Heterotic Line Bundle Models on Smooth Calabi-Yau Manifolds

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Motivation: an Algorithmic Approach to String Pheno

- Higher dim geometry $\longrightarrow$ String compactification $\longrightarrow$ 4d effective theory
- Many constraints from phenomenology: $\mathcal{N} = 1$ SUSY (broken), SM-like particle spectrum, detailed particle phenomenology (Higgs doublets, stable proton, Yukawa couplings etc.), moduli stabilisation. Any particular model is bound to fail.
- Systematic approaches:
  - monad bundles over CICYs (Anderson, Gray, He, Lukas)
  - monad bundles over toric CYs (He, Lee, Lukas)
  - line bundle sums over CICYs (Anderson, AC, Gray, Lukas, Mishra, Palti)
  - line bundle sums over toric CYs (He, Lee, Lukas, Sun)
  - models with no intermediate GUT (Anderson, AC, Lee, Lukas)
- Similar systematic scans for orbifold models (Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, ...) and free fermionic models (Faraggi et al.)

Heterotic Line Bundle Models

We begin with the $E_8 \times E_8$ heterotic string in 10 dimensions, specified by a metric and a non-abelian gauge field. To compactify, we need to specify a space X and a bundle V over that space.

The required geometric data consists of:

- a smooth Calabi-Yau threefold X
- a holomorphic poly-stable line bundle sum $V = L_1 \oplus \ldots \oplus L_5$ on X with $c_1(V) = 0$, such that the structure group is $S(U(1)^5) \subset SU(5) \subset E_8$
- vanishing slopes: $\mu(L_a) = \int_X c_1(L_a) \wedge J^2 = 0$, simultaneously for all $a = 1, \ldots, 5$
- Green-Schwarz anomaly cancellation condition: $\mathrm{ch}_2(V) + \mathrm{ch}_2(V') - \mathrm{ch}_2(TX) = [W]$; in practice, require: $c_2(TX) - c_2(V) \in$ Mori cone of X

The result: Effective field theory: $\mathcal{N} = 1$, 4-dimensional GUT with gauge group $SU(5) \times S(U(1)^5)$ and matter in $\mathbf{10}$, $\overline{\mathbf{10}}$, $\mathbf{5}$, $\overline{\mathbf{5}}$, $\mathbf{1}$

Heterotic Line Bundle Models – continued

GUT $\longrightarrow$ Standard Model. The required geometric data consists of:

- a freely-acting discrete symmetry Γ, such that X/Γ is non-simply connected;
- an equivariant structure on V, such that $V \longrightarrow X$ descends to a bundle $\widetilde{V} \longrightarrow X/\Gamma$
- complete the bundle $\widetilde{V}$ with a discrete Wilson line to $\widetilde{V} \oplus W$ in order to break the GUT group

The result: Standard-like model with gauge group $G_{\text{SM}} \times S(U(1)^5)$

The 4D Effective Field Theory

Gauge group: $SU(5) \times S(U(1)^5)$. Extra $U(1)$s G-S anomalous in general.

Matter multiplets: $\mathbf{10}_a$, $\overline{\mathbf{10}}_a$, $\mathbf{5}_{a,b}$, $\overline{\mathbf{5}}_{a,b}$, $\mathbf{1}_{a,b}$

multiplet	$S(U(1)^5)$ charge	bundle	total number	required
$\mathbf{10}_a$	e_a	V	$\sum_a h^1(X, L_a)$	$3\|\Gamma\|$
$\overline{\mathbf{10}}_a$	$-\mathrm{e}_a$	V^*	$\sum_a h^1(X, L_a^*)$	0
$\overline{\mathbf{5}}_{a,b}$	$\mathrm{e}_a + \mathrm{e}_b$	$\wedge^2 V$	$\sum_{a<b} h^1(X, L_a \otimes L_b)$	$3\|\Gamma\| + n_H$
$\mathbf{5}_{a,b}$	$-\mathrm{e}_a - \mathrm{e}_b$	$\wedge^2 V^*$	$\sum_{a<b} h^1(X, L_a^* \otimes L_b^*)$	n_H
$\mathbf{1}_{a,b}$	$\mathrm{e}_a - \mathrm{e}_b$	$V \otimes V^*$	$\sum_{a,b} h^1(X, L_a \otimes L_b^*)$	n_H

Recall: $V = \bigoplus_{a=1}^5 L_a$

$\mathbf{1}_{a,b}$: singlets under $SU(5)$ (G_{SM} after quotienting); bundle moduli

$\langle \mathbf{1}_{a,b} \rangle = 0$: line bundle sum; $\langle \mathbf{1}_{a,b} \rangle \neq 0$: non-Abelian bundle

