Lectures on Higgs/Composite Higgs
Part II

Eduardo Pontón

IFT-UNESP & ICTP-SAIFR

Outline

Lecture I: The Standard Model and Electroweak Symmetry Breaking (EWSB)

- The Standard Model (SM): Setting up the Stage
- The 125 GeV Resonance: Properties
- The Hierarchy Problem and Higgs Compositeness
- Useful Lessons from the QCD Sector of the SM

Lecture II: Realistic Higgs Compositeness Scenarios

- Introduction: The Big Picture
- Illustrating the main ideas in the simplest (realistic) case
  - Pseudo-Nambu Goldstone Bosons
  - Custodial Symmetry
  - Partial compositeness
  - The Higgs Potential

Composite Higgs

BSM: Standard Model + Strongly coupled sector

Strong dynamics

(e.g. asymptotically free gauge theory)

New fermions (and scalars, if SUSY)

Composite Higgs

BSM: Standard Model + Strongly coupled sector

Strong dynamics

(e.g. asymptotically free gauge theory)

New fermions (and scalars, if SUSY)

Sufficiently large global symmetry G

($SU(3)_C \times SU(2)_L \times U(1)_Y \subset G$)

Composite Higgs

BSM: Standard Model + Strongly coupled sector

Strong dynamics

(e.g. asymptotically free gauge theory)

New fermions (and scalars, if SUSY)

Sufficiently large global symmetry G

($SU(3)_C \times SU(2)_L \times U(1)_Y \subset G$)

(E.g. $\langle \psi \bar{\psi} \rangle \neq 0$)

Dynamically generated scale Λ

Set of resonances

Composite Higgs

BSM: Standard Model + Strongly coupled sector

Strong dynamics
(e.g. asymptotically free gauge theory)

New fermions (and scalars, if SUSY)

Sufficiently large global symmetry G

($SU(3)_C \times SU(2)_L \times U(1)_Y \subset G$)

(E.g. $\langle \psi \bar{\psi} \rangle \neq 0$)

Dynamically generated scale Λ

$G \rightarrow \mathcal{H}$

Set of resonances

Composite Higgs

BSM: Standard Model + Strongly coupled sector

Strong dynamics
(e.g. asymptotically free gauge theory)

New fermions (and scalars, if SUSY)

Sufficiently large global symmetry G

($SU(3)_C \times SU(2)_L \times U(1)_Y \subset G$)

(E.g. $\langle \psi\bar{\psi} \rangle \neq 0$)

Dynamically generated scale Λ

$G \to \mathcal{H}$

Set of resonances

- $\boxed{SU(2)_L \times U(1)_Y \not\subset \mathcal{H}}$ $\longrightarrow$ **``Technicolor", no need of Higgs (ruled out)**
- $\boxed{SU(2)_L \times U(1)_Y \subset \mathcal{H}}$ $\longrightarrow$ **SM group unbroken at Λ**

  Amongst resonances: state with Higgs quantum numbers

Composite Higgs

BSM: Standard Model + Strongly coupled sector

Strong dynamics

(e.g. asymptotically free gauge theory)

New fermions (and scalars, if SUSY)

Sufficiently large global symmetry G

($SU(3)_C \times SU(2)_L \times U(1)_Y \subset G$)

(E.g. $\langle \psi \bar{\psi} \rangle \neq 0$)

Dynamically generated scale Λ

$G \to \mathcal{H}$

Set of resonances

- $SU(2)_L \times U(1)_Y \subset \mathcal{H}$

SM group unbroken at Λ

Amongst resonances: state with Higgs quantum numbers

Composite Higgs

BSM: Standard Model + Strongly coupled sector

Strong dynamics
(e.g. asymptotically free gauge theory)

New fermions (and scalars, if SUSY)

Sufficiently large global symmetry G

($SU(3)_C \times SU(2)_L \times U(1)_Y \subset G$)

(E.g. $\langle \psi \bar{\psi} \rangle \neq 0$)

Dynamically generated scale Λ $\longrightarrow$ $G \to \mathcal{H}$

Set of resonances

- $\boxed{SU(2)_L \times U(1)_Y \subset \mathcal{H}}$ $\longrightarrow$ SM group unbroken at Λ

Amongst resonances: state with Higgs quantum numbers

SM gauge fields and fermions should not be composite $\longrightarrow$ more later...

(though third family may have some degree of compositeness)

The Higgs as a pNGB

Elementary sector:
SM gauge bosons, quarks & leptons

SM gauge interactions ⟷ **Mixing**

New strong sector:
resonances + Higgs bound state

But why would the Higgs resonance be lighter than the rest?

Natural to interpret the composite Higgs as a (pseudo) Nambu-Goldstone boson

Higgs in $G/\mathcal{H}$

Georgi & Kaplan '84
Agashe et. al '03

Inspiration from pions in QCD (with 2 flavors): $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$

$\pi^0, \pi^\pm$ **are NGB's of spontaneous breaking**

Acquire masses from explicit breaking:

- $m_q \neq 0 \;\Rightarrow\; m_\pi^2 \simeq m_q B_0$
- $e \neq 0 \;\Rightarrow\; m_{\pi^\pm}^2 - m_{\pi^0}^2 \sim \dfrac{e^2}{16\pi^2}\Lambda^2$

What we will (not) do

Our aim here is not to build specific UV models, that have a certain set of global symmetries and whose dynamics breaks such symmetries in a given manner. This would be a very difficult question, in general requiring new tools to deal with the associated strong interactions.

[In some cases one can have some control, either because the situation is sufficiently close to QCD, or because of enhanced symmetries, such as supersymmetry]

Instead, we will *assume* that there exists a UV model with the given $G \to \mathcal{H}$ symmetry breaking pattern. Since we will be interested in the "low-energy" properties of such a hypothetical theory, we expect that only the light modes (i.e. NGB's or pNGB's) will be crucial, plus perhaps a few additional resonances (akin to the QCD rho meson). The physics of these light states will be largely determined by the symmetries, so we can proceed in a model-independent way (apart from the choice of the symmetry groups).

The effective low-energy Lagrangian will contain a number of free parameters that would be, in principle, determined by the more microscopic parameters. One should also remember that certain observables may be sensitive to undetermined coefficients whose size we can at best estimate. However, there will be other observables that are IR dominated, hence they will correspond to predictions of the theory.

Of the various possible symmetry breaking patterns, we will choose the simplest one to illustrate the main ideas...

