Resurrecting the minimal renormalizable supersymmetric SU(5)model

work in progress with B. Bajc and S. Lavignac

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Planck 2014

Beyond the Standard Model

Standard Model of *electroweak* and *strong interactions* despite its astonishing *phenomenological success* has some difficulties:

* *massive neutrinos*
* *dark matter*
* *hierarchy problem*
* *electric charge quantization*

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$\longrightarrow$ *new physics* beyond the Standard Model?

Supersymmmetric Grand Unification is one of the most promising candidates, at the same time predicting also *proton decay*, existence of *magnetic monopole(s)* and some yet unobserved *new particles*.

The simplest GUT model is the *minimal renormalizable SU(5)*.

Motivation

The *minimal renormalizable supersymmetric SU(5) model* *excluded* according to *Murayama-Pierce '01* , . . .

- *gauge coupling unification* (no unification within MSSM $\longrightarrow$ $m_T \lesssim 1.4 \cdot 10^{15}\,\mathrm{GeV}$)
- *proton decay* ($m_T \gtrsim 2.0 \cdot 10^{17}\,\mathrm{GeV}$)

Assumption:

low-energy supersymmetry spectrum - 3^{rd} generation *sparticles* at the $\mathcal{O}(1\,\mathrm{TeV})$ scale, and *gauginos* around the EW scale ($M_2 \approx 200\,\mathrm{GeV}$, $M_3/M_2 \simeq 3.5$).

Can all the *phenomenological constraints* be fulfilled by relaxing that request, allowing for some *more general superpartner mass spectrum*?

Can we bound the parameter space of allowed soft supersymmetry breaking terms in MSSM (*soft masses* and *trilinear a-terms*) and learn something about SUSY breaking?

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Our starting points

1. Why minimal renormalizable SUSY SU(5)?
   - *predictiveness* - probably the only way to ever test the high scale Yukawas (no SU(5) singlets used, small number of parameters $\longrightarrow$ the masses are calculable)
   - *smallness of terms* $W \supset C\,\frac{Q_i Q_j Q_k L_l}{M_P}$; $\quad C \lesssim 10^{-7}$ experimental fact
2. *perturbativity* (of couplings) at least up to the unification scale
3. *soft terms* at the GUT scale *SU(5) invariant* (supergravity mediation)
4. studying the mass scales of the theory (*the effects of running*), not its flavor structure [the only constraint is *small FCNCs*]
5. correcting the *down-sector quark masses* by generation dependent *supersymmetric thresholds* (a-terms)

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6. neutrino masses (R-parity violation)
7. dark matter (gravitino)

Scales in the theory

Decomposition

$$\mathbf{SU(5)} \supset \overbrace{(\text{SM particles} \oplus \underbrace{\textbf{superpartners}}_{\textbf{light thresholds}})}^{\textbf{MSSM}} \oplus \textbf{heavy thresholds}$$

Matching scales

$$\text{weak scale} \sim \mathcal{O}(m_Z) \longrightarrow \mathbf{m}_{\text{susy}} \equiv \sqrt{\mathbf{m}_{\tilde{\mathbf{t}}_1}\mathbf{m}_{\tilde{\mathbf{t}}_2}} \approx \sqrt{\mathbf{m}_{\tilde{\mathbf{u}}_3^c}\mathbf{m}_{\tilde{\mathbf{Q}}_3}} \quad \textbf{(SM)}$$

$$\mathbf{m}_{\text{susy}} \sim \mathcal{O}(\mathbf{1}-\mathbf{100}\,\text{TeV}) \longrightarrow \mathbf{M}_{\text{GUT}} \quad \textbf{(MSSM)}$$

$$\mathbf{M}_{\text{GUT}} \sim \mathcal{O}(\mathbf{10^{16}}\,\text{GeV}) \longrightarrow M_{\text{Planck}} \quad \textbf{(SU(5))}$$

