(gT)$ ('soft')
- Physical interpretation: collective phenomena in the QGP
- Genuinely leading order effects that must be resummed to all orders, via reorganizations of the perturbative expansion

# HTL–resummed entropy




- ‘2-particle-irreducible’ resummation (HTL–dressed propagators) *(J.-P. Blaizot, A. Rebhan, E. I., 2000)*
- Physical picture: weakly coupled quasiparticles.
- Good agreement with the lattice data (Bielefeld) for $T \gtrsim 2.5T_c$.

# Quarkonia melting

- Recall : a hadron whose size is larger than the Debye radius $R_D = 1/m_D$ cannot survive in the plasma
- Quarkonia : bound states of heavy quarks (charm $c$ or bottom $b$)

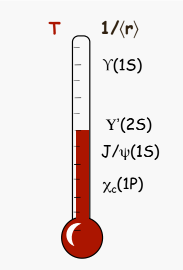
  ▷ small size $R \sim 1/m_Q \Longrightarrow$ can survive up to higher temperatures
- Two families (including excited states) :

  ▷ $c\bar{c}$ (charmonium, $m_c = 1.3$ GeV) : $J/\psi(1S)$, $\psi(2S)$, $\chi_c(1P)$

  ▷ $b\bar{b}$ (bottomium, $m_b = 4.2$ GeV) : $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$
- Sequential suppression :

  ▷ excited states are larger and melt before the low energy ones

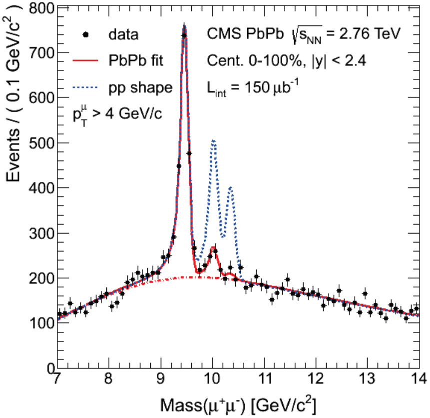
  ▷ bottomium family melts after the charmonium one
- Quarkonia melting acts as a thermometer !



- The $\Upsilon$ family is better suited since less subjected to ambiguities

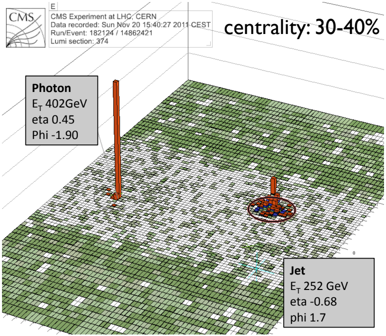
  ▷ no recombination since less $b\bar{b}$ pairs than $c\bar{c}$

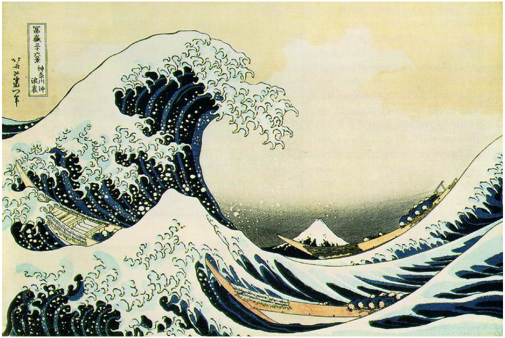
  (at LHC : $\sim 100\, c\bar{c}$ pairs in central Pb–Pb collisions $\Longrightarrow$ recombination)