The Choice of Group (I)

Requiring that G and $\mathcal{H}$ be large enough to contain the SM $SU(2)_L \times U(1)_Y$ and that they generate at least four NGB's that have the quantum numbers of the SM Higgs would naturally suggest that we focus on:

$$\underbrace{G = SU(3)}_{\substack{3^2-1=8 \\ \text{generators}}} \longrightarrow \underbrace{\mathcal{H} = SU(2)_L \times U(1)_Y}_{\substack{3+1=4 \\ \text{generators}}}$$

$$T_L^a = \left(\begin{array}{cc|c} \multicolumn{2}{c|}{\tau_L^a} & 0 \\ & & 0 \\ \hline 0 & 0 & 0 \end{array}\right) \qquad Y = \frac{1}{6}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Thus, $\mathrm{Dim}(G/\mathcal{H}) = 4$, and there will be four (real) Nambu Goldstone bosons from the spontaneous breaking, with the desired quantum numbers:

$$T^{\hat{a}} \longleftrightarrow \left(\begin{array}{cc|c} & & H^+ \\ & & H^0 \\ \hline H^- & H^{0*} & 0 \end{array}\right) \begin{array}{l} \left.\vphantom{\begin{array}{c} H^+ \\ H^0 \end{array}}\right\} SU(2)_L \text{ doublet} \\ \\ \end{array}$$

$\underbrace{H^- \quad H^{0*}}_{SU(2)_L \text{ anti-doublet}}$

Recall that the quantum numbers are read from

$$[Y, T^{\hat{a}}] = \pm\frac{1}{2}T^{\hat{a}} \qquad [T_L^3, T^{\hat{a}}] = \pm\frac{1}{2}T^{\hat{a}} \qquad \text{etc.}$$

However, the unbroken group does not contain a custodial symmetry, so we would expect large corrections to the T-parameter from the strong dynamics... we need a more elaborate scheme!

The Choice of Group (II)

It turns out that the smallest group (and simplest breaking pattern) such that

- It contains $SU(2)_L \times U(1)_Y$
- It contains $SU(2)_L \times SU(2)_R$
- Generates exactly four NGB's that can be identified as the SM Higgs

is

$$G = SO(5) \times U(1)_X \longrightarrow \mathcal{H} = [SO(4) \simeq SU(2)_L \times SU(2)_R] \times U(1)_X \supset SU(2)_L \times U(1)_Y$$

$\frac{1}{2}\, 5\,(5-1) = 10$ generators

$\frac{1}{2}\, 4\,(4-1) = 3 + 3 = 6$ generators

Thus, $\mathrm{Dim}(G/\mathcal{H}) = 4$, and there will be four (real) Nambu Goldstone bosons from the spontaneous breaking:

$$\underbrace{i\sqrt{2}\, h^a T^{\hat{a}}}_{\text{broken generators}} = \begin{pmatrix} 0 & 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & 0 & h_2 \\ 0 & 0 & 0 & 0 & h_3 \\ 0 & 0 & 0 & 0 & h_4 \\ -h_1 & -h_2 & -h_3 & -h_4 & 0 \end{pmatrix} \qquad \text{with} \qquad Q_X = 0$$

NGB Quantum Numbers

It is useful to have an explicit basis that displays the subgroups of interest. The 10 generators of SO(5) can be arranged as follows: (see next slide for an explicit representation)

$$\{T_L^i, T_R^j, \hat{T}^a\} \qquad \text{with} \quad \mathrm{Tr}(T^A T^B) = \delta^{AB} \qquad \begin{array}{c} i,j = 1,2,3 \\ a = 1,2,3,4 \end{array}$$

We are especially interested in $SU(2)_L \times SU(2)_R$ and indeed:

$$[T_L^i, T_L^j] = i\epsilon^{ijk} T_L^k \qquad [T_R^i, T_R^j] = i\epsilon^{ijk} T_R^k \qquad [T_L^i, T_R^j] = 0$$

NGB Quantum Numbers

It is useful to have an explicit basis that displays the subgroups of interest. The 10 generators of SO(5) can be arranged as follows: (see next slide for an explicit representation)

$$\{T_L^i, T_R^j, \hat{T}^a\} \qquad \text{with} \quad \mathrm{Tr}(T^A T^B) = \delta^{AB} \qquad \begin{array}{l} i, j = 1, 2, 3 \\ a = 1, 2, 3, 4 \end{array}$$

We are especially interested in $SU(2)_L \times SU(2)_R$ and indeed:

$$[T_L^i, T_L^j] = i\epsilon^{ijk} T_L^k \qquad [T_R^i, T_R^j] = i\epsilon^{ijk} T_R^k \qquad [T_L^i, T_R^j] = 0$$

We can label the fields according to their (T_L^3, T_R^3) charges. For instance, for the broken generators:

$$\hat{T}_{(+,+)} = \frac{1}{\sqrt{2}}(\hat{T}^1 + i\,\hat{T}^2) \;,\; \hat{T}_{(-,+)} = \frac{1}{\sqrt{2}}(\hat{T}^3 - i\,\hat{T}^4) \;,\; \hat{T}_{(+,-)} = \frac{1}{\sqrt{2}}(\hat{T}^3 + i\,\hat{T}^4) \;,\; \hat{T}_{(-,-)} = \frac{1}{\sqrt{2}}(\hat{T}^1 - i\,\hat{T}^2)$$

where, for example, $[T_L^3, \hat{T}_{(-,+)}] = -\frac{1}{2}\hat{T}_{(-,+)}$ and $[T_R^3, \hat{T}_{(-,+)}] = +\frac{1}{2}\hat{T}_{(-,+)}$, etc.

NGB Quantum Numbers

It is useful to have an explicit basis that displays the subgroups of interest. The 10 generators of SO(5) can be arranged as follows: (see next slide for an explicit representation)

$$\{T^i_L, T^j_R, \hat{T}^a\} \qquad \text{with} \quad \mathrm{Tr}(T^A T^B) = \delta^{AB} \qquad \begin{array}{l} i,j = 1,2,3 \\ a = 1,2,3,4 \end{array}$$

We are especially interested in $SU(2)_L \times SU(2)_R$ and indeed:

$$[T^i_L, T^j_L] = i\epsilon^{ijk} T^k_L \qquad [T^i_R, T^j_R] = i\epsilon^{ijk} T^k_R \qquad [T^i_L, T^j_R] = 0$$