Running of model parameters between matching scales (RGEs)

single scale effective theory **2-loop** RGEs + **1-loop** thresholds

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Content

1. **Higgs sector:**

adjoint representation: $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

$$\mathbf{24_H} = \underbrace{(\mathbf{8}, \mathbf{1}, \mathbf{0})}_{m_8(m_\Sigma)} \oplus \underbrace{(\mathbf{1}, \mathbf{3}, \mathbf{0})}_{m_3(m_\Sigma)} \oplus \underbrace{(\mathbf{1}, \mathbf{1}, \mathbf{0})}_{m_1} \oplus \underbrace{(\mathbf{3}, \mathbf{2}, -\frac{\mathbf{5}}{\mathbf{6}}) \oplus (\overline{\mathbf{3}}, \mathbf{2}, \frac{\mathbf{5}}{\mathbf{6}})}_{m_V}$$

fundamental & *antifundamental representation:* $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em}$

$$\mathbf{5_H} = \underbrace{(\mathbf{3}, \mathbf{1}, -\frac{\mathbf{1}}{\mathbf{3}})}_{m_T} \oplus \underbrace{(\mathbf{1}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}})}_{m_H} \quad , \qquad \overline{\mathbf{5}}_\mathbf{H} = \underbrace{(\overline{\mathbf{3}}, \mathbf{1}, \frac{\mathbf{1}}{\mathbf{3}})}_{m_T} \oplus \underbrace{(\mathbf{1}, \mathbf{2}, -\frac{\mathbf{1}}{\mathbf{2}})}_{m_H}$$

Content

2. **Gauge sector:**

$$\mathbf{24_g} = (\mathbf{8},\mathbf{1},\mathbf{0}) \oplus (\mathbf{1},\mathbf{3},\mathbf{0}) \oplus (\mathbf{1},\mathbf{1},\mathbf{0}) \oplus \underbrace{(\mathbf{3},\mathbf{2},-\tfrac{\mathbf{5}}{\mathbf{6}}) \oplus (\overline{\mathbf{3}},\mathbf{2},\tfrac{\mathbf{5}}{\mathbf{6}})}_{m_V}$$

3. **Matter (Yukawa) sector:**

$$\mathbf{10_i} = \underbrace{(\mathbf{3},\mathbf{2},\tfrac{\mathbf{1}}{\mathbf{6}})}_{m_{\tilde{Q}_i}} \oplus \underbrace{(\overline{\mathbf{3}},\mathbf{1},-\tfrac{\mathbf{2}}{\mathbf{3}})}_{m_{\tilde{u}^c_i}} \oplus \underbrace{(\mathbf{1},\mathbf{1},\mathbf{1})}_{m_{\tilde{e}^c_i}} \quad , \qquad \overline{\mathbf{5}}_\mathbf{i} = \underbrace{(\overline{\mathbf{3}},\mathbf{1},\tfrac{\mathbf{1}}{\mathbf{3}})}_{m_{\tilde{d}^c_i}} \oplus \underbrace{(\mathbf{1},\mathbf{2},-\tfrac{\mathbf{1}}{\mathbf{2}})}_{m_{\tilde{L}_i}}$$

Structure of the theory (high energy)

Higgs sector superpotential

$$W_H = \frac{\mu}{2} Tr\, \mathbf{24_H}^2 + \sqrt{30}\,\frac{\lambda}{3} Tr\, \mathbf{24_H}^3 + \eta\, \mathbf{\overline{5}_H} \left(\mathbf{24_H} + 3\frac{\langle\sigma\rangle}{\sqrt{30}} \right) \mathbf{5_H}$$

