Evaluation of stability in Charge Sensitive Amplifiers (CSA)

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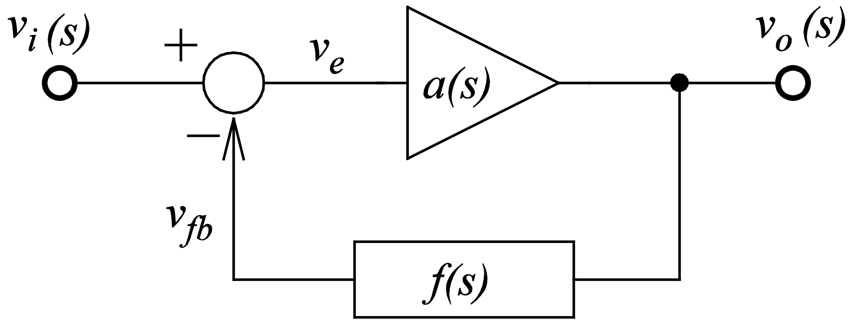
Negative feedback - why?

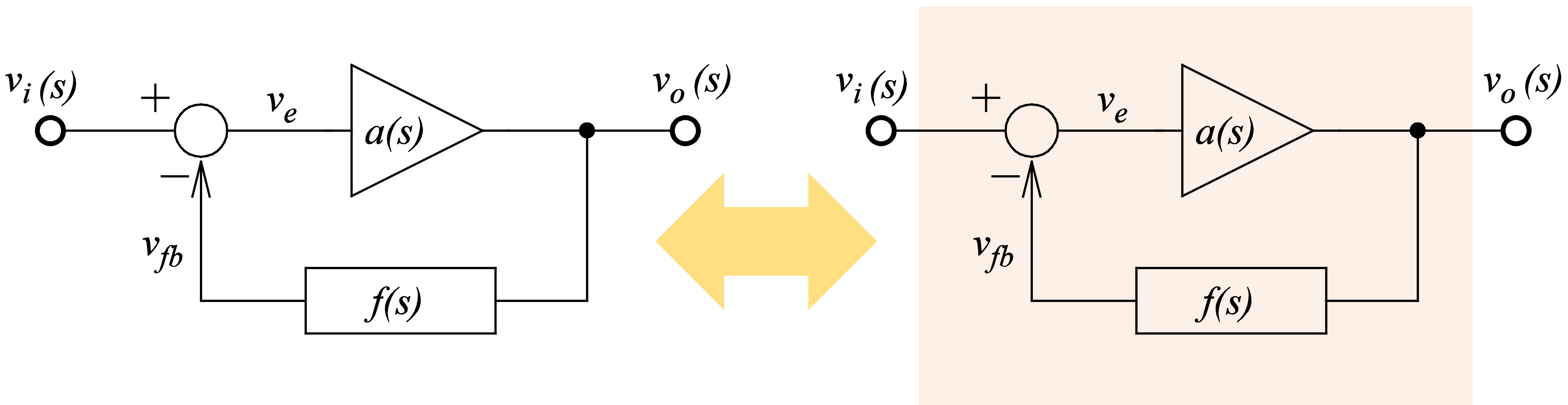


$$T(j\omega) = \frac{v_o(s)}{v_i(s)} = \frac{a(j\omega)}{1 + a(j\omega)f(j\omega)} \approx \frac{1}{f(j\omega)} \quad if \quad a(j\omega) \gg 1$$

Using negative feedback provides:

- ❑ Stabilisation of gain (process variation, temperature)
- ❑ Improvement of PSRR and distortion
- ❑ Bandwidth-gain trading
- ❑ **Negative feedback can be used to change the input or output impedances (allows for low input impedance CSA)**

Feedback amplifier - descriptions



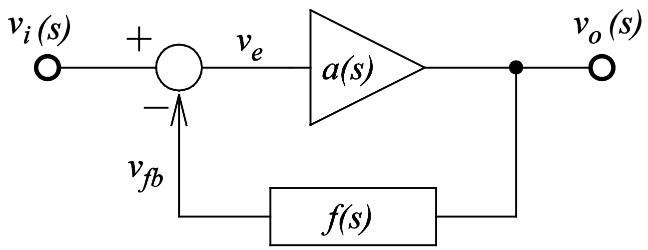
$$\mathrm{T}(j\omega) = \frac{v_o(s)}{v_i(s)} = \frac{a(j\omega)}{1 + a(j\omega)f(j\omega)}$$

Feedback amplifier described by amplifier and feedback network functions in frequency domain

$$\mathrm{T(s)} = \frac{v_o(s)}{v_i(s)} = H\,\frac{b_0\,s^n + b_1\,s^{n-1} + ..\,b_{n-1}\,s + b_n}{c_0\,s^n + c_1\,s^{n-1} + ..\,c_{n-1}\,s + c_n}$$

Network function in Laplace domain

Stability criterion for open loop analysis



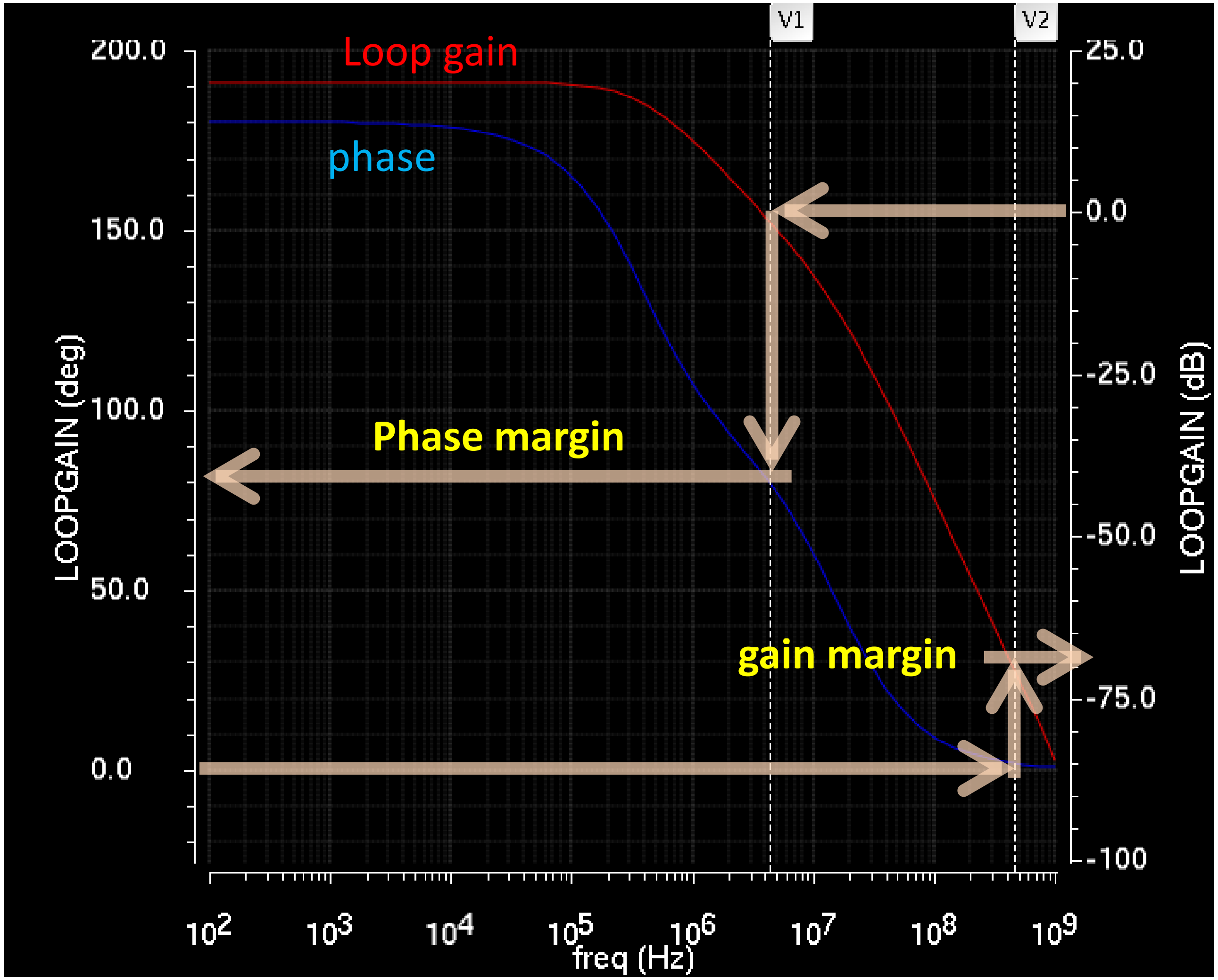
$$T(j\omega) = \frac{v_o(s)}{v_i(s)} = \frac{a(j\omega)}{1 + a(j\omega)f(j\omega)}$$

Assumption that a($j\omega$)>>1 valid for frequencies below bandwidth limitation, at high frequency when loop gain approaches 1 we have to check the phase to prevent denominator becomes 0!

Barkhausen stability criterion → $a(j\omega)f(j\omega) \neq -1$

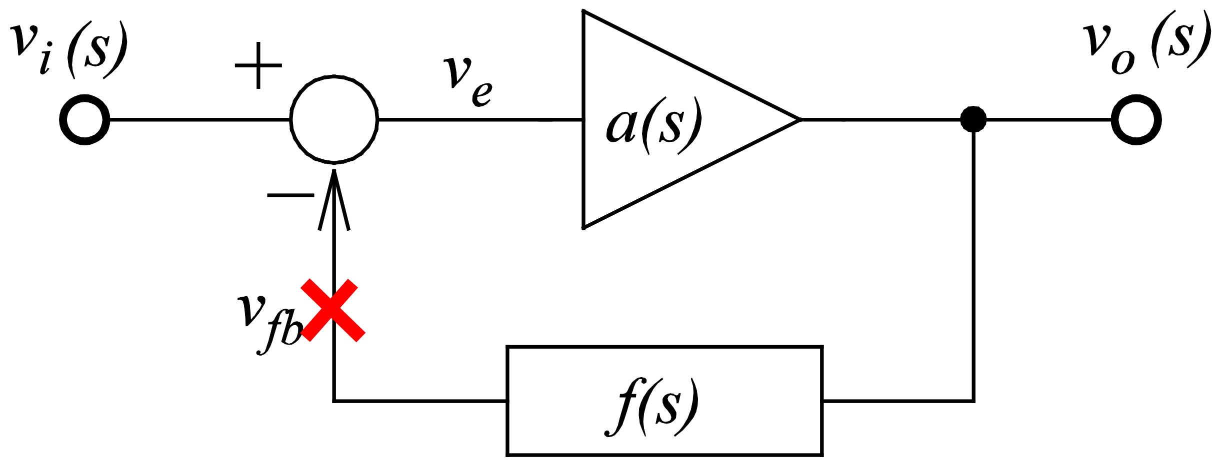
Check amplitude and phase of the loop gain ($a \cdot f$)

Stability measure for open loop analysis: phase and gain margins



Loop gain amplitude and phase of two pole system

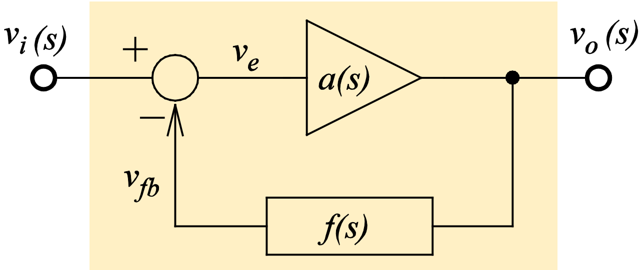
CAD simulation for open loop analysis



Test loop gain amplitude and phase:

- AC simulation of open loop circuit (sensing signal at the output of the open loop)
- STB simulation of closed loop circuit in Spectre (method based on Middlebrook double injection method)

Stability criteria for close loop analysis



After factorization

Zeroes (reals or conjugated complex pairs)

$$T(s) = \frac{v_o(s)}{v_i(s)} = H\frac{b_0\, s^n + b_1\, s^{n-1} + ..\, b_{n-1}\, s + b_n}{c_0\, s^n + c_1\, s^{n-1} + ..\, c_{n-1}\, s + c_n} = K\frac{(s-z_n)(s-z_{n-1})\dots(s-z_2)(s-z_1)}{(s-p_n)(s-p_{n-1})\dots(s-p_2)(s-p_1)}$$

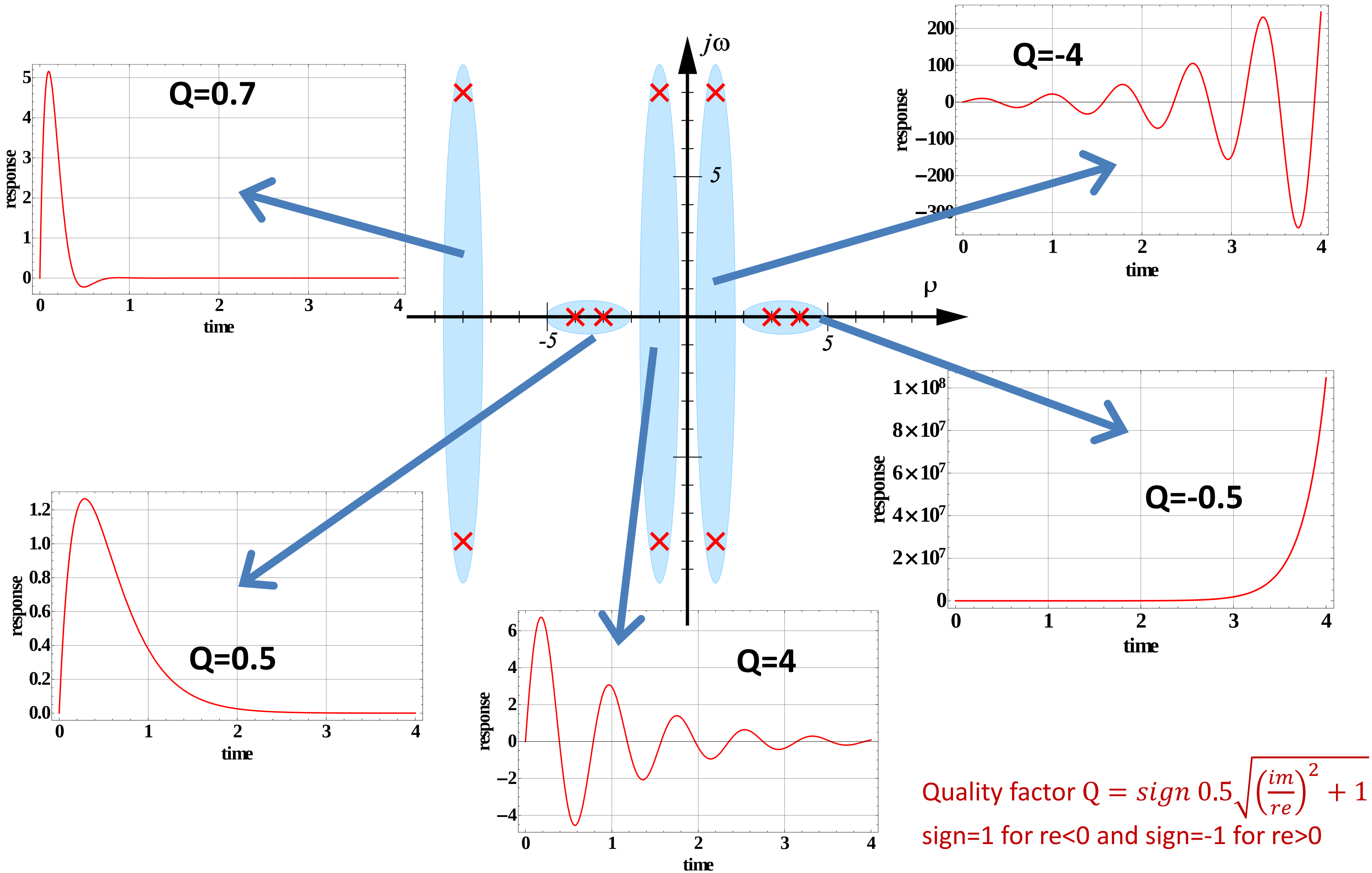
Poles (reals or conjugated complex pairs)

