

Effect of interaction terms on particle production due to time-varying mass

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Outline

1. Introduction
2. How to calculate particle number
3. Calculation of particle number
4. Summary

1. Introduction

■ Particle production from vacuum

■ It is known that a varying background causes production of particles

■ Oscillating Electric field → pair production of electrons

[E. Brezin and C. Itzykson, *Phys. Rev. D* **2**, 1191 (1970)]

■ Changing metric → gravitational particle production

[L. Parker, *Phys. Rev.* **183**, 1057 (1969)]

[L. H. Ford, *Phys. Rev. D* **35**, 2955 (1987)]

■ Oscillating inflaton → (p)reheating

[L. Kofman, A. D. Linde, A. A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994)]

[L. Kofman, A. D. Linde, A. A. Starobinsky, *Phys. Rev. D* **56**, 3258 (1997)]

■ etc...

■ Example of scalar particle production

g : coupling

■ Let us consider : $\mathcal{L}_{int} = -\frac{1}{2}g^2|\phi|^2\chi^2$

ϕ : complex scalar field
(classical)

χ : real scalar particle
(quantum)

■ If ϕ goes near the origin, χ particles are produced

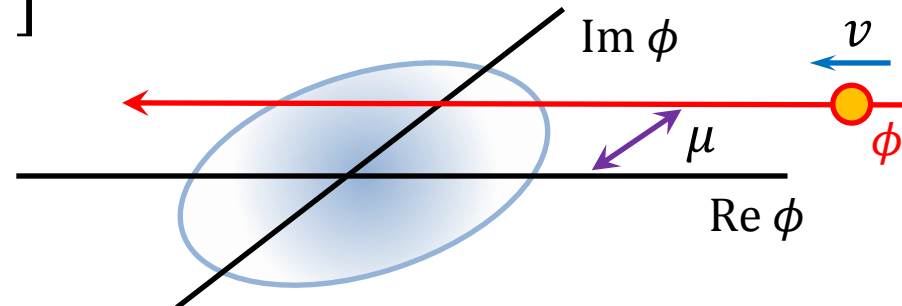
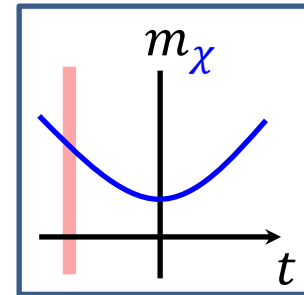
■ Because

→ mass of χ ($m_\chi = g\phi$) becomes small around $|\phi| = 0$

→ kinetic energy of ϕ converts to χ particles

■ produced occupation number :