Yukawa sector superpotential

$$W_Y = \mathbf{\overline{5}_i}\, {Y_5}^{i,j}\, \mathbf{10_j}\, \mathbf{\overline{5}_H} + \frac{1}{8}\, \mathbf{10_i}\, Y_{10}^{i,j}\, \mathbf{10_j}\, \mathbf{5_H} \quad , \quad \mathrm{i=1,2,3}$$

$$\left.\begin{aligned} m_T &= \frac{5}{\sqrt{30}}\eta\langle\sigma\rangle \\ m_\Sigma &= m_8 = m_3 = 5\mu = 5\lambda\langle\sigma\rangle \\ m_1 &= \mu = \lambda\langle\sigma\rangle \\ m_V &= \frac{5}{\sqrt{30}}\, g_{\mathrm{GUT}}\,\langle\sigma\rangle \end{aligned}\right\} \Longrightarrow \textit{perturbativity}\ (m_T, m_\Sigma \lesssim m_V)$$

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Structure of the theory (low energy)

MSSM superpotential

$$W_{\mathrm{MSSM}} = \mathbf{u^c} Y_u \mathbf{Q H_u} + \mathbf{d^c} Y_d \mathbf{Q H_d} + \mathbf{e^c} Y_e \mathbf{L H_d} + \mu \mathbf{H_u H_d}$$

soft Lagrangian ($\not{\mathrm{SUSY}}$)

$$\begin{aligned} -\mathcal{L}_{soft} = {} & m^2_{H_u} H_u^\dagger H_u + m^2_{H_d} H_d^\dagger H_d \\ & + \tilde{Q}^\dagger m^2_{\tilde{Q}} \tilde{Q} + \tilde{u}^c m^2_{\tilde{u}^c} \tilde{u}^{c\dagger} + \tilde{e}^c m^2_{\tilde{e}^c} \tilde{e}^{c\dagger} + \tilde{L}^\dagger m^2_{\tilde{L}} \tilde{L} + \tilde{d}^c m^2_{\tilde{d}^c} \tilde{d}^{c\dagger} \\ & + \frac{1}{2}\left(M_1 \tilde{b}\tilde{b} + M_2 \tilde{w}\tilde{w} + M_3 \tilde{g}\tilde{g} \right) + \mathrm{h.c.} \\ & + \tilde{u}^c A_u \tilde{Q} H_u + \tilde{d}^c A_d \tilde{Q} H_d + \tilde{e}^c A_e \tilde{L} H_d + b H_u H_d + \mathrm{h.c.} \end{aligned}$$

SU(5)-invariant boundary conditions at the GUT scale

$$\alpha_1(M_{\rm GUT}) \;=\; \alpha_2(M_{\rm GUT}) \;=\; \alpha_3(M_{\rm GUT}) \;\equiv\; \alpha_{\rm GUT}$$

$$\begin{aligned} Y_u(M_{\rm GUT}) &= Y_u^T(M_{\rm GUT}) \\ Y_d(M_{\rm GUT}) &= Y_e^T(M_{\rm GUT}) \end{aligned}$$

$$\begin{aligned} A_u(M_{\rm GUT}) &= A_u^T(M_{\rm GUT}) \\ A_d(M_{\rm GUT}) &= A_e^T(M_{\rm GUT}) \end{aligned}$$

$$\begin{aligned} M_1(M_{\rm GUT}) \;=\; M_2(M_{\rm GUT}) \;=\; M_3(M_{\rm GUT}) &\equiv M_{1/2} \\ m_{\tilde{Q}_i}(M_{\rm GUT}) \;=\; m_{\tilde{u}_i^c}(M_{\rm GUT}) \;=\; m_{\tilde{e}_i^c}(M_{\rm GUT}) &\equiv \tilde{m}_{10_i} \\ m_{\tilde{L}_i}(M_{\rm GUT}) \;=\; m_{\tilde{d}_i^c}(M_{\rm GUT}) &\equiv \tilde{m}_{\bar{5}_i} \end{aligned} \qquad (i = 1, 2, 3)$$

All the *splittings* within SU(5) representations are only due to *running* !

