a glimpse on QCD

(Part I)

Stéphane Peigné
peigne@subatech.in2p3.fr

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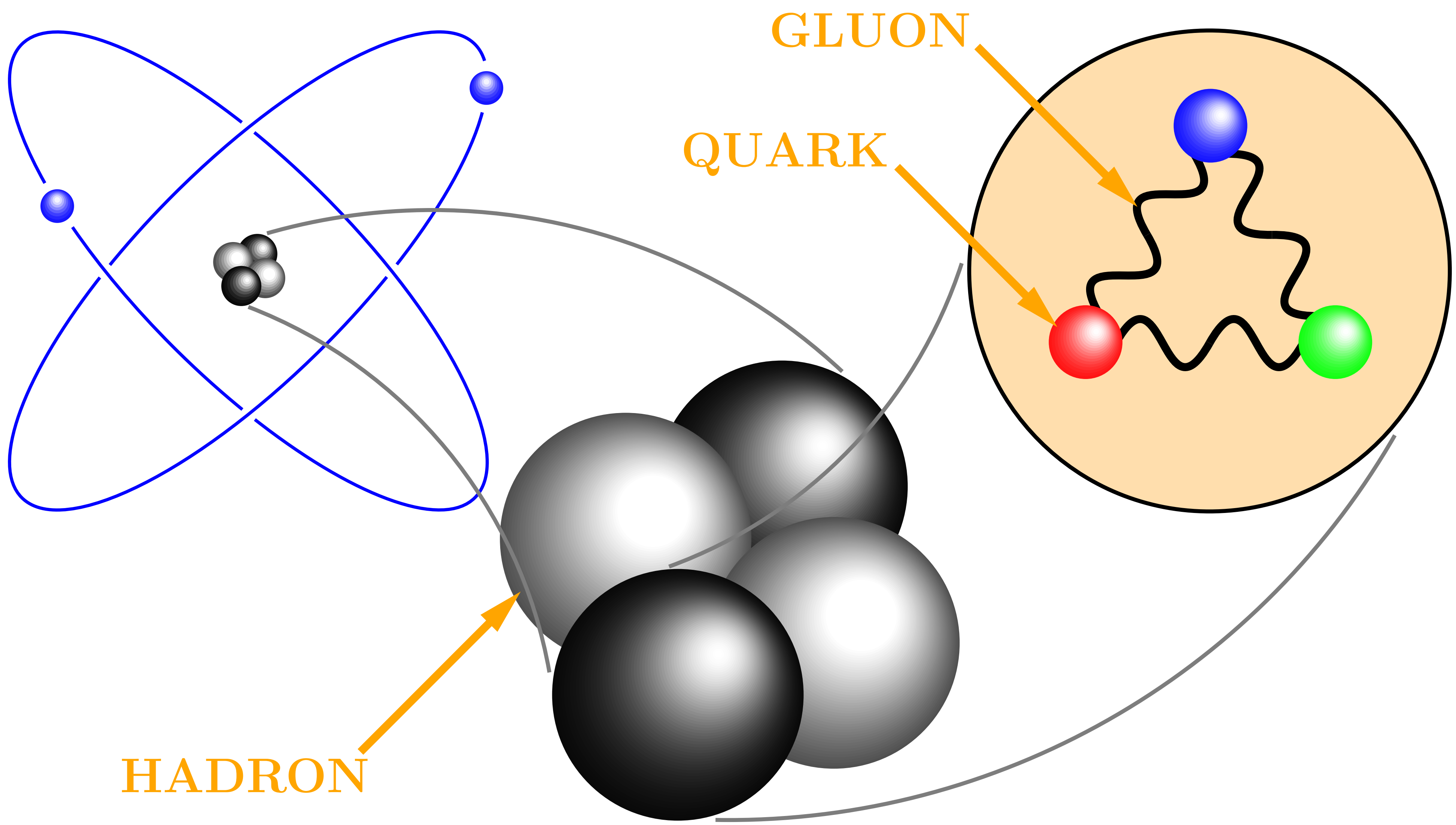
reminder: matter structure



$$\text{hadrons} = \text{mesons } (q\bar{q}) + \text{baryons } (qqq, \bar{q}\bar{q}\bar{q})$$

- Standard Model of Particle Physics

  $\longrightarrow$ non-gravitational interactions

- electromagnetic, weak, strong interactions

  - obey Quantum Mechanics $+$ Special Relativity

    $\Rightarrow$ Quantum Field Theories (QFT)

  - are all described by GAUGE THEORIES

    gauge groups $\underbrace{\mathrm{SU}(2) \otimes \mathrm{U}(1)}_{\textit{electroweak}} \otimes \underbrace{\mathrm{SU}(3)}_{\textit{strong}}$

gauge theory?

- simplest case: electromagnetic interaction

'textbook' construction of $\mathcal{L}_{\rm QED}$ from gauge principle:

$$\mathcal{L}_{\rm Dirac} = \bar{\psi}(x)\,(i\gamma^\mu\partial_\mu - m)\,\psi(x) \qquad \text{(free electron)}$$

$\gamma^\mu =$ Dirac matrices; $\quad \partial_\mu = \partial/\partial x^\mu \quad (\mu = 0, 1, 2, 3)$

$\mathcal{L}_{\rm Dirac}$ invariant under $\psi \to e^{ie\Lambda}\psi$ only if $\Lambda =$ cst.

"gauge principle": *physics must be insensitive to local transformations* $\psi(x) \to e^{ie\Lambda(x)}\psi(x)$

$\Rightarrow$ modify $\mathcal{L}_{\rm Dirac}$ by: $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$ such that:

$$D_\mu\psi \to e^{ie\Lambda(x)}D_\mu\psi \quad \text{when} \quad \psi \to e^{ie\Lambda(x)}\psi$$

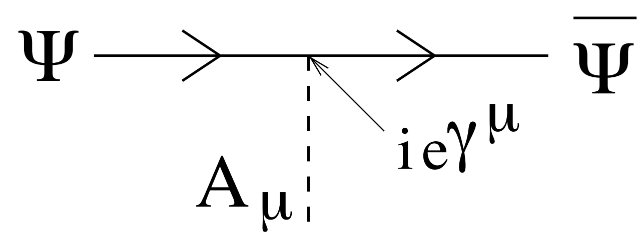
$$\Rightarrow \quad A_\mu \to A_\mu - \partial_\mu\Lambda$$

$\longrightarrow$ *gauge invariant* lagrangian:

$$\Rightarrow \mathcal{L}' = \bar{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi - e\,\bar{\psi}\gamma^\mu\psi\,A_\mu$$

- one 'gauge parameter' $\Lambda(x) \rightarrow$ one field $A_\mu \leftrightarrow$ photon
- gauge principle dictates form of electron-photon interaction :

$$-e\,\bar{\psi}\gamma^\mu\psi\,A_\mu \longrightarrow$$



- kinetic term for A_μ: $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$

$$\Rightarrow \boxed{\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi}$$

- QED is remarkably successful: accuracy $\sim 10^{-8}$!

from *simplest gauge invariant lagrangian built with* ψ, A_μ

$\longrightarrow$ validates gauge principle

QCD = Quantum ChromoDynamics

= theory of strong interaction

based on gauge symmetry

in color space

menu (day 1)

I) Birth of QCD

- Yukawa theory
- notion of quark
- color quantum number
- parton model
- idea of non-abelian gauge invariance

II) Recreation: color factors

III) QCD asymptotic freedom

- running coupling in QED
- zero charge phenomenon
- unitarity: an obstacle to asymptotic freedom?
- how the obstacle is overcome in QCD

menu (day 2)

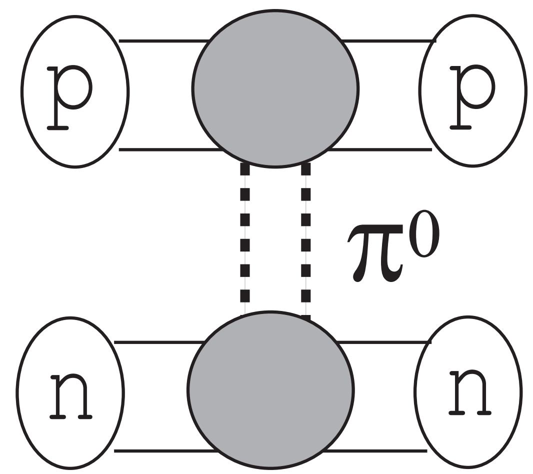
IV) Medium-induced parton energy loss in perturbative QCD

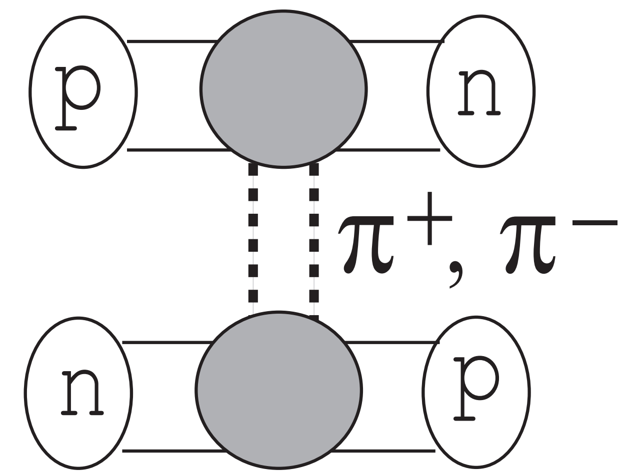
- parton energy loss in phenomenology
- QCD bremsstrahlung
  - radiation induced by single scattering
  - medium-induced gluon radiation
- J/Ψ suppression in p-A collisions from parton energy loss

1) Birth of QCD

Yukawa theory (1935)

π meson
exchange





$$\mathcal{L}_{\mathrm{Y}} = \bar{N}\left(i\gamma^{\mu}\partial_{\mu} - m_N\right)N + \frac{1}{2}\left[\left(\partial_{\mu}\vec{\pi}\right)^2 - m_{\pi}^2\vec{\pi}^2\right] - ig_{\pi NN}\bar{N}\gamma^5\vec{\tau}N\cdot\vec{\pi}$$

- strong interaction between nucleons *attractive*

$\Longrightarrow$ pion spin is even $\Longrightarrow$ assume scalar pion

- finite range $\sim \mathcal{O}\left(1\mathrm{fm}\right) \Rightarrow m_{\pi} \sim \left(1\,\mathrm{fm}\right)^{-1} \simeq 200\,\mathrm{MeV}$

$$m_{\mathrm{electron}} \ll m_{\pi} \ll m_{\mathrm{proton}}$$

- strong interaction binds protons $\Rightarrow g_{\pi NN} \gg e$

Yukawa theory confirmed:
pion observed in cosmic rays (1947)