$$n_{\chi k} = V \cdot \exp \left[-\pi \frac{k^2 + g^2 \mu^2}{g v} \right]$$



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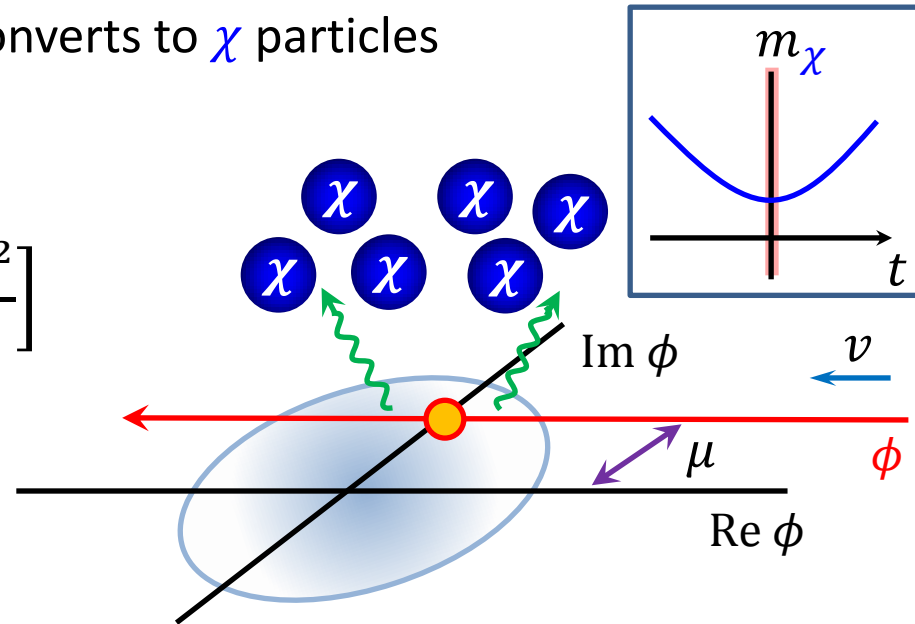
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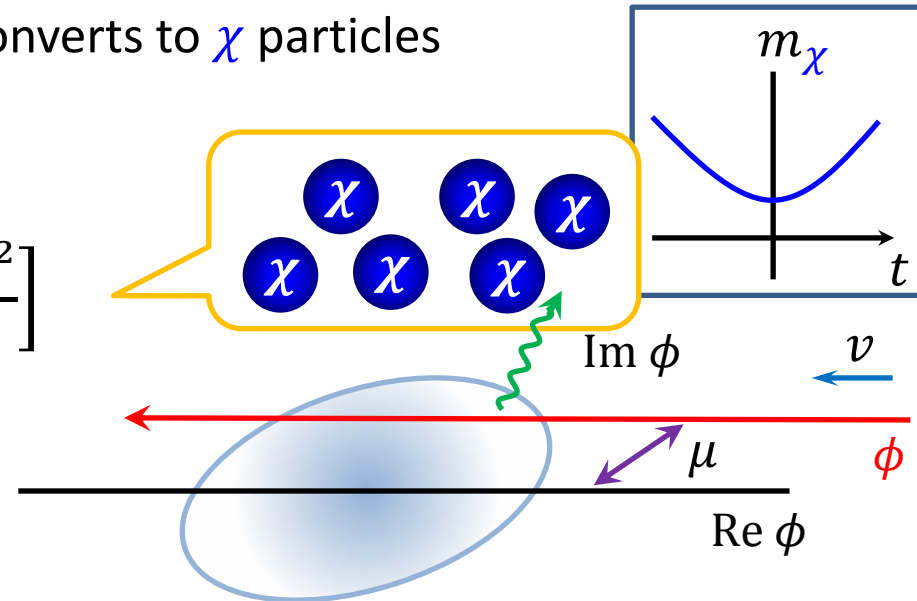
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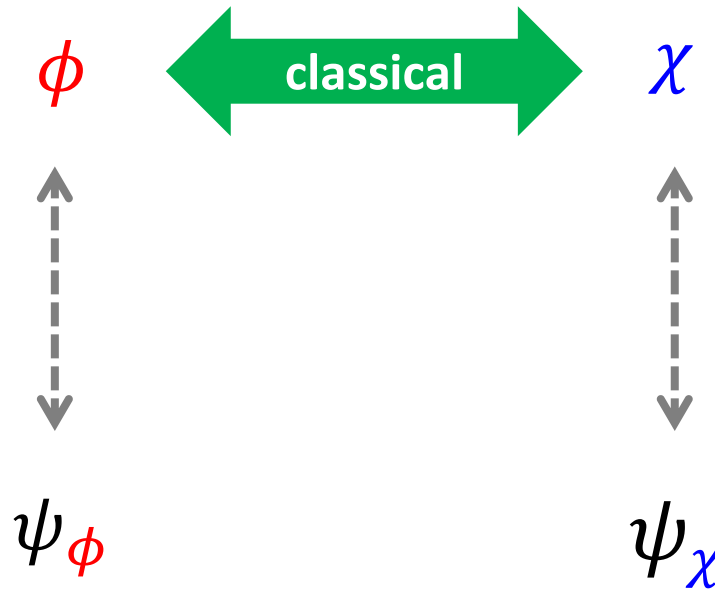
■ Our interests

1. How about supersymmetric model?
 - What is the role of the superpartner of the background field?
2. How do (quantum) interaction terms affect particle production?
 - Usually production rates are calculated in the purely classical background
 - We would like to estimate the contribution of the quantum interaction term



