Precision tests of unitarity in leptonic mixing

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LB, O. Fischer and J.J. van der Bij, EPL 105 (2014) 11001 [arXiv:1310.2057]

Outline

Introduction

LEP and several other (high and low energy) experiments delivered an impressive amount of very precise data

Assuming the SM, predictions are possible: top and Higgs masses (different degree of accuracy)

With the discovery of the Higgs boson, all parameters in the SM are fixed

$\Rightarrow$ No free degrees of freedom in fit to precision observables

It is universally accepted that the SM provides a satisfactory fit

However. . .

some anomalies: Γ_Z^{inv}, $\sin^2\theta_{\text{eff}}^{\text{lept}}$ vs. $\sin^2\theta_{\text{eff}}^{\text{hadr}}$, $((g-2)_\mu)$, . . .

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Example: Γ_Z^{inv}

	Experiment	SM
$\Gamma_Z^{\rm inv}/\Gamma_{\rm lept}$	5.942(16)	5.9721(2)

2σ discrepancy, deficit (similar deficit in NuTeV)

Loinaz et al. Phys.Rev. D67 (2003) 073012: postulate universal suppression $1-\varepsilon$ in $Z\nu\nu$ coupling (also $W\ell\nu$ suppressed)

- direct reduction of $\Gamma_Z^{\rm inv} \propto \varepsilon$
- effect on all other observables: $G_\mu^2 = G_F^2(1-2\varepsilon)$, from $\mu \to e\nu_e\nu_\mu$

Result of the fit was $\varepsilon \sim \mathcal{O}(10^{-3})$, but large T parameters were also needed: perhaps a heavy SM Higgs (now excluded)

Generalisation to flavour: $G_\mu^2 = G_F^2(1-\varepsilon_e-\varepsilon_\mu)$ [Phys.Rev. D70 (2004) 113004]

Impact on CKM $\propto \varepsilon_\mu$ only (e-flavour Cabibbo-suppressed)

Lepton unitarity violation

What can lead to such suppression?
Mixing of active neutrinos with n *sterile states* Loinaz et al.
$\nu_i,\ i = 1, ..., 3+n$ mass eigenstates: PMNS matrix non-unitary
New unitary diagonalisation matrix is $\mathcal{U}$:

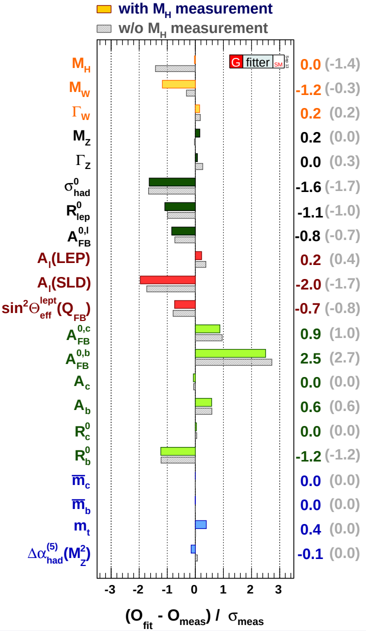
$$\begin{pmatrix} \nu_1 \\ \vdots \\ \nu_{3+n} \end{pmatrix} = \begin{pmatrix} \mathcal{U}_{PMNS} & \mathcal{W} \\ \mathcal{W}^\dagger & \mathcal{V} \end{pmatrix} \begin{pmatrix} \nu_{L_e} \\ \vdots \\ N_n \end{pmatrix}$$

As a submatrix, $\mathcal{U}_{PMNS}$ non unitary (for $n \neq 0$)
The amount of unitarity violation is quantified by ($\alpha, \beta = 1..3$)

$$\varepsilon_\alpha = \sum_{i>3} |\mathcal{U}_{\alpha i}|^2 = 1 - \sum_\beta |\mathcal{U}_{\alpha\beta}|^2$$

Neutrino oscillation bound: $\varepsilon \lesssim \mathcal{O}(10^{-2})$ Antusch et al., NPB 810, 369 (2009)
MEG experiment bound: off-diagonal elements of $\mathcal{U}\mathcal{U}^\dagger$ are negligible

GFitter [Eur. Phys. J. C 72, 2205 (2012)]



How is a fit done?