- π = pseudo-scalar
- $m_{\pi^+} \simeq 140\,\mathrm{MeV}$
- $\dfrac{g_{\pi NN}^2}{4\pi} \simeq 14 \gg \dfrac{e^2}{4\pi} \simeq \dfrac{1}{137}$

difficulties:

- hundreds of other hadrons were found
- Yukawa theory in QFT framework is problematic:
  - perturbation only tool at disposal in QFT...
  - ...but $g_{\pi NN}$ increases with energy

$\longrightarrow$ $g_{\pi NN} \gg 1$ at all energies

- until the advent of QCD, it was believed that g_{eff} increases with energy in *any* QFT

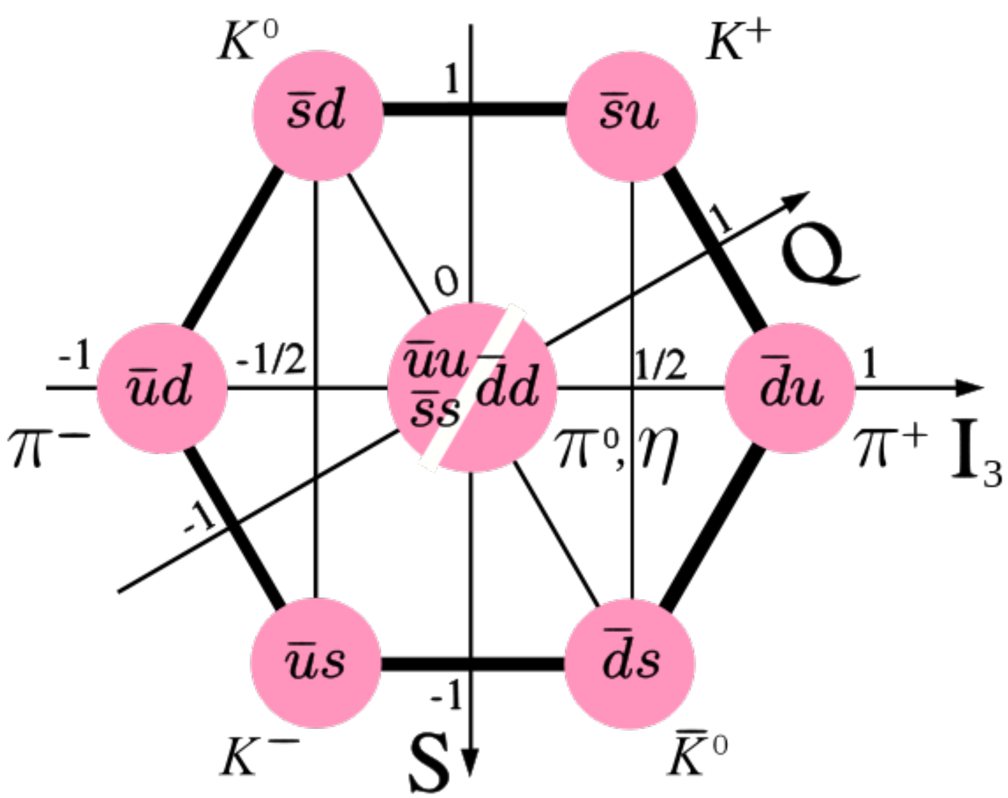
-- see III) to see why it was believed so and why an opposite behavior may arise in non-abelian gauge theories --

- this led to conclude that Lagrangian methods should be abandoned to apprehend the strong interaction

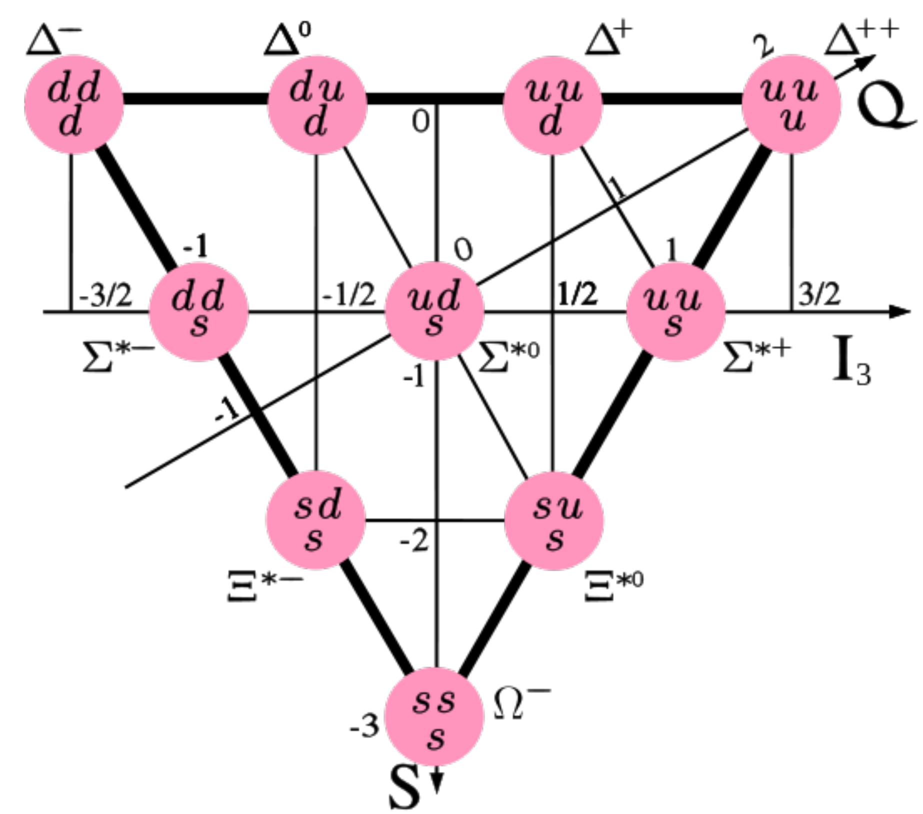
notion of quark

- in the 50's: hadrons with given J^P and *similar masses* may be grouped in octets and decuplets

a meson octet:



a baryon decuplet:



⟹ suggests $SU(3)_f$ as underlying symmetry assumptions of Gell-Mann and Zweig (1964) :

- ∃ three *quark flavors* u, d, s
- triplet $Q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ belongs to fundamental representation of SU(3)$_f$:

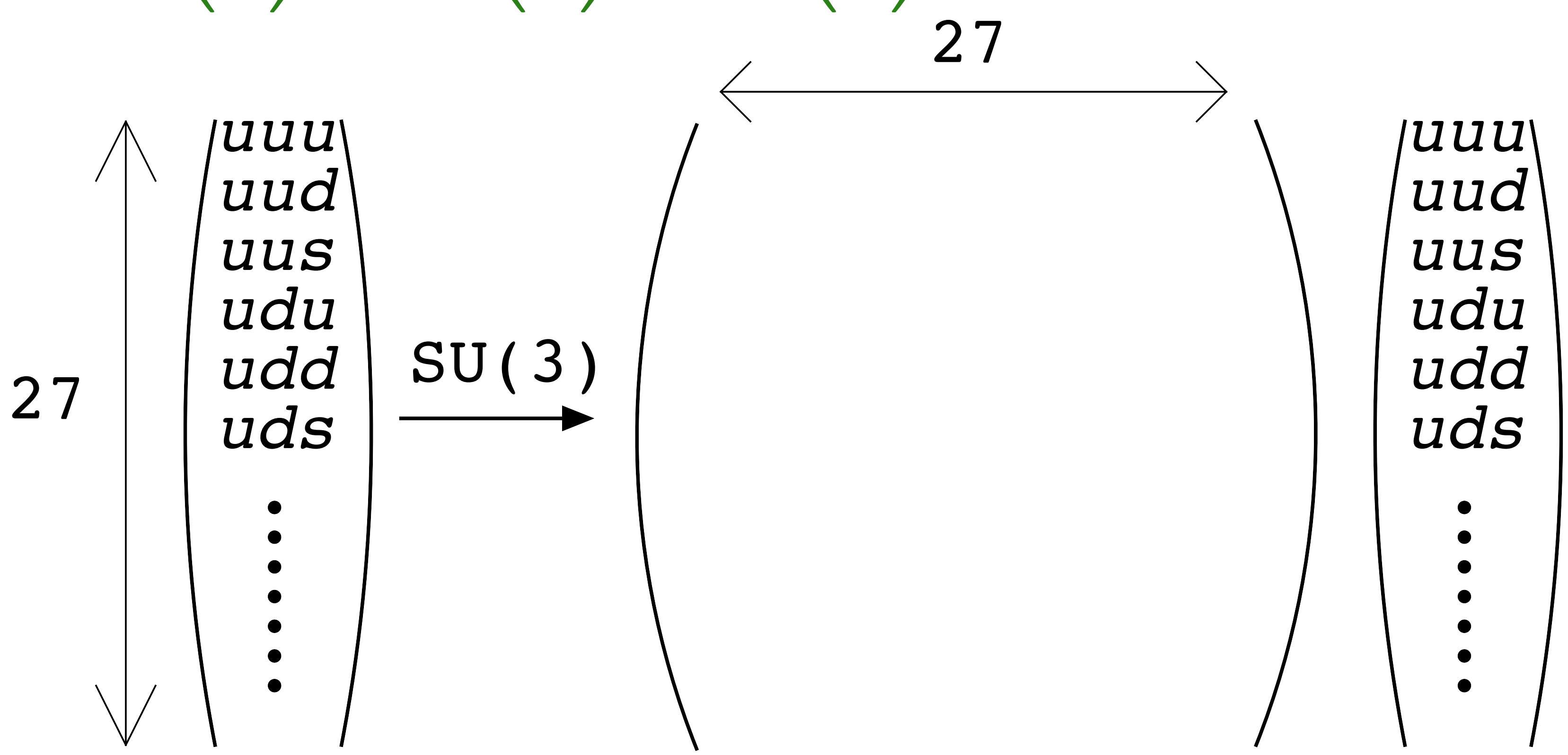
$$Q \to U\,Q \text{ under } U \in \mathrm{SU}(3)$$