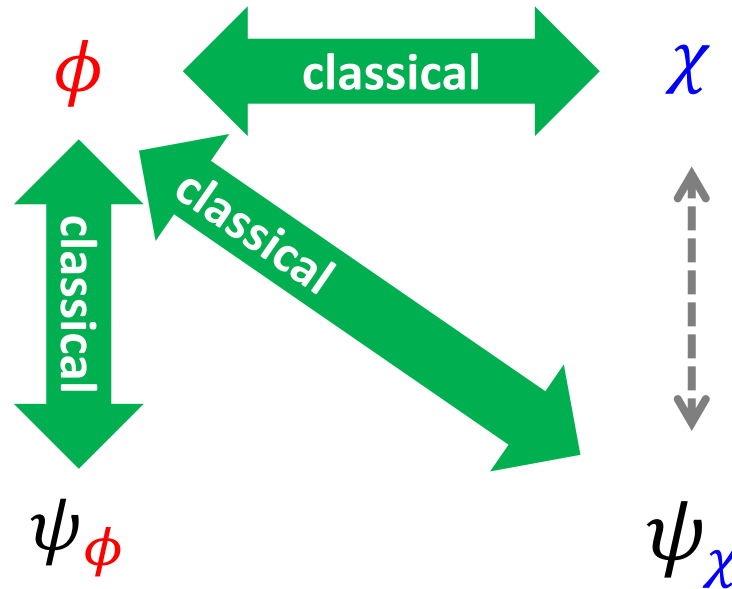
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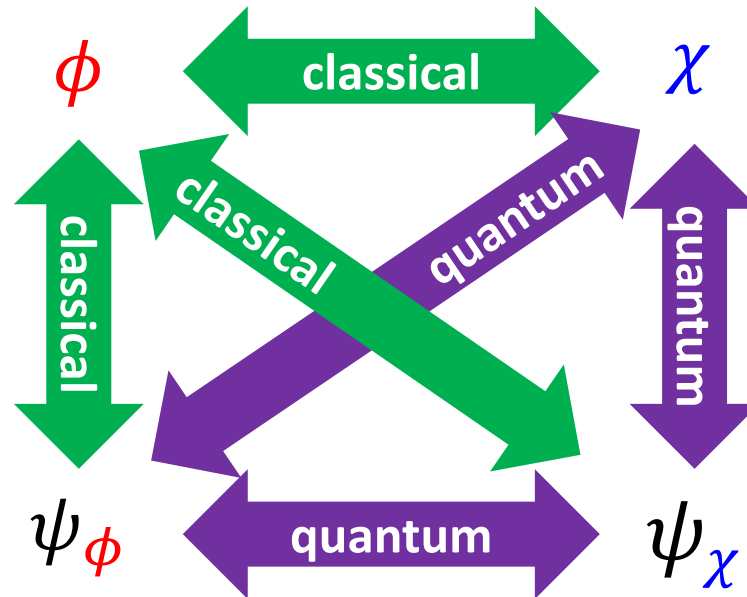
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Model in this talk

Super potential :

$$W = \frac{1}{2} g \Phi X^2$$

$$\Phi = \phi + \sqrt{2}\theta\psi_\phi + \theta^2 F_\phi$$

$$X = \chi + \sqrt{2}\theta\psi_\chi + \theta^2 F_\chi$$

g : coupling

→ Interaction terms in components :

$$\mathcal{L}_{int} = -g^2 |\phi|^2 |\chi|^2 - \frac{1}{4} g^2 |\chi|^4 - g \left(\frac{1}{2} \phi \psi_\chi \psi_\chi + \psi_\phi \psi_\chi \chi + (h.c.) \right)$$

Stationary point :

- $\chi = \psi_\phi = \psi_\chi = 0$, but ϕ can have **any** value

Masses

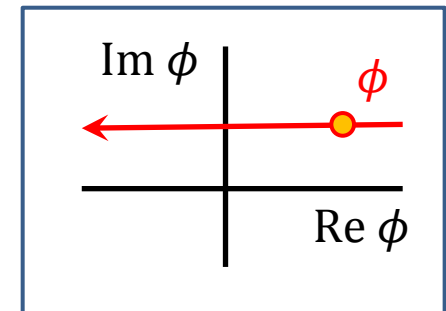
Production
is possible

impossible

- $\phi \neq 0 \rightarrow \chi, \psi_\chi$: mass = $g\phi$, ψ_ϕ : massless

- However, ψ_ϕ 's mass may be influenced by ϕ through loop effects...?

- Is quantum part of ϕ also influenced?