Theoretical and experimental constraints

- *Higgs mass* ($m_h \simeq 125.7\,\mathrm{GeV}$)

- correct *down-sector fermion mass* relations (δm_d, δm_s, δm_b)
- *vacuum* (meta)*stability* (UFB 1,2,3 and CCB 1,2,3)

- gauge coupling *unification*
- *perturbativity* ($m_T, m_\Sigma \lesssim m_V \ll M_{\mathrm{Planck}}$)
- *proton lifetime* bounds $\tau_p^{exp}(p \to K^+\overline{\nu}) > 2.3 \times 10^{33}\,\mathrm{yrs} \longrightarrow m_T \gtrsim \ldots$,
  $\tau_p^{exp}(p \to \pi^0 e^+) > 13 \times 10^{33}\,\mathrm{yrs} \longrightarrow m_V \gtrsim \ldots$

- *LEP* and *LHC* bounds on sfermion and gaugino masses
  ($m_{\tilde{Q}_{1,2}}, m_{\tilde{g}} \gtrsim 1\,\mathrm{TeV}$; $m_{\tilde{Q}_3}, m_{\tilde{\chi}} \gtrsim 300\,\mathrm{GeV}$)

Mass of the light Higgs

matching scale between SM and MSSM RGEs

$$m_{\rm susy} \equiv \sqrt{m_{\tilde{t}_1}(m_{\rm susy}) m_{\tilde{t}_2}(m_{\rm susy})} \approx \sqrt{m_{\tilde{u}_3}(m_{\rm susy}) m_{\tilde{Q}_3}(m_{\rm susy})}$$

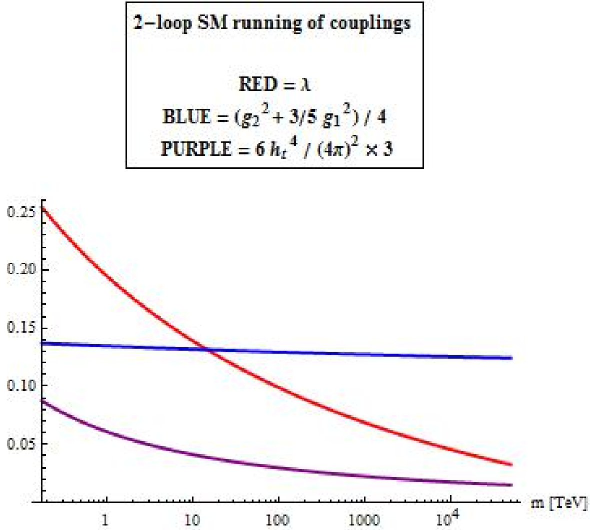
is determined by (2-loop) SM running of the *Higgs quartic coupling*

$$V_{\rm Higgs} = -m_h^2 H_u^\dagger H_u + \frac{\lambda}{2}\left(H_u^\dagger H_u\right)^2$$

with the *matching condition*

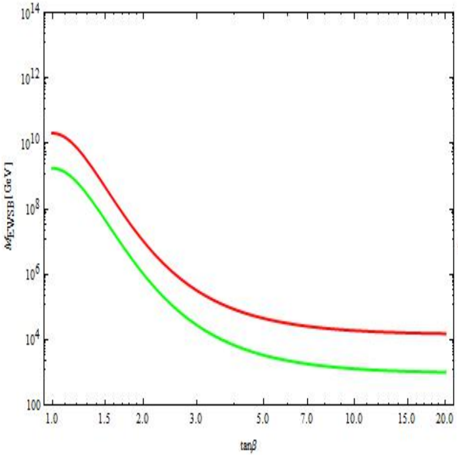
$$\lambda(m_{\rm susy}) = \overbrace{\left(\frac{3}{5} g_1^2(m_{\rm susy}) + g_2^2(m_{\rm susy})\right)\frac{\cos^2(2\beta)}{4}}^{tree-level} + \\ + \underbrace{\frac{6\,(\lambda_t \sin\beta)^4}{(4\pi)^2}\left(\frac{X_t}{m_{\rm susy}}\right)^2\left[1 - \frac{1}{12}\left(\frac{X_t}{m_{\rm susy}}\right)^2\right]}_{\text{1-loop stop-mixing contribution to Higgs mass } \leq \frac{6\,(\lambda_t \sin\beta)^4}{(4\pi)^2}\times 3} + \dots$$