We can label the fields according to their (T^3_L, T^3_R) charges. For instance, for the broken generators:

$$\hat{T}_{(+,+)} = \frac{1}{\sqrt{2}}(\hat{T}^1 + i\hat{T}^2) \;,\; \hat{T}_{(-,+)} = \frac{1}{\sqrt{2}}(\hat{T}^3 - i\hat{T}^4) \;,\; \hat{T}_{(+,-)} = \frac{1}{\sqrt{2}}(\hat{T}^3 + i\hat{T}^4) \;,\; \hat{T}_{(-,-)} = \frac{1}{\sqrt{2}}(\hat{T}^1 - i\hat{T}^2)$$

where, for example, $[T^3_L, \hat{T}_{(-,+)}] = -\frac{1}{2}\hat{T}_{(-,+)}$ and $[T^3_R, \hat{T}_{(-,+)}] = +\frac{1}{2}\hat{T}_{(-,+)}$, etc. Then:

$$i\sqrt{2}\, h^a \hat{T}^a \equiv H^+ \hat{T}_{(+,+)} + H^0 \hat{T}_{(-,+)} + H^{0*} \hat{T}_{(+,-)} + H^- \hat{T}_{(-,-)}$$

$$= \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 & H^+ + H^- \\ & 0 & 0 & 0 & i\,(H^+ - H^-) \\ & & 0 & 0 & H^0 + H^{0*} \\ & & & 0 & -i\,(H^0 - H^{0*}) \\ & & & & 0 \end{pmatrix} \quad \longleftrightarrow \quad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} H^{0*} & H^+ \\ -H^- & H^0 \end{pmatrix} \updownarrow SU(2)_L$$

$$\longleftrightarrow \; SU(2)_R$$

The electric charge is given by $Q = T^3_L + T^3_R + Q_X 1_{5\times 5}$

SO(5) Generators

$SO(4) \simeq SU(2)_L \times SU(2)_R$ **generators:** $$(T^{\alpha}_{L,R})_{jk} = -\frac{i}{2}\left[\frac{1}{2}\epsilon_{\alpha\beta\gamma}\left(\delta^{\beta}_{j}\delta^{\gamma}_{k} - \delta^{\beta}_{k}\delta^{\gamma}_{j}\right) \pm \left(\delta^{\alpha}_{j}\delta^{4}_{k} - \delta^{\alpha}_{k}\delta^{4}_{j}\right)\right]$$

$$T^1_L = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad T^2_L = \begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad T^3_L = \begin{pmatrix} 0 & -\frac{i}{2} & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T^1_R = \begin{pmatrix} 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad T^2_R = \begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad T^3_R = \begin{pmatrix} 0 & -\frac{i}{2} & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$SO(5)/SO(4)$ **generators:** $$(\hat{T}^{a})_{jk} = -\frac{i}{\sqrt{2}}\left(\delta^{a}_{j}\delta^{5}_{k} - \delta^{a}_{k}\delta^{5}_{j}\right)$$

$$\hat{T}^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \hat{T}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix} \quad \hat{T}^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 \end{pmatrix} \quad \hat{T}^4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 \end{pmatrix}$$

Other Symmetry Breaking Patterns

To satisfy your curiosity, there are other games in town

G	$\mathcal{H}$	N_G	NGBs rep.$[\mathcal{H}]$ = rep.$[\mathrm{SU}(2) \times \mathrm{SU}(2)]$
SO(5)	SO(4)	4	$\mathbf{4} = (\mathbf{2}, \mathbf{2})$
SO(6)	SO(5)	5	$\mathbf{5} = (\mathbf{1}, \mathbf{1}) + (\mathbf{2}, \mathbf{2})$
SO(6)	SO(4) × SO(2)	8	$\mathbf{4}_{+2} + \bar{\mathbf{4}}_{-2} = 2 \times (\mathbf{2}, \mathbf{2})$
SO(7)	SO(6)	6	$\mathbf{6} = 2 \times (\mathbf{1}, \mathbf{1}) + (\mathbf{2}, \mathbf{2})$
SO(7)	G_2	7	$\mathbf{7} = (\mathbf{1}, \mathbf{3}) + (\mathbf{2}, \mathbf{2})$
SO(7)	SO(5) × SO(2)	10	$\mathbf{10}_0 = (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{2}, \mathbf{2})$
SO(7)	$[\mathrm{SO}(3)]^3$	12	$(\mathbf{2}, \mathbf{2}, \mathbf{3}) = 3 \times (\mathbf{2}, \mathbf{2})$
Sp(6)	Sp(4) × SU(2)	8	$(\mathbf{4}, \mathbf{2}) = 2 \times (\mathbf{2}, \mathbf{2}), (\mathbf{2}, \mathbf{2}) + 2 \times (\mathbf{2}, \mathbf{1})$
SU(5)	SU(4) × U(1)	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+5} = 2 \times (\mathbf{2}, \mathbf{2})$
SU(5)	SO(5)	14	$\mathbf{14} = (\mathbf{3}, \mathbf{3}) + (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$
SO(9)	SO(8)	8	$\mathbf{8} = (\mathbf{2}, \mathbf{2})_1 + (\mathbf{2}, \mathbf{2})_{-1}$

Modified from Mrazek et. al 2011

but we will be satisfied with studying a particular example from here on...

The SO(5)/SO(4) Pion Lagrangian

We have already seen two examples of how to describe NGB's:

- The pions of QCD are parameterized by $U = e^{2i\pi^a \tau^a / f_\pi}$
- In the EW theory, the (eaten) NGB's are written as $\Sigma(x) = e^{-i\pi^a(x)\tau^a/v}$

  which parameterize the massless fluctuations about the Higgs vev: $\Sigma(x) \times \begin{pmatrix} 0 \\ v \end{pmatrix}$

It turns out that the SO(5)/SO(4) case can be treated in a similar manner!

[Other symmetry breaking patterns may require the general CCWZ formalism] **Callan, Coleman, Wess & Zumino, Phys. Rev. 177, 5, p.2247**

The breaking of $SO(5) \to SO(4)$ can be thought as arising from the vev of a "Higgs field in the fundamental representation of SO(5)": $\Phi_0 = \{0, 0, 0, 0, 1\}^T$

$$\Phi(x) \equiv e^{i\Pi(x)/f_\pi} \times \Phi_0 \qquad\qquad \Pi(x) = \Pi^a(x)\hat{T}^a$$

Including *external sources* for the SO(5) currents, the effective Lagrangian at 2-derivative order is:

$$\mathcal{L} = \frac{1}{2} f_\pi^2 |D_\mu \Phi|^2 \qquad\qquad D_\mu \Phi = \partial_\mu \Phi - i\rho_\mu^A T^A \Phi$$

Non-linear Realizations

This Lagrangian describes the low-energy physics of the SO(5)/SO(4) NGB's regardless of whether the breaking arises from strong dynamics or from a "fundamental" Higgs field!