■ Equations of Motion for field operators :

$$\phi : 0 = (\partial^2 + g^2|\chi|^2)\phi + \frac{1}{2}g\psi_\chi^\dagger\psi_\chi^\dagger$$

$$\chi : 0 = \left(\partial^2 + g^2|\phi|^2 + \frac{1}{2}g^2|\chi|^2\right)\chi + g\psi_\phi^\dagger\psi_\chi^\dagger$$

$$\psi_\phi : 0 = \bar{\sigma}^\mu\partial_\mu\psi_\phi + ig\chi^*\psi_\chi^\dagger$$

$$\psi_\chi : 0 = \bar{\sigma}^\mu\partial_\mu\psi_\chi + ig\phi^*\psi_\chi^\dagger + ig\chi^*\psi_\phi^\dagger$$



How do we calculate produced particle number including interaction term ?

2. How to calculate particle number

■ Definition of (occupation) number :

$$n_k = \langle 0^{\text{in}} | a_{\mathbf{k}}^{\text{out}\dagger} a_{\mathbf{k}}^{\text{out}} | 0^{\text{in}} \rangle$$

→ Information about in-state (@ far past) and out-state (@ far future) of field needs for the calculation

■ How are they related to each other?

(in-state)  (out-state)

→ Asymptotic field expansion

■ Example in a simple case (scalar field)

■ Operator field equation : $0 = (\partial^2 + M^2)\phi + J[\phi]$

■ Commutation relation : $[\phi(\mathbf{x}), \dot{\phi}(\mathbf{y})] = i\hbar\delta^3(\mathbf{x} - \mathbf{y})$

→ Formal solution (**Yang-Feldman equations**)

$$\begin{aligned}\phi(x) &= \sqrt{Z}\phi^{\text{as}}(x) - i \int_{t^{\text{as}}}^{x^0} dy^0 \int d^3y [\phi^{\text{as}}(x), \phi^{\text{as}}(y)] J[\phi(y)] \\ &= \sqrt{Z}\phi^{\text{as}}(x) - i \int_{t^{\text{as}}}^{x^0} dy^0 \int d^3y [\phi^{\text{as}}(x), \phi^{\text{as}}(y)] J[\sqrt{Z}\phi^{\text{as}}(y)] + \dots\end{aligned}$$

Z : some const.

ϕ^{as} : asymptotic field
 $0 = (\partial^2 + M^2)\phi^{\text{as}}$

■ If we take $t^{\text{as}} = t^{\text{in}} = -\infty$ or $t^{\text{as}} = t^{\text{out}} = +\infty$,

$$\phi^{\text{out}}(x^{\text{out}}) = \phi^{\text{in}}(x^{\text{out}}) - \frac{i}{\sqrt{Z}} \int d^4y [\phi^{\text{in}}(x^{\text{out}}), \phi^{\text{in}}(y)] J[\sqrt{Z}\phi^{\text{in}}(y)] + \dots$$

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$a_{\mathbf{k}}^{\text{out}}$

$a_{\mathbf{k}}^{\text{in}}$

field
as

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■ Example in a simple case (scalar field)

- ϕ^{as} is free particle, so we can expand with plane waves as

$$\phi^{\text{as}}(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\underbrace{\phi_k^{\text{as}}(x^0)}_{\text{plane wave}} \underbrace{a_{\mathbf{k}}^{\text{as}}}_{\text{(time dependent) wave func.}} + \underbrace{\phi_k^{\text{as}*}(x^0)}_{\text{(time dependent) wave func.}} \underbrace{a_{-\mathbf{k}}^{\text{as}\dagger}}_{\text{creation/annihilation op.}} \right)$$

plane wave

(time dependent) wave func.

creation/annihilation op.

$$0 = \ddot{\phi}_k^{\text{as}} + (\mathbf{k}^2 + M^2)\phi_k^{\text{as}}$$

- inner product relation : $\dot{\phi}_k^{\text{as}*} \phi_k^{\text{as}} - \phi_k^{\text{as}*} \dot{\phi}_k^{\text{as}} = i/Z$

$$\left(\begin{array}{l} \text{which comes from conditions} \\ [a_{\mathbf{k}}^{\text{as}}, a_{\mathbf{k}'}^{\text{as}\dagger}] = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}'), \quad Z[\phi^{\text{as}}(\mathbf{x}), \dot{\phi}^{\text{as}}(\mathbf{y})]_{t \rightarrow t^{\text{as}}} = i\hbar \delta^3(\mathbf{x} - \mathbf{y}) \end{array} \right)$$

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$$a_{\mathbf{k}}^{\text{as}} = -iZ \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} (\dot{\phi}_k^{\text{as}*} \phi^{\text{as}} - \phi_k^{\text{as}*} \dot{\phi}^{\text{as}})$$