- strong interaction is invariant under SU(3)$_f$

$$[H_s, U] = 0$$

build all possible baryons made of u, d, s

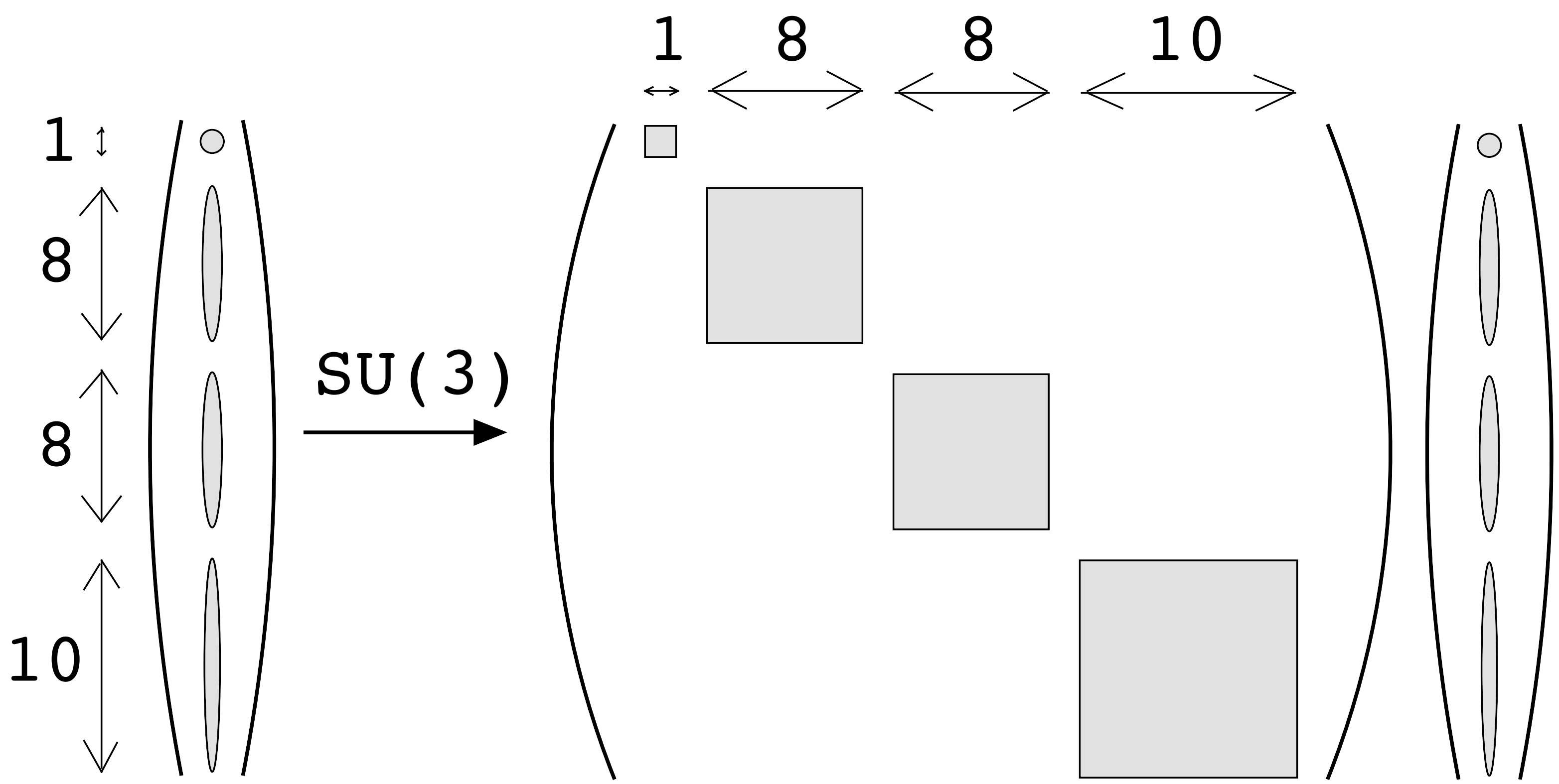
$$\begin{pmatrix}u\\d\\s\end{pmatrix}\otimes\begin{pmatrix}u\\d\\s\end{pmatrix}\otimes\begin{pmatrix}u\\d\\s\end{pmatrix}\underset{\mathrm{SU(3)}}{\sim}\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$$



representation theory ➡ 27×27 matrix

can be diagonalized by blocks of *minimal* sizes

1, 8, 8, 10: $\quad 3\otimes 3\otimes 3 = 1\oplus 8\oplus 8\oplus 10$



∃ four subspaces invariant under SU(3)

consider baryon |B⟩ in subspace of dimension 10

$$H_s|B\rangle = m_B|B\rangle \Rightarrow UH_s|B\rangle = H_sU|B\rangle = m_BU|B\rangle$$

$U|B\rangle \neq |B\rangle$ eigenstate of H_s with eigenvalue m_B

⟹ decuplet of baryons with similar masses

(Gell-Mann & Zweig 'quark' idea
allowed to predict hyperon $\Omega^- = sss$)

the *color* quantum number

- correct description of *meson* spectrum

$\longrightarrow$ quark $=$ fermion (spin 1/2)

- correct description of *baryon* spectrum

$\longrightarrow$ baryon wave functions totally *symmetric*

in contradiction with Pauli principle!

- to rescue Pauli principle, 'color' is introduced

Greenberg (1964)
Han & Nambu (1965)

given quark flavour carries color index

$$q_f^i \qquad (i = 1, 2, 3)$$

→ totally *antisymmetric* baryon wave function:

$$\text{baryon} \sim \epsilon_{ijk}\, q^i_{f_1} q^j_{f_2} q^k_{f_3}$$

- latter combination is *color singlet* $\Leftrightarrow$ invariant under rotations in *color* space:

$$\begin{pmatrix} q^1_f \\ q^2_f \\ q^3_f \end{pmatrix} \to U \begin{pmatrix} q^1_f \\ q^2_f \\ q^3_f \end{pmatrix} \;; \quad U \in \mathrm{SU}(3)_{\mathrm{color}}$$

POSTULATE: ONLY COLOR SINGLET SYSTEMS CAN CORRESPOND TO PHYSICAL PARTICLES

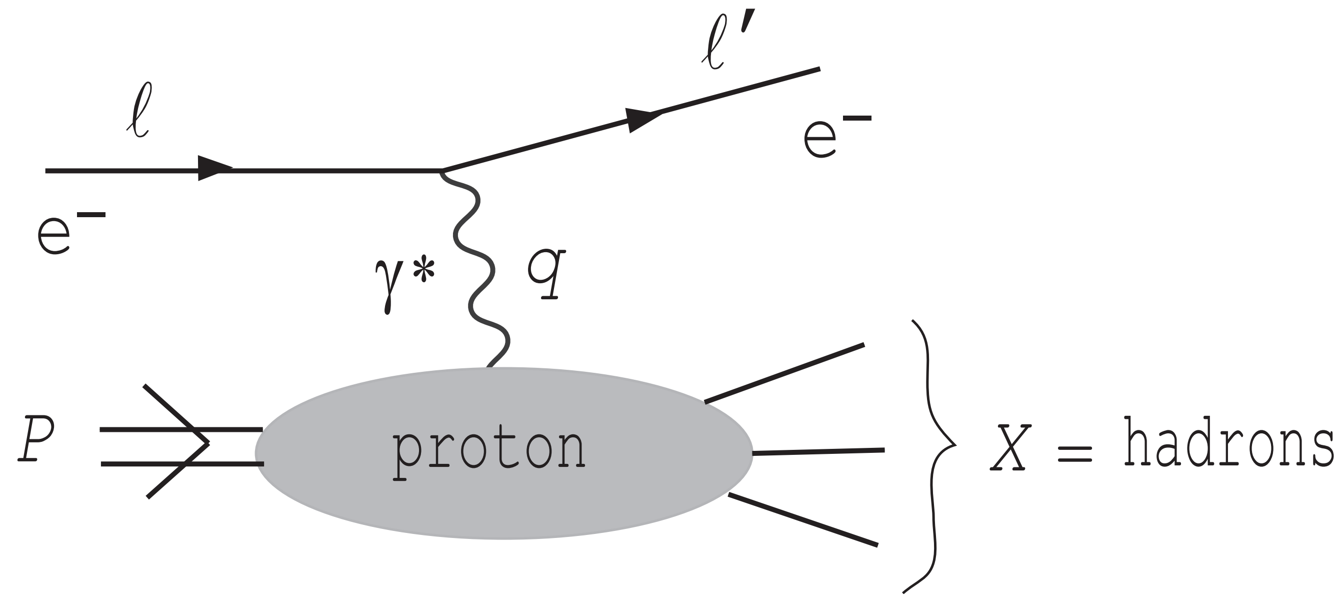
→ *colored* quarks are *confined* into *colorless* hadrons

parton model

- existence of colored quarks established by the DIS experiments at SLAC in the late 60's