The important point is that it is fully SO(5) invariant, with

- The SO(4) symmetries being linearly realized
- The SO(5)/SO(4) symmetries being non-linearly realized

This can be readily understood in our formalism as follows:

$$\Phi(x) \to g\,\Phi(x) = \underbrace{g\,e^{i\Pi(x)/f_\pi}}_{g' = e^{i\alpha^a \hat{T}^a} e^{i\beta^j T^j}} \times \Phi_0 = e^{i\Pi'(x)/f_\pi} \times \Phi_0$$

an element of G written in "standard form"

Thus, our Lagrangian is invariant when the pions transform according to $\Pi(x) \to \Pi'(x)$, where

$$e^{i\Pi'(x)/f_\pi} = g\,e^{i\Pi(x)/f_\pi} h(\Pi(x), g)^\dagger \qquad \text{and} \qquad h(\Pi(x), g) = e^{i\beta^j T^j} \in \mathcal{H}$$

[Note that when $g = h \in \mathcal{H}$, then we simply have $\Pi'(x) = h\,\Pi(x) h^\dagger$, i.e. $\mathcal{H}$ is *linearly* realized]

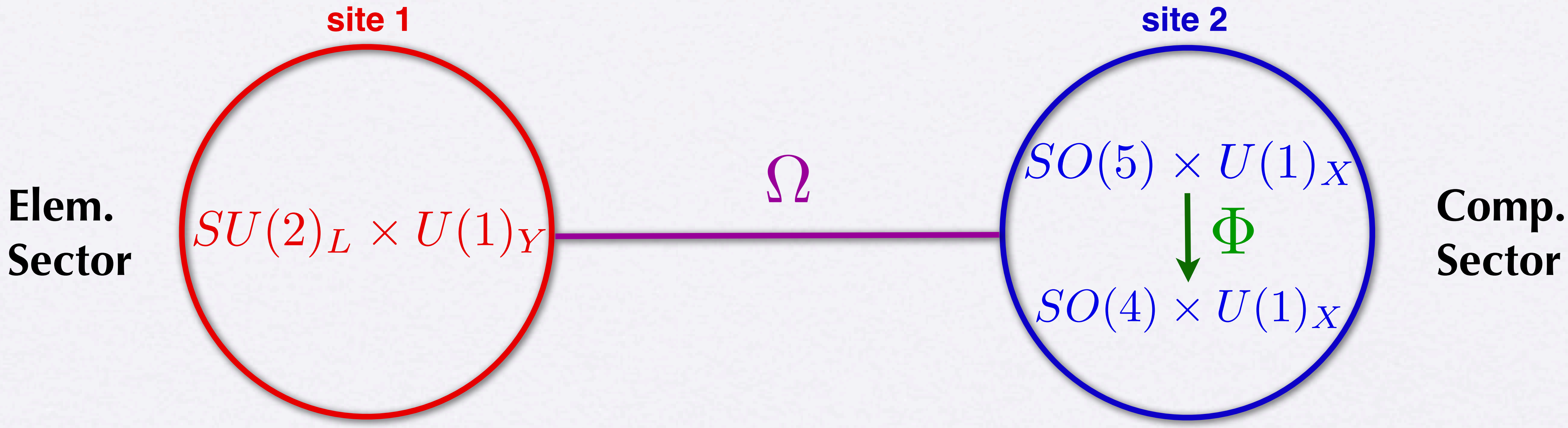
For the most part we do not need the explicit form of the general non-linear transformation. But we know that if we write the most general $\mathcal{H}$-invariant theory in terms of Φ, it will also be G-invariant!

Resonances

In order to describe the idea of an underlying strong dynamics, we can include some resonances. According to our “big picture”, these should fall into complete multiplets of SO(5). We will first see how to include spin-1 resonances (similar to the rho meson in QCD), and later we will comment on fermionic resonances.

Resonances

The construction we will be interested in is a “2-site model”:



Start with a theory symmetric under $G_L \times G_R \equiv [SO(5) \times U(1)_X]^{\mathrm{global}} \times [SO(5) \times U(1)_X]^{\mathrm{global}}$

- The purpose of the “link field” $\Omega(x) \to g_L \Omega(x) g_R^\dagger$ is to break $G_L \times G_R \to G_{L+R}$, i.e.

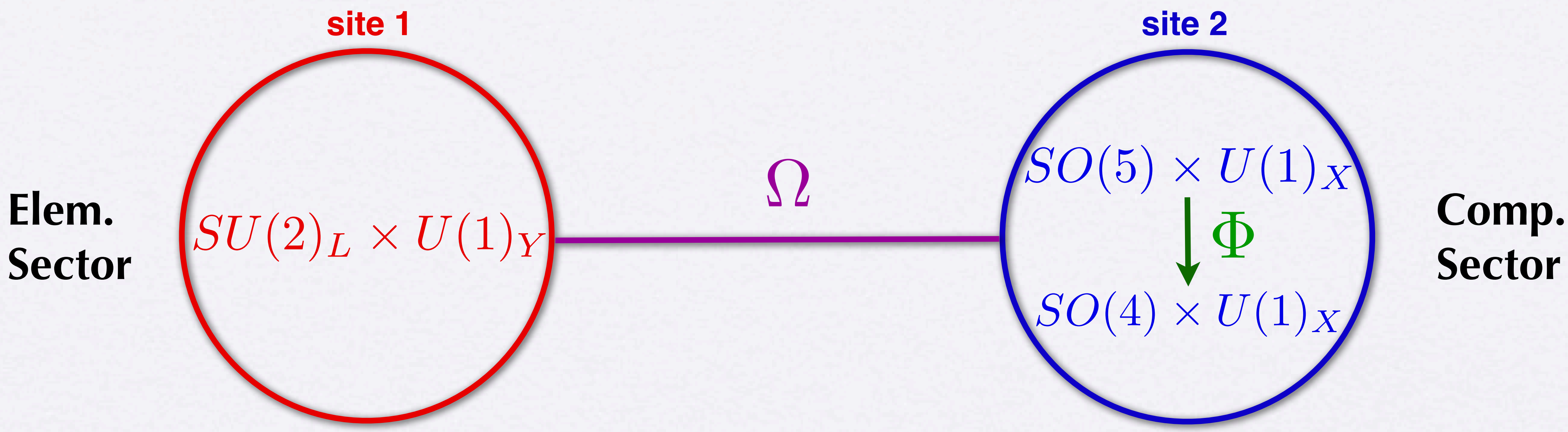
$$\Omega(x) = e^{i\tilde{\Pi}(x)^\alpha \hat{T}^\alpha / f_\Omega} \in G_L \times G_R / G_{L+R}$$

- The vector resonances are introduced by gauging the composite site: $G_L^{\mathrm{global}} \times G_R^{\mathrm{gauge}}$
  Since G_R is broken by the link field, the corresponding gauge fields (ρ_μ) eat the $\tilde{\Pi}$’s.
  We still have a theory with an exact global SO(5), namely $G_L^{\mathrm{global}} \times G_R^{\mathrm{gauge}} \to G_{L+R}^{\mathrm{global}}$

- On the elementary site we gauge $SU(2)_L \times U(1)_Y \subset G_L$: explicit breaking of global SO(5)!