■ Example in a simple case (scalar field)

■ Relation between $a_{\mathbf{k}}^{\text{in}}$ and $a_{\mathbf{k}}^{\text{out}}$

$$a_{\mathbf{k}}^{\text{out}} = -iZ \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} (\dot{\phi}_k^{\text{out}*} \phi^{\text{out}} - \phi_k^{\text{out}*} \dot{\phi}^{\text{out}})$$

$$\phi^{\text{out}}(x^{\text{out}}) = \phi^{\text{in}}(x^{\text{out}}) - \frac{i}{\sqrt{Z}} \int d^4y [\phi^{\text{in}}(x^{\text{out}}), \phi^{\text{in}}(y)] J[\sqrt{Z}\phi^{\text{in}}(y)] + \dots$$

$$a_{\mathbf{k}}^{\text{out}} = \alpha_k a_{\mathbf{k}}^{\text{in}} + \beta_k a_{-\mathbf{k}}^{\text{in}\dagger} - \frac{i}{\sqrt{Z}} \int d^4x e^{-i\mathbf{k}\cdot\mathbf{x}} (\alpha_k \phi_k^{\text{in}*} - \beta_k \phi_k^{\text{in}}) J[\sqrt{Z}\phi^{\text{in}}] + \dots$$

(usual) Bogoliubov tf low

Interaction effects

$$\left(\begin{array}{l} \alpha_k \equiv -iZ (\dot{\phi}_k^{\text{out}*} \phi_k^{\text{in}} - \phi_k^{\text{out}*} \dot{\phi}_k^{\text{in}}) \\ \beta_k \equiv -iZ (\dot{\phi}_k^{\text{out}*} \phi_k^{\text{in}*} - \phi_k^{\text{out}*} \dot{\phi}_k^{\text{in}*}) \end{array} \right) \Rightarrow \left(\begin{array}{l} \phi_k^{\text{in}} = \alpha_k \phi_k^{\text{out}} + \beta_k \phi_k^{\text{out}*} \\ \phi_k^{\text{out}} = \alpha_k^* \phi_k^{\text{in}} - \beta_k^* \phi_k^{\text{in}*} \\ |\alpha_k|^2 - |\beta_k|^2 = 1 \end{array} \right)$$

■ Example in a simple case (scalar field)

■ Produced (occupation) number :

$$\begin{aligned}n_k &= \langle 0^{\text{in}} | a_{\mathbf{k}}^{\text{out}\dagger} a_{\mathbf{k}}^{\text{out}} | 0^{\text{in}} \rangle \\&= \left| \left(\beta_k a_{-\mathbf{k}}^{\text{in}\dagger} - \frac{i}{\sqrt{Z}} \int d^4x e^{-i\mathbf{k}\cdot\mathbf{x}} (\alpha_k \phi_k^{\text{in}*} - \beta_k \phi_k^{\text{in}}) J[\sqrt{Z}\phi^{\text{in}}] + \dots \right) | 0^{\text{in}} \rangle \right|^2 \\&= \begin{cases} V \cdot |\beta_k|^2 + \dots & \left[\beta_k \neq 0 \right] \\ 0 + \frac{1}{Z} \left| \int d^4x e^{-i\mathbf{k}\cdot\mathbf{x}} \phi_k^{\text{in}*} J[\sqrt{Z}\phi^{\text{in}}] | 0^{\text{in}} \rangle \right|^2 + \dots & \left[\beta_k = 0 \right] \end{cases}\end{aligned}$$

→ Particles can be produced even if $\beta_k = 0$!

3. Calculation of particle number

Equation of Motion (again) :

$$\phi : 0 = (\partial^2 + g^2|\chi|^2)\phi + \frac{1}{2}g\psi_\chi^\dagger\psi_\chi^\dagger$$

$$\chi : 0 = \left(\partial^2 + g^2|\phi|^2 + \frac{1}{2}g^2|\chi|^2\right)\chi + g\psi_\phi^\dagger\psi_\phi^\dagger$$

$$\psi_\phi : 0 = \bar{\sigma}^\mu\partial_\mu\psi_\phi + ig\chi^*\psi_\chi^\dagger$$

$$\psi_\chi : 0 = \bar{\sigma}^\mu\partial_\mu\psi_\chi + ig\phi^*\psi_\chi^\dagger + ig\chi^*\psi_\phi^\dagger$$