Deep Inelastic Scattering

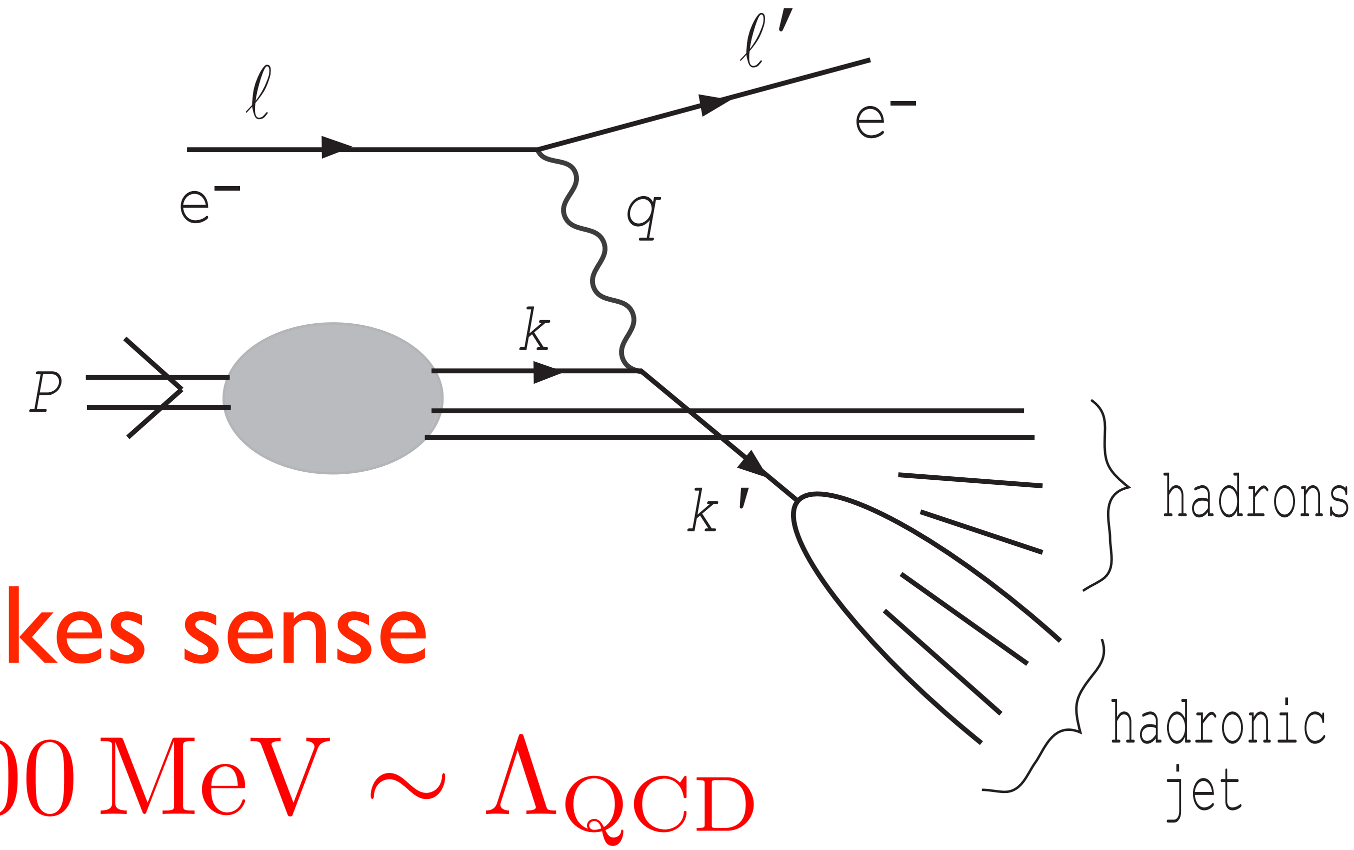
$e^- + p \to e^- + X$



- *deep*: virtuality $q^2 = -Q^2 < 0$ of γ^* is large: γ^* has spatial resolution $1/Q \ll R_{\text{proton}} \sim 1\,\text{fm}$

⟹ DIS probes *deep* inner proton structure

$\Rightarrow \gamma^*$ couples to proton charged constituents



speaking of quarks makes sense when $Q \gg R_p^{-1} \sim 200\,\mathrm{MeV} \sim \Lambda_{\mathrm{QCD}}$

- *inelastic:* in a typical event, proton is broken and final state consists of many hadrons

DIS cross section in parton model?

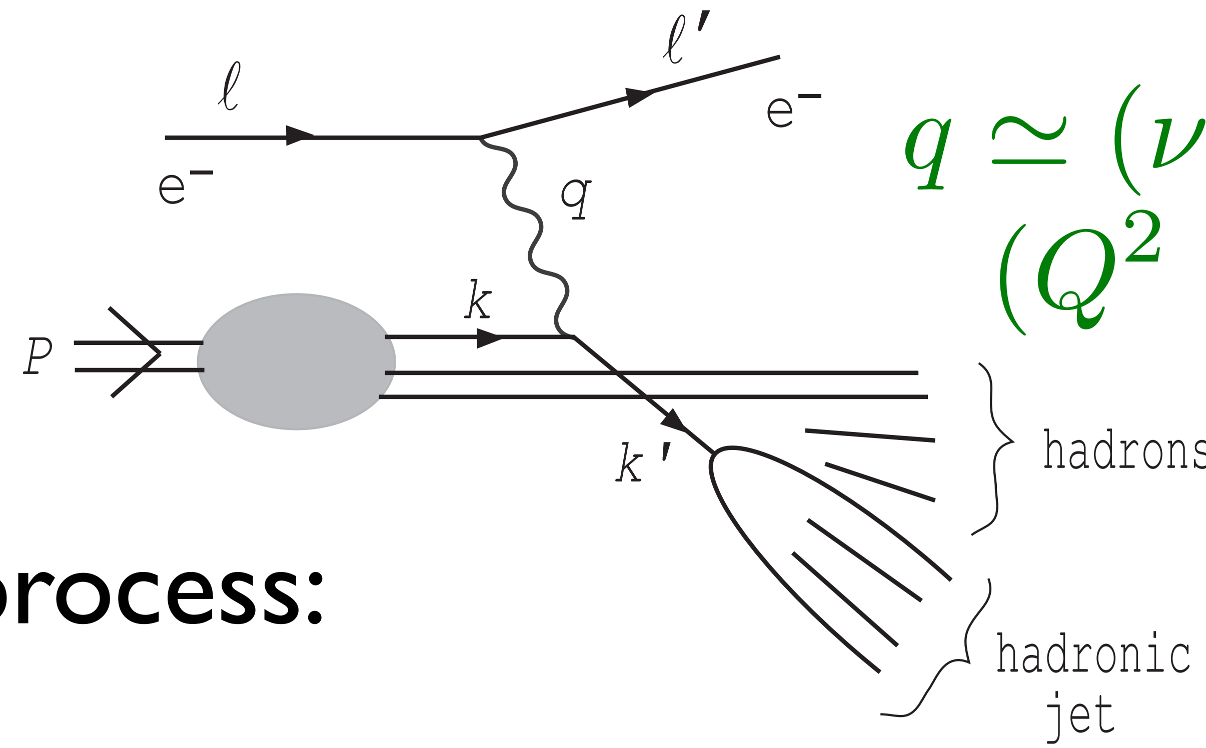
Bjorken (1969)
Feynman (1969)

choose infinite-momentum frame:

$$\boxed{P^0, \nu \to \infty}$$

$$\ell \simeq (P^0, \vec{0}_\perp, -P^0)$$

$$P \simeq (P^0, \vec{0}_\perp, P^0)$$



$$q \simeq (\nu, \vec{q}_\perp, -\nu)$$
$$(Q^2 \simeq q_\perp^2)$$

DIS ~ 3-step process:

i) incoming proton evolves in partonic configuration in typical time $(1/\Lambda)\cdot(P^0/\Lambda)$ $\quad k = \xi P$

ii) γ^*scatters off parton in time $\Delta t \sim 1/\nu \to 0$

partons quasi-free during γ^* interaction

frozen target approximation

$\Longrightarrow$ $\sigma_{\rm DIS}^{\rm proton} \sim$ *incoherent* sum of $\sigma_{\rm DIS}^{\rm parton}$

iii) struck quark *hadronizes* into hadronic jet in time

$$t_{\rm hadro} \sim (1/\Lambda) \cdot (k'^{\,0}/\Lambda)$$

within parton model, $\sigma_{\rm DIS}$ factorizes:

$$\sigma(e^- p \to e^- X) = \sum_a \int_0^1 d\xi f_a(\xi) \hat{\sigma}(e^-(\ell) + a(\xi P) \to e^-(\ell') + a(k'))$$

$f_a(\xi) =$ parton distribution function (PDF)

$$k'^2 = (k+q)^2 = 2\xi P \cdot q - Q^2 = 0 \Rightarrow \boxed{\xi = \frac{Q^2}{2P \cdot q} \equiv x_B}$$

for spin 1/2 parton:

$$\frac{d\hat{\sigma}(e^- a \to e^- a)}{d\hat{t}} = \frac{2\pi\alpha^2 e_a^2}{\hat{t}^2}\left[1 + \frac{\hat{u}^2}{\hat{s}^2}\right]$$

$$\hat{s} = (\ell + k)^2; \quad \hat{t} = (\ell - \ell')^2 = q^2 = -Q^2; \quad \hat{u} = (\ell - k')^2$$

$$\left.\begin{array}{l} y = \dfrac{P\cdot q}{P\cdot \ell} \\ \hat{s}+\hat{t}+\hat{u}=0 \\ \hat{u}/\hat{s} = y-1 \end{array}\right\} \Rightarrow \frac{d\sigma_{DIS}}{dx dQ^2} = \underbrace{\left(\sum_a f_a(x) e_a^2\right)}_{\text{characteristic of proton structure}} \cdot \underbrace{\frac{2\pi\alpha^2}{Q^4}[1+(1-y)^2]}_{\text{partonic cross section}}$$

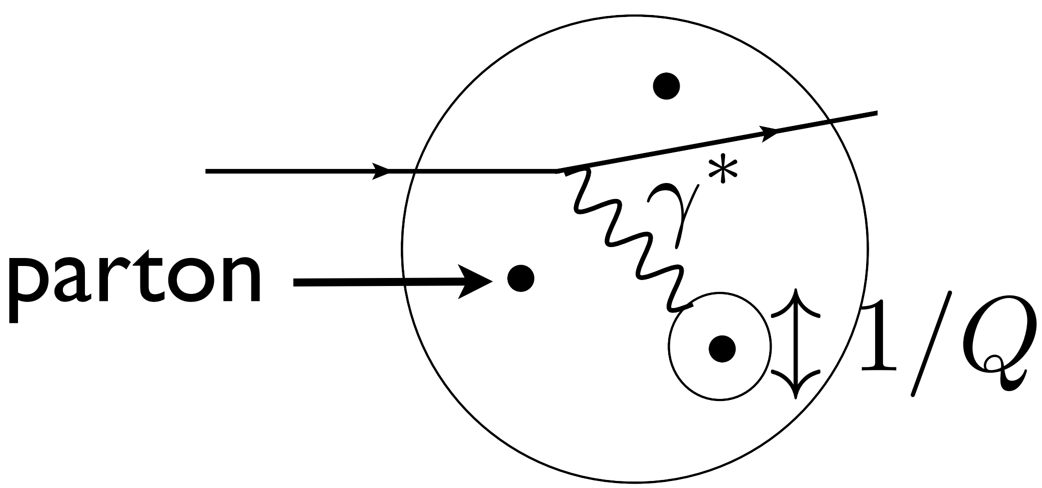
In Bjorken limit,

$$\boxed{Q^2 \to \infty,\ P\cdot q \to \infty \text{ at fixed } x_B = \frac{Q^2}{2P\cdot q}}$$

'probability density' $f_a(x)$ 'scales' with Q^2

(Bjorken scaling)

