On the gauge dependence of the SM vacuum instability scale

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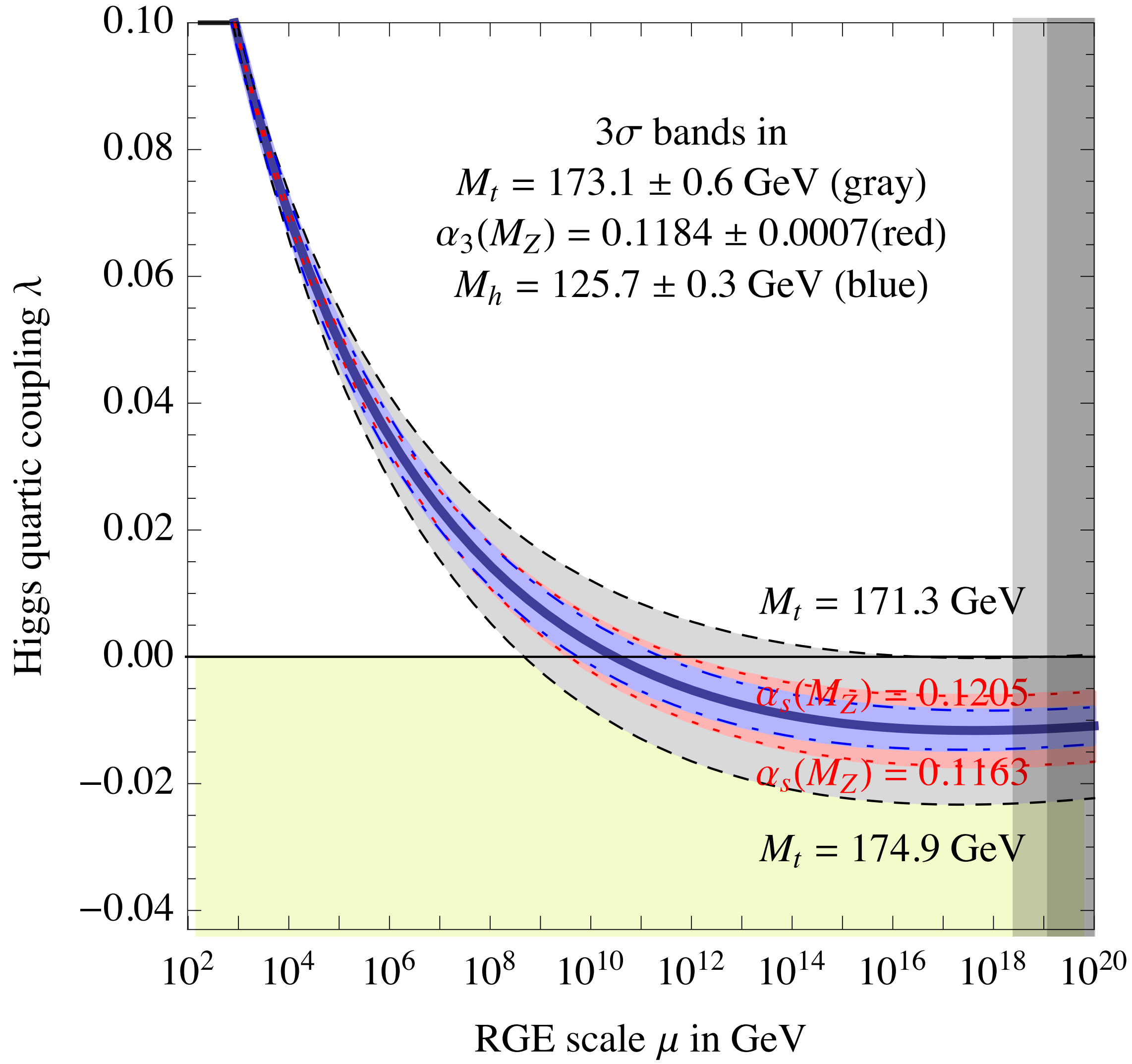
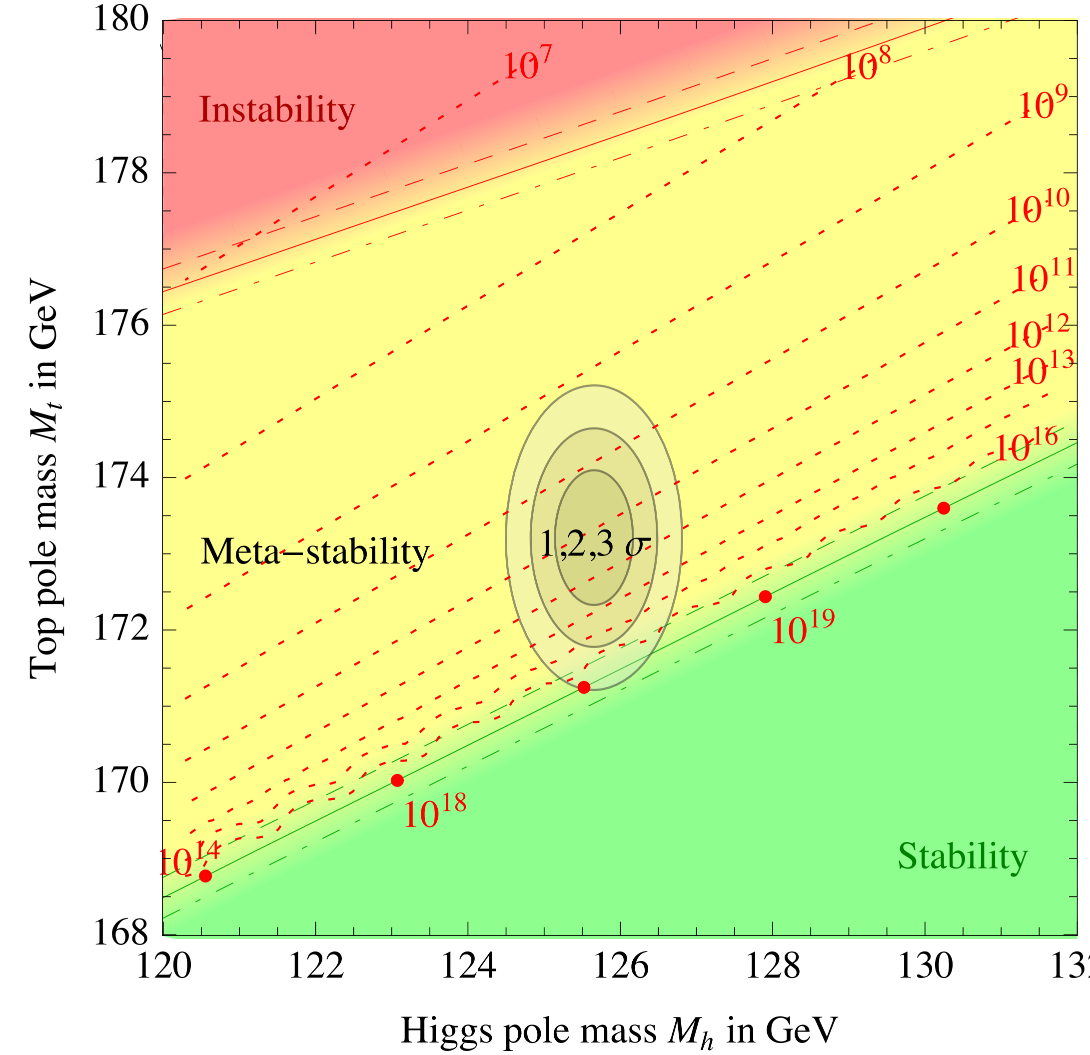
Outline