ϕ : macroscopic

$\chi, \psi_\phi, \psi_\chi$: microscopic

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EOM for asymptotic fields (as = in, out):

$$\phi^{\text{as}} : 0 = \partial^2\phi^{\text{as}}$$

$$\chi^{\text{as}} : 0 = (\partial^2 + g^2|\langle\phi^{\text{as}}\rangle|^2)\chi^{\text{as}}$$

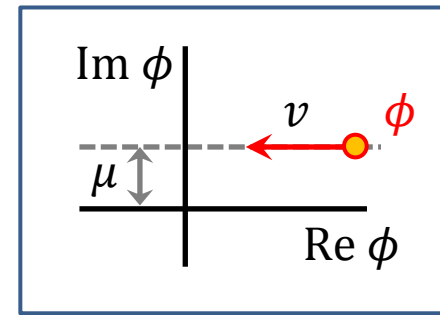
$$\psi_\phi^{\text{as}} : 0 = \bar{\sigma}^\mu\partial_\mu\psi_\phi^{\text{as}}$$

$$\psi_\chi^{\text{as}} : 0 = \bar{\sigma}^\mu\partial_\mu\psi_\chi^{\text{as}} + ig\langle\phi^*\rangle\psi_\chi^{\text{as}\dagger}$$

Solutions for Asymptotic fields

Assuming $\langle \phi^{\text{in}} \rangle = \langle \phi^{\text{in}} \rangle(t)$ for simplicity, then

$$\phi^{\text{as}} = \langle \phi^{\text{as}} \rangle + \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\phi_k^{\text{as}} a_{\phi\mathbf{k}}^{\text{as}} + \phi_k^{\text{as}*} b_{\phi-\mathbf{k}}^{\text{as}\dagger} \right)$$



$$0 = \partial^2 \phi^{\text{as}}$$

$$\sqrt{Z_\phi} \langle \phi^{\text{in}} \rangle = vt + i\mu, \quad \sqrt{Z_\phi} \phi_k^{\text{in}} = \sqrt{Z_\phi} \phi_k^{\text{out}} = \frac{1}{\sqrt{2|\mathbf{k}|}} e^{-i|\mathbf{k}|t}$$

$$\rightarrow \beta_{\phi k} = 0$$

$$\phi_k^{\text{out}} = \alpha_k^* \phi_k^{\text{in}} - \beta_k^* \phi_k^{\text{in}*}$$

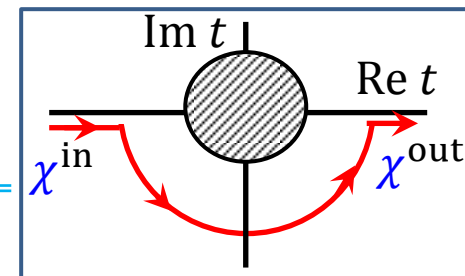
$$\chi^{\text{as}} = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\chi_k^{\text{as}} a_{\chi\mathbf{k}}^{\text{as}} + \chi_k^{\text{as}*} b_{\chi-\mathbf{k}}^{\text{as}\dagger} \right)$$

$$\sqrt{Z_\chi} \chi_k^{\text{in}} \sim \frac{1}{\sqrt{2\omega_k}} \exp \left[-i \int^t dt' \omega_k(t') \right] \quad (\text{valid for } |t| \gtrsim 1/\sqrt{g\nu})$$

$$= (\partial^2 + g^2 |\langle \phi^{\text{as}} \rangle|^2) \chi^{\text{as}}$$

$$\left(\omega_k \equiv \sqrt{\mathbf{k}^2 + Z_\phi g^2 |\langle \phi^{\text{in}} \rangle|^2} \right)$$

$$(\text{analytic continuation}) \rightarrow \beta_{\chi k} \sim -i \exp \left[-\pi \frac{k^2 + g^2 \mu^2}{2g\nu} \right]$$