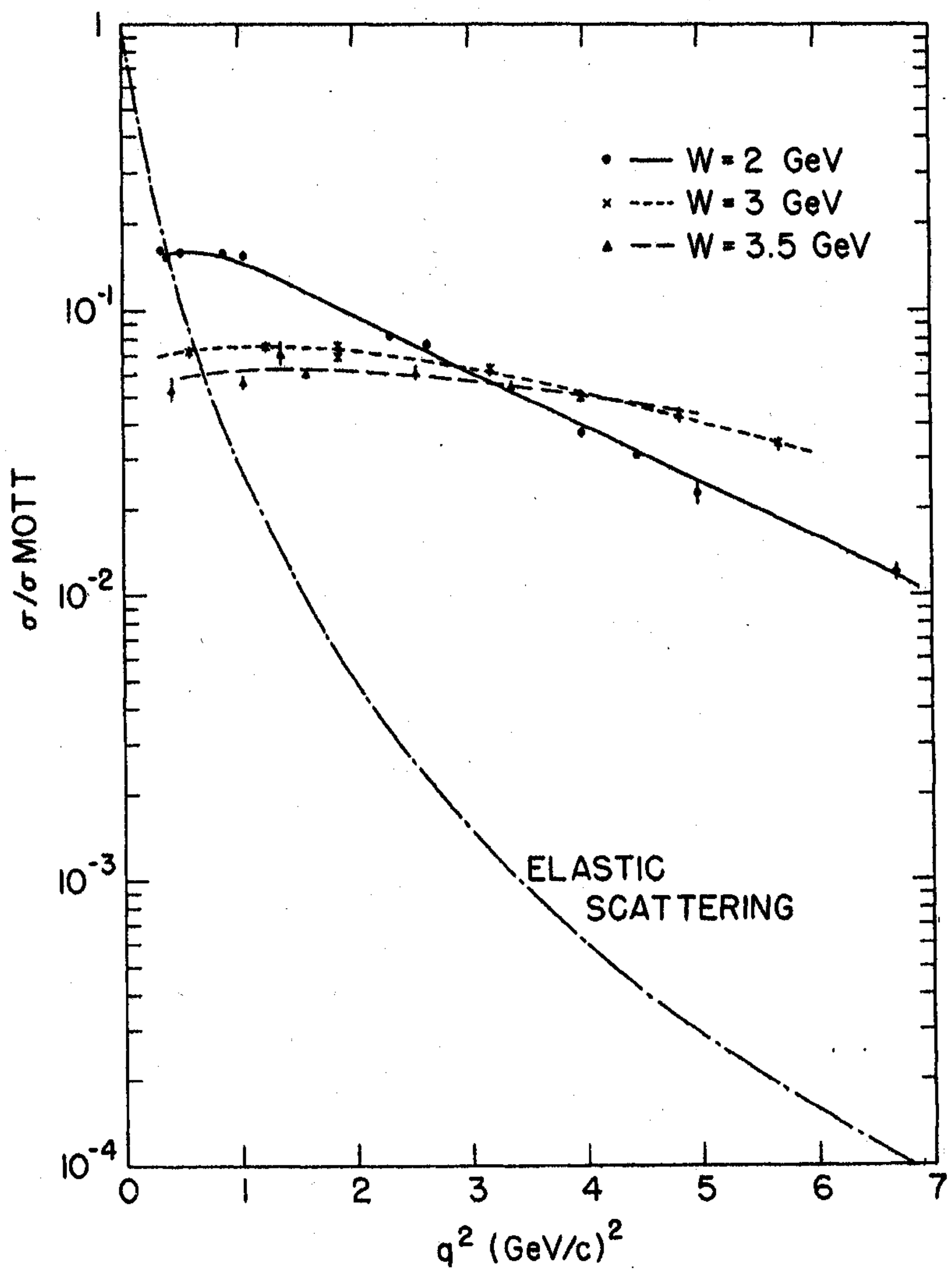
- follows from assumption of *pointlike* partons:



formally: $\rho(\vec{q}) = \text{cst.} \Leftrightarrow$

$$\rho(\vec{r}) \sim \int d^3\vec{q}\, e^{i\vec{q}\cdot\vec{r}} \rho(\vec{q}) \sim \delta^3(\vec{r})$$

- Bjorken scaling confirmed by experiment



Breidenbach *et al* (1969)

- factor $[1+(1-y)^2]$ $\Longrightarrow$ charged partons have spin 1/2

charged partons identified to Gell-Mann & Zweig quarks

quarks are real elementary particles, not just abstractions used to build the hadron spectrum

DIS revealed color

Brachychiton discolor

idea of non-abelian gauge invariance

- Yang & Mills (1954) introduce non-abelian gauge invariance to describe strong interaction

  $m_p \simeq m_n$ suggests invariance under

$$\begin{pmatrix} p \\ n \end{pmatrix} \to U(x) \begin{pmatrix} p \\ n \end{pmatrix} \; ; \quad U(x) \in \mathrm{SU}(2)_{\mathrm{isospin}}$$

  but not phenomenologically successful

- YM idea is however very fruitful for theory:
  - quantization of YM theory by preserving unitarity is highly non-trivial $\Longrightarrow$ *ghosts*

    Faddeev & Popov (1967)
  - YM idea applied to weak interaction

Glashow (61) Salam & Ward (64) Weinberg (67)

$\Longrightarrow$ electro-magnetic and weak interactions unified

$$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$$

$$\mathrm{SU}(2) \leftrightarrow e^{i\alpha^a(x)\tau^a} \qquad \mathrm{U}(1) \leftrightarrow e^{i\alpha(x)}$$

4 gauge parameters $\Longrightarrow$ 4 gauge fields A_μ

$$\begin{cases} \text{photon} \\ \text{weak bosons } W^+, W^-, Z^0 \end{cases}$$

$\Longrightarrow$ electroweak theory unitary and renormalizable

't Hooft (71) Lee & Zinn-Justin (72) 't Hooft & Veltman (72)

$\Longrightarrow$ electroweak theory beautifully confirmed

discovery of Z^0, W^+, W^- ... and Higgs boson!

1973: YM idea for strong interaction is revived

Weinberg; Gross & Wilczek; Gell-Mann & Leutwyler

- strong interaction: gauge principle in *color space*
  - quark field $\psi_i(x)$ carries color index: $i = 1, 2, 3$
  - color not observable $\Rightarrow$ physics invariant under:

$$\psi_i(x) \to U_{ij}(x)\,\psi_j(x); \quad U(x) = e^{ig_s\Lambda^a(x)T^a} \in \mathrm{SU}(3)$$

- $\mathcal{L}_{\text{Dirac, quark}} = \sum_{i=1}^{3} \bar{\psi}_i \left(i\gamma^\mu \partial_\mu - m\right) \psi_i$

is made invariant under $\psi(x) \to U(x)\,\psi(x)$ by shifting:

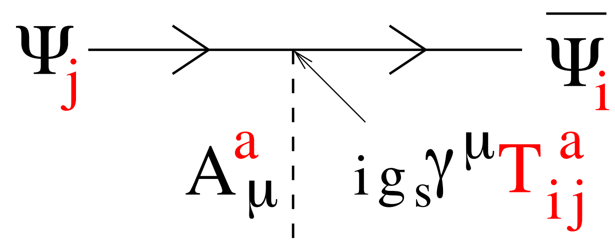
$$\delta_{ij}\partial_\mu \to (D_\mu)_{ij} = \delta_{ij}\partial_\mu + ig_s A_\mu^a T_{ij}^a$$

- eight parameters $\Lambda^a(x) \to$ eight fields $A_\mu^a \leftrightarrow$ gluons

$$\Rightarrow \boxed{\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} + \bar{\psi}_i\left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j}$$

- quark-gluon interaction :

$$-g_s\,\bar{\psi}_i\gamma^\mu A^a_\mu\, T^a_{ij}\,\psi_j \longrightarrow$$



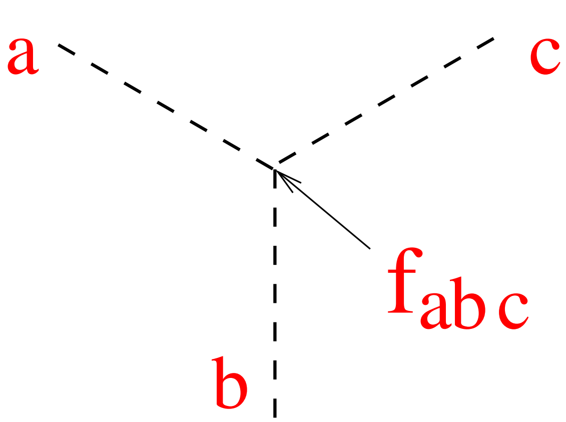
T^a_{ij} = color factor

gluon carries color charge

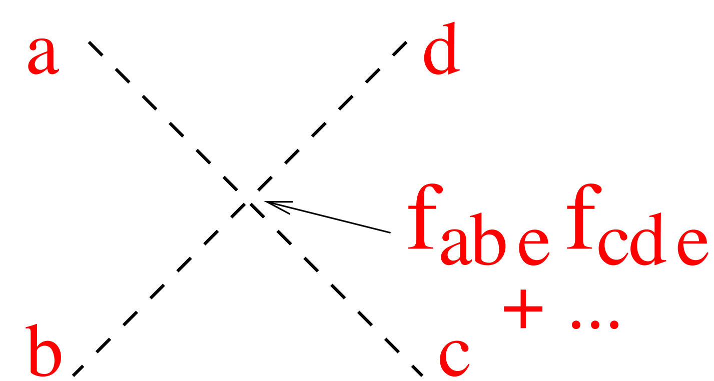
- arises from non-abelian $\mathrm{SU}(3)$: $\left[T^a, T^b\right] = i f_{abc} T^c$

$$\Rightarrow F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$$

$\Rightarrow F^a_{\mu\nu}F^{\mu\nu,a}$ contains gluon self-interactions:



3–gluon coupling



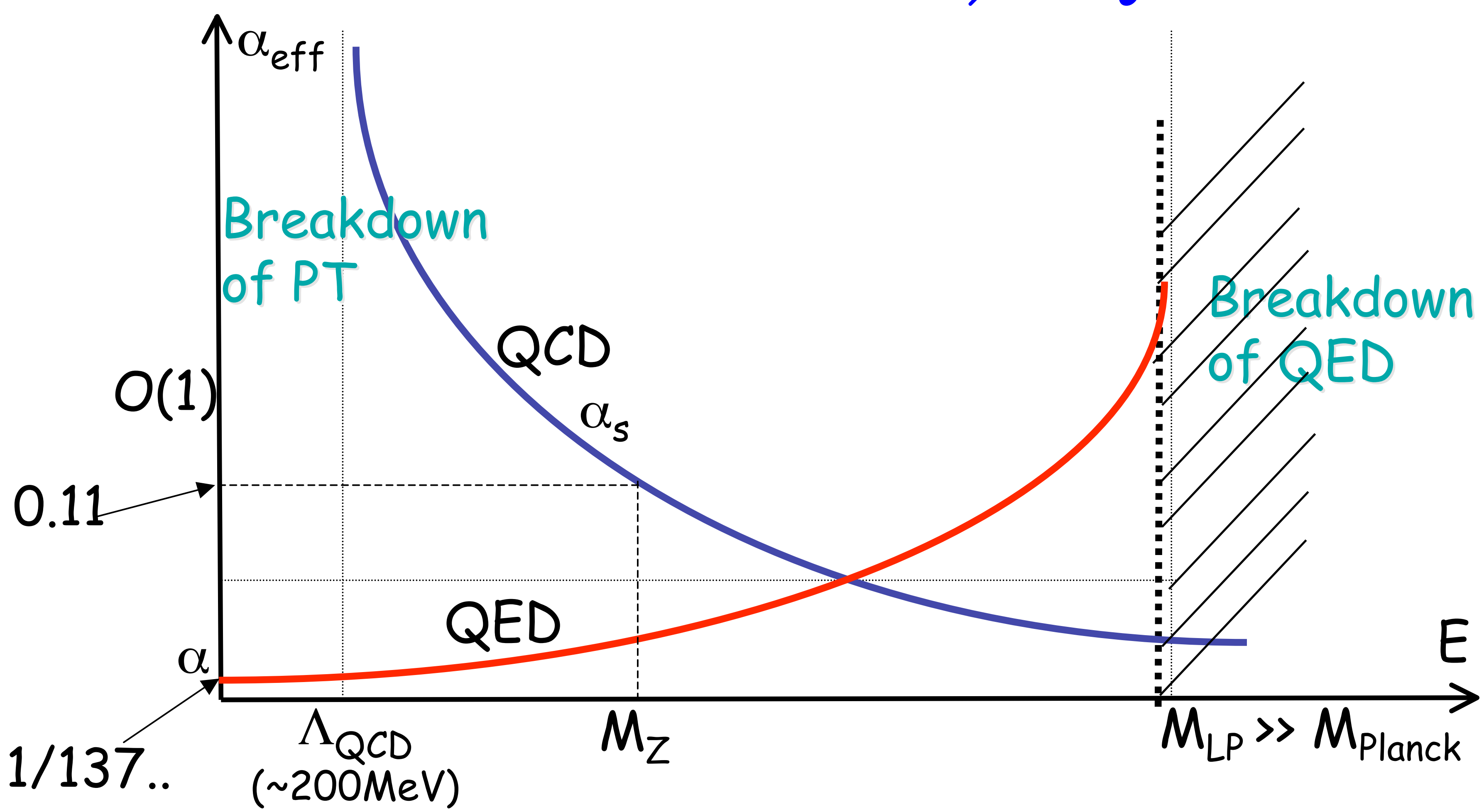
4–gluon coupling

- a crucial prediction of QCD: *asymptotic freedom*

Gross & Wilczek; Politzer (1973)

effective QCD coupling $\alpha_s(Q) = \dfrac{g^2(Q)}{4\pi}$

decreases with $\nearrow Q$



at high Q, QCD can be treated *perturbatively*

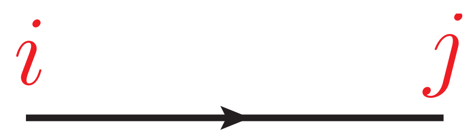
II) Recreation: color factors

pictorial representation of color factors

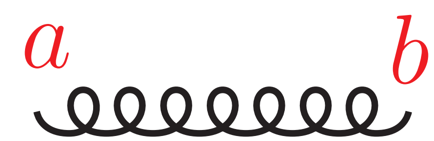
- in general, a given QCD Feynman diagram

$$= \sum_{\text{terms}} (\text{color factor}) \cdot (\text{Lorentz factor})$$

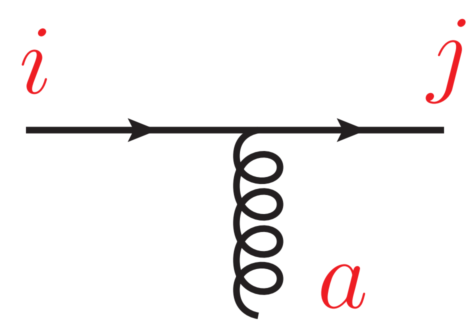
- each color factor is a contraction over color indices between products of the 'bricks':



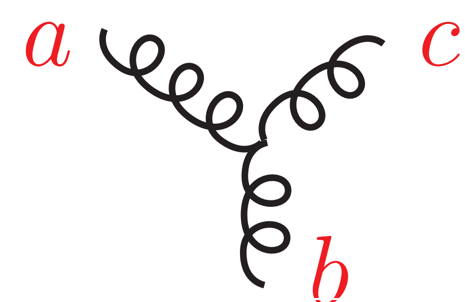
$= \delta_{ij}$



$= \delta_{ab}$



$= T^a_{ji}$

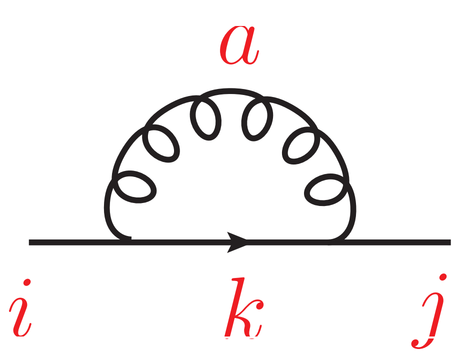


$= i f_{abc}$

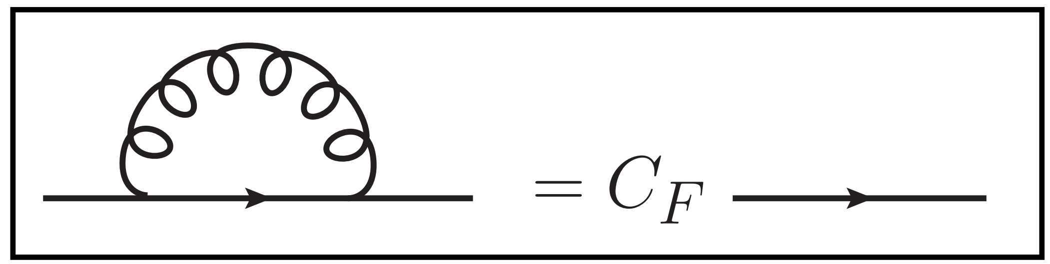
- conveniently derived using pictorial rules

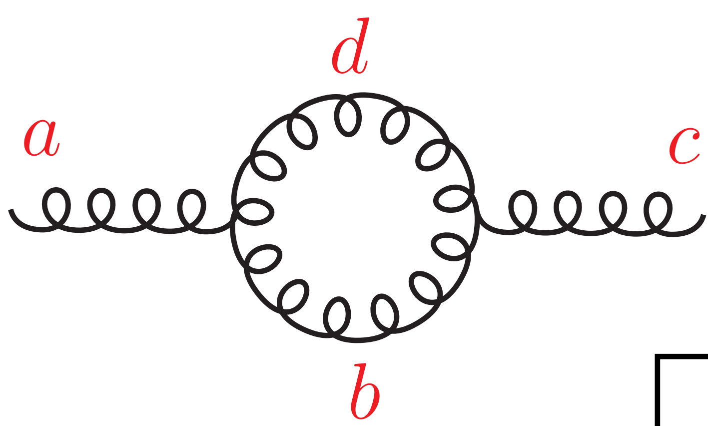
- consider N colors $\quad$ ($\mathrm{SU}(N)$ gauge group)

some examples

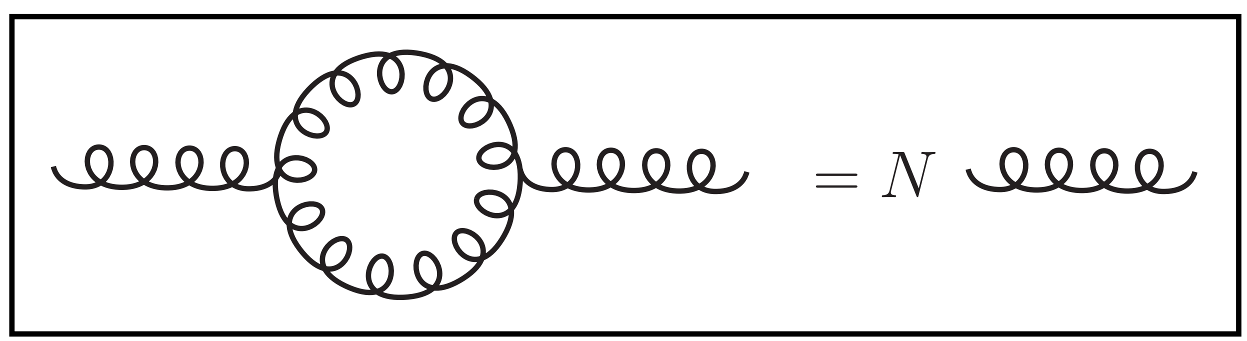


$$= T^a_{jk} T^a_{ki} = (T^a T^a)_{ji} = C_F \delta_{ij} \qquad C_F = \frac{N^2-1}{2N}$$