Time response bounded (stable system) → all poles with negative real parts (all poles on the left half s-plane)

Checking of pole locations for polynomial which are not factorized (rough results of analytical calculations):

- ❑ Routh-Hurwitz criterion (checking coefficients of Ruth array – tedious algorithm for simplified circuits only)
- ❑ Run PZ (pole zero) analysis in Spectre or HSpice for full or simplified circuit

Time responses of two-pole system, s-plane



Quality factor $Q = sign\ 0.5\sqrt{\left(\frac{im}{re}\right)^2 + 1}$

sign=1 for re<0 and sign=-1 for re>0

Close loop analysis

Analysis of the location of poles:

- ❑ Analytical analysis of the simplified circuits
- ❑ PZ analysis in HSpice or Spectre (list of poles and zeroes in the circuit)

Analytical analysis :

- ❑ Simplifying the circuit
- ❑ Finding network function (building and solving equations)
- ❑ Factorization of denominator (poles) and numerator (zeroes) of network function based on the assumption that all poles and zeroes are well separated (and are on the left half plane – circuit is stable)

$$\left(1+\frac{s}{P1}\right)\left(1+\frac{s}{P2}\right)\left(1+\frac{s}{P3}\right) \approx 1+\frac{s}{P1}+\frac{s^2}{P1\,P2}+\frac{s^3}{P1\,P2\,P3} \quad \text{if} \quad \text{P1} \ll \text{P2} \ll \text{P3}$$

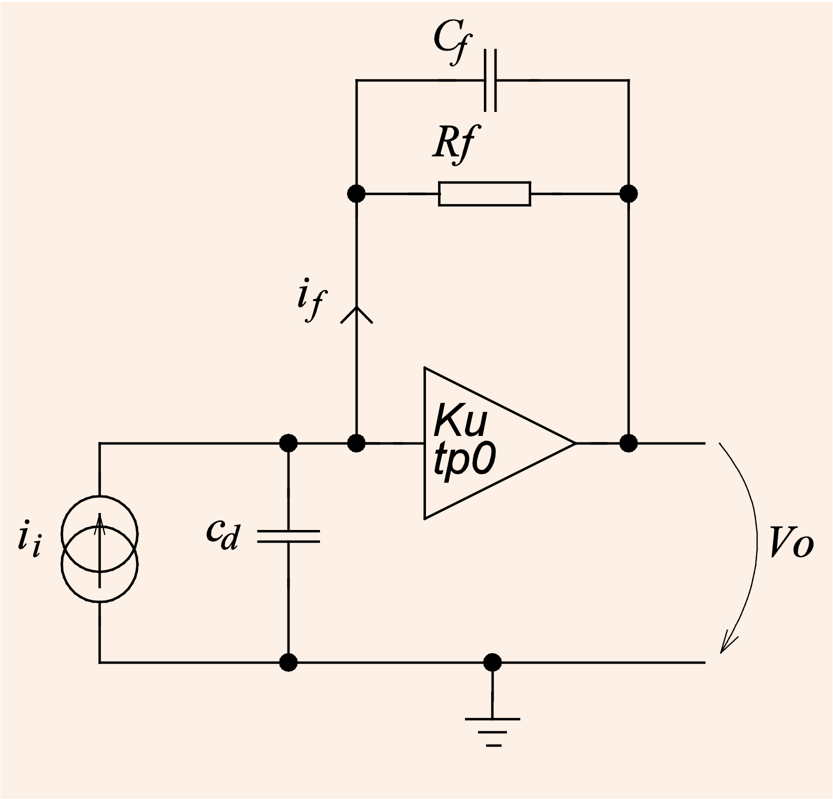
Open loop analysis versus close loop analysis (phase margin PM versus pole quality Q)

Q=0.5 (two real poles/asymptotic response) → **phase margin from ~83° and above** (*)
Q=0.7 (pair of complex poles, response with~5% undershoot) → **phase margin ~70°**

(*) The conversion between pole quality and phase margin is really ambiguous (one can find system with PM=85° and Q=0.55).

- ❑ Phase margin depends on position of poles as well as type of feedback.
- ❑ Time response depends on location of zeroes and poles not on phase margin

Ideal CSA (single pole amplifier, ideal output buffer) – closed loop analysis – analytical approach



$$T(s) = \frac{Vo}{ii} = \frac{Kus\ Zf\ Zi}{Zf + Zi + Kus\ Zi}$$

$$P1 \cong \frac{1}{Rf\ Cf} \qquad P2 \cong \frac{Ku}{\frac{cd}{Cf}\ tp0}$$

GBP

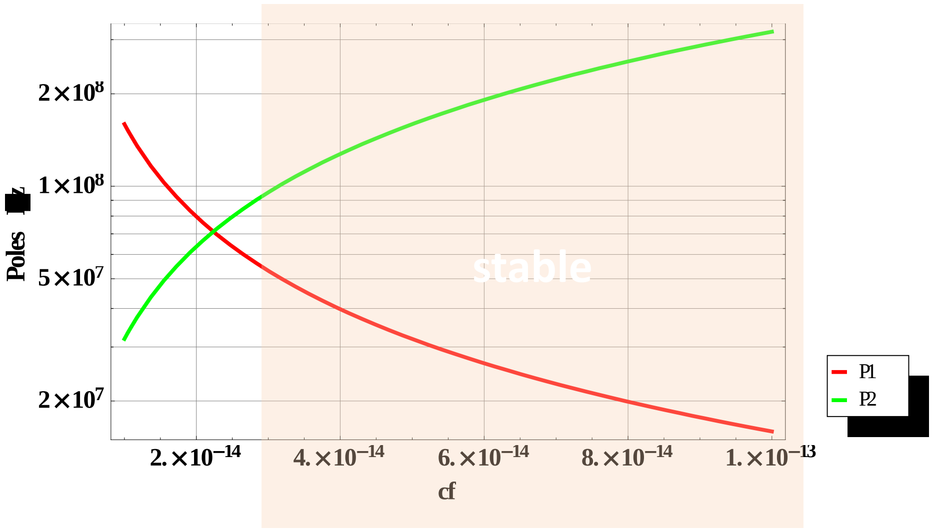
Stable circuit → finding parameters of the feedback for which P1<<P2 (real and well separated poles)

$$Zi = \frac{1}{s\ cd} \qquad Kus = \frac{Ku}{1 + s\ tp0} \qquad Zf = \frac{Rf}{1 + s\ tf} \qquad tf = Rf\ Cf$$

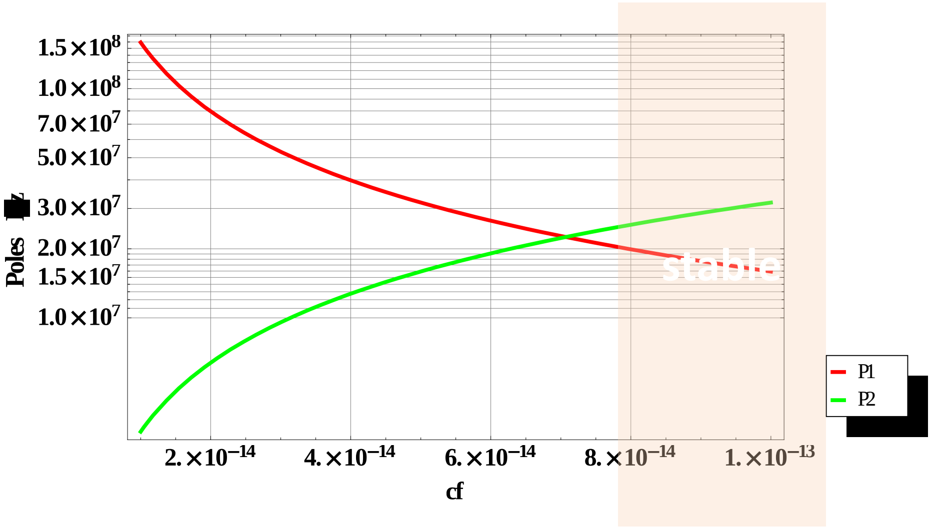
Ideal CSA (2 pole system)

For good phase margin P1<<P2 (P1 and P2 real)

$$P1 \cong \frac{1}{Rf\ Cf} \qquad P2 \cong \frac{GBP}{\frac{cd}{Cf}}$$



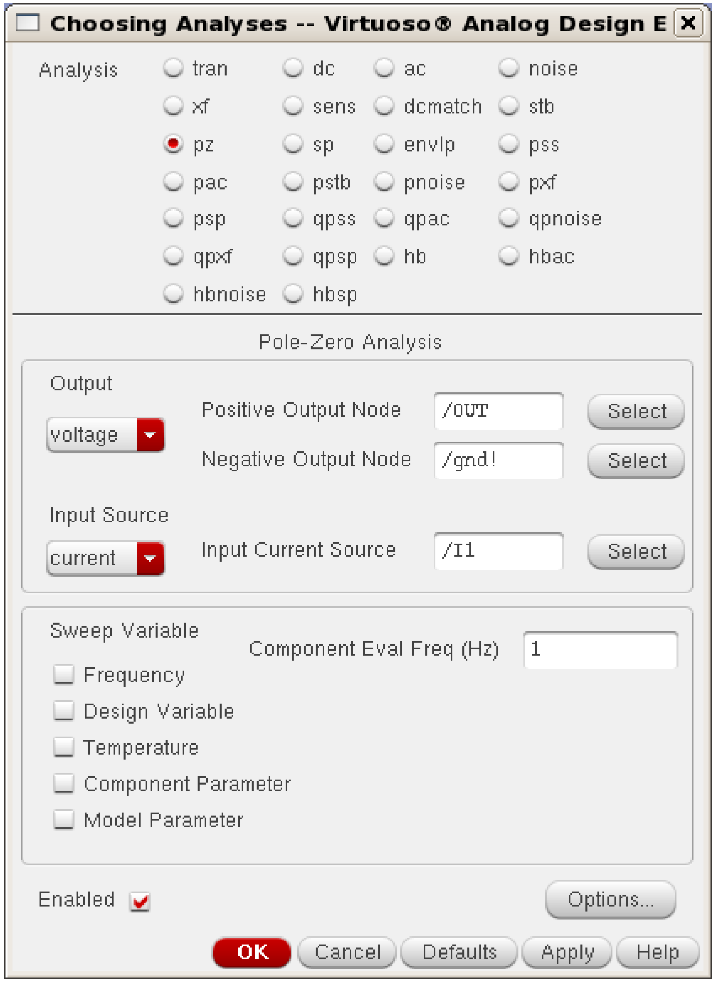
Cd=1pF

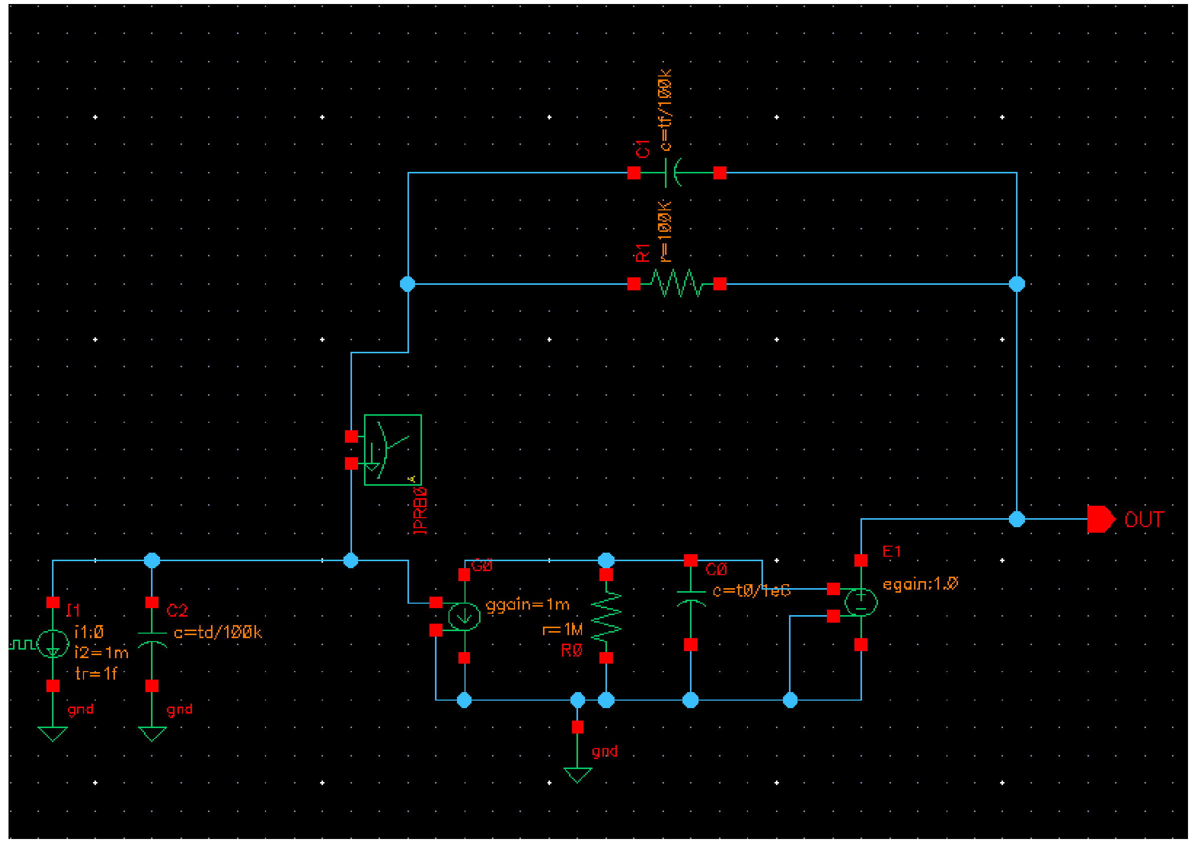


Cd=10pF

Rf=100k, tp0=50ns, Ku=60dB (GBP ~2GHz)