- Very clean successive suppression pattern for the $\Upsilon$'s

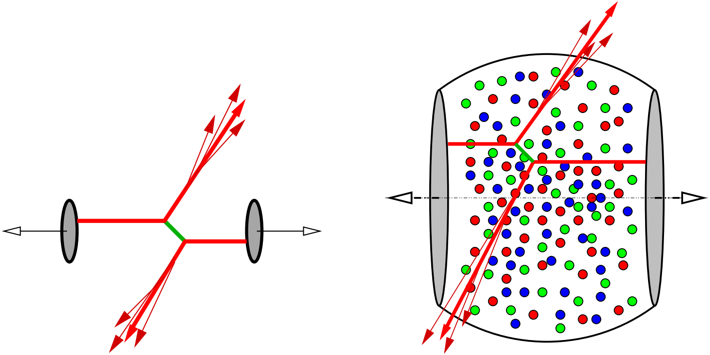
# Di-jet asymmetry & wave turbulence



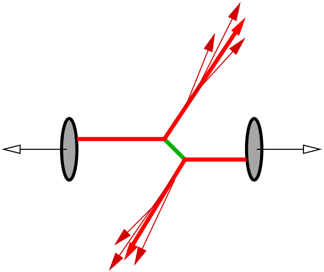
# Jet quenching

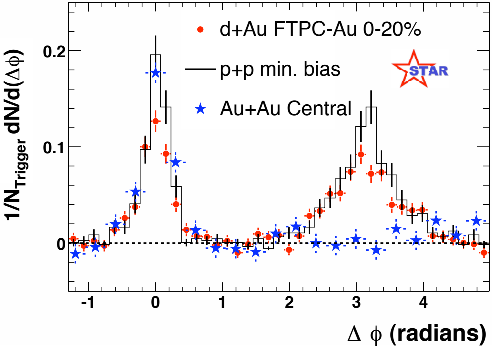
- How the probe the ephemeral QGP phase created at the intermediate stages of a heavy ion collision (at RHIC or the LHC) ?



- Shut a hard parton and measure its interactions !
- Hard partons are typically created in pairs which propagate back–to–back in the transverse plane
- In–medium interactions may alter this azimuthal correlation

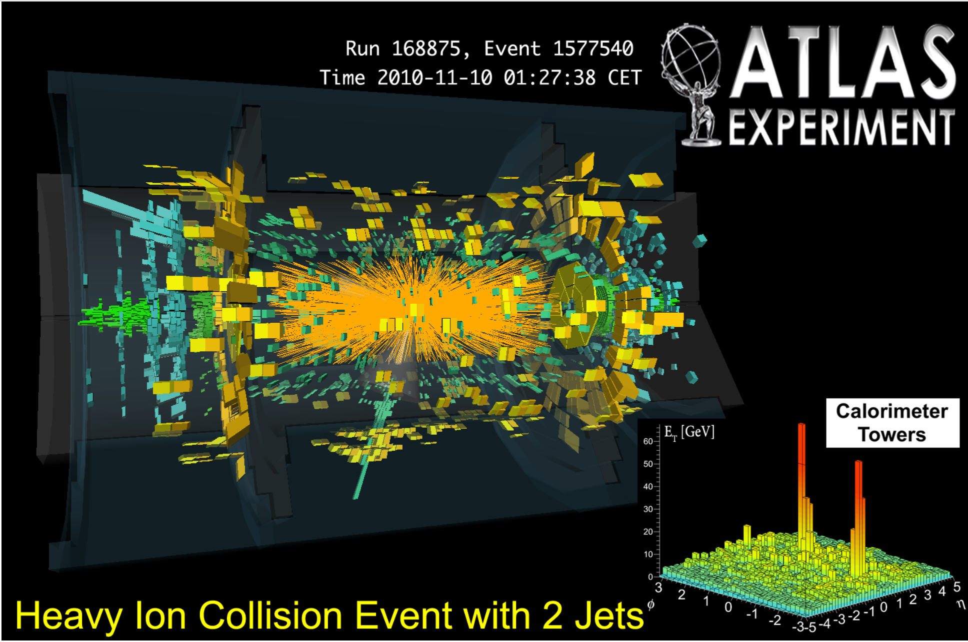
- How the probe the ephemeral QGP phase created at the intermediate stages of a heavy ion collision (at RHIC or the LHC) ?