Solutions for Asymptotic fields

eigen spinor for helicity op. :

$$k^i \bar{\sigma}^i e_{\mathbf{k}}^{\pm} = \pm |\mathbf{k}| \bar{\sigma}^0 e_{\mathbf{k}}^{\pm}$$

$$\psi_{\phi}^{\text{as}} = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(e_{\mathbf{k}}^{+} \psi_{\phi k}^{+, \text{as}} a_{\psi_{\phi} \mathbf{k}}^{+ \text{as}} + e_{\mathbf{k}}^{-} \psi_{\phi k}^{-, \text{as}*} a_{\psi_{\phi} -\mathbf{k}}^{- \text{as}\dagger} \right)$$

$$\sqrt{Z_{\phi}} \psi_{\phi k}^{\pm, \text{in}} = \sqrt{Z_{\phi}} \psi_{\phi k}^{\pm, \text{out}} = e^{-i|\mathbf{k}|t}$$

$$0 = \bar{\sigma}^{\mu} \partial_{\mu} \psi_{\phi}^{\text{as}}$$

$$\Rightarrow \underline{\underline{\beta_{\psi_{\phi} k}^{\pm} = 0}}$$

$$\psi_{\chi}^{\text{as}} = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{s=\pm} \left(e_{\mathbf{k}}^s \psi_{\chi k}^{(+), s, \text{as}} a_{\psi_{\chi} \mathbf{k}}^s - \sigma^0 e_{-\mathbf{k}}^{s\dagger} \psi_{\chi k}^{(-), s, \text{as}*} a_{\psi_{\chi} -\mathbf{k}}^{s\dagger} \right)$$

$$\sqrt{Z_{\chi}} \psi_{\chi k}^{(\pm), s, \text{in}} \sim \frac{1}{2} \left(\sqrt{1 + \frac{gvt}{\omega_k}} \pm \sqrt{1 - \frac{gvt}{\omega_k}} e^{i\theta_k^s} \right) \exp \left[-i \int^t dt' \omega_k(t') \right]$$

$$0 = \bar{\sigma}^{\mu} \partial_{\mu} \psi_{\chi}^{\text{as}} + ig \langle \phi^* \rangle \psi_{\chi}^{\text{as}\dagger}$$

(valid for $|t| \gtrsim 1/\sqrt{g\nu}$)

$$\left(\omega_k \equiv \sqrt{\mathbf{k}^2 + Z_{\phi} g^2 |\langle \phi^{\text{in}} \rangle|^2}, \quad e^{i\theta_k^s} \equiv \frac{s|\mathbf{k}| - ig\mu}{\sqrt{\mathbf{k}^2 + g^2 \mu^2}} \right)$$

$$\text{(analytic continuation)} \Rightarrow \underline{\underline{\beta_{\psi_{\chi} k}^s \sim -s \cdot \exp \left[-\pi \frac{k^2 + g^2 \mu^2}{2g\nu} \right]}}$$

■ Produced Particle number

$$\blacksquare n_{\chi k} = |\beta_{\chi k}|^2 + \dots = \underline{\underline{\exp\left[-\pi \frac{k^2 + g^2 \mu^2}{gv}\right]}} + \dots$$

$$\blacksquare n_{\psi_{\chi} k} = \sum_{s=\pm} |\beta_{\psi_{\chi} k}^s|^2 + \dots = \underline{\underline{2 \times \exp\left[-\pi \frac{k^2 + g^2 \mu^2}{gv}\right]}} + \dots$$

→ leading term is obtained

$$\blacksquare n_{\phi k} = |\beta_{\phi k}|^2 + \dots = \underline{\underline{0}} + \dots$$

$$\blacksquare n_{\psi_{\phi} k} = \sum_{s=\pm} |\beta_{\psi_{\phi} k}^s|^2 + \dots = \underline{\underline{0}} + \dots$$

 Focus on!