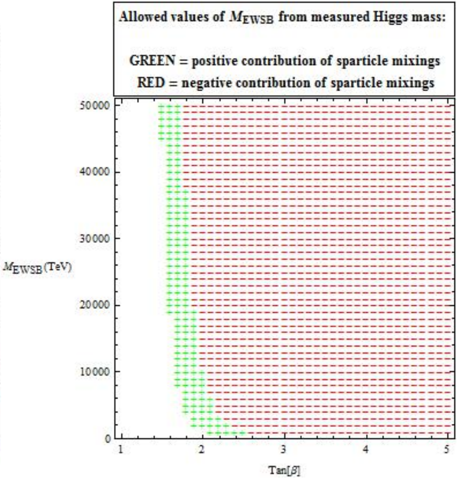
Mass of the light Higgs



Mass of the light Higgs

For each *tan β* exist a *minimal* m_{susy} which fits the measured Higgs mass





Mismatch between measured and SU(5)-respecting masses

SM running between m_Z and $m_{\rm susy}$ and MSSM running between $m_{\rm susy}$ and $M_{\rm GUT}$ for *leptonic* and *down sector Yukawas*

$$\mathbf{y_e}, \mathbf{y}_\mu, \mathbf{y}_\tau : \quad m_Z \xrightarrow{SM} m_{\rm susy} \xrightarrow{MSSM} M_{\rm GUT} \quad \begin{pmatrix} \text{no susy threshold corr.} \\ \alpha_2 \text{ instead of } \alpha_3 \text{ depen.} \end{pmatrix}$$

SU(5) symmetry + *minimal renormalizable model* $\longrightarrow$ *charged lepton* and *down-type quark* masses (Yukawas) equal at the GUT scale

minimal renormalizable SU(5) model $\longrightarrow \left\{ \begin{array}{rcl} m_e(M_{\rm GUT}) & = & m_d(M_{\rm GUT}) \\ m_\mu(M_{\rm GUT}) & = & m_s(M_{\rm GUT}) \\ m_\tau(M_{\rm GUT}) & = & m_b(M_{\rm GUT}) \end{array} \right.$

$$\mathbf{y_d}, \mathbf{y_s}, \mathbf{y_b} : \quad M_{\rm GUT} \xrightarrow{MSSM} m_{\rm susy} \xrightarrow{SM} m_Z$$

$\longrightarrow$ *wrong mass relations* when run down to the electroweak scale

$$\left. \begin{array}{l} m_e(m_Z)/m_d(m_Z) \\ m_\mu(m_Z)/m_s(m_Z) \\ m_\tau(m_Z)/m_b(m_Z) \end{array} \right\} = \textbf{wrong} \longrightarrow \textbf{threshold corrections needed}$$

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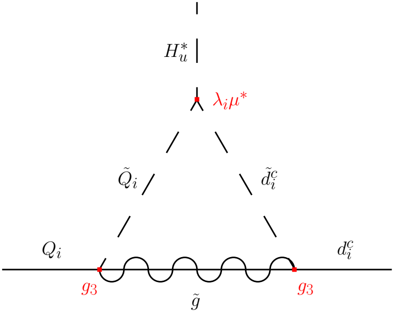
Correcting light fermion masses with a-terms



Diagrams for the finite corrections to the quark Yukawa couplings.