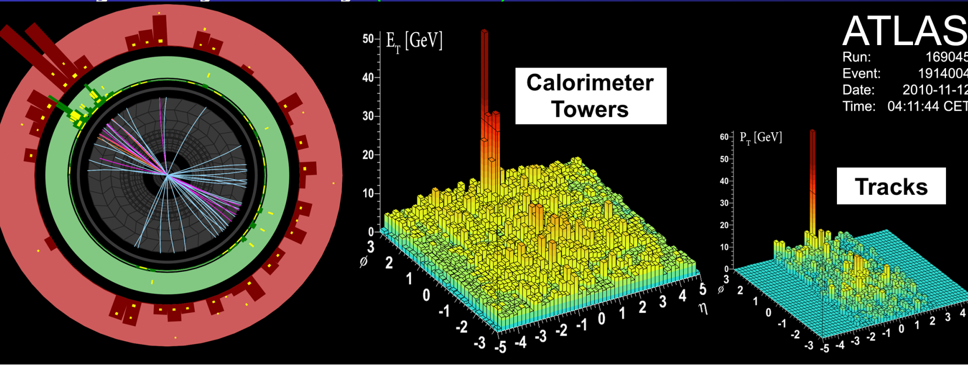



- Au+Au collisions at RHIC: the away peak is washed out
- 'Jet quenching' ... although jets were not really measured at RHIC
  - energy loss & transverse momentum broadening for the leading particle

# The LHC gives us access to real jets

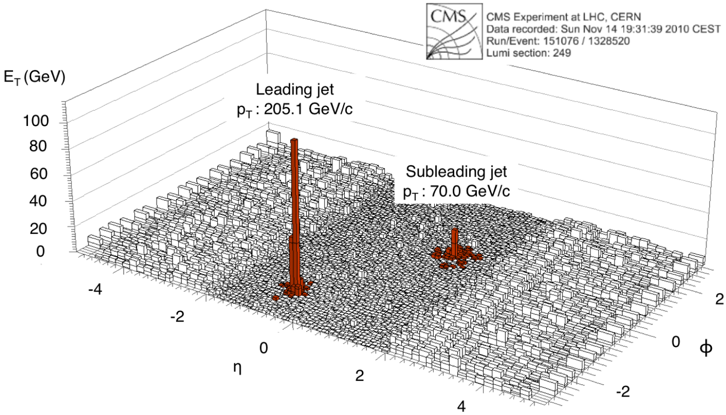


# Di–jet asymmetry *(ATLAS)*




- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV
- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Remarkably large if compared to the typical scale in the medium: the temperature $T \sim 1$ GeV

# Di–jet asymmetry *(CMS)*

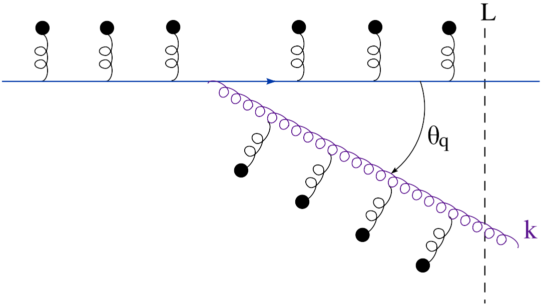



- Central Pb+Pb: the secondary jet is barely visible
- Detailed studies show that the 'missing energy' is carried by many soft ($p_\perp < 4$ GeV) hadrons propagating at large angles
- Can we understand that from first principles ?

# Medium–induced gluon radiation

*Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov (BDMPS-Z) ∼ 1995*

- Collisions with the plasma constituents provide acceleration (transverse momentum kicks) and thus allow for additional radiation




- Gluon missions can occur anywhere inside the medium (with size $L$)
  ... but they are not instantaneous : formation time $\tau_f$
- 2 key ingredients: formation time & transverse momentum broadening

# The formation time

- By the uncertainty principle, it takes some time to emit a gluon !