Also: explore the moduli space of non-Abelian bundles by explicitly constructing bundles which split into a sum of line bundles

The 4D Effective Field Theory - continued

The $U(1)$ symmetries constrain the superpotential

$$W = \mu\, H\bar{H} + Y^{(d)}_{pq}\, H\, \bar{\mathbf{5}}^p \mathbf{10}^q + Y^{(u)}_{pq}\, \bar{H}\, \mathbf{10}^p \mathbf{10}^q + \\ + \rho_p\, \bar{H} L^p + \lambda_{pqr}\, \bar{\mathbf{5}}^q \bar{\mathbf{5}}^q \mathbf{10}^r + \\ + \lambda'_{pqrs}\, \bar{\mathbf{5}}^p \mathbf{10}^q \mathbf{10}^r \mathbf{10}^s + \ldots$$

Example: $\mu = \mu_0 + \mu_{1,\alpha} \mathbf{1}^{\alpha}_{a,b} + \mu_{2,\alpha,\beta} \mathbf{1}^{\alpha}_{a,b} \mathbf{1}^{\beta}_{c,d} + \ldots + \mu_{\mathrm{np}}$

$\mu_0 = 0$ by construction; $\mu_1 = 0$ due to the $U(1)$s

A Comprehensive Scan

The manifolds: complete intersection CY threefolds (CICYs) – common zero locus of homogeneous polynomials in products of projective spaces (Candelas, Green, Hübsch, Lütken)

Select those that are known to admit a freely-acting discrete symm (Braun) and are favourable: 68 CICYs with $h^{1,1}(X) < 7$.

The bundles: Line bundles are classified by their first Chern class:

$$c_1(L) = k^i J_i$$

with $1 \leq i \leq h^{1,1}(X)$ and $k^i \in \mathbb{Z}$. Describe a rank 5 line bundle sum

$$V = \bigoplus_{a=1}^{5} L_a = \bigoplus_{a=1}^{5} \mathcal{O}(\vec{k}_a), \qquad \text{where} \quad \vec{k}_a = (k_a^1, \ldots, k_a^{h^{1,1}(X)})$$

by $5 \times h^{1,1}(X)$ integers. For $-k_{\text{max}} \leq k_a^i \leq k_{\text{max}}$, one has many choices:

$$(2k_{\text{max}} + 1)^{h^{1,1}(X)}$$

A Comprehensive Scan – Results

We have scanned over $\sim 10^{40}$ bundles. This was possible only to the fact that many constraints (e.g.: from stability, index constraints from the spectrum) can be imposed along the way, before even constructing the whole line bundle sum.

Imposing the constraints for a consistent susy string vacuum and the index constraints that lead to a correct chiral asymmetry we found:

$h^{1,1}(X)$	1	2	3	4	5	6	All
No. models	0	0	6	552	21731	41036	63325

In addition, requiring the absence of $\overline{\mathbf{10}}$-multiplets and the presence of at least one $H - \overline{H}$ pair, led to:

$$34,989 \text{ models}$$

Roughly, the number of models per CY increases by one order of magnitude for each additional Kähler parameter.

A heterotic Standard Model on the Tetraquadric CY

Calabi-Yau data:

$$X = \begin{array}{c} \mathbb{CP}^1 \\ \mathbb{CP}^1 \\ \mathbb{CP}^1 \\ \mathbb{CP}^1 \end{array} \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \end{array} \right]^{4,68}_{-128} \quad ; \quad \Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$$

Bundle data:

$$(k_a^i) = \left[\begin{array}{ccccc} -1 & -1 & 0 & 1 & 1 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 2 & -1 & -1 & 0 \\ 1 & 2 & 0 & -1 & -2 \end{array} \right] \quad + \quad \text{equivariant structure}$$

Cohomologies and the GUT spectrum:

$$\begin{array}{lclclcl}
h^\bullet(X, L_2) & = & (0,8,0,0) & , & h^\bullet(X, L_5) & = & (0,4,0,0) \\
h^\bullet(X, L_2 \otimes L_4) & = & (0,4,0,0) & , & h^\bullet(X, L_2 \otimes L_5) & = & (0,3,3,0) \\
h^\bullet(X, L_4 \otimes L_5) & = & (0,8,0,0) & , & h^\bullet(X, L_1 \otimes L_2^*) & = & (0,0,12,0) \\
h^\bullet(X, L_1 \otimes L_5^*) & = & (0,0,12,0) & , & h^\bullet(X, L_2 \otimes L_3^*) & = & (0,20,0,0) \\
h^\bullet(X, L_2 \otimes L_4^*) & = & (0,12,0,0) & , & h^\bullet(X, L_3 \otimes L_5^*) & = & (0,0,4,0)
\end{array}$$