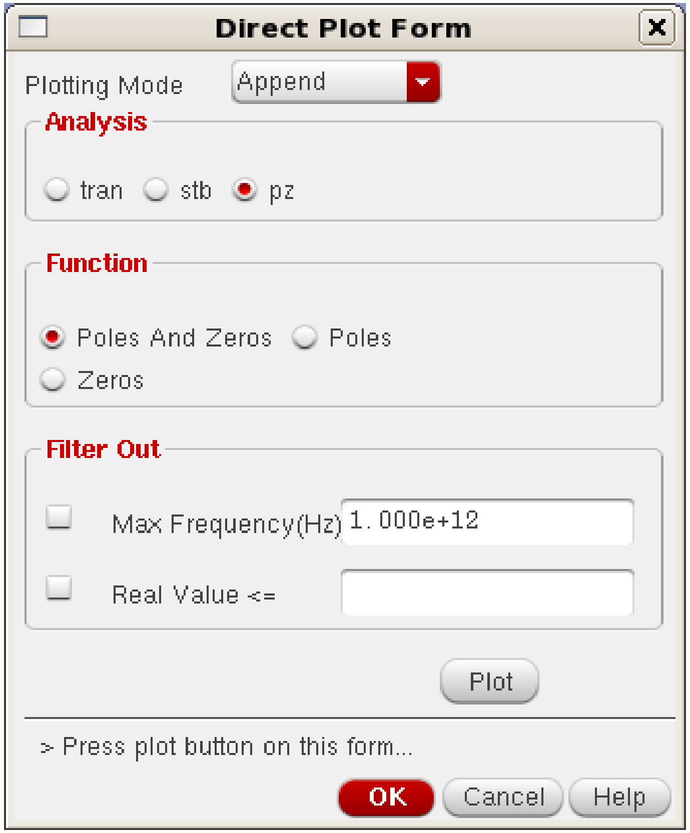
Spectre PZ analysis of ideal CSA

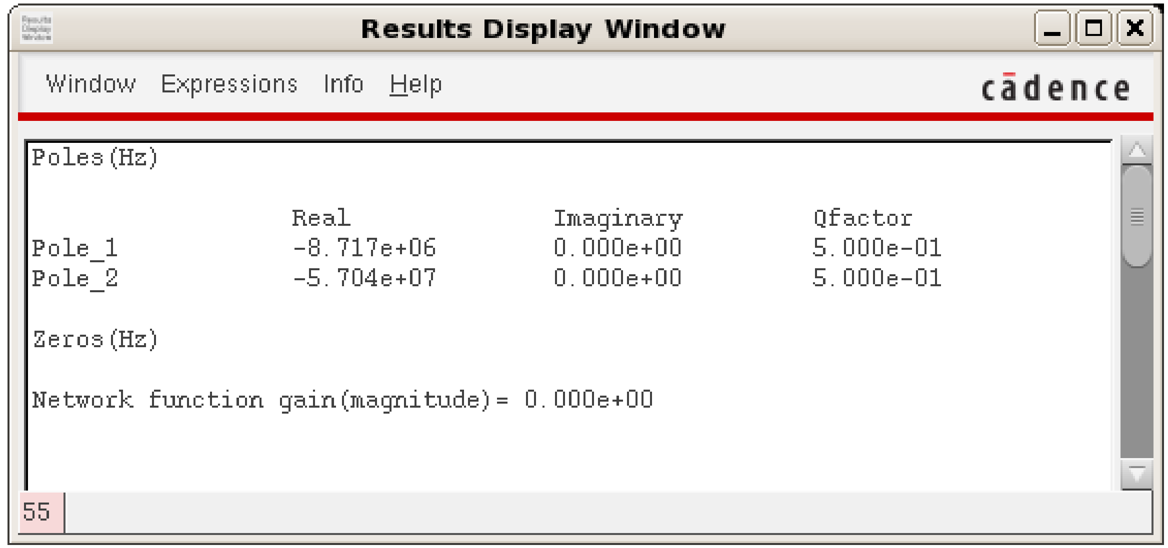




- Circuit in close loop configuration
- Definition of input and output nodes

Spectre PZ analysis of ideal CSA

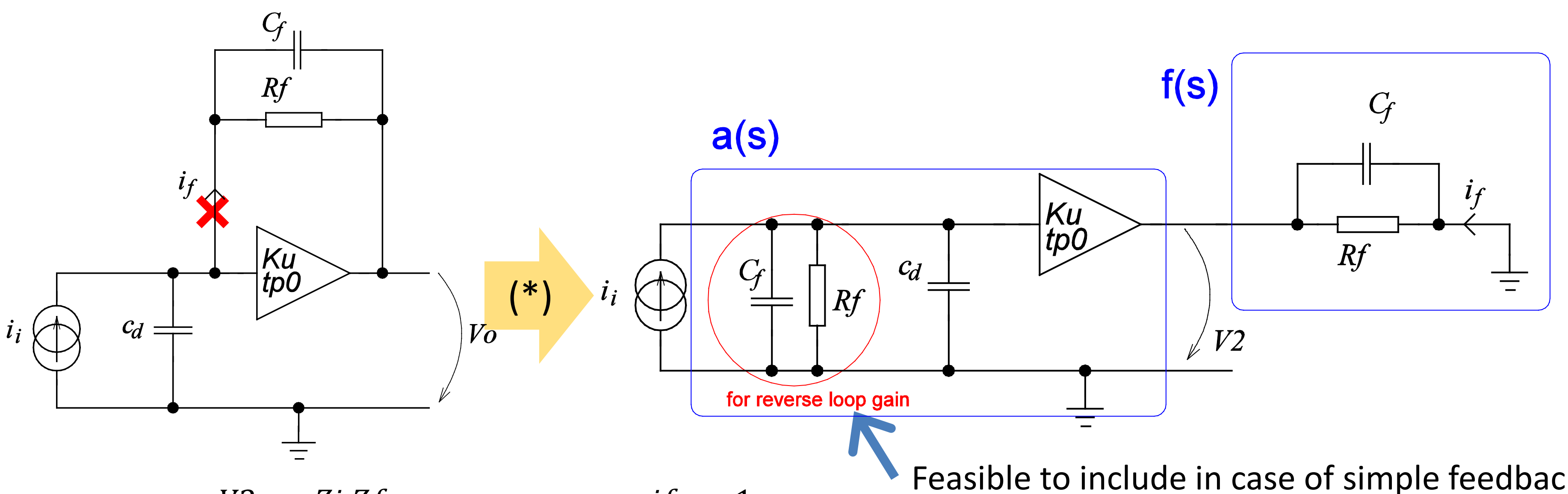




Access to results through direct plot form or print summary

Ideal CSA with Rf=100k, tp0=50ns, Ku=60dB (GBP ~2GHz), tf=20ns, cd=10p, PM=86°

CSA; shunt-shunt feedback type. Open loop analysis.



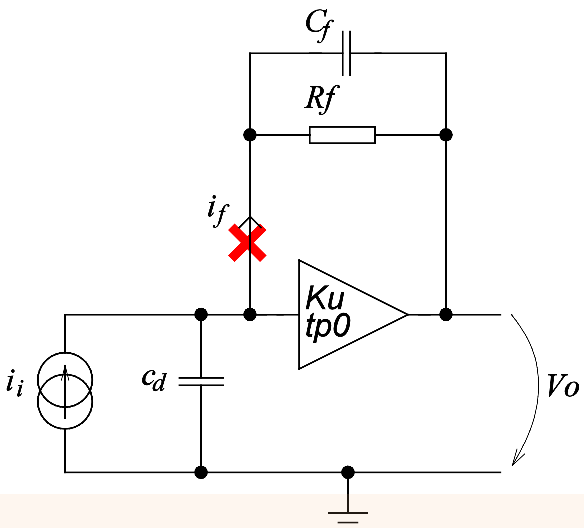
$$a(s) = \frac{V2}{ii} = \frac{Zi\,Zf}{Zi + Zf}\,Kus \qquad f(s) = \frac{if}{V2} = \frac{1}{Zf}$$

$$A_{closed\ loop}(s) = \frac{a(s)}{1 + a(s)f(s)} = T(s) = \frac{Kus\,Zf\,Zi}{Zf + Zi + Kus\,Zi}$$

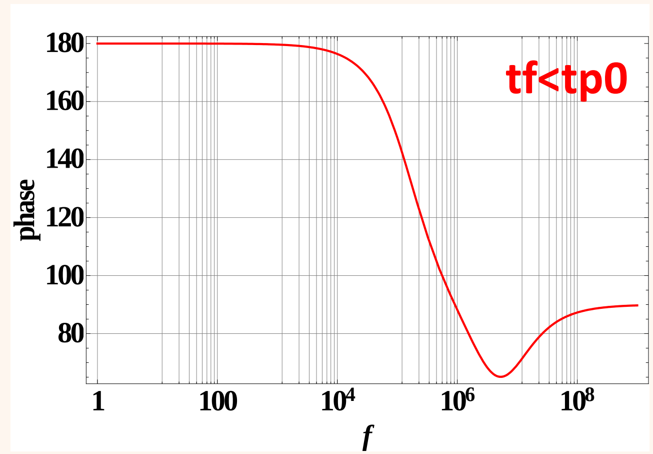
Loop gain: ratio of i_f/i_i → amplitude and phase of i_f (for i_i=1)

(*) Grey, Meyer, Analysis and Design of Analog Integrated Circuits, chapter 8.5.1

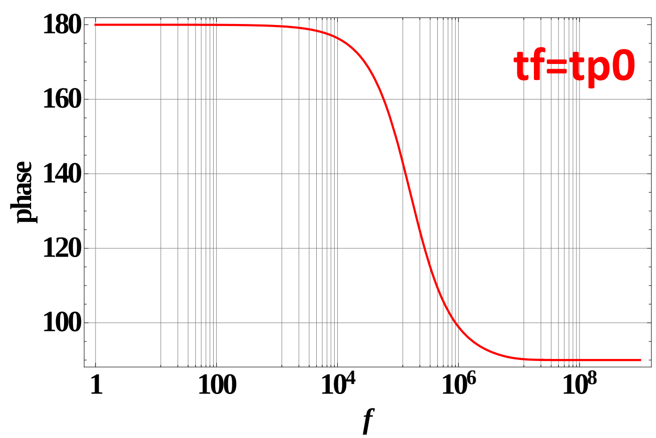
Open loop analysis of ideal CSA – examples of loop gain phase behaviour

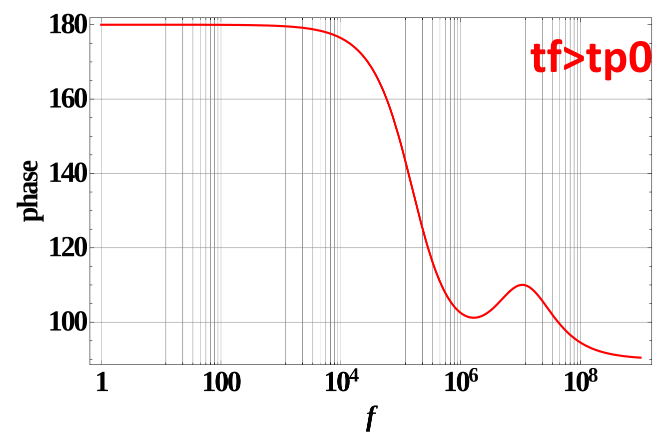


$$Loop\ Gain = Ku \frac{1 + s\ tf}{\big(1 + s\ (tf + Rf\ cd)\big)(1 + s\ tp0)}$$

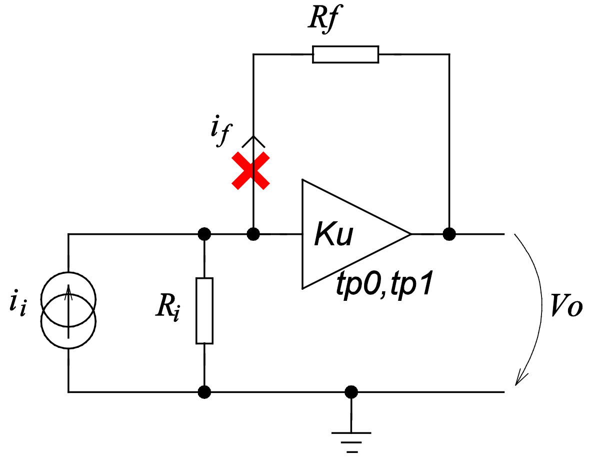


Rf=100k, tp0=50ns, Ku=60dB (GBP ~2GHz), tf=20ns, cd=10p

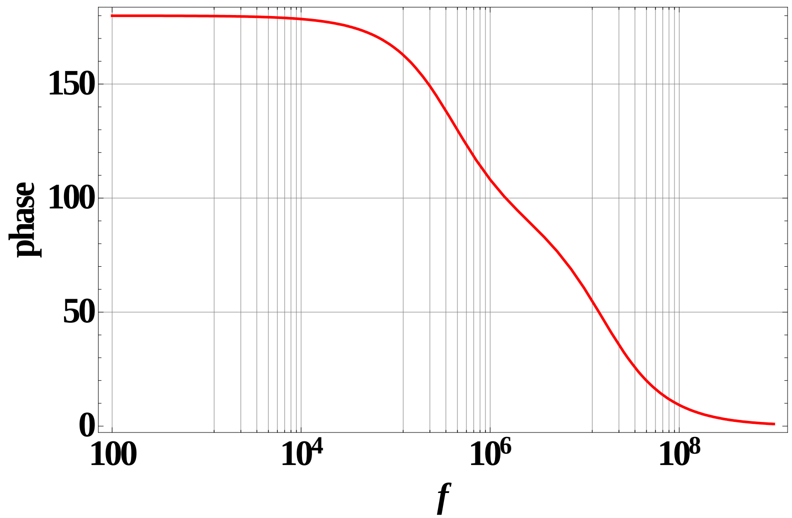




Loop gain phase of shunt-shunt feedback amplifier (for comparison) – always monotonic