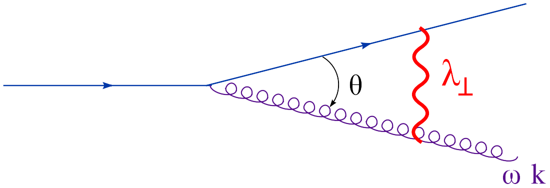
  ▷ the gluon must lose quantum coherence with respect to its source

- Gluon with energy $\omega$ and transverse momentum $k_\perp$ :

  ▷ the quark–gluon transverse separation $b_\perp$ at the formation time $\tau_f$ must be larger than the gluon transverse wavelength $\lambda_\perp$

$$b_\perp \simeq \theta \tau_f \gtrsim \lambda_\perp \simeq 1/k_\perp$$

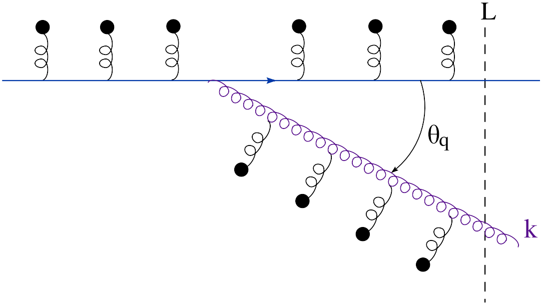
$$k_\perp \simeq \omega \theta$$




$$\tau_f \simeq \frac{\omega}{k_\perp^2} \simeq \frac{1}{\omega\theta^2}$$

# Transverse momentum broadening

- The gluon receives random kicks from the plasma constituents
- Parton mean free path $\ell$
- Average (momentum)$^2$ transfer per scattering $m_D^2$




$$\frac{\mathrm{d}\langle k_\perp^2 \rangle}{\mathrm{d}t} \simeq \frac{m_D^2}{\ell} \equiv \hat{q}$$ 'jet quenching parameter'

# In–medium formation time

- The gluon acquires a (momentum)$^2 \sim \hat{q}$ per unit time ...
- ... and hence a momentum $k_f^2 \simeq \hat{q}\,\tau_f$ during its formation.
- The condition of quantum decoherence requires $\tau_f \simeq \omega/k_f^2$

$$\tau_f \simeq \sqrt{\frac{\omega}{\hat{q}}}\,, \qquad \theta_f \equiv \frac{k_f}{\omega} \simeq \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- N.B. : small $\omega \Longrightarrow$ small $\tau_f$ & large $\theta_f$
- Maximal $\omega$ for this mechanism : $\tau_f \simeq L \;\Rightarrow\; \omega_c = \hat{q}L^2$
- Minimal emission angle: $\theta_c \equiv \theta(\omega_c) \sim 1/\sqrt{\hat{q}L^3}$
- Some typical value (consistent with the phenomenology) :

$$\hat{q} \simeq (1 \div 2)\ \text{GeV}^2/\text{fm},\;\; L \simeq 5\ \text{fm},\;\; \omega_c \simeq 40\ \text{GeV},\;\; \theta_c \simeq 0.1$$

# Emission probability

- Spectrum : Bremsstrahlung $\times$ average number of emissions

$$\omega\,\frac{\mathrm{d}N}{\mathrm{d}\omega}\,\simeq\,\alpha_s\,\frac{L}{\tau_f(\omega)}\,\simeq\,\alpha_s\sqrt{\frac{\omega_c}{\omega}}\qquad(\omega_c=\hat{q}L^2)$$

- Energy loss by the leading particle :

$$\Delta E=\int^{\omega_c}\mathrm{d}\omega\;\omega\,\frac{\mathrm{d}N}{\mathrm{d}\omega}\;\sim\;\alpha_s\omega_c\;\sim\;\alpha_s\hat{q}L^2$$

  - integral dominated by its upper limit $\omega=\omega_c$

- One is naturally led to distinguish between 2 types of emissions

# Emission probability

- Spectrum : Bremsstrahlung $\times$ average number of emissions

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega_c = \hat{q}L^2)$$

- Energy loss by the leading particle :

$$\Delta E = \int^{\omega_c} \mathrm{d}\omega\, \omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \sim \alpha_s \omega_c \sim \alpha_s \hat{q} L^2$$