$8\,\mathbf{10}_2\,,\ 4\,\mathbf{10}_5\,,\ 4\,\overline{\mathbf{5}}_{2,4}\,,\ 3\,\overline{\mathbf{5}}_{2,5}\,,\ 8\,\overline{\mathbf{5}}_{4,5}\,,\ 3\,\mathbf{5}_{2,5}\,,\ 12\,\mathbf{1}_{2,1}\,,\ 12\,\mathbf{1}_{5,1}\,,\ 20\,\mathbf{1}_{2,3}\,,\ 12\,\mathbf{1}_{2,4}\,,\ 4\,\mathbf{1}_{5,3}$

After quotienting and including an appropriate WIlson line:

$2\,\mathbf{10}_2\,,\ \mathbf{10}_5\,,\ \overline{\mathbf{5}}_{2,4}\,,\ 2\,\overline{\mathbf{5}}_{4,5}\,,\ H_{2,5}\,,\ \overline{H}_{2,5}\,,\ 3\,\mathbf{1}_{2,1}\,,\ 3\,\mathbf{1}_{5,1}\,,\ 5\mathbf{1}_{2,3}\,,\ 3\,\mathbf{1}_{2,4}\,,\ \mathbf{1}_{5,3}$

Allowed Operators

The superpotential in GUT notation:

$$W = \lambda_{IJK}\mathbf{5}_{2,5}^{(I)}\mathbf{10}_{2}^{(J)}\mathbf{10}_{5}^{(K)} + \rho_{IJK}\mathbf{1}_{2,4}^{(I)}\overline{\mathbf{5}}_{4,5}^{(J)}\mathbf{5}_{2,5}^{(K)} ,$$

and at the Standard Model level:

$$W = \lambda_i \overline{H}_{2,5}(Q_2^{(i)}u_5 + Q_5 u_2^{(i)}) + \rho_{\alpha i}\mathbf{1}_{2,4}^{(\alpha)}L_{4,5}^{(i)}\overline{H}_{2,5} ,$$

where $i = 1, 2$ labels the two $\mathbf{10}_2$ families and the two lepton doublets $L_{4,5}$ from the two $\overline{\mathbf{5}}_{4,5}$ multiplets and $\alpha = 1, 2, 3$ labels the three singlets $\mathbf{1}_{2,4}$.

Features of the Model

- SM gauge group with 4 (global) $U(1)$s
- exact MSSM spectrum with no exotics charged under G_{SM}
- one pair of Higgs doublets which remains massless everywhere in the moduli space, except for the directions $\langle \mathbf{1}_{2,4} \rangle \neq 0$
- hierarchy of Yukawa couplings (at the perturbative level):

$$Y^u = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad Y^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- no dim 4 or 5 proton decay operators - valid throughout the moduli space

Non-Abelian Deformations

We have explicitly constructed non-Abelian deformations of V to explore the entire moduli space. The EFT expectations regarding the fate of the Higgs doublets are confirmed when looking at the spectra of the non-Abelian models.

Singlets for the model:

$$3\,\mathbf{1}_{2,1}\,,\;3\,\mathbf{1}_{5,1}\,,\;5\mathbf{1}_{2,3}\,,\;3\,\mathbf{1}_{2,4}\,,\;\mathbf{1}_{5,3}$$

Look more closely into the region $\langle \mathbf{1}_{2,4} \rangle = 0$. The bundles in this region can be described by monads. They lead to models with gauge group $G_{\text{SM}} \times U_X(1)$:

- $U_X(1)$ leads to a (global) $U_{B-L}(1)$
- Higgs doublets still massless
- dim 4 proton decay operators forbidden by $U_{B-L}(1)$
- dim 5 proton decay operators forbidden by the existence of the line bundle locus!

Conclusions and Outlook

- Interesting phenomenology can be achieved with line bundle models. These also provide an accessible window in the larger moduli space of non-Abelian bundles. $U(1)$ symmetries constrain the Lagrangian.
- The scan exhausted the class of line bundle models with an underlying $SU(5)$ GUT: $35,000$ models. We expect a much larger number of SMs.
- Much work remains to be done. Technical difficulties related to computing line bundle cohomology and enumerating all possible equivariant structures for a given (X, V, Γ).
- The Higgs fields can remain massless away from the Abelian locus.
- We observed the unexpected absence of operators due to the enhanced symmetry locus. How can this be explained? Does string theory produce EFTs with non-generic couplings?
- Why is the number of poly-stable bundles with $c_1(V) = 0$, $c_2(V)$ constrained by the anomaly cancellation condition and $c_3(V)$ fixed by the number of families, finite? Can this be related to some type of Donaldson-Thomas invariants?