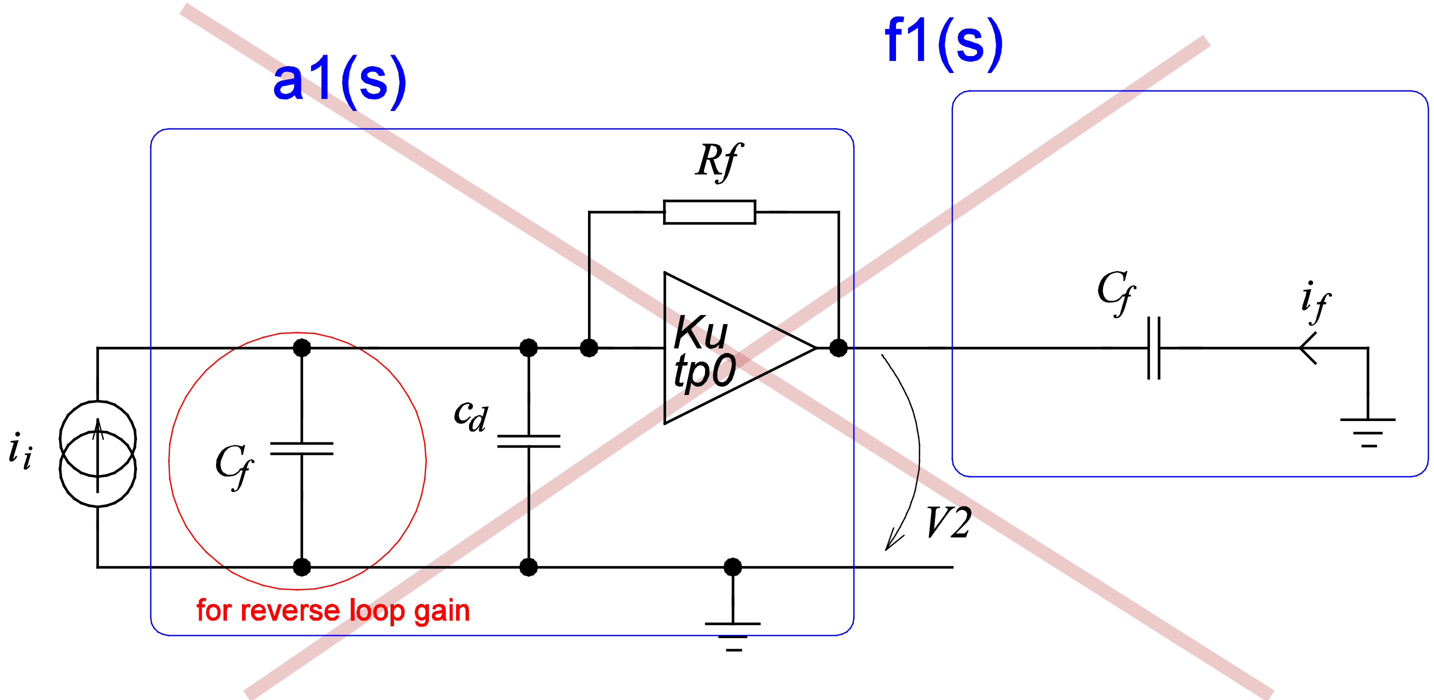


$$Loop\ Gain = Ku\,\frac{Ri}{Ri+Rf}\,\frac{1}{(1+s\,tp0)(1+s\,tp1)}$$



Rf=1M,Ri=10k, tp0=10ns, tp1=400ns, Ku=60dB

CSA- loop partially open



$$A_{closed\ loop}(s) = \frac{a1(s)}{1 + a1(s)f1(s)} = T(s) = \frac{Kus\ Zf\ Zi}{Zf + Zi + Kus\ Zi}$$

The loop is not fully open – the loop gain and its phase is not significant for the stability estimation

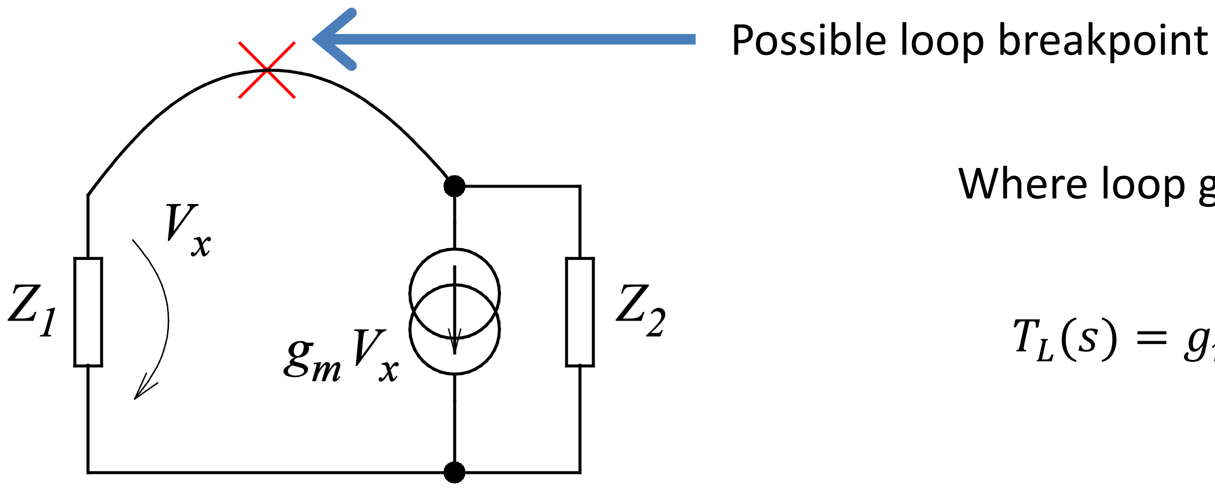
Open loop analysis

Physical opening of the loop has some drawbacks:

- Problems with DC operating point of the circuits → neccessity of use replicas for generation of bias voltages, tricks for compensation of leakage (gate or base) currents, creation of special schematics (not the same as used for final circuit) etc.
- Taking into account the reverse loop gain is difficult for active feedbacks

Calculation (or measurement) of loop gain using Middlebrook Double Injection method – principles

Any single loop feedback circuit can be represented by following scheme:



Where loop gain $T_L(s)$ is

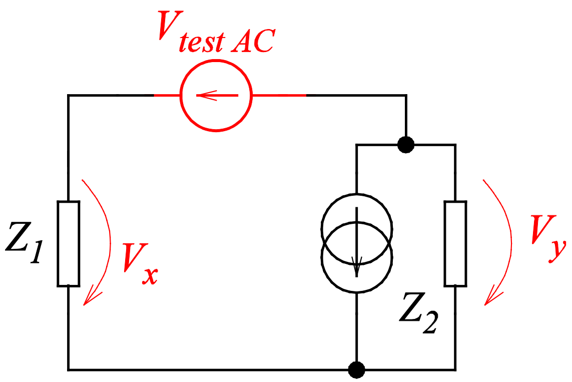
$$T_L(s) = g_m \frac{Z_1 Z_2}{Z_1 + Z_2}$$

R.D. Middlebrook, *Measurement of loop gain in feedback systems*, Int.J.Electronics, 1975, Vol.38, No.4, 485-512

P.J. Hurst, *Determination of Stability Using Return Ratios in Balanced Fully Differential Feedback Circuits*, IEEE Trans. on Circ. and Systems, Vol.42, No.12, Dec. 1995

Sol Rosenstark, *Feedback amplifier principles*, Macmillan Publishing Company New York, 1985, ISBN 0-02-947810-3

Middlebrook Double Injection method

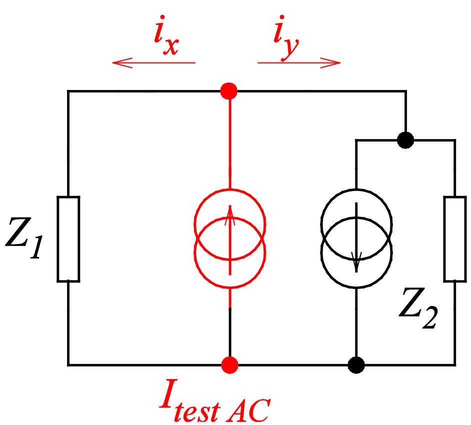


$$\frac{V_y}{V_x} \equiv T_V = g_m\, Z_2 + \frac{Z_2}{Z_1}$$

Loop Gain: $$T_L = g_m \frac{Z_1 Z_2}{Z_1 + Z_2}$$

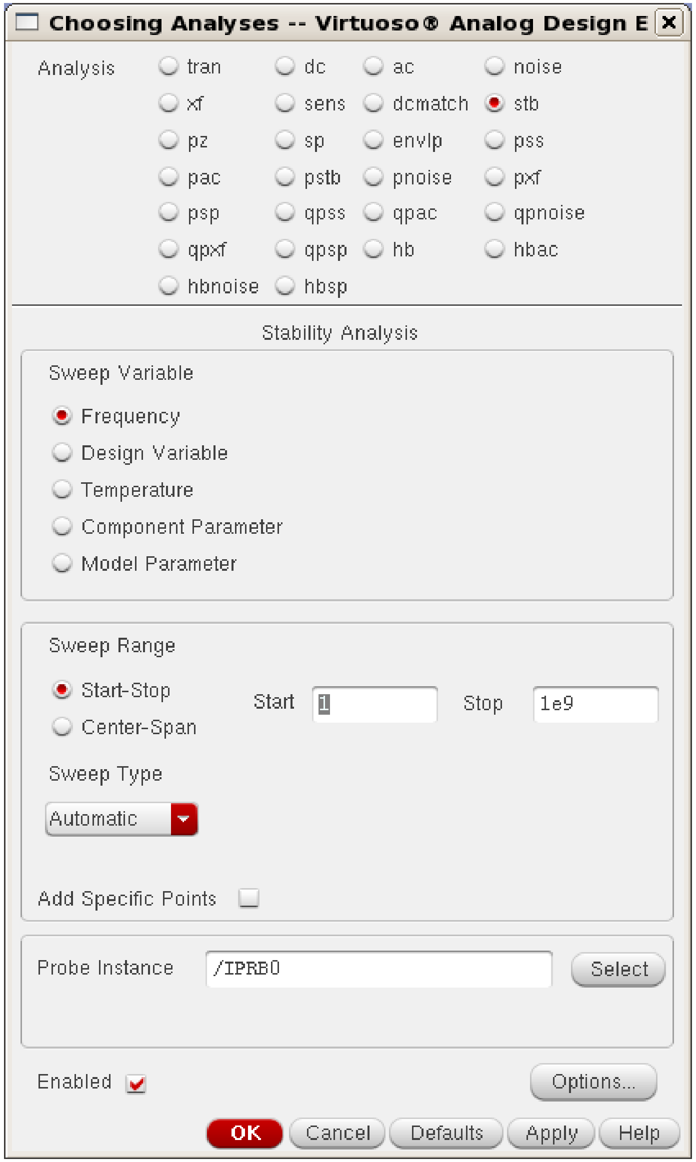
$$\frac{i_y}{i_x} \equiv T_i = g_m\, Z_1 + \frac{Z_1}{Z_2}$$

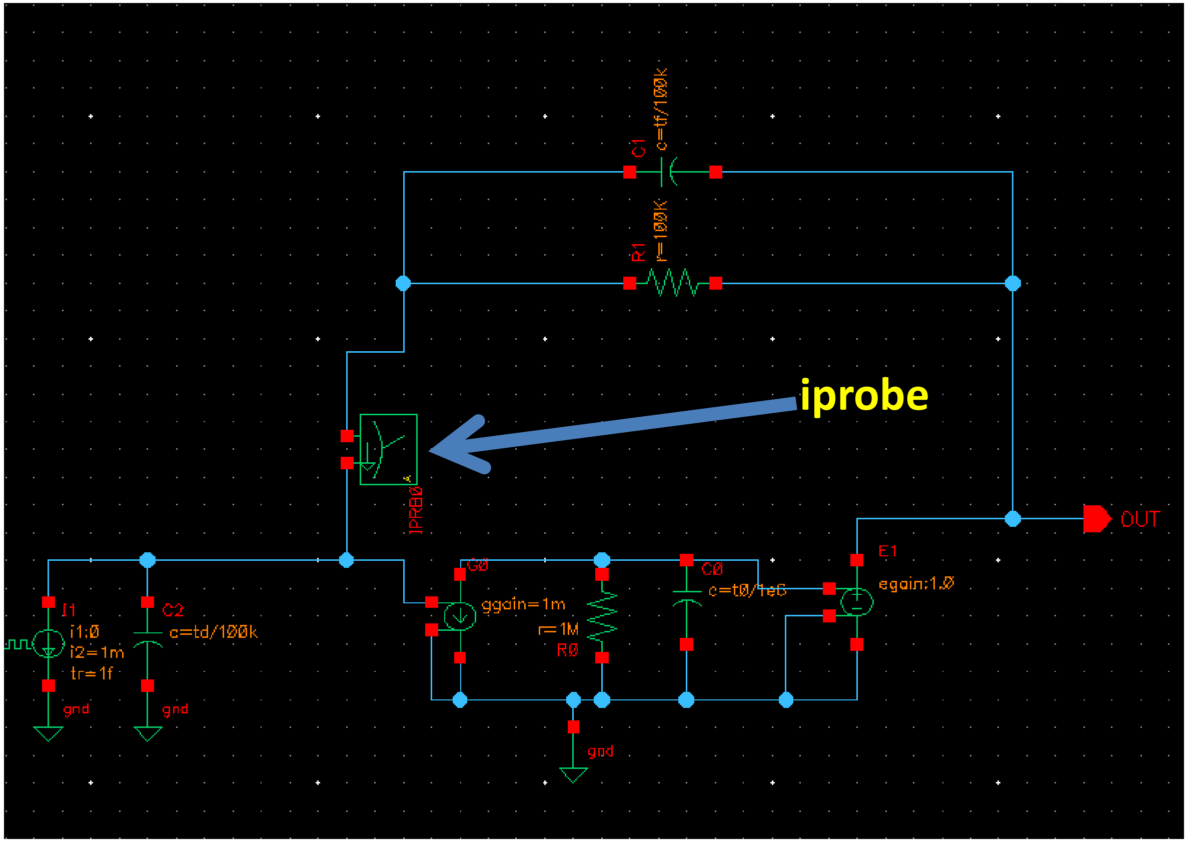
$$T_L = \frac{T_V\ T_i - 1}{T_V + T_i + 2}$$



- ❑ No DC break in the loop, all loading effects included
- ❑ Measure/calculate T_V and T_i, then calculate T_L