– weighting of data with uncertainty^{-2}
– data with large uncertainties give little contribution to total χ^2

- most of measures do not matter in the fit $\rightarrow$ dilution of $\chi^2/d.o.f.$
- Higgs boson mass now known (M_W agreement gets worse)
- only positive pull from $A^{0,b}_{FB} \sim \sin^2\theta^{\text{hadr}}_{\text{eff}}$

Data

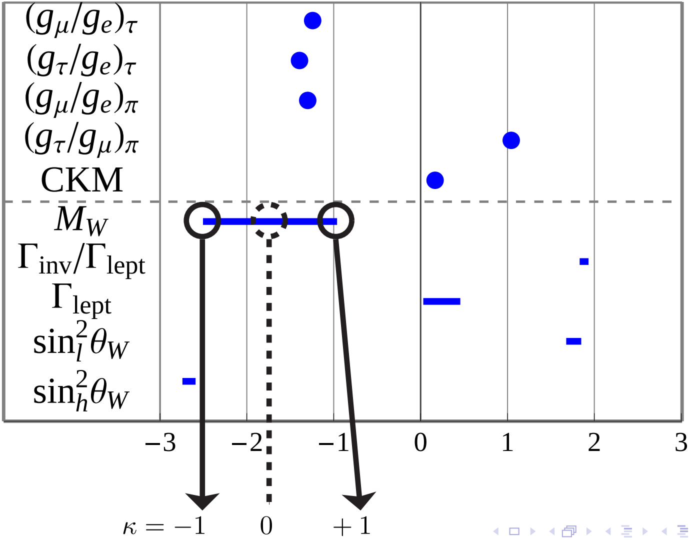
We are after $\mathcal{O}(10^{-3})$ effect, we need measures with comparable accuracy

Observable	Experiment	SM
$(g_\mu/g_e)_\tau$	1.0020(16)	1.0
$(g_\tau/g_e)_\tau$	1.0029(21)	1.0
$(g_\mu/g_e)_\pi$	1.0021(16)	1.0
$(g_\tau/g_\mu)_\pi$	0.9965(33)	1.0
CKM	0.9999(6)	1.0
M_W (GeV)	80.385(15)	80.359(11)
$\Gamma_{\rm inv}/\Gamma_{\rm lept}$	5.942(16)	5.9721(2)
$\Gamma_{\rm lept}$ (MeV)	83.984(86)	84.005(15)
$s^{2,\rm lept}_{\rm eff}$	0.23113(21)	0.23150(1)
$s^{2,\rm hadr}_{\rm eff}$	0.23222(27)	0.23150(1)

All other measures (NuTeV, W/k decays, ...) have larger uncertainties

Also: MEG ($\mu \to e\gamma$) and $\beta\beta0\nu$ bounds accounted for

$(\sigma_{th} - \sigma_{exp} + \kappa\, \delta_{th})/\delta_{exp}$



Fit of SM with T parameter

Observable	χ^2_{SM}	χ^2_T
Total χ^2	21.34	20.32
$(g_\mu/g_e)_\tau$	19.80	18.78
$(g_\tau/g_e)_\tau$	20.28	19.26
$(g_\mu/g_e)_\pi$	19.65	18.63
$(g_\tau/g_\mu)_\pi$	19.99	18.97
CKM	21.31	20.29
M_W (GeV)	19.39	19.35
$\Gamma_{\rm inv}/\Gamma_{\rm lept}$	17.80	16.85
$\Gamma_{\rm lept}$ (MeV)	21.36	20.19
$s^{2,\rm lept}_{\rm eff}$	18.25	18.14
$s^{2,\rm hadr}_{\rm eff}$	14.24	10.46

Removing a data point: a posteriori justification if considerable change of χ^2 (in statistics, such a point is an *outlier*)