Correcting light fermion masses with a-terms

$$\delta m_i \equiv m_i^{exp} - m_i^{SU(5)} = -\frac{2\alpha_3}{3\pi} v \cos\beta \, m_{\tilde{g}} \, I_3\Big[\overbrace{m^2_{\tilde{d}_{R_i}}}^{\substack{\tilde{m}^2_{\bar{5}_i} \\ \uparrow \\ m^2_{\tilde{d}^c_i}}}, \overbrace{m^2_{\tilde{d}_{L_i}}}^{\substack{\tilde{m}^2_{10_i} \\ \uparrow \\ m^2_{\tilde{Q}_i}}}, m^2_{\tilde{g}}\Big] \lambda_i X_i$$

$$\approx -\frac{2\alpha_3}{3\pi} v \cos\beta \underbrace{\frac{m_{\tilde{g}}}{\tilde{m}_i} I_1\left[\left(\frac{m_{\tilde{g}}}{\tilde{m}_i}\right)^2\right]}_{\substack{\lessgtr \\ 0.57 \text{ at } \frac{m_{\tilde{g}}}{\tilde{m}_i} = 2.12}} \frac{\lambda_i X_i}{\tilde{m}_i} \propto \frac{\alpha_3(m_{\rm susy})\, a_i}{\tilde{m}_i}$$

$$X_t \equiv \frac{a_t}{\lambda_t} - \frac{\mu}{\tan\beta} \approx \frac{a_t}{\lambda_t}$$

$$X_b \equiv \frac{a_b}{\lambda_b} - \mu\tan\beta \approx \frac{a_b}{\lambda_b}$$

$$X_\tau \equiv \frac{a_\tau}{\lambda_\tau} - \mu\tan\beta \approx \frac{a_\tau}{\lambda_\tau}$$

Problem is accommodating the measured *b quark mass* - apart from using large *a-terms* other sources can not account for more than 10 % of required corrections.

Vacuum (meta)stability

1. *absolute vacuum stability* $\longrightarrow$ our vacuum is a *global minimum*

UFB 1,2,3 $\longrightarrow$ $m^2_{H_u} > 0$

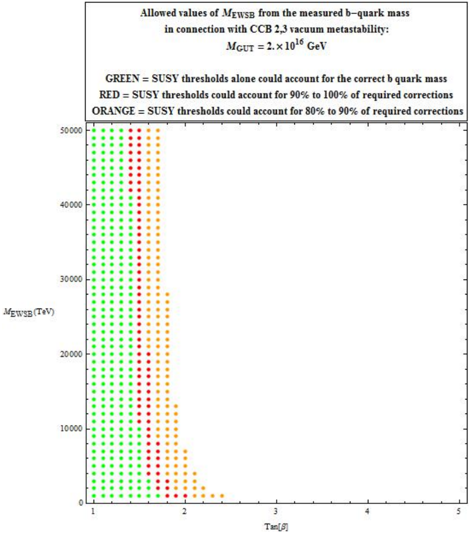
CCB 1,2,3 $\longrightarrow$ $\frac{|a_i|}{\lambda_i} \sim |X_i| \not\lesssim \sqrt{3\left(m^2_{H_d} + m^2_{\tilde{Q}_i} + m^2_{\tilde{d}^c_i}\right)}$

2. *vacuum metastability* $\longrightarrow$ our vacuum only a *local minimum*, but its *lifetime* longer than the age of the Universe

CCB 1,2*,3* $\longrightarrow$ $|a_i| \lesssim \sqrt{m^2_{H_d} + m^2_{\tilde{Q}_i} + m^2_{\tilde{d}^c_i}}$