Resonances

The construction we will be interested in is a “2-site model”:



$$\mathcal{L} = \mathcal{L}_{\rm SM}(\psi_L^{\rm el}, \tilde{\psi}_R^{\rm el}, A_\mu^{\rm el}) + \frac{1}{4} f_\Omega^2\, {\rm Tr}|D_\mu \Omega|^2 - \frac{1}{4g_\rho^2}(F_{\mu\nu}^{\rm cp})^2 + \frac{1}{2} f_\pi^2 |D_\mu \Phi|^2$$

$$D_\mu \Omega = \partial_\mu \Omega - i A_\mu^A T^A \Omega + i \Omega \rho_\mu^A T^A \qquad\qquad D_\mu \Phi = \partial_\mu \Phi - i \rho_\mu^A T^A \Phi$$

We can use the local G_R to choose a gauge where $\Omega(x) = 1$. In this (unitary) gauge:

$$\frac{1}{4} f_\Omega^2 {\rm Tr}|D_\mu \Omega|^2 = \frac{1}{4} f_\Omega^2 (A_\mu^B - \rho_\mu^B)^2 \quad \longrightarrow \quad m_\rho^2 = \tfrac{1}{2} g_\rho^2 f_\Omega^2$$

Furthermore, $\langle \Phi \rangle$ splits the vector (ρ_μ^j) from the axial ($a_\mu^{\hat{a}}$) resonances: $\begin{cases} m_\rho^2 = \frac{1}{2} g_\rho^2 f_\Omega^2 \\ m_a^2 = \frac{1}{2} g_\rho^2 (f_\Omega^2 + f_\pi^2) \end{cases}$

Partial Compositeness

Consider the spin-1 sector, in particular the generators of $SU(2)_L \times U(1)_Y$. For instance,

$$-\frac{1}{4} W^i_{L\,\mu\nu} W^{i\,\mu\nu}_L - \frac{1}{4} \rho^i_{L\,\mu\nu} \rho^{i\,\mu\nu}_L + \frac{1}{4} f^2_\Omega (g W^i_{L\,\mu} - g_\rho \rho^i_{L\,\mu})^2 \qquad i = 1, 2, 3$$

is diagonalized in the basis

$$\tilde{\rho}^i_{L\,\mu} = c_\theta \rho^i_{L\,\mu} - s_\theta W^i_{L\,\mu} \;, \qquad \tilde{W}^i_{L\,\mu} = c_\theta W^i_{L\,\mu} + s_\theta \rho^i_{L\,\mu} \qquad \text{where} \quad t_\theta = \frac{g}{g_\rho}$$

$$m^2_{\tilde{\rho}} = \tfrac{1}{2}(g^2_\rho + g^2) f^2_\Omega \qquad m^2_{\tilde{W}} = 0$$

Thus, the actual SM gauge bosons (recall we have not discussed EWSB yet) are an admixture of the original gauge bosons on site 1 and the composite gauge bosons on site 2. This is reminiscent of the photon-rho mixing phenomenon in QCD, and illustrates the idea of ***partial compositeness***

Partial compositeness in the fermion sector will provide an appealing alternative to the technicolor paradigm in regards to the generation of SM fermion masses:

$$|q^{\rm SM}\rangle = c_{\theta_q} |q^{\rm elem.}\rangle + s_{\theta_q} |Q^{\rm comp.}\rangle$$

Partial Compositeness

Go back to the UV theory and consider the spin-1 resonances again. The idea of partial compos. relies on the existence of spin-1 resonances in the strongly coupled sector, some of which have the quantum numbers of the SM gauge fields (so that they can mix at the quadratic level).

But this is actually guaranteed given our assumption about the global symmetries of the UV theory which, via Noether's theorem, leads to symmetry currents J_A^μ that create appropriate spin-1 states. Since $SU(2)_L \times U(1) \subset G$, some of these states will have the desired quantum numbers.

In the UV theory, the couplings to the external gauge fields are just the expected ones:

$$\mathcal{L} \supset A_\mu^A J_A^\mu$$

When it comes to fermion compositeness, we need to assume that the UV theory contains *fermionic operators* that create (some) states with SM quantum numbers. We can then write couplings to the elementary fermion fields of the form:

$$\mathcal{L} \supset \psi_L^{\mathrm{elem}} \mathcal{O}_R \qquad \text{or} \qquad \mathcal{L} \supset \psi_R^{\mathrm{elem}} \mathcal{O}_L$$

We will assume that this is the case, and focus on the effective theory describing the fermionic resonances produced by these fermionic operators, the same way we focused on the effective description of the ρ_μ^A .

Fermionic Resonances

Since the strongly coupled sector is G invariant, the fermionic resonances fill complete multiplets of the global symmetry. For instance, in the SO(5) case, we can have

$$\psi^{\mathrm{cp}} = 1, 4, 5, 10, 14, \ldots$$

Since the strong dynamics breaks SO(5) spontaneously to SO(4), the actual fermionic mass eigenstates fall into SO(4) representations. However, this is taken into account by use of the Φ field (after EWSB, they will further split, but by a smaller amount).

The effective Lagrangian for the fermionic resonances is

coupling to ρ_μ

couplings to Φ
(discuss later)

$$\mathcal{L}^{\mathrm{eff}}_{\mathrm{cp}} = \bar{\psi}^{\mathrm{cp}}(i\not{D}^{\mathrm{cp}} - M_{\mathrm{cp}})\psi^{\mathrm{cp}} + \mathcal{L}_{\mathrm{Yukawa}}$$

Fermionic Resonances

Since the strongly coupled sector is G invariant, the fermionic resonances fill complete multiplets of the global symmetry. For instance, in the SO(5) case, we can have

$$\psi^{\rm cp} = 1, 4, 5, 10, 14, \ldots$$

Since the strong dynamics breaks SO(5) spontaneously to SO(4), the actual fermionic mass eigenstates fall into SO(4) representations. However, this is taken into account by use of the Φ field (after EWSB, they will further split, but by a smaller amount).

The effective Lagrangian for the fermionic resonances is

coupling to ρ_μ

couplings to Φ (discuss later)

$$\mathcal{L}^{\rm eff}_{\rm cp} = \bar{\psi}^{\rm cp}(i\not{D}^{\rm cp} - M_{\rm cp})\psi^{\rm cp} + \mathcal{L}_{\rm Yukawa}$$

In addition, the mixing with the elementary fermions takes the form

$$\mathcal{L}_{\rm mix} = \Delta\, \bar{\psi}^{\rm el}_L\, \Omega\, \mathcal{P}_\psi \psi^{\rm cp}_R + \tilde{\Delta}\, \bar{\tilde{\psi}}^{\rm el}_R\, \Omega\, \mathcal{P}_{\tilde{\psi}} \tilde{\psi}^{\rm cp}_L$$

$\mathcal{P}_\psi$ projects the composite state components with the appropriate SM quantum numbers

$\Delta, \tilde{\Delta}$: explicit breaking of SO(5) $\longrightarrow$ small numbers from small mixings

$$\tan\theta_\psi = \Delta / M_{\rm cp}$$

The MCHM_5

MCHM = Minimal Composite Higgs Model

When all the strong fermionic resonances are in the fundamental of SO(5), the effective description of the fermionic sector, including the mixing with the elementary sector is:

$$\mathcal{L}_f = \sum_{\psi=q_L,u_R,d_R} \bar{\psi} i \not{D} \psi + \overbrace{\bar{q}_L \Delta_{q_u} Q_R^u + \bar{q}_L \Delta_{q_d} Q_R^d + \bar{u}_R \Delta_u U_L + \bar{d}_R \Delta_d D_L}^{\textbf{Mixing terms}} + \text{h.c.}$$

$$+ \sum_{\Psi=Q^u,Q^d,U,D} \bar{\Psi}(i\not{D} - M_\Psi)\Psi + m_{y_u} \bar{Q}_L^u U_R + m_{y_d} \bar{Q}_L^d D_R + \text{h.c.}$$

$$+ y_u (\bar{Q}_L^u \Phi)(\Phi^\dagger U_R) + y_d (\bar{Q}_L^d \Phi)(\Phi^\dagger D_R) + \text{h.c.} \quad \longleftarrow \quad \textbf{Yukawa terms}$$

In this model, the resonances that mix with the SU(2) doublets arise from two different fields, Q_L^u and Q_L^d . In other "non-minimal" models, a single Q can be sufficient.