Impact of unitarity violation

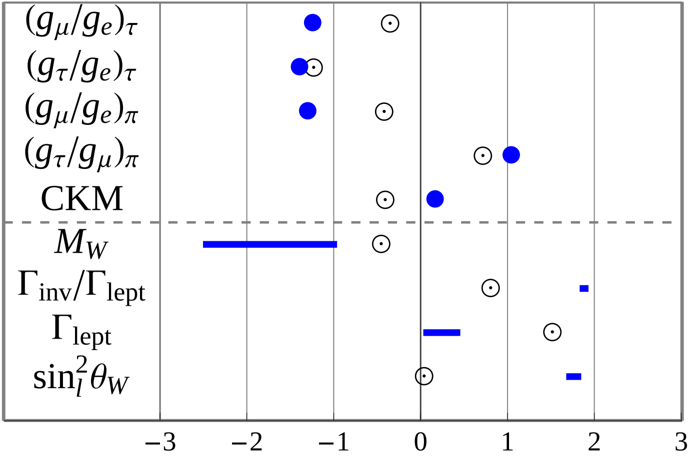
$$
\begin{aligned}
\frac{g_\alpha}{g_\beta} &= 1 - \frac{\epsilon_\alpha - \epsilon_\beta}{2} \\
CKM &= 1 + \epsilon_\mu \\
\frac{M_W}{[M_W]_{\text{SM}}} &= 1{+}0.11\,(\epsilon_e + \epsilon_\mu){+}0.0056T \\
\frac{\Gamma_{\text{inv}}/\Gamma_{\text{lept}}}{\left[\Gamma_{\text{inv}}/\Gamma_{\text{lept}}\right]_{\text{SM}}} &= 1{-}0.76\,(\epsilon_e + \epsilon_\mu){-}0.67\,\epsilon_\tau{-}0.0015T \\
\frac{\Gamma_{\text{lept}}}{\left[\Gamma_{\text{lept}}\right]_{\text{SM}}} &= 1{+}0.60\,(\epsilon_e + \epsilon_\mu){+}0.0093T \\
\frac{\sin^2\theta_{\text{eff}}}{[\sin^2\theta_{\text{eff}}]_{\text{SM}}} &= 1{-}0.72\,(\epsilon_e + \epsilon_\mu){-}0.011T \\
\text{BR}(\mu \to e\gamma) &\propto \epsilon_e\epsilon_\mu
\end{aligned}
$$

Result of the fit

Observable	χ^2_{SM}	χ^2_T	χ^2_ϵ	$\chi^2_{\epsilon+T}$
Total χ^2	21.34	20.32	18.03	18.00
$(g_\mu/g_e)_\tau$	19.80	18.78	17.49	17.36
$(g_\tau/g_e)_\tau$	20.28	19.26	14.00	13.46
$(g_\mu/g_e)_\pi$	19.65	18.63	17.39	17.24
$(g_\tau/g_\mu)_\pi$	19.99	18.97	17.29	17.27
CKM	21.31	20.29	15.87	15.20
M_W (GeV)	19.39	19.35	16.87	11.65
$\Gamma_{\rm inv}/\Gamma_{\rm lept}$	17.80	16.85	15.76	15.41
$\Gamma_{\rm lept}$ (MeV)	21.36	20.19	17.55	17.54
$s^{2,\rm lept}_{\rm eff}$	18.25	18.14	16.15	16.03
$s^{2,\rm hadr}_{\rm eff}$	14.24	10.46	5.34	5.31

Table: The χ^2 for the standard model (χ^2_{SM}), the minimum with T parameter only (χ^2_T), with unitarity violation only (χ^2_ε), with unitarity violation and the T parameter ($\chi^2_{\varepsilon+T}$), are evaluated *excluding* the entry on each line. The total χ^2 (considering all entries) is given for reference.

$(\sigma_{th} - \sigma_{exp} + \kappa\,\delta_{th})/\delta_{exp}$



We are not after a single major improvement, but a coherent one!