**more complicated situation, numerical analysis required*

Vacuum (meta)stability



Combining corrections to *b-quark mass* and *vacuum metastability* constraints

Gauge coupling Unification

No unification in MSSM $\longrightarrow$ high-energy thresholds m_T, m_8, m_3, m_V required

single scale (m_{susy}) MSSM **2-loop** RGEs $+$ **1-loop** thresholds

Gauge coupling Unification

No unification in MSSM $\longrightarrow$ high-energy thresholds m_T, m_8, m_3, m_V required

single scale ($m_{\rm susy}$) MSSM **2-loop** RGEs + **1-loop** thresholds

$$\Big[\overbrace{m_V^2}^{m_T^2 \;\gtrsim}(m_3 m_8)^{1/2}\Big]^{1/3} = M_{\rm GUT} \times \exp\left[\frac{\pi}{18}\left(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}\right)_{2-loop}(M_{\rm GUT})\right]$$

$$\times \left(\frac{m_{\rm susy}^2}{m_{\tilde{w}} m_{\tilde{g}}}\right)^{1/9} \underbrace{\prod_{i=1}^{3}\left(\frac{m_{\tilde{u}_i^c} m_{\tilde{e}_i^c}}{m_{\tilde{Q}_i}^2}\right)^{1/36}}_{\simeq\ \mathcal{O}(1)}$$

Gauge coupling Unification

No unification in MSSM $\longrightarrow$ high-energy thresholds m_T, m_8, m_3, m_V required

single scale ($m_{\rm susy}$) MSSM **2-loop** RGEs + **1-loop** thresholds

$$m_T^0 \simeq 10^{15}\,\mathrm{GeV} \times \left(\frac{m_{\rm susy}}{1\,\mathrm{TeV}}\right)^{5/6}$$

$$m_T = \overbrace{M_{\rm GUT} \times \exp\left[\frac{5\pi}{6}\left(-\alpha_1^{-1} + 3\alpha_2^{-1} - 2\alpha_3^{-1}\right)_{2-loop}(M_{\rm GUT})\right]}^{\equiv\, m_T^0}$$

$$\times \underbrace{\left(\frac{m_3}{m_8}\right)^{5/2}}_{=\,1} \underbrace{\left(\frac{m_{\tilde{w}}}{m_{\tilde{g}}}\right)^{5/3}}_{\uparrow\; m_{\tilde{g}} \approx m_{\rm susy}} \underbrace{\prod_{i=1}^{3}\left(\frac{m^4_{\tilde{Q}_i}}{m^3_{\tilde{u}^c_i} m_{\tilde{e}^c_i}} \frac{m^2_{\tilde{L}_i}}{m^2_{\tilde{d}^c_i}}\right)^{1/12}}_{\lesssim\, \mathcal{O}(1)} \left(\frac{m^4_{\tilde{h}} m_A}{m^5_{\rm susy}}\right)^{1/6}$$

Light m_T mediates too fast proton decay.

Large $m_{\rm susy}$ poses the opposite problem: m_T can be too heavy (perturbativity).

Gauge coupling Unification

$$|\mu|^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2} \quad \text{(tree-level EWSB condition at } m_{\rm susy}\text{)}$$

$$m_{\tilde{h}} = |\mu|$$

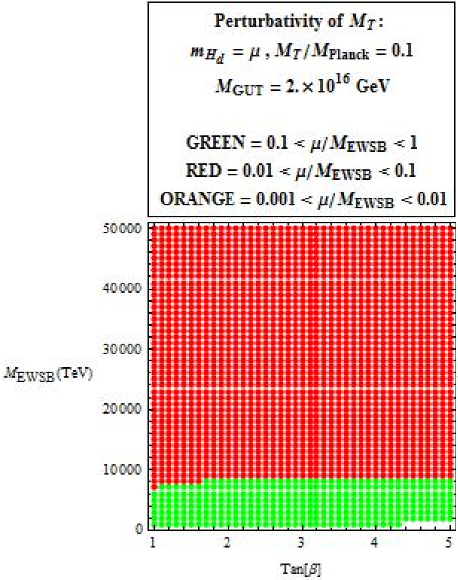
$$m_A = \sqrt{(\mu^2 + m_{H_d}^2)(1 + 1/\tan^2\beta)} \approx \sqrt{(m_{H_d}^2 - m_{H_u}^2)\frac{\tan^2\beta + 1}{\tan^2\beta - 1}}$$