Thus, the new parameters in the model are: $f_\pi, g_\rho, M_\rho \quad M_\Psi, m_{y_u}, m_{y_d}, y_u, y_d, \quad \Delta_{q_u}, \Delta_{q_d}$

EWSB

We have not discussed EWSB so far. What we have described is the low-energy limit (NGB's + a few massive resonances) of the (unspecified) UV theory that results from the *spontaneous* breaking of SO(5) to SO(4) by the underlying strong dynamics.

But we are also (weakly) coupling this theory to an external, "elementary", sector. The effects of these couplings should be a perturbation on top of the strong dynamics, but they can nevertheless be physically important. In particular, the vev of the field Φ can change its "SO(5) direction" slightly in the full theory compared to the limit where the couplings to the external fields are turned off.

If so, the initially unbroken group $SU(2)_L \times U(1)_Y \supset \mathcal{H}$ can get broken and the SM W and Z gauge bosons can get a non-zero mass. Thus, this is a question of "vacuum alignment":

- **Before EWSB:** $\Phi = \frac{1}{h}\sin(h/f_\pi)\,\{h_1, h_2, h_3, h_4, h\cot(h/f_\pi)\}^T \qquad h \equiv \sqrt{h_1^2 + h_2^2 + h_3^2 + h_4^2}$

- **After EWSB:** $\langle h_3 \rangle = v \qquad \langle \Phi \rangle = (0, 0, \epsilon, 0, \sqrt{1-\epsilon^2})^T$

New parameter: $\epsilon = \sin(v/f_\pi)$

The interesting point is that this is a calculable, dynamical outcome of the construction!

The Eff. Theory for the Sources

An elegant way of analyzing the theory is to integrate out the heavy states. To illustrate the procedure, let us focus on the spin-1 resonances. In momentum space, we have

$$\mathcal{L} \supset \underbrace{-\frac{1}{2g_\rho^2} p^2 \rho_\mu^B \rho_B^\mu - \frac{1}{2g_{X_2}^2} p^2 X_{2\,\mu} X_2^\mu}_{\text{Kinetic terms}} \underbrace{+ \frac{1}{4} f_\Omega^2 (A_\mu^B - \rho_\mu^B)^2 + \frac{1}{4} f_{\Omega_X}^2 (X_{1\,\mu} - X_{2\,\mu})^2}_{\text{link terms (unitary gauge)}} + \underbrace{\frac{1}{2} f_\pi^2 |D_\mu \Phi|^2}_{SO(5) \to SO(4)}$$

A very useful trick is to turn on all external sources (at the end we can leave those corresponding to the SM subgroup). Since we start from an SO(5) symmetric theory, and integrate out complete SO(5) multiplets, we must obtain a theory for the sources that is SO(5) invariant:

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{2} \Pi_A^{(0)} \mathrm{Tr}(A_\mu A^\mu) + \frac{1}{2} \Pi_A^{(2)} \Phi^T A_\mu A^\mu \Phi + \frac{1}{2} \Pi_X^{(0)} X_{1\,\mu} X_1^\mu$$

We can find the correlators by analyzing the limit with $\Phi = \Phi_0 = \{0, 0, 0, 0, 1\}^T$:

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{2} \Pi_A^{(0)} A_\mu^j A^{j\,\mu} + \frac{1}{2} \left(\Pi_A^{(0)} + \frac{1}{2} \Pi_A^{(2)} \right) A_\mu^{\hat{a}} A^{\hat{a}\,\mu} + \frac{1}{2} \Pi_X^{(0)} X_{1\,\mu} X_1^\mu$$

The EOM for the heavy resonances (in this limit) in the full theory are simply:

$$\rho_\mu^j = -\frac{m_\rho^2}{p^2 - m_\rho^2} A_\mu^j \qquad a_\mu^{\hat{a}} = -\frac{m_\rho^2}{p^2 - m_a^2} A_\mu^{\hat{a}} \qquad X_{2\,\mu} = -\frac{m_{X_2}^2}{p^2 - m_{X_2}^2} X_{1\,\mu}$$

The Eff. Theory for the Sources

We can find the correlators by analyzing the limit with $\Phi = \Phi_0 = \{0,0,0,0,1\}^T$:

$$\mathcal{L}_{\rm eff} \supset \frac{1}{2}\Pi_A^{(0)} A_\mu^j A^{j\,\mu} + \frac{1}{2}\left(\Pi_A^{(0)} + \frac{1}{2}\Pi_A^{(2)}\right) A_\mu^{\hat a} A^{\hat a\,\mu} + \frac{1}{2}\Pi_X^{(0)} X_{1\,\mu} X_1^\mu$$

The EOM for the heavy resonances (in this limit) in the full theory are simply:

$$\rho_\mu^j = -\frac{m_\rho^2}{p^2 - m_\rho^2} A_\mu^j \qquad a_\mu^{\hat a} = -\frac{m_\rho^2}{p^2 - m_a^2} A_\mu^{\hat a} \qquad X_{2\,\mu} = -\frac{m_{X_2}^2}{p^2 - m_{X_2}^2} X_{1\,\mu}$$

* $m_\rho^2 = \frac{1}{2} g_\rho^2 f_\Omega^2 \qquad m_a^2 = \frac{1}{2} g_\rho^2 (f_\Omega^2 + f_\pi^2) \qquad m_{X_2}^2 = \frac{1}{2} g_{X_2}^2 f_{\Omega_X}^2$

The Eff. Theory for the Sources

We can find the correlators by analyzing the limit with $\Phi = \Phi_0 = \{0,0,0,0,1\}^T$:

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{2}\Pi_A^{(0)} A_\mu^j A^{j\,\mu} + \frac{1}{2}\left(\Pi_A^{(0)} + \frac{1}{2}\Pi_A^{(2)}\right) A_\mu^{\hat{a}} A^{\hat{a}\,\mu} + \frac{1}{2}\Pi_X^{(0)} X_{1\,\mu} X_1^\mu$$