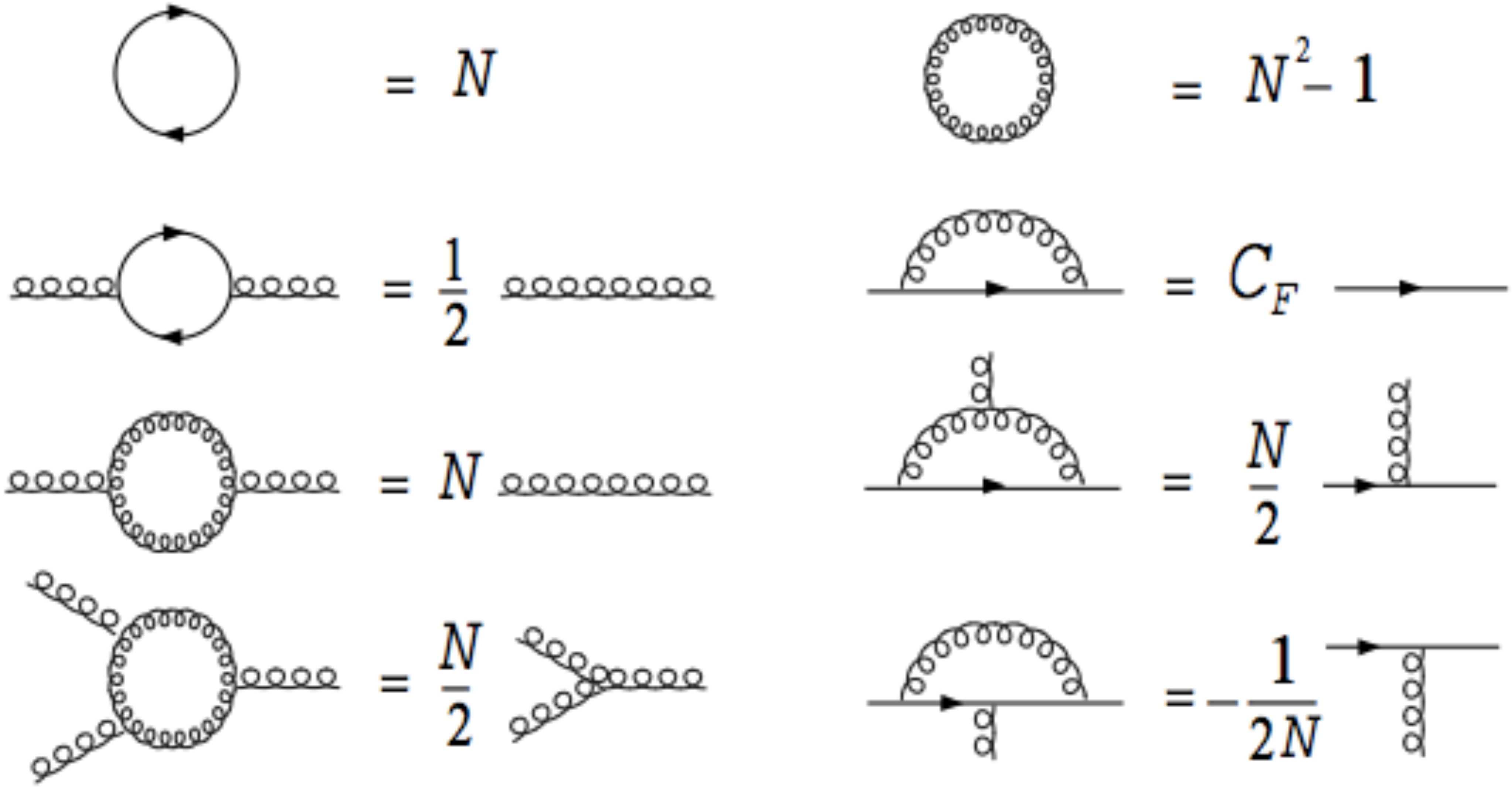




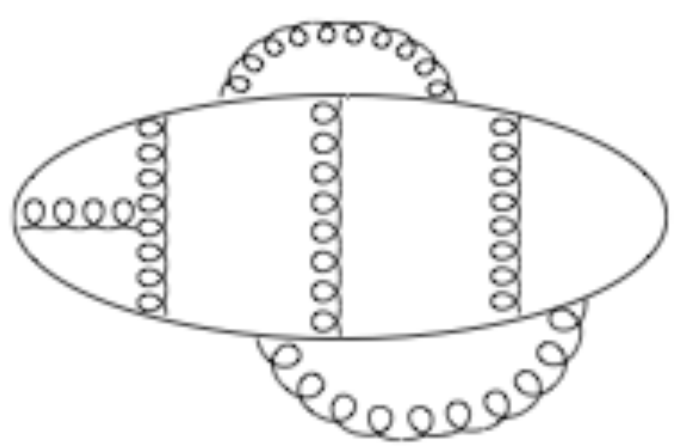
$$= i f_{abd}\, i f_{bcd} = f_{abd} f_{cbd} = N \delta_{ac}$$



using basic identities involving T^a and f_{abc},
one can obtain other rules when needed



exercise: verify that

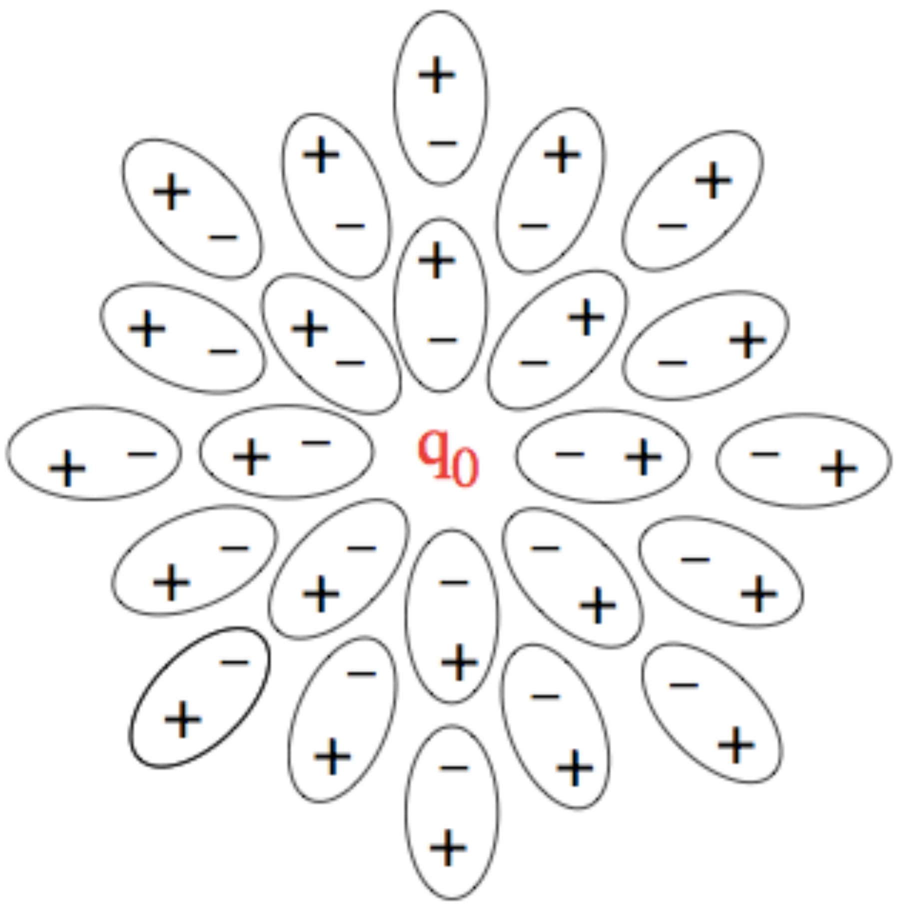


$$= -\frac{C_F^2}{16N}$$

III) QCD asymptotic freedom

running coupling in QED

- heuristically: test charge q_0 in QED



$r \gg 1/m \Rightarrow$ charge screening (vacuum polarization)

$\alpha_{\text{eff}} \searrow$ when $r \nearrow$

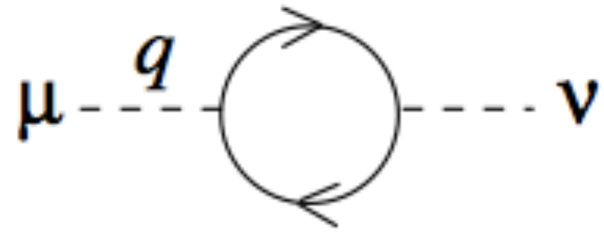
probe distance r
$\leftrightarrow$ momentum transfer $Q \sim 1/r$

$\Rightarrow$ in QED: $\alpha_{\text{eff}} \nearrow$ when $Q \nearrow$

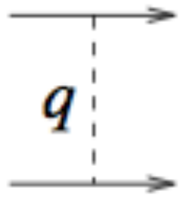
"QED coupling is *running* with energy"

- theoretically:

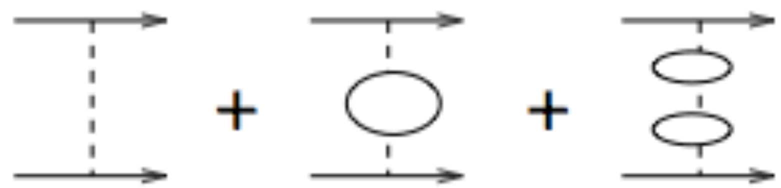
$$\Pi^{\mu\nu}(q) =$$



$$= \Pi(q^2)\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)$$



$$\propto \frac{\alpha_0}{q^2} \longrightarrow$$



$$+ \cdots \propto \frac{\alpha_0}{q^2(1-\Pi(q^2))}$$

$$\Rightarrow \alpha(q^2) = \frac{\alpha_0}{1-\Pi(q^2)}$$

$\Pi(q^2)$ is UV divergent $\rightarrow$ renormalization

$$\Rightarrow \beta(\alpha) \equiv \frac{d\alpha}{d\log Q^2} = \frac{\alpha^2}{3\pi} > 0$$

α increases logarithmically with energy

zero charge phenomenon

Suppose QED is a meaningful theory up to Q_{max}

$$\frac{1}{3\pi}\log\left(\frac{Q^2_{\rm max}}{Q^2}\right) = \frac{1}{\alpha(Q^2)} - \frac{1}{\alpha(Q^2_{\rm max})} < \frac{1}{\alpha(Q^2)}$$

- *continuum limit* $Q_{\rm max} \to \infty$ possible only if $\alpha(Q^2) = 0$

the only self-consistent theory is the trivial one!