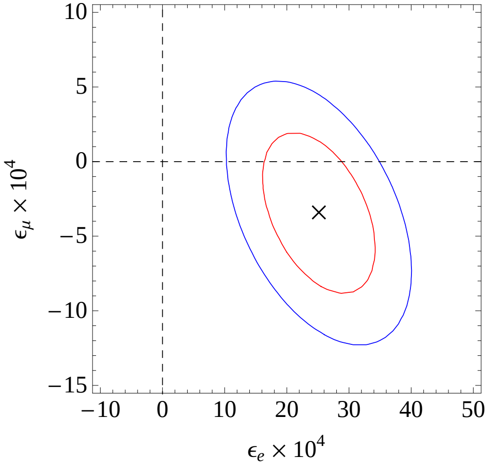
Best fit point excluding $\sin^2\theta_{\text{eff}}^{\text{hadr}}$

$$\chi^2_\varepsilon = 5.34$$

$$\begin{aligned} \varepsilon_e &= (25.2 \pm 8.1) \times 10^{-4} \\ \varepsilon_\mu &= (-3.4 \pm 4.8) \times 10^{-4} \\ \varepsilon_\tau &= (18.4 \pm 27.6) \times 10^{-4} \end{aligned}$$

with a correlation matrix of

$$\rho = \begin{pmatrix} 1 & -0.32 & -0.12 \\ -0.32 & 1 & -0.04 \\ -0.12 & -0.04 & 1 \end{pmatrix}$$



Conclusions

Measurement of the Higgs boson mass fixes radiative corrections
Standard Model cannot explain discrepancies in precision data

Hint of existence of sterile neutrinos from Γ_{inv}^{Z}

Slight improvement by neglecting $\sin^2\theta_{\text{eff}}^{\text{hadr}}$ or by considering unitarity violation

Major improvement with both: $\varepsilon_e \sim \mathcal{O}(10^{-3})$ at 3σ, $\chi^2 \sim 5$
(additional oblique corrections unnecessary)

See-saw model? ε_e too large for type-I, unless *strong* cancellations occur
Akhmedov et al., JHEP 1305 (2013) 081

Outlook: LHC and MESA, improving by a factor two M_W and $\sin^2\theta_{\text{eff}}$
assuming the same central values $\to 5.3\sigma$ effect

No observation of $\mu \to e\gamma$ in near future

Backup slides

Best fit point excluding $\sin^2\theta_{\text{eff}}^{\text{hadr}}$ and Γ_{inv}^{Z}

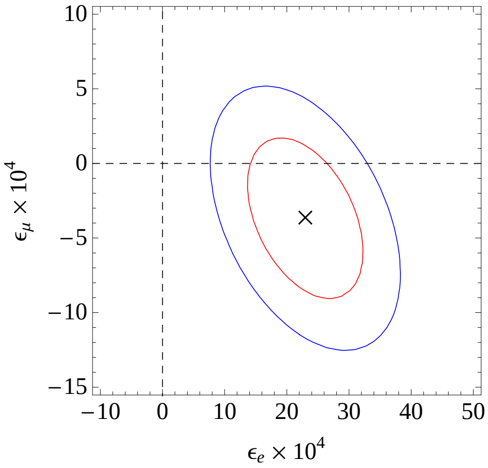
If $m_{h_n} < M_Z/2$, Γ_{inv}^{Z} stays unaffected: has to be removed from the fit

$$\chi^2_\varepsilon = 4.4$$

$$\begin{aligned} \varepsilon_e &= (23.0 \pm 8.5) \times 10^{-4} \\ \varepsilon_\mu &= (-3.7 \pm 4.9) \times 10^{-4} \\ \varepsilon_\tau &= (-7.7 \pm 38.3) \times 10^{-4} \end{aligned}$$

with a correlation matrix of

$$\rho = \begin{pmatrix} 1 & -0.32 & 0.08 \\ -0.32 & 1 & 0.03 \\ 0.08 & 0.03 & 1 \end{pmatrix}$$



If $\varepsilon_\mu \equiv \varepsilon_\tau \equiv 0$: $\chi^2_\varepsilon = 4.9$ and $\varepsilon_e = 20.5 \pm 8.5$ (2.4σ)