The EOM for the heavy resonances (in this limit) in the full theory are simply:

$$\rho_\mu^j = -\frac{m_\rho^2}{p^2 - m_\rho^2} A_\mu^j \qquad a_\mu^{\hat{a}} = -\frac{m_\rho^2}{p^2 - m_a^2} A_\mu^{\hat{a}} \qquad X_{2\,\mu} = -\frac{m_{X_2}^2}{p^2 - m_{X_2}^2} X_{1\,\mu}$$

Replacing back in the full action and comparing to the expected form of the eff. theory, we get

$$\Pi_A^{(0)} = \hat{\Pi}_A \qquad \Pi_A^{(2)} = 2(\hat{\Pi}_{\hat{A}} - \hat{\Pi}_A) \qquad \Pi_X^{(0)} = \hat{\Pi}_X$$

where

$$\hat{\Pi}_A = \frac{p^2 m_\rho^2}{g_\rho^2\left(p^2 - m_\rho^2\right)} \qquad \hat{\Pi}_{\hat{A}} = \frac{m_\rho^2\left(p^2 + m_\rho^2 - m_a^2\right)}{g_\rho^2\left(p^2 - m_a^2\right)} \qquad \hat{\Pi}_X = \frac{p^2 m_X^2}{g_{X_2}^2\left(p^2 - m_X^2\right)}$$

* $m_\rho^2 = \frac{1}{2} g_\rho^2 f_\Omega^2 \qquad m_a^2 = \frac{1}{2} g_\rho^2 (f_\Omega^2 + f_\pi^2) \qquad m_{X_2}^2 = \frac{1}{2} g_{X_2}^2 f_{\Omega_X}^2$

The Eff. Theory for the Sources

We can find the correlators by analyzing the limit with $\Phi = \Phi_0 = \{0,0,0,0,1\}^T$:

$$\mathcal{L}_{\rm eff} \supset \frac{1}{2}\Pi_A^{(0)} A_\mu^j A^{j\,\mu} + \frac{1}{2}\left(\Pi_A^{(0)} + \frac{1}{2}\Pi_A^{(2)}\right) A_\mu^{\hat a} A^{\hat a\,\mu} + \frac{1}{2}\Pi_X^{(0)} X_{1\,\mu} X_1^\mu$$

The EOM for the heavy resonances (in this limit) in the full theory are simply:

$$\rho_\mu^j = -\frac{m_\rho^2}{p^2 - m_\rho^2} A_\mu^j \qquad a_\mu^{\hat a} = -\frac{m_\rho^2}{p^2 - m_a^2} A_\mu^{\hat a} \qquad X_{2\,\mu} = -\frac{m_{X_2}^2}{p^2 - m_{X_2}^2} X_{1\,\mu}$$

Replacing back in the full action and comparing to the expected form of the eff. theory, we get

$$\Pi_A^{(0)} = \hat\Pi_A \qquad \Pi_A^{(2)} = 2(\hat\Pi_{\hat A} - \hat\Pi_A) \qquad \Pi_X^{(0)} = \hat\Pi_X$$

where

$$\hat\Pi_A = \frac{p^2 m_\rho^2}{g_\rho^2\left(p^2 - m_\rho^2\right)} \qquad \hat\Pi_{\hat A} = \frac{m_\rho^2\left(p^2 + m_\rho^2 - m_a^2\right)}{g_\rho^2\left(p^2 - m_a^2\right)} \qquad \hat\Pi_X = \frac{p^2 m_X^2}{g_{X_2}^2\left(p^2 - m_X^2\right)}$$

We can now turn on the Higgs vev in our effective theory, and keep only the SM sources: **

$$\Pi_{W_L^i} = \Pi_A^{(0)} + \frac{1}{4}\Pi_A^{(2)}\sin^2(h/f_\pi) \qquad \Pi_B = \Pi_X^{(0)} + \frac{g_0'^2}{g_0^2}\left(\Pi_A^{(0)} - \Pi_X^{(0)} + \frac{1}{4}\Pi_A^{(2)}\sin^2(h/f_\pi)\right)$$

* $m_\rho^2 = \frac{1}{2}g_\rho^2 f_\Omega^2 \qquad m_a^2 = \frac{1}{2}g_\rho^2(f_\Omega^2 + f_\pi^2) \qquad m_{X_2}^2 = \frac{1}{2}g_{X_2}^2 f_{\Omega_X}^2$

** $W_{R\,\mu}^3 = \frac{g_0'}{g_0}B_\mu \qquad X_{1\,\mu} = \frac{\sqrt{g_0^2 - g_0'^2}}{g_0}B_\mu$

The Eff. Theory for the Sources

The fermions are treated in an analogous manner:

a) Keep all the fermion sources and write the most general SO(5) invariant action, allowing for Φ

b) Integrate out the heavy fermionic states in the full theory and compare to the effective theory, both in the limit $\Phi = \Phi_0$. Thus, identify the correlators, which can be classified according to the SO(4) representations contained in the original SO(5) ones:

$$\mathcal{L}_{\text{eff}} = \sum_{\psi=q_L,u_R,d_R} \sum_{r_4} \bar{\psi}^{(r_4)} \not{p} (1 + \hat{\Pi}_{\psi}^{(r_4)}) \psi^{(r_4)} + \sum_{\psi=u,d} \sum_{r_4} \bar{q}_L^{(r_4)} \hat{M}_{\psi^{(r_4)}} \psi_R^{(r_4)} + \text{h.c.}$$

c) Now turn on an arbitrary Higgs configuration, and turn on the SM fermions sources to read off the corresponding correlators (in terms of the SO(4) ones)

The Eff. Theory for the Sources

For the MCHM5, this procedure results in

$$\mathcal{L}_{\rm eff} = \bar{u}_L \not{p}(1+\Pi_{u_L})u_L + \bar{d}_L \not{p}(1+\Pi_{d_L})d_L + \bar{u}_R \not{p}(1+\Pi_{u_R})u_R + \bar{d}_R \not{p}(1+\Pi_{d_R})d_R$$
$$+ \bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \text{h.c.}$$

where

$$\Pi_{u_L} = \Pi^0_{q^u} + \Pi^0_{q^d} + \Pi^1_{q^u}\frac{s_h^2}{2}\ , \qquad \Pi_{d_L} = \Pi^0_{q^u} + \Pi^0_{q^d} + \Pi^1_{q^d}\frac{s_h^2}{2}\ ,$$
$$\Pi_{u_R} = \Pi^0_u + \Pi^1_u c_h^2\ , \qquad \Pi_{d_R} = \Pi^0_d + \Pi^1_d c_h^2\ ,$$
$$M_u = m^1_u \frac{s_h c_h}{\sqrt{2}}\ , \qquad M_d = m^1_d \frac{s_h c_h}{\sqrt{2}}$$