Landau, Abrikosov, Khalatnikov (1956)

- in reality $\alpha(Q^2) \neq 0 \Rightarrow Q_{\rm max} \to \infty$ limit not defined

$\alpha(Q^2)$ singular at $Q = \Lambda_{\rm UV}$ (Landau pole)

$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \frac{1}{3\pi}\alpha(Q_0^2)\log\left(\frac{Q^2}{Q_0^2}\right)} = \frac{3\pi}{\log(\Lambda_{\mathrm{UV}}^2/Q^2)}$$

watch the sign arising from sign of $\Pi(q^2)$

- QED is not a consistent theory!
- in practice Λ_{UV} is irrelevant

$$\Lambda_{\mathrm{UV}} \equiv Q_0 \exp\left[3\pi/2\alpha(Q_0^2)\right] \sim 10^{277}\,\mathrm{GeV} \gg M_{\mathrm{universe}} \sim 10^{80} m_e$$

- QED tested to excellent accuracy

(including QED *running* $\alpha(Q^2) \nearrow$ with Q^2)

at accessible energies where $\alpha(Q^2) \ll 1$

an apparent obstacle to asymptotic freedom: unitarity

- QED: behaviour of $\alpha(Q^2)$ arises from sign of $\Pi(Q^2)$
- sign of $\Pi(Q^2)$ seems to be constrained by unitarity

unitarity of S-matrix, $SS^\dagger = 1$, implies:

- probability conservation

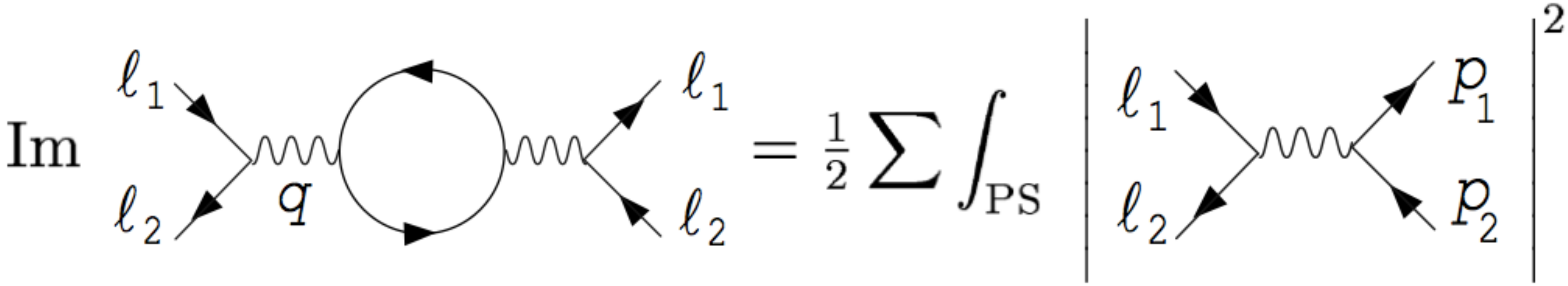
$$S_{ik} = \langle k|S|i\rangle \Rightarrow \sum_k S_{ik} S^\dagger_{kj} = \delta_{ij} \quad \underset{i=j}{\Longrightarrow} \quad \sum_k |\langle k|S|i\rangle|^2 = 1$$

- optical theorem

$$S = 1 + iT \Rightarrow i(T - T^\dagger) = -TT^\dagger \quad \Rightarrow \quad i(T - T^*)_{jj} = -\sum_k |T_{jk}|^2$$

$$\Rightarrow 2\mathrm{Im}\,\mathcal{M}_{jj} = \sum_k |\mathcal{M}_{jk}|^2 \qquad |k\rangle = \text{physical state}$$

optical theorem holds order by order in perturbation



(✧)

$\mathrm{Im} \neq 0 \Leftrightarrow$ ∃ intermediate *on-shell* physical state

E.g., $\mathrm{Im}\,\Pi(q^2) \neq 0 \Leftrightarrow q^2 > 4m^2$

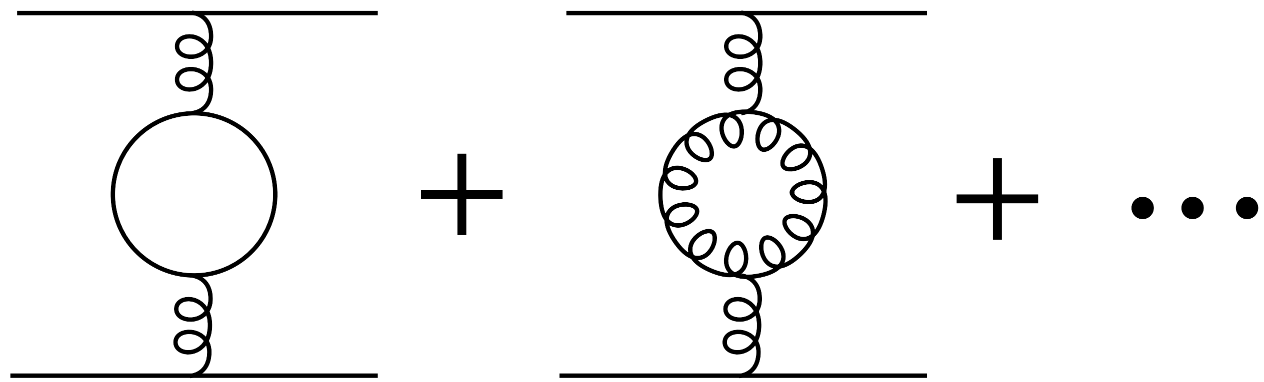
r.h.s. of (✧) is *positive* $\Longrightarrow$ fixes sign of $\Pi(q^2)$

$\beta_{\mathrm{QED}} > 0$ is a direct consequence of unitary!

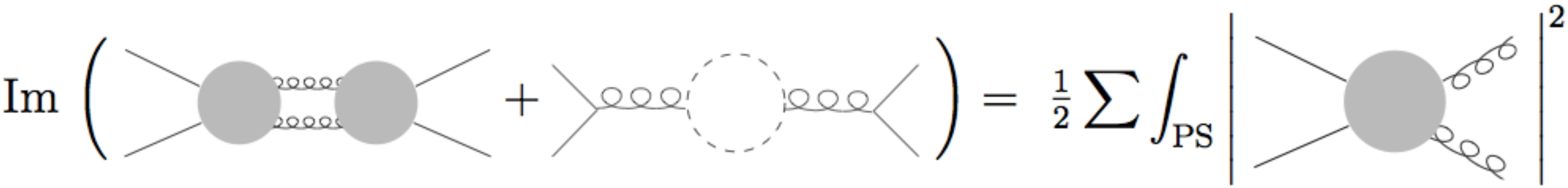
$\Longrightarrow$ difficult to imagine a *unitary* theory with $\beta < 0$

how the obstacle is overcome in non-abelian gauge theory

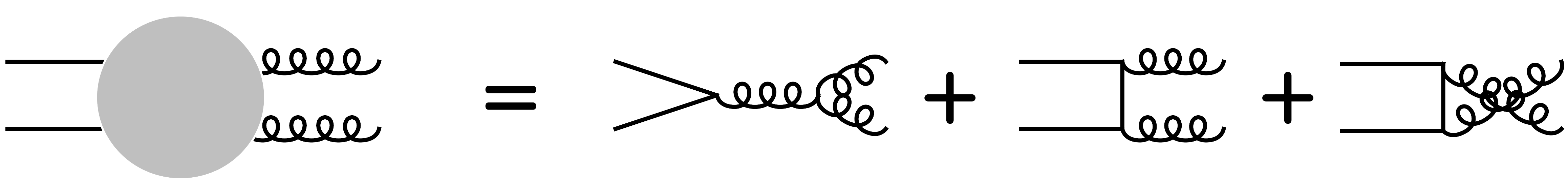
- running of QCD coupling given by



- is sign of gluon loop constrained by optical theorem?



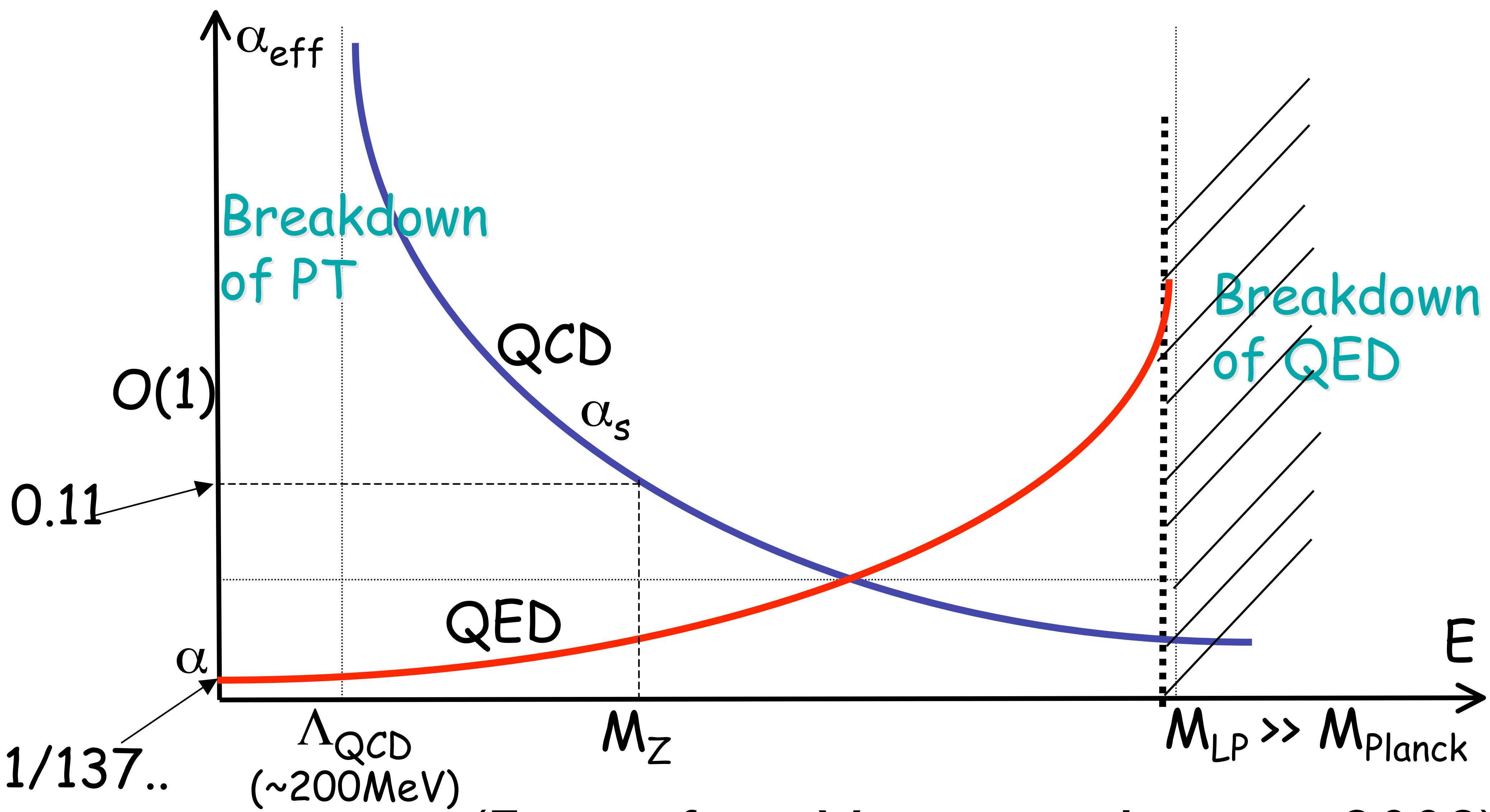
r.h.s. is positive but does not constrain sign of gluon loop



$\Longrightarrow$ no obstacle to $\beta_{\mathrm{QCD}} < 0$

- confirmed by calculation:

asymptotic freedom = QCD prediction



(Figure from Veneziano lecture, 2008)

asymptotic freedom ⟵ gluon self-couplings

⟵ non-abelian gauge theory