Spectre STB analysis of ideal CSA

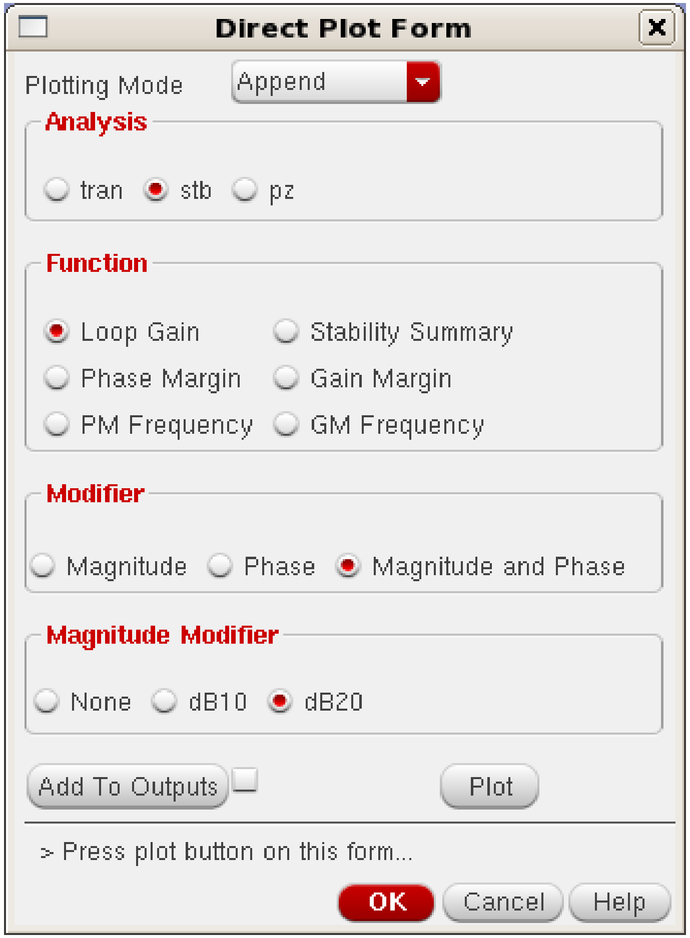


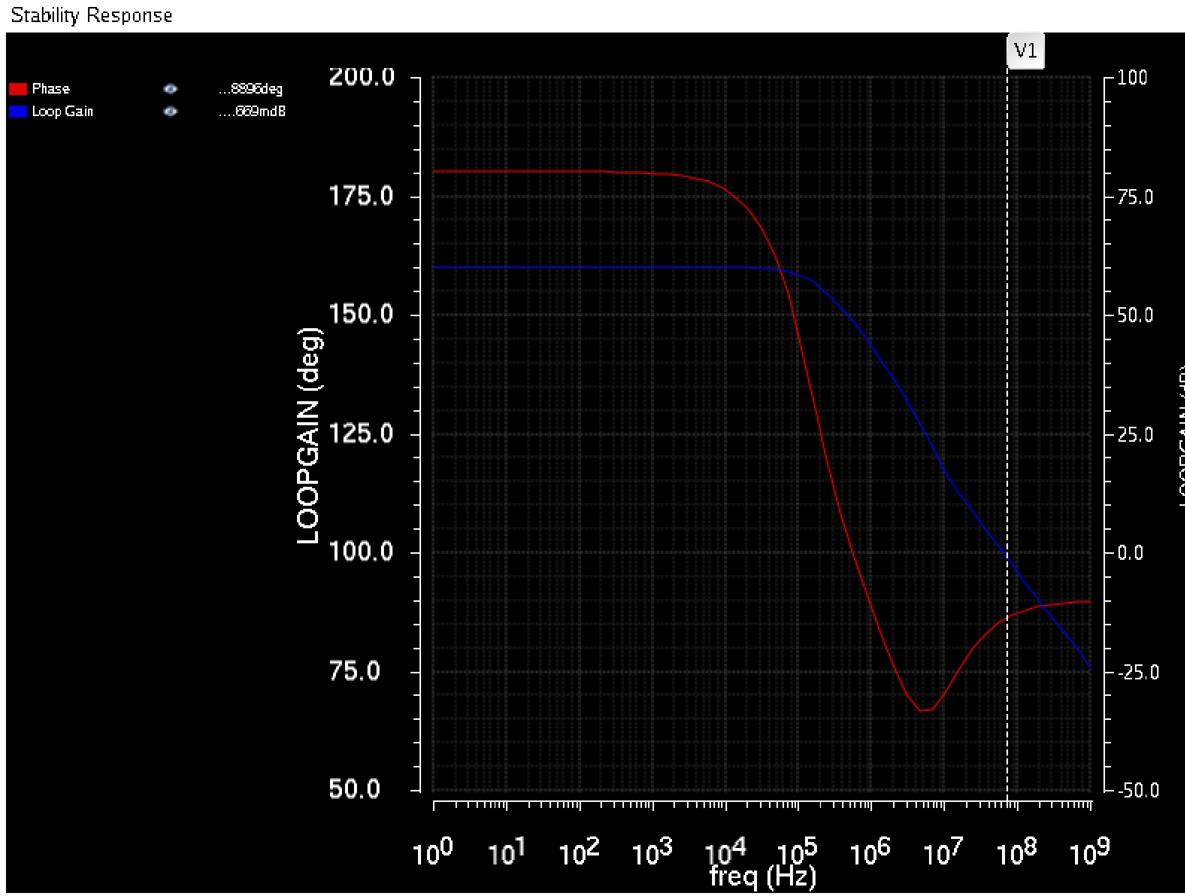


Based on Middlebrook double injection method

- ❑ Circuit in close loop configuration
- ❑ iprobe component from analogLib defining the loop breakpoint

Spectre STB analysis of ideal CSA

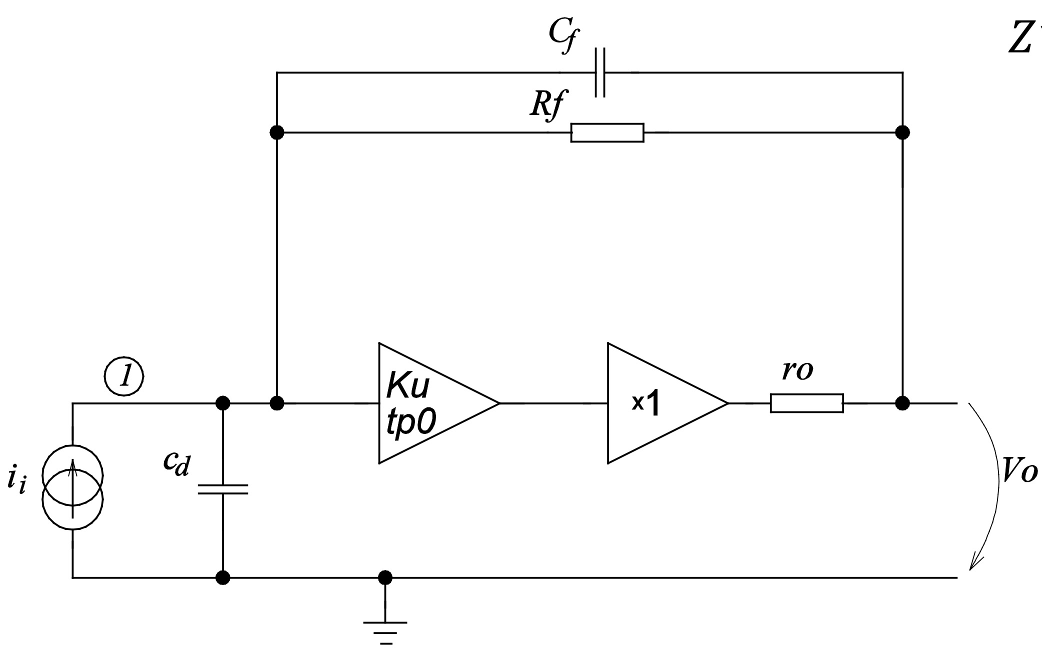




Access to results through direct plot form or print summary

Ideal CSA with Rf=100k, tp0=50ns, Ku=60dB (GBP ~2GHz), tf=20ns, cd=10p, PM=86°, two real poles

CSA with finite impedance output buffer – close loop analysis



$$Z1,2 \cong \mp \sqrt{\frac{Ku}{ro\ Cf\ tp0}}$$ **GBP**

$$P1 \cong \frac{1}{Rf\ Cf}$$

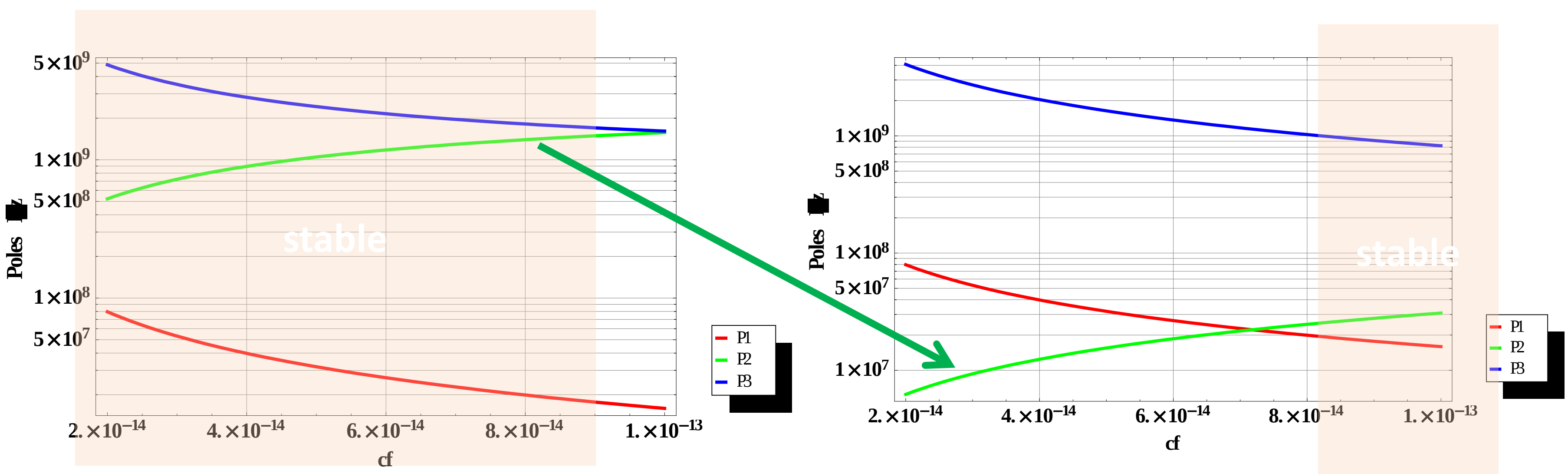
GBP

$$P2 \cong \frac{Ku}{\left(\frac{cd}{Cf}+1\right)\ tp0}$$

$$P3 \cong \frac{1}{Cf\ ro}$$

CSA with finite impedance output buffer – close loop analysis

$$P1 \cong \frac{1}{Rf\ Cf} \qquad P2 \cong \frac{GBP}{(\frac{cd}{Cf}+1)} \qquad P3 \cong \frac{1}{Cf\ ro}$$

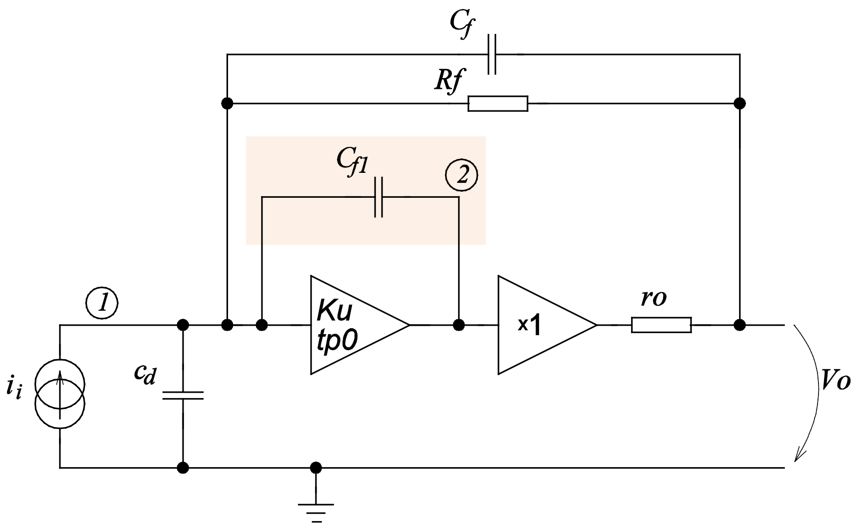


Cd=.1pF

Cd=10pF

Rf=100k, tp0=50ns, Ku=60dB (GBP ~2GHz), ro=2k

CSA with finite impedance output buffer and compensation – close loop analysis



$$Z_{1,2} \cong -\frac{Cf1}{2\,ro\,Cf\,(Cf1+cl)} \mp \sqrt{\frac{GBP_{mod}}{ro\,Cf}}$$

Zeroes in LFP and RHP asymmetric – visible undershoot

$$GBP = \frac{gm}{cl}, \qquad GBP_{mod} = \frac{gm}{cl + Cf1}$$

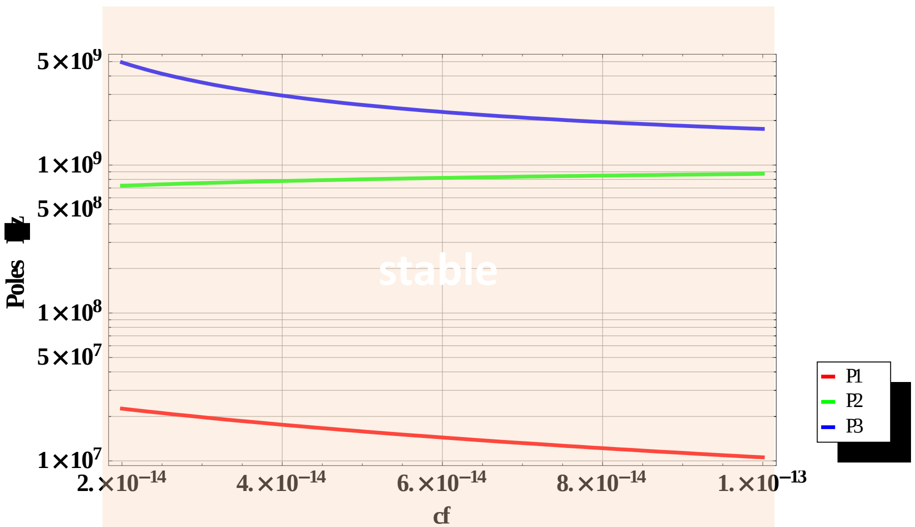
$$P_1 \approx \frac{1}{Rf\,(Cf + Cf1)}$$ → unchanged (*)

$$P_2 \approx \frac{GBP_{mod}}{\frac{cd}{Cf + Cf1} + \frac{Cf\,Cf1}{Cf + Cf1}\,GBP_{mod}\;ro}$$ → moved down with Cf1