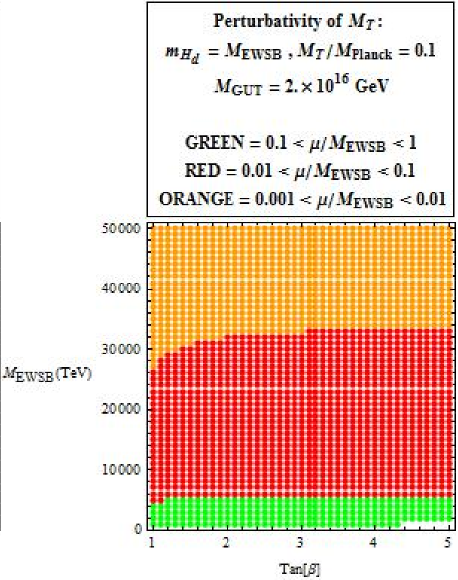
$$\frac{m_{\tilde{w}}}{m_{\tilde{g}}} \approx \frac{\alpha_2(m_{\tilde{g}})}{\alpha_3(m_{\tilde{g}})} \frac{\alpha_3(M_{\rm GUT})}{\alpha_2(M_{\rm GUT})}$$

$$m_T = \overbrace{M_{\rm GUT} \times \exp\left[\frac{5\pi}{6}\left(-\alpha_1^{-1} + 3\alpha_2^{-1} - 2\alpha_3^{-1}\right)_{2-loop}(M_{\rm GUT})\right]}^{m_T^0 \;\simeq\; 10^{15}\,\mathrm{GeV} \,\times\, \left(\frac{m_{\rm susy}}{1\,\mathrm{TeV}}\right)^{5/6}}$$

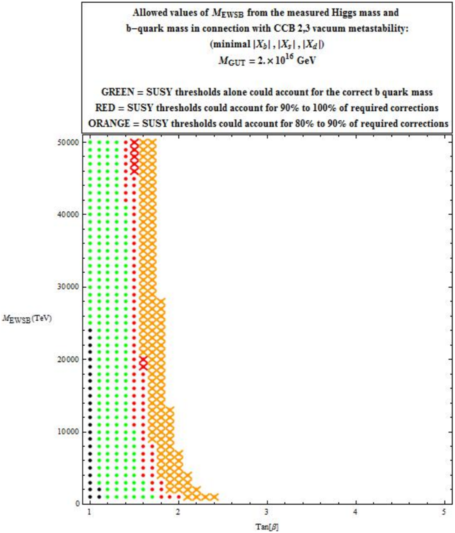
$$\times \underbrace{\left(\frac{m_3}{m_8}\right)^{5/2}}_{=\,1} \underbrace{\left(\frac{m_{\tilde{w}}}{m_{\tilde{g}}}\right)^{5/3}}_{\uparrow\; m_{\tilde{g}} \,\approx\, m_{\rm susy}} \underbrace{\prod_{i=1}^{3}\left(\frac{m_{\tilde{Q}_i}^4}{m_{\tilde{u}_i^c}^3 m_{\tilde{e}_i^c}} \frac{m_{\tilde{L}_i}^2}{m_{\tilde{d}_i^c}^2}\right)^{1/12}}_{\simeq\, \mathcal{O}(1)} \left(\frac{m_{\tilde{h}}^4 m_A}{m_{\rm susy}^5}\right)^{1/6}$$

Perturbativity





Results & Conclusions



POINTS TO TAKE HOME:

- ∗ $m_h + \delta m_b +$ *vacuum metastability*
- ∗ strong correlation between $\tan\beta$ and allowed $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_1}}$
- ∗ importance of *2-loop* running for λ

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work in progress . . .

TO DO list:

- ∗ numerical analysis of the *CCB 2,3* vacuum metastability
- ∗ *RGEs* for *a-terms* and *soft masses* $\longrightarrow$ check of *SU(5)* boundary conditions

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Thank you for your attention!