with $s_h = \sin(h/f_\pi)$, $c_h = \cos(h/f_\pi)$ and

$$\Pi^0_{q^u} = \hat{\Pi}_{q^{u(4)}}\ , \qquad \Pi^0_{q^d} = \hat{\Pi}_{q^{d(4)}}\ , \qquad \Pi^0_u = \hat{\Pi}_{u^{(4)}}\ , \qquad \Pi^0_d = \hat{\Pi}_{d^{(4)}}\ ,$$
$$\Pi^1_{q^u} = \hat{\Pi}_{q^{u(1)}} - \hat{\Pi}_{q^{u(4)}}\ , \qquad \Pi^1_{q^d} = \hat{\Pi}_{q^{d(1)}} - \hat{\Pi}_{q^{d(4)}}\ , \qquad \Pi^1_u = \hat{\Pi}_{u^{(1)}} - \hat{\Pi}_{u^{(4)}}\ , \qquad \Pi^1_d = \hat{\Pi}_{d^{(1)}} - \hat{\Pi}_{d^{(4)}}\ ,$$
$$m^0_u = \hat{M}_{u^{(4)}}\ , \qquad m^0_d = \hat{M}_{d^{(4)}}\ ,$$
$$m^1_u = \hat{M}_{u^{(1)}} - \hat{M}_{u^{(4)}}\ , \qquad m^1_d = \hat{M}_{d^{(1)}} - \hat{M}_{d^{(4)}}$$

The Higgs Potential

We are finally ready to address the question of EWSB. So far we allowed ourselves to turn on an arbitrary Higgs background. But which one is preferred by the theory?

At tree level there is no potential for *h* due to its NGB nature. The explicit breaking of the global symmetry arises from the couplings to the external sources, and their effects enter first at 1-loop order. This is just the Coleman-Weinberg potential, which can be written in terms of the correlators of the effective theory for the sources as

$$V(h) = \int \frac{d^4p}{(2\pi)^4} \left[\frac{9}{2} \log \Pi_W(h) - 2N_c \sum_\psi \log[p^2 \Pi_{\psi_L}(h) \Pi_{\psi_R}(h) - \Pi^2_{\psi_L \psi_R}(h)] \right]$$

Note that we were careful to work out the correlators to all orders in momentum, so we have all the information needed to compute the above integral. However, we must include the “bare” kinetic terms for the sources (which make them propagating degrees of freedom)

The *h*-dependent pieces of this integral are actually finite. For instance,

$$\frac{\Pi_W}{-\frac{p^2}{g_0^2} + \Pi_A^{(0)}} = 1 + \frac{1}{4} \frac{\Pi_A^{(2)}}{-\frac{p^2}{g_0^2} + \Pi_A^{(0)}} \sin^2(h/f_\pi) \quad \text{and} \quad \frac{\Pi_A^{(2)}}{-\frac{p^2}{g_0^2} + \Pi_A^{(0)}} = \frac{2g_0^4 m_\rho^4 \left(m_\rho^2 - m_a^2\right)}{p^2 \left(p^2 - m_a^2\right) \left[g_\rho^2 \left(p^2 - m_\rho^2\right) - g_0^4 m_\rho^2\right]}$$

The finiteness follows from the absence of possible counterterms.

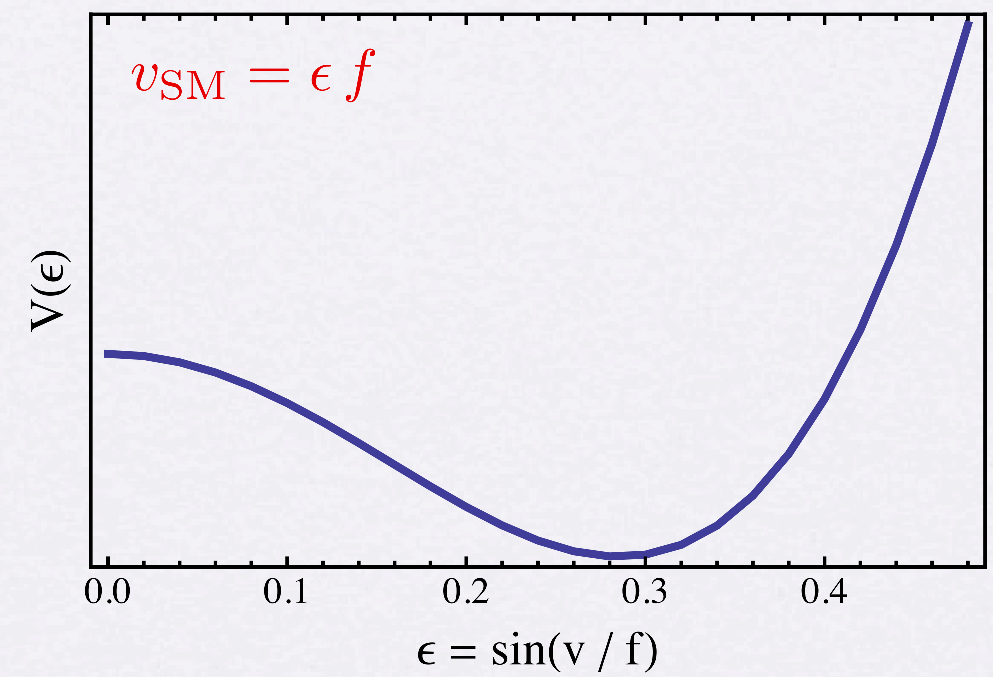
Summary

We have reviewed how to implement the pNGB idea for EWSB. By construction, this leads to a light Higgs boson, which in some region of parameter space exhibits SM-like properties.

However, it is accompanied by a set of resonances that mix with the SM ("external") degrees of freedom, thereby generating dynamically a Higgs potential:

In the limit $g, g' \to 0$ & $\Delta, \tilde{\Delta} \to 0$ the Higgs is an exact NGB. Thus the Higgs potential is generated by the gauge and "Yukawa" interactions!

- **Gauge interactions: prefer ``vacuum alignment" (no EWSB)**
- **Yukawa interactions (dominated by top): can induce EWSB**



One can then analyze models such as the 2-site model to investigate how one can get 125 GeV, and what type of deviations from the SM can be expected... enjoy your journey!

Further Reading

A nice set of lectures on composite Higgs is

“The Higgs as a Composite Nambu Goldstone Boson”, Roberto Contino, arXiv:1005.4269

For the connection to Extra Dimensions (which I did not have time to cover), see e.g.

“TASI 2004 lectures: To the fifth dimension and back”, Raman Sundrum, hep-th/0508134

“TASI 2011: Four Lectures on TeV Scale Extra Dimensions”, (lecture 4)
Eduardo Pontón, arXiv:1207.3827

The original MCHM5 model was proposed in

“The Minimal Composite Higgs Model”, K. Agashe, R. Contino and A. Pomarol, hep-ph/0412089