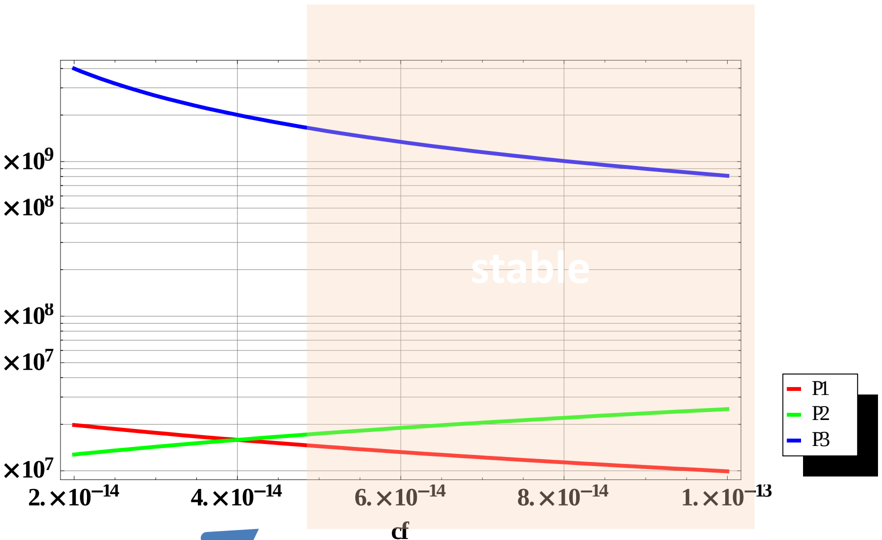
$$P_3 \sim \frac{1}{Cf\,ro}$$ → moved up (Cf now is a fraction of Cf')

(*) assumed effective value of Cf'=Cf+Cf1

CSA with finite impedance output buffer and compensation – close loop analysis



Cd=0.1pF

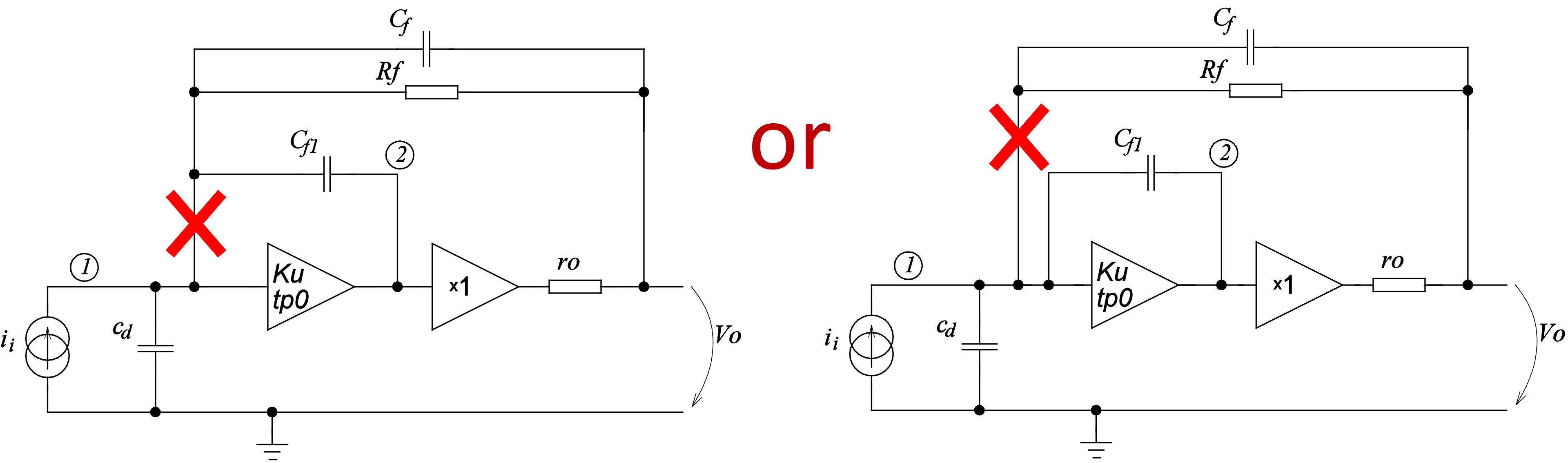


Cd=10pF

Stable range of Cf starts lower due to the fact that effective feedback capacitance is now Cf+Cf1

Rf=100k, tp0=50ns, Ku=60dB (GBP ~2GHz), ro=2k, Cf1=50f

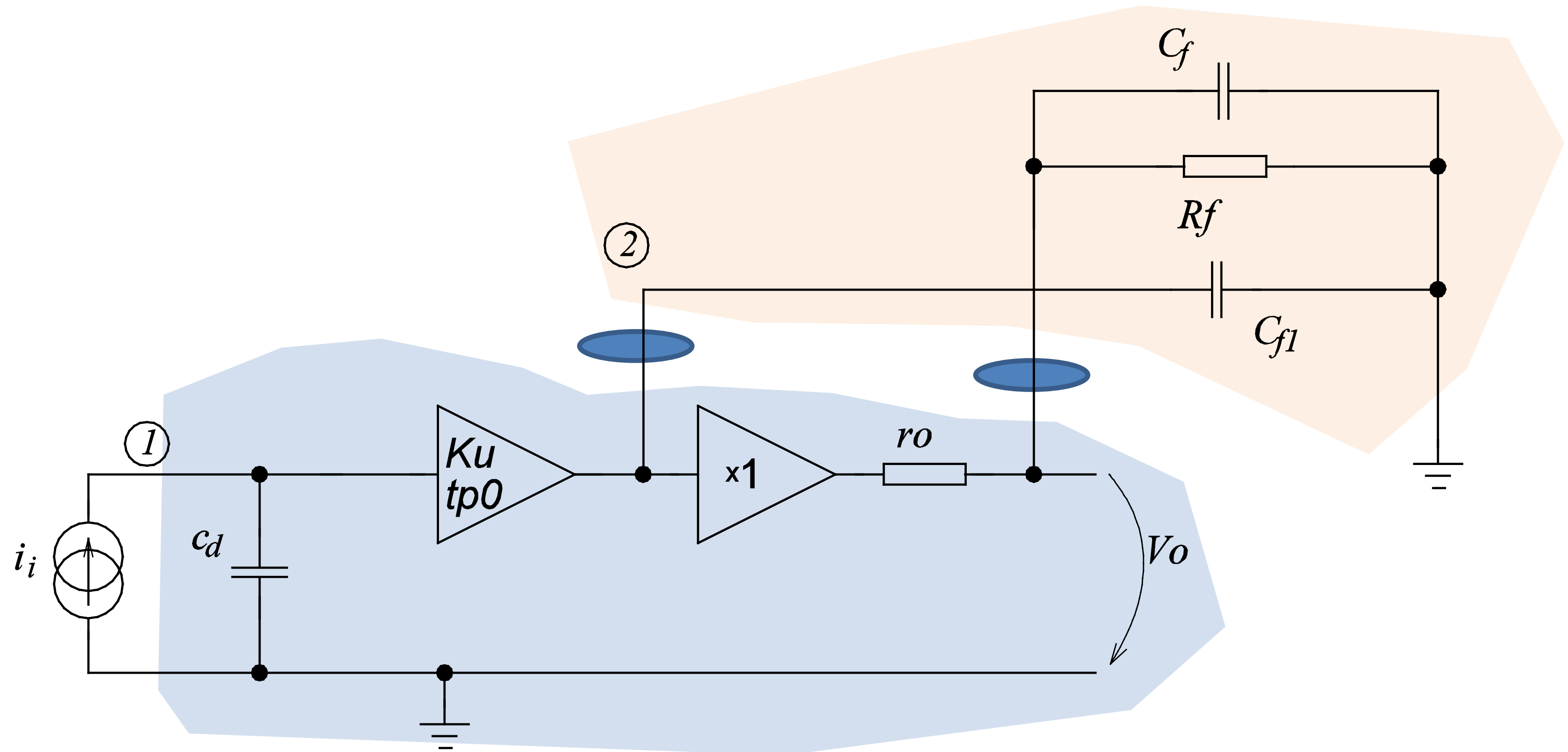
CSA with finite impedance output buffer and compensation – open loop analysis: where to open the loop?



Criterion:

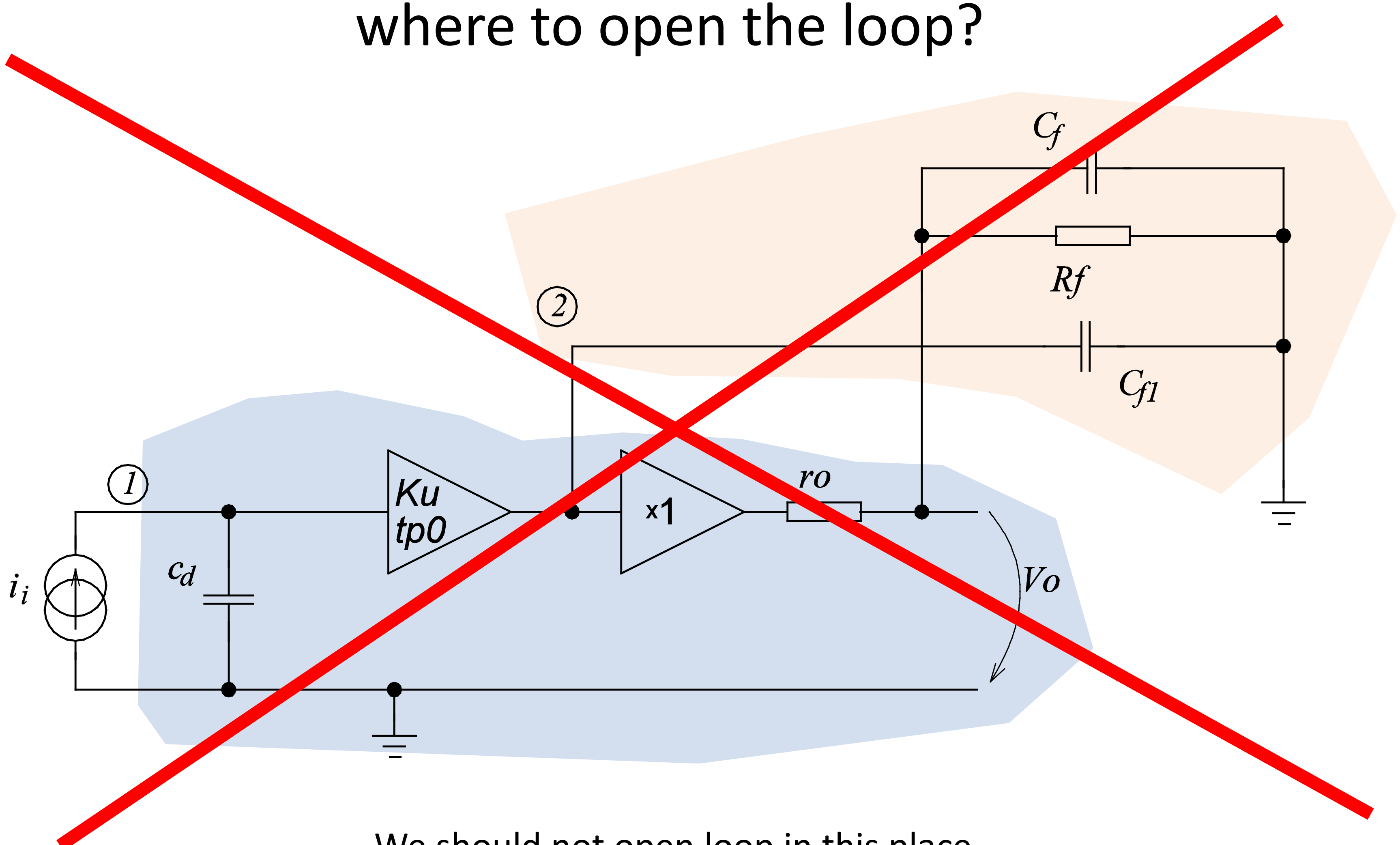
- ❑ loop fully open
- ❑ calculate amplifier and feedback transmittances (*a* and *f*) and *T=a/(1+af)* and compare it to transmittance of circuit in close loop configuration

CSA with finite impedance output buffer and compensation – open loop analysis: where to open the loop?