→ need to calculate next to leading order

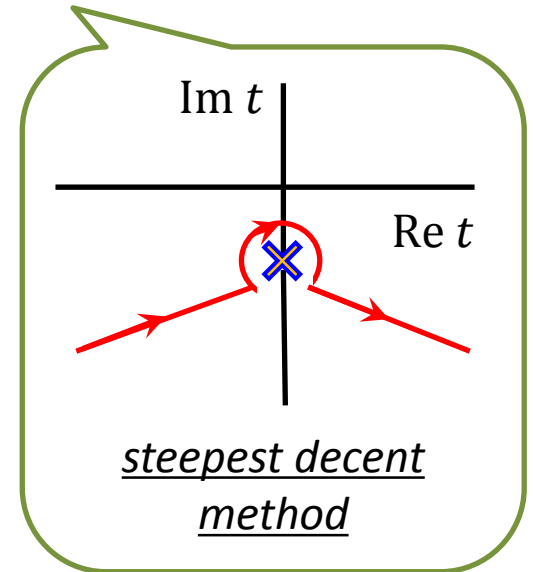
■ Calculation of $n_{\psi_{\phi k}}$

$$\begin{aligned}
 n_{\psi_{\phi k}} &= \sum_{s=\pm} \left\langle 0^{\text{in}} \left| a_{\psi_{\phi k}}^{s,\text{out}\dagger} a_{\psi_{\phi k}}^{s,\text{out}} \right| 0^{\text{in}} \right\rangle \\
 &= \frac{g^2 Z_{\chi}^2}{Z_{\phi}} \sum_{s=\pm} \left| \int d^4 x e^{-i\mathbf{k}\cdot\mathbf{x}} \psi_{\phi k}^{\text{in}*} \cdot \chi^{\text{in}*} \cdot e_{\mathbf{k}}^{s\dagger} \psi_{\chi}^{\text{in}\dagger} \right| 0^{\text{in}} \rangle \Big|^2 + \dots \\
 &\sim V \cdot \frac{g^2 Z_{\chi}^2}{Z_{\phi}} \int \frac{d^3 p}{(2\pi)^3} \sum_{s,r} \left(1 - sr \frac{\mathbf{p}\cdot\mathbf{k}}{pk} \right) \left| \int dt \psi_{\phi k}^{\text{in}*} \chi_{|\mathbf{k}+\mathbf{p}|}^{\text{in}} \psi_{\chi p}^{(s)r,\text{in}} \right|^2
 \end{aligned}$$

$$\sqrt{Z_{\phi}} \psi_{\phi k}^{\text{in}} = e^{-i|\mathbf{k}|t}$$

$$\sqrt{Z_{\chi}} \chi_k^{\text{in}} \sim \frac{1}{\sqrt{2\omega_k}} e^{-i \int^t dt' \omega_k(t')}$$

$$\sqrt{Z_{\chi}} \psi_{\chi k}^{(\pm)s,\text{in}} \sim \frac{1}{2} \left(\sqrt{1 + \frac{gvt}{\omega_k}} \pm \sqrt{1 - \frac{gvt}{\omega_k}} e^{i\theta_k^s} \right) e^{-i \int^t dt' \omega_k(t')}$$



■ This integral is too complicated to perform

→ We estimated in special case $\mathbf{k} = 0$ with ***steepest decent method***

■ Result

$$n_{\psi_{\phi} k=0}/V = \frac{C}{Z_{\phi}^2} \cdot \frac{g^2}{4\pi} \cdot \exp \left[-\pi \frac{g^2 \mu^2}{g v} \right]$$

$$\left(C = \left(\frac{5}{9\pi e} \right)^{5/6} \frac{\Gamma(1/3)^2}{3^{1/3} \cdot 2\pi} \sim 0.06765 \right)$$

$$\text{(c.f.) } n_{\chi k}/V \sim \exp \left[-\pi \frac{k^2 + g^2 \mu^2}{g v} \right], \quad n_{\psi_{\chi} k}/V \sim 2 \times \exp \left[-\pi \frac{k^2 + g^2 \mu^2}{g v} \right]$$

■ Produced number of ψ_{ϕ} is suppressed by factor g^2 comparing with χ or ψ_{χ}

■ This results is consistent with perturbativity

■ Z_{ϕ} can be removed by redefinition : $g \rightarrow g Z_{\phi}, \mu \rightarrow \mu / Z_{\phi}, v \rightarrow v / Z_{\phi}$

4. Summary

1. We constructed the Bogoliubov transformation taking into account interaction effects
2. We calculated produced particle's (occupation) number

■ $n_{\chi k}/V \sim \exp\left[-\pi \frac{k^2 + g^2 \mu^2}{g v}\right], \quad n_{\psi \chi k}/V \sim 2 \times \exp\left[-\pi \frac{k^2 + g^2 \mu^2}{g v}\right]$

■ $n_{\psi \phi k=0}/V \sim 0.07 \cdot \frac{g^2}{4\pi} \cdot \exp\left[-\pi \frac{g^2 \mu^2}{g v}\right] \neq 0$

- Massless particle can be produced, however the production is suppressed by the coupling

■ $n_{\phi k=0}/V \dots$ now in progress

- In the case of exact SUSY one may find the same result
- It would be interesting to see modification induced by supersymmetry breaking terms

- this is issue under investigation