- Relatively hard emissions with $\omega \sim \omega_c$ :
  - large formation times: $\tau_f \sim L$
  - rare events : probability of $\mathcal{O}(\alpha_s)$
  - control the energy loss by the leading particle
  - small emission angle $\theta_c \Rightarrow$ the energy remains inside the jet

# Emission probability

- Spectrum : Bremsstrahlung × average number of emissions

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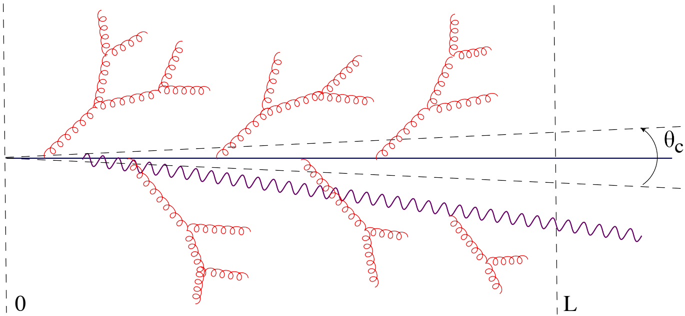
- Relatively soft emissions with $\omega \ll \omega_c$ :
  - small formation times : $\tau_f \ll L$
  - quasi–deterministic : probability of $\mathcal{O}(1)$ for $\omega \lesssim \alpha_s^2\,\omega_c$
  - a relatively smaller contribution to the energy loss : $\Delta E_{\rm soft} \sim \alpha_s^2 \omega_c$
  - ... but this can be lost at very large angles
- Potentially relevant for the di–jet asymmetry ☺

# In–medium jet evolution

- When probability of $\mathcal{O}(1) \Longrightarrow$ multiple branchings become important



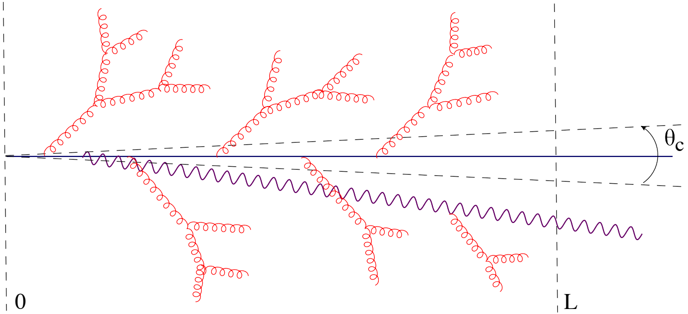

- Successive emissions can be treated as independent from each other

  ▷ non–trivial ! not true for jet evolution in the vacuum

  ▷ possible interference effects are washed out by scattering with the medium

  *(Blaizot, Dominguez, E.I., Mehtar-Tani, 2012)*

- When probability of $\mathcal{O}(1) \Longrightarrow$ multiple branchings become important



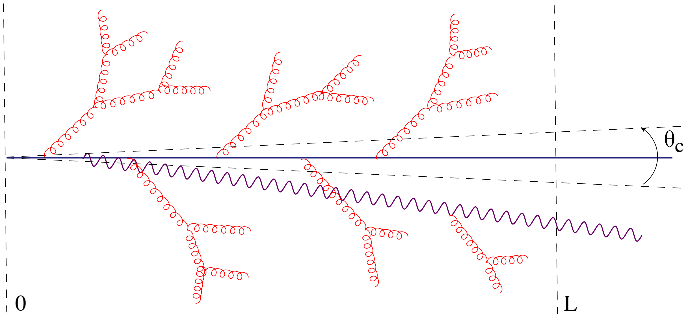

- Successive emissions can be treated as independent from each other
- The branchings of the soft gluons are quasi–democratic