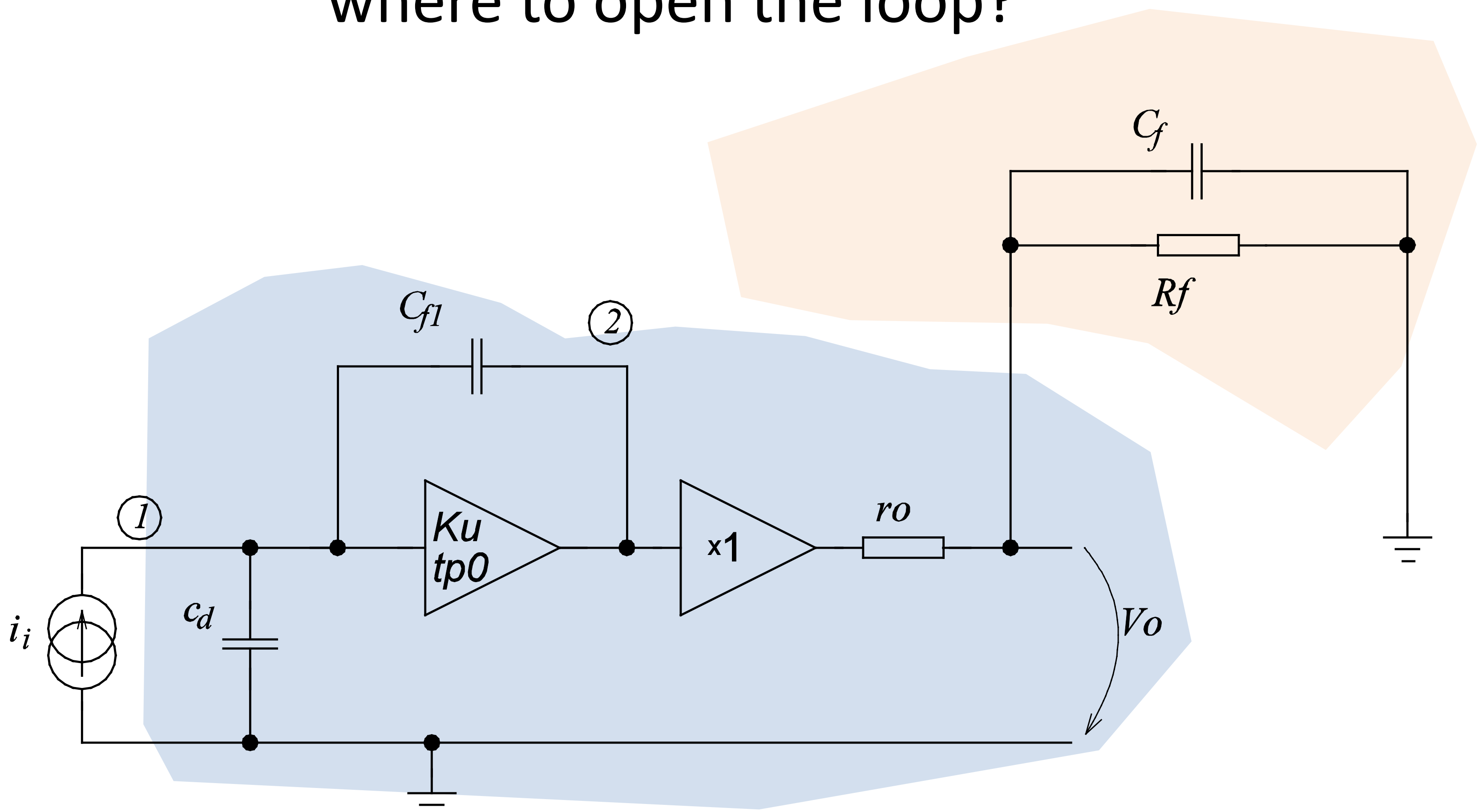
- ❑ *a* and *f* not two-port devices → not possible to calculate transmittances: *a, f* and *a·f*
- ❑ Cf1 acts as internal compensation (acts on P2 only)→ after opening of Cf1 the *a* changes significantly → errors in estimation of PM up to 50%

CSA with finite impedance output buffer and compensation – open loop analysis: where to open the loop?



We should not open loop in this place

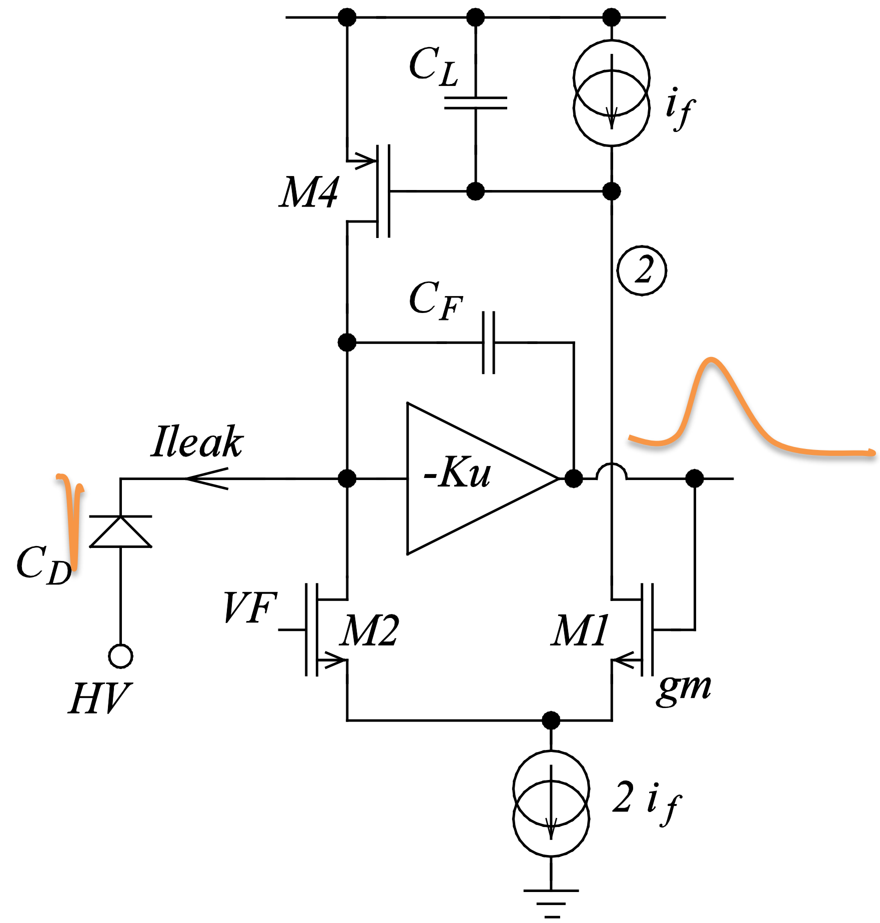
CSA with finite impedance output buffer and compensation – open loop analysis: where to open the loop?



a and *f* well defined, transmittance *t=a/(1+af)* the same as transmittance of circuit in close loop configuration → proper breakpoint for the loop

Krummenacher feedback – dual branch active feedback example

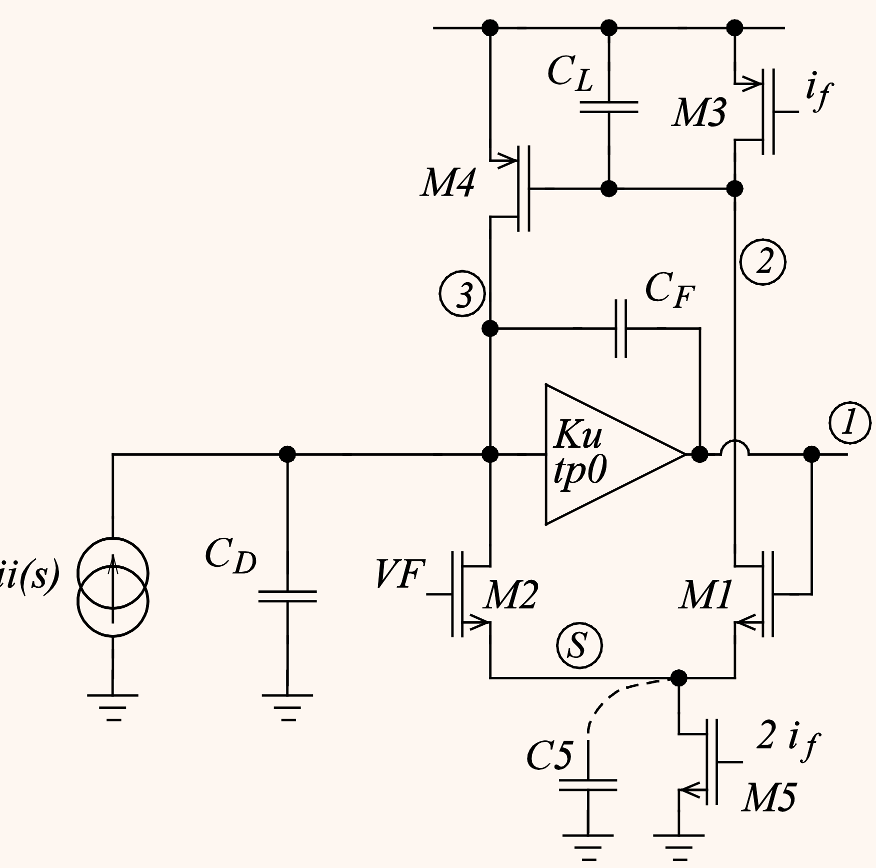
- Fast feedback (signal discharge); M1 biased with i_f equivalent to feedback resistor of value $1/g_m$ and feedback capacitor C_F
- Slow feedback (leakage compensation); excess current from M1 generated by leakage attempting to flow from M2 causes lowering potential on N2 what in consequence increase the current through M4 (variation of voltage is filtered by C_l)
  - slow feedback is forcing detector leakage to flow through M4
  - VF control voltage is restored at the output



Krummenacher feedback – close loop analysis

$$Z_1 = \frac{gds1 + gds3}{Cl}$$

$$Z_2 = 2\frac{gm12}{C5}$$



$$P_1 \approx \frac{2\ gm4}{Cl}$$

Very low frequency pole related to leakage compensation filtering (for gm1=gm2=gm12)

$$P_2 \approx \frac{gm12}{2\ Cf}$$

Low frequency pole related to fast feedback: 2/gm12 ~ Rf

$$P_3 \approx 2\frac{gm12}{C5}$$

Med. frequency pole related to parasitic on node S → C5 should be minimized: C5<<Cf !!

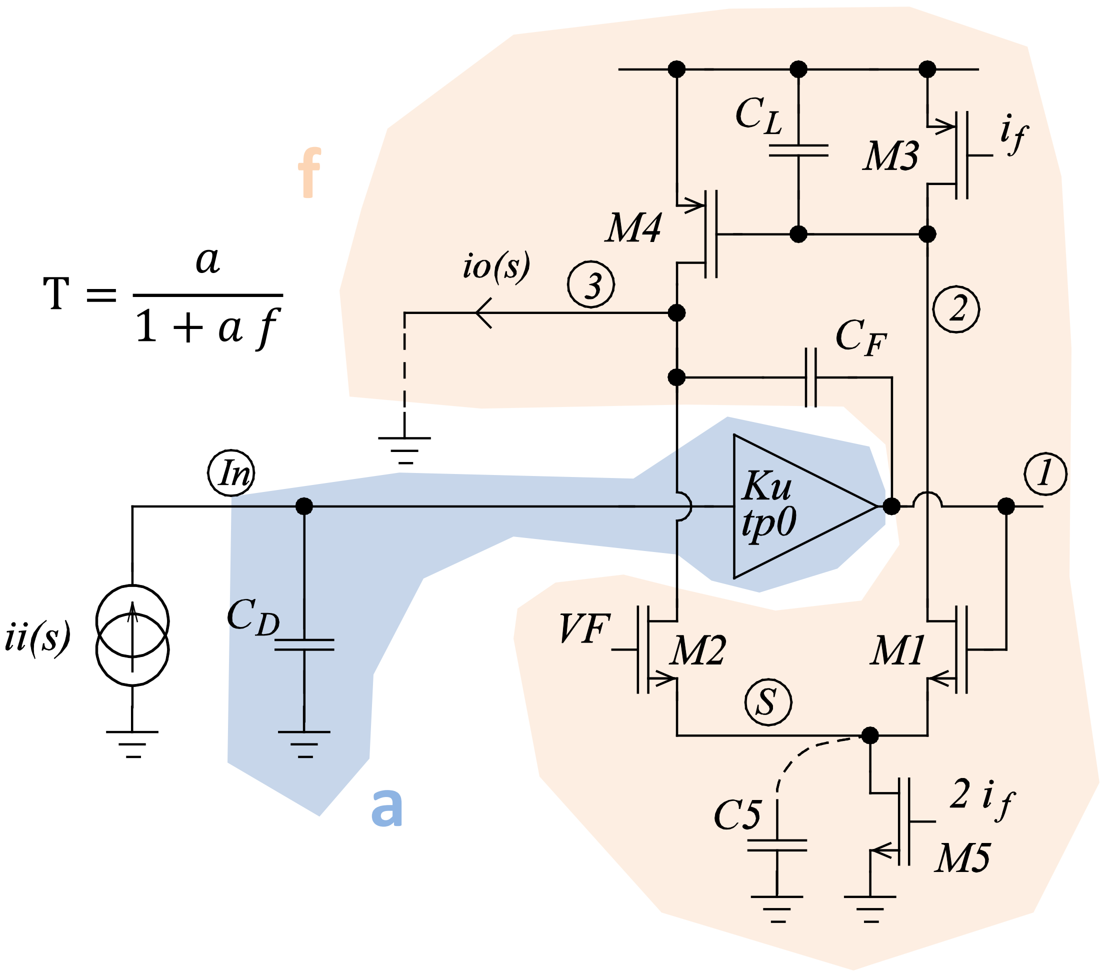
$$P_4 \approx \frac{Cf}{Cd}\frac{Ku}{\tau_{P0}} = \frac{Cf}{Cd}GBP$$

High frequency pole related to dominant pole of amplifier → GBP should be maximized

Good stability provided for well separated poles: P1<<P2<<P3<<P4

Critical points: separation of P2 and P3 by minimizing C5 (or increase of Cf), separation of P1 and P2 if gm4>>gm1 (high leakage) by proper value of Cl (Cl>>Cf)

Krummenacher feedback – open loop analysis



- Breaking loop at common point of two feedbacks at the input
- After calculation of *a*, *f* and *T=a/(1+af)* the position of zeroes and poles are the same as in the close loop analysis → *a* and *f* are unaffected after opening the loop → classical or STB open loop gain analysis can be performed

Krummenacher open loop: comparison of classical (no reverse loop gain) and STB approach

Poles and zeroes of the loop gain (a f):

$gm1 = gm2 = gm12$

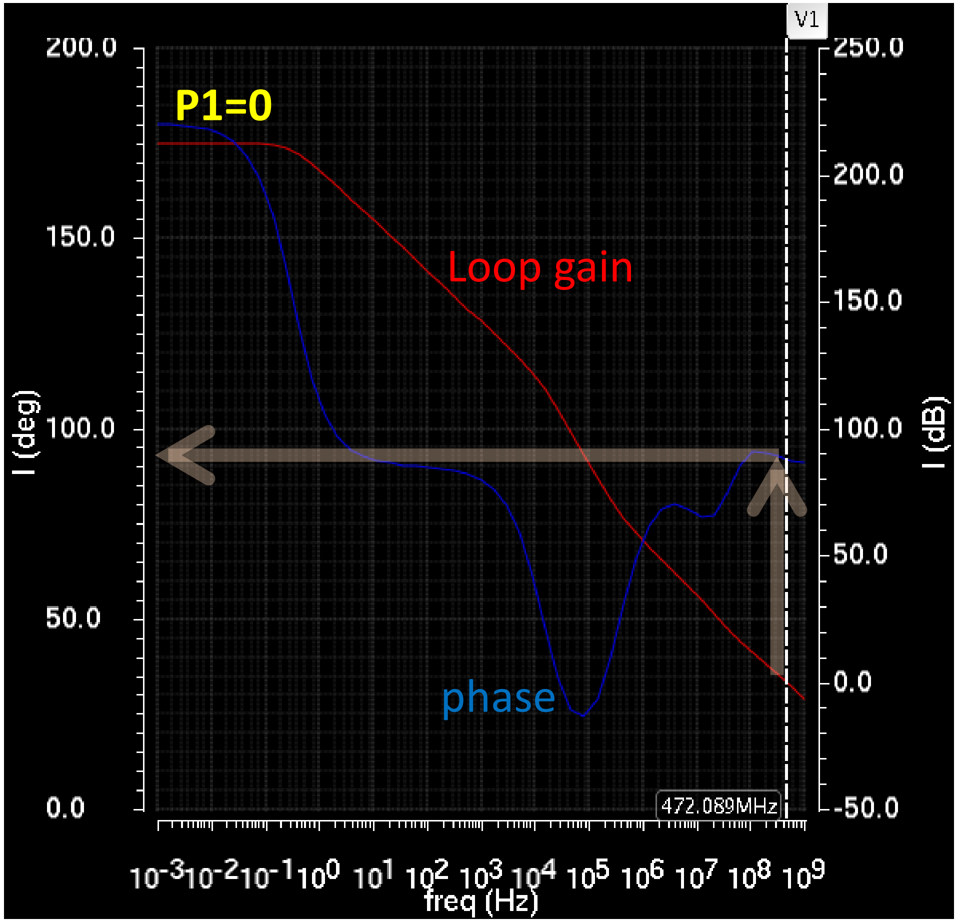
$$Z_1 = \frac{gm4}{Cl} \qquad Z_2 = \frac{gm12}{2\ Cf} \qquad Z_3 = 2\ \frac{gm12}{C5}$$

$$\boldsymbol{P_1 \approx 0\ \ or\ \ P_1 \approx P_2\ (STB)} \qquad P_2 \approx \frac{gds1 + 2\ gds3}{2\ Cl} \qquad P_3 \approx 2\ \frac{gm12}{C5} \qquad P_4 \approx \frac{1}{\tau_{P0}}$$

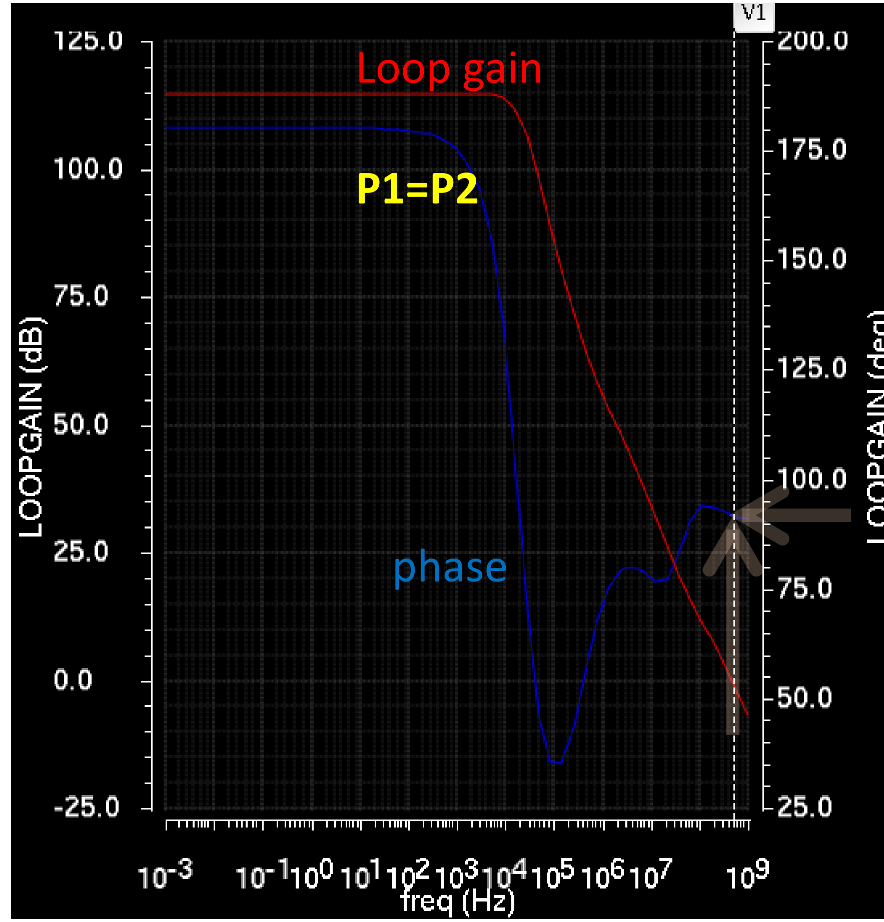
Differences only for low frequencies → PM measurement not affected

Krummenacher open loop: AC open loop vs. STB

65nm design for CMS CTPix ASIC, simplified circuit (all transistors and amplifier built with VCCS)

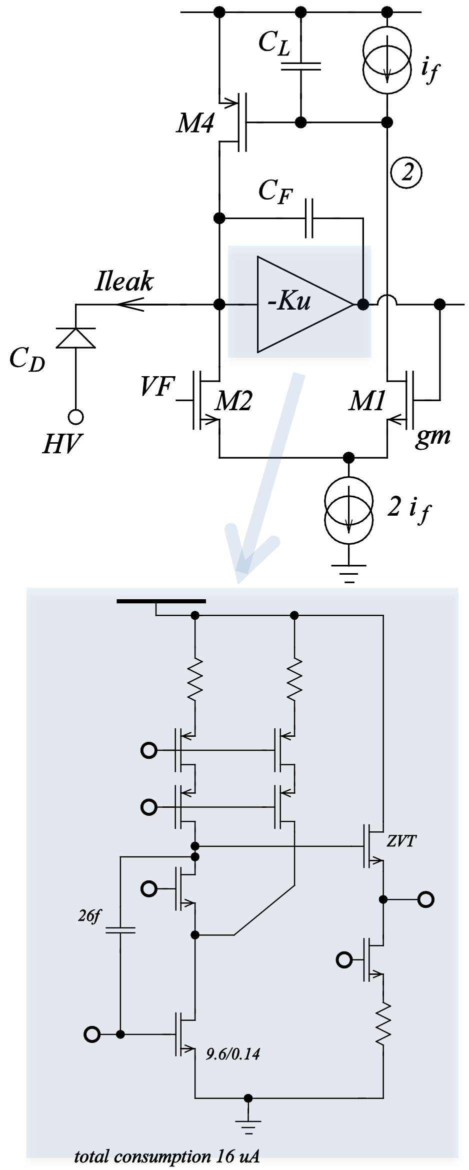


AC analysis of open loop circuit
PM 92°



STB analysis
PM 92°

CTPix preamp: STB and PZ analysis

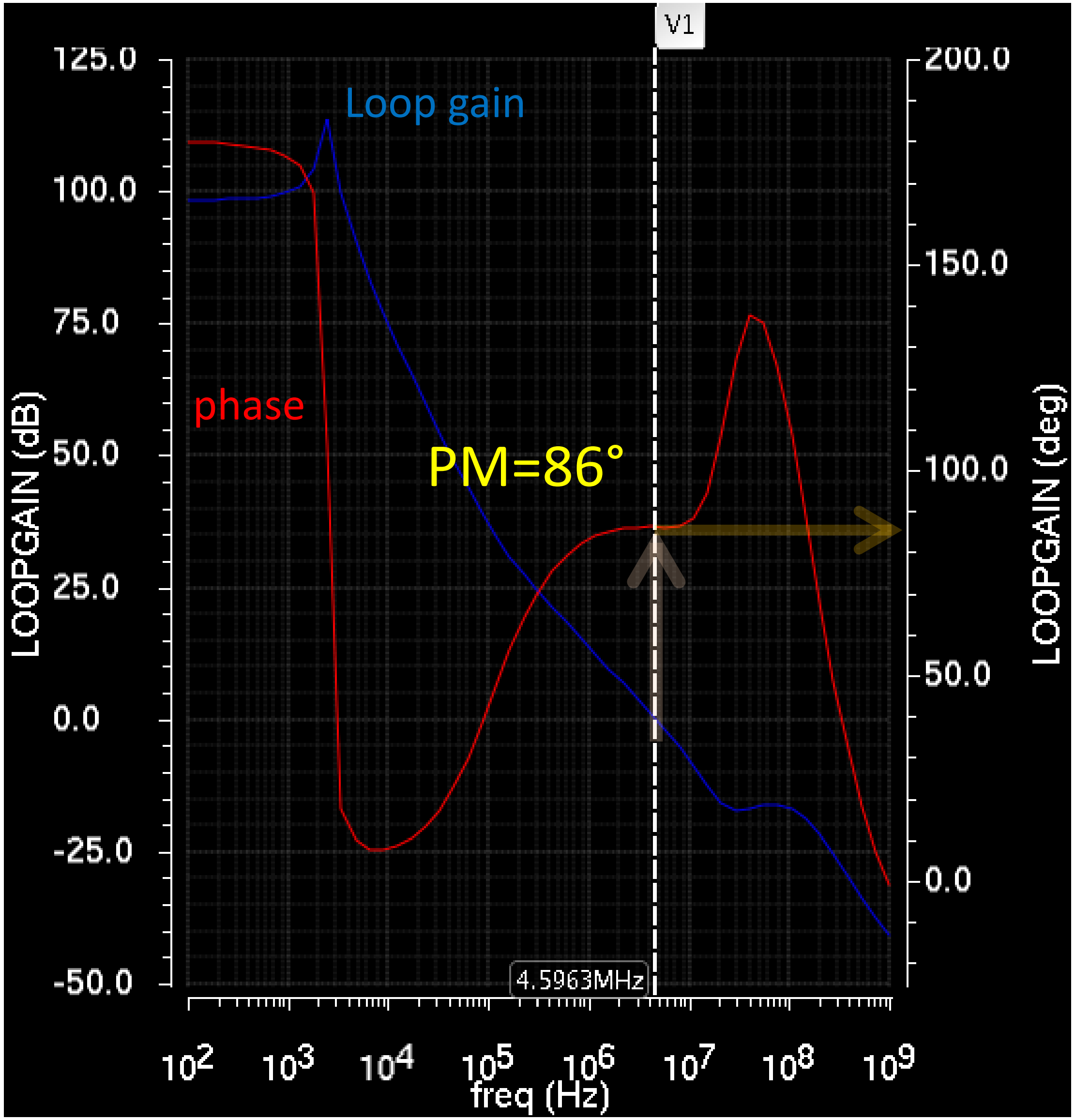


Input stage:

- Designed for long pixels (1500x100um), cd=280fF
- Telescopic cascode with degenerated PMOS sources
- Input transistor: NMOS 9.6um/140nm
- Cf=6.4fF, Cf1=26fF
- Consumption: 16uA (12uA input transistor)
- Open loop gain: ~54dB
- GBP: 2GHz (for compensated cascode)

Krummenacher feedback (leakage compensation for n+ on p- detectors, If=80nA)

CTPix preamp: STB analysis



CTPix preamp: PZ analysis

Results Display Window

Window Expressions Info Help cādence

Poles(Hz)

	Real	Imaginary	Qfactor
Pole_1	-1.140e+05	0.000e+00	5.000e-01
Pole_2	-5.375e+06	0.000e+00	5.000e-01
Pole_3	-1.493e+07	0.000e+00	5.000e-01
Pole_4	-4.268e+07	0.000e+00	5.000e-01
Pole_5	-4.877e+07	0.000e+00	5.000e-01
Pole_6	-1.219e+08	0.000e+00	5.000e-01
Pole_7	-1.329e+08	0.000e+00	5.000e-01
Pole_8	-1.436e+08	8.149e+07	5.749e-01
Pole_9	-1.436e+08	-8.149e+07	5.749e-01
Pole_10	-2.088e+08	2.734e+06	5.000e-01
Pole_11	-2.088e+08	-2.734e+06	5.000e-01
Pole_12	-2.882e+08	0.000e+00	5.000e-01
Pole_13	-3.124e+08	0.000e+00	5.000e-01
Pole_14	-3.124e+08	0.000e+00	5.000e-01
Pole_15	-3.918e+08	0.000e+00	5.000e-01
Pole_16	-5.041e+08	0.000e+00	5.000e-01
Pole_17	-5.689e+08	0.000e+00	5.000e-01
Pole_18	-6.013e+08	0.000e+00	5.000e-01
Pole_19	-8.141e+08	0.000e+00	5.000e-01
Pole_20	-1.012e+09	0.000e+00	5.000e-01
Pole_21	-1.206e+09	0.000e+00	5.000e-01
Pole_22	-1.415e+09	0.000e+00	5.000e-01
Pole_23	-1.665e+09	0.000e+00	5.000e-01
Pole_24	-2.008e+09	5.199e+08	5.165e-01
Pole_25	-2.008e+09	-5.199e+08	5.165e-01
Pole_26	-2.293e+09	0.000e+00	5.000e-01
Pole_27	-4.331e+09	0.000e+00	5.000e-01
Pole_28	-4.331e+09	0.000e+00	5.000e-01
Pole_29	-9.056e+11	0.000e+00	5.000e-01
Pole_30	-9.690e+11	0.000e+00	5.000e-01

Zeros(Hz)

	Real	Imaginary	Qfactor
Zero_1	-3.243e+02	0.000e+00	5.000e-01
Zero_2	-1.912e+07	0.000e+00	5.000e-01
Zero_3	-4.208e+07	0.000e+00	5.000e-01
Zero_4	-5.876e+07	0.000e+00	5.000e-01
Zero_5	-1.330e+08	0.000e+00	5.000e-01
Zero_6	-1.975e+08	0.000e+00	5.000e-01
Zero_7	-2.086e+08	0.000e+00	5.000e-01
Zero_8	-3.876e+08	0.000e+00	5.000e-01
Zero_9	-5.043e+08	0.000e+00	5.000e-01
Zero_10	-5.656e+08	0.000e+00	5.000e-01
Zero_11	-6.002e+08	0.000e+00	5.000e-01
Zero_12	7.045e+08	0.000e+00	-5.000e-01
Zero_13	-8.124e+08	0.000e+00	5.000e-01
Zero_14	-1.012e+09	0.000e+00	5.000e-01

11 HelpAction

Dominant poles → reals
Three high freq. complex pairs at 150MHz, 200MHz and 2GHz with Q < 0.6

Summary

- Open loop gain analysis (classical or STB) and close loop analysis should be both used for establish the stability margins of the design
- Time response (asymptotic or oscillatory) defined by position of poles and zeroes not phase margin → keep in mind ambiguities between quality of the poles and phase margin!
- Stable design should have all poles on LHP with quality below 0.7 and phase margin above 70°
  - lower pole quality and higher phase margin are very welcome!
  - higher quality poles at frequencies ~ft and above probably acceptable (if inevitable)
- Open loop gain analysis and STB check only the feedback under test (do not check feedbacks related to internal amplifier architecture like regulated cascode, more exact transistor models etc.) → PZ analysis has big advantage from this viewpoint
- We discussed only single ended feedbacks → differential feedbacks possible to analyse in STB however the differential iprobe does not exists in analogLib → PZ analysis more straightforward for differential circuits