  ▷ the daughter gluons carry comparable energy fractions: $x \sim 1/2$

  ▷ non–trivial ! bremsstrahlung is strongly asymmetric : $x \ll 1$

# In–medium jet evolution

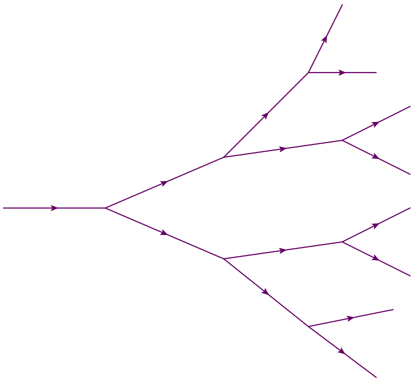
- When probability of $\mathcal{O}(1) \Longrightarrow$ multiple branchings become important




- Successive emissions can be treated as independent from each other
- The branchings of the soft gluons are quasi–democratic
- The quasi–democratic cascade develops wave turbulence

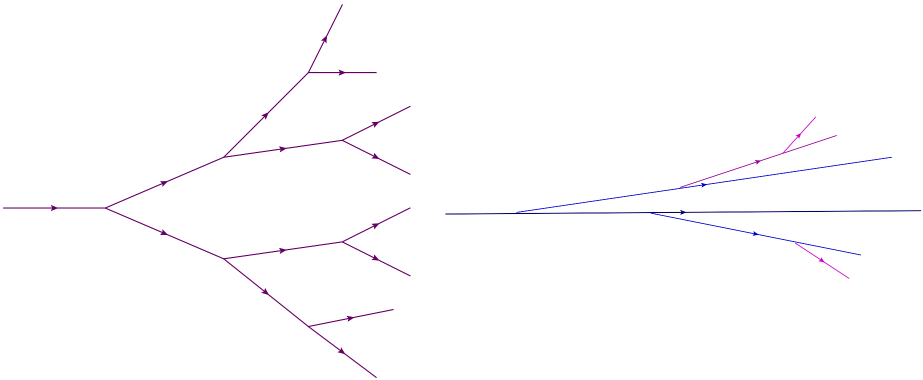
  *(Blaizot, E.I., Mehtar-Tani, 2013)*

# Wave turbulence



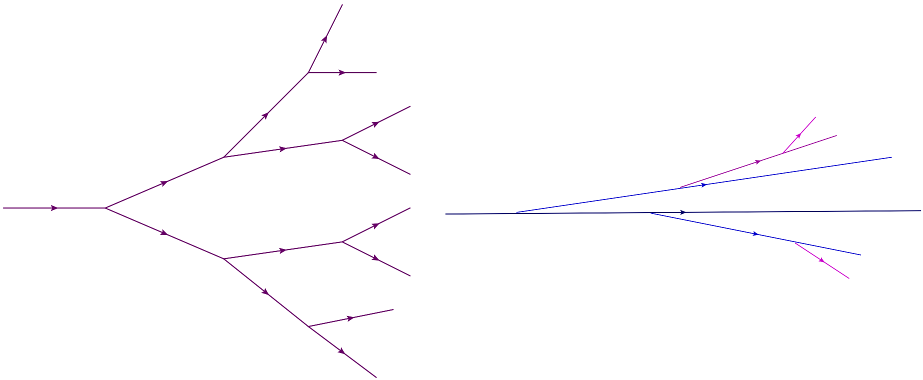
- The rate for energy transfer from one parton generation to the next one is independent of the generation (i.e. of $x$)
  - via successive branchings, the energy flows from large $x$ to small $x$, without accumulating at any intermediate value of $x$

# Wave turbulence



- The rate for energy transfer from one parton generation to the next one is independent of the generation (i.e. of $x$)
- This is not what happens for a jet in the vacuum (DGLAP equation)
  - splittings are typically asymmetric and the energy remains in the partons with the largest values of $x$

# Wave turbulence



- The rate for energy transfer from one parton generation to the next one is independent of the generation (i.e. of $x$)
- The most efficient mechanism to transport energy between 2 widely separated scales *(Richardson, '21; Kolmogorov, '41; Zakharov, '92 ...)*
  - here, from large $x$ to small $x$, and hence to large angles