- SM phase diagram
- Gauge dependent SM effective potential
- Physical observables in the vacuum stability analysis
- Gauge dependence of the SM vacuum instability scale

SM phase diagram



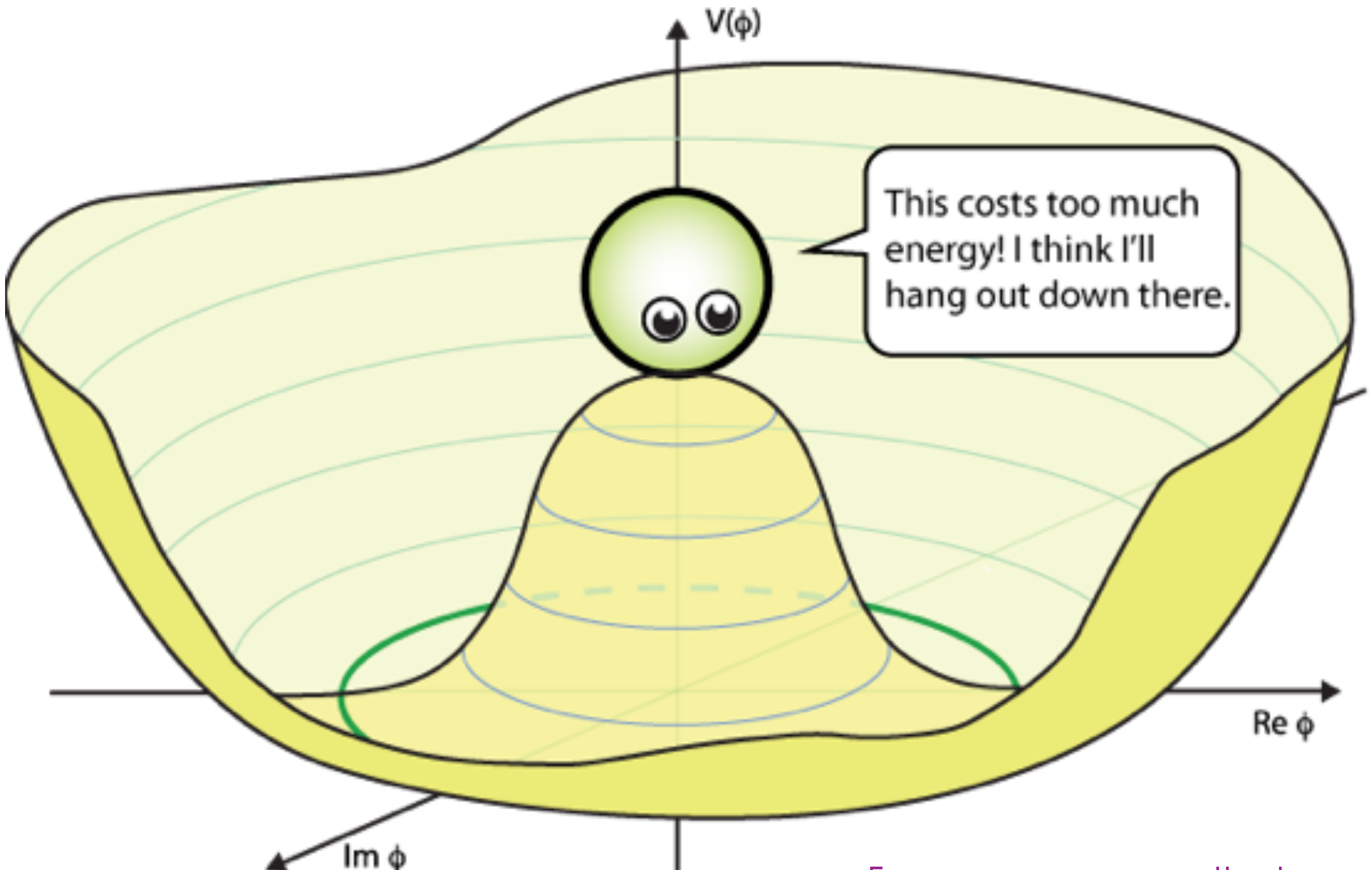


[Buttazzo et al. (2013)]

Effective Potential (EP)

- Provides a geometrical language for the survey of the vacuum structure of a QFT

[Schwinger (1951), Jona-Lasinio (1964), Coleman, Weinberg (1973)]



[www.quantumdiaries.org]

Background field method (BFM)

- The EP is most conveniently computed with the BFM [Jackiw (1974)]

1. Change of variable in the path-integral expression of the generating functional

$$e^{iW[j]} = \int \mathcal{D}\Phi \, \exp\left(i \int d^4x \, (\mathcal{L}(\Phi) + j\Phi)\right)$$

$$= \int \mathcal{D}\phi \, \exp\left(i \int d^4x \left(\mathcal{L}(\phi_c) + j\phi_c + \phi \underbrace{\left(\left.\frac{\partial \mathcal{L}}{\partial \phi}\right|_{\phi_c} + j \right)}_{= 0 \text{ e.o.m.}} + \frac{1}{2}\phi^2 \underbrace{\left.\frac{\partial^2 \mathcal{L}}{\partial \phi^2}\right|_{\phi_c}}_{i\mathcal{D}^{-1}\{\phi_c, x\}} + \dots \right)\right)$$

↑ $\Phi(x) = \phi_c + \phi(x)$

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$\Phi(x) = \phi_c + \phi(x)$

2. Gaussian path integral (one-loop approx.)

$$e^{iW[j]} \approx \exp\left(i \int d^4x \, (\mathcal{L}(\phi_c) + j\phi_c)\right) \left(\det i\mathcal{D}^{-1}\{\phi_c, x - y\}\right)^{-\frac{1}{2}}$$

Background field method (BFM)

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3. Effective action

$$\Gamma[\phi_c] = W[j] - \int d^4x \, j\phi_c \approx \int d^4x \, \mathcal{L}(\phi_c) + \frac{i}{2} \log \det i\mathcal{D}^{-1}\{\phi_c, x-y\}$$

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4. Effective potential

$$\Gamma[\phi_c] = \int d^4x\, [-V_{\text{eff}}(\phi_c) + \ldots]$$

$$V_{\text{eff}}^{\text{1-loop}}(\phi_c) = V(\phi_c) + i\sum_n \eta \int \frac{d^4k}{(2\pi)^4} \log\det i\tilde{\mathcal{D}}_n^{-1}\{\phi_c; k\}$$

$\eta = -\frac{1}{2}(1)$ for bosons (fermions)

Origin of the gauge dependence

- EP is gauge dependent [Jackiw (1974)]

C. Discussion

The observation that $V(\hat{\phi})$ is gauge-dependent for a gauge theory raises a question concerning the physical significance of any mathematical properties of $V(\hat{\phi})$. I have already remarked that

Origin of the gauge dependence

- EP is gauge dependent [Jackiw (1974)]

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The observation that $V(\hat{\phi})$ is gauge-dependent for a gauge theory raises a question concerning the physical significance of any mathematical properties of $V(\hat{\phi})$. I have already remarked that

- 4-boson scattering amplitude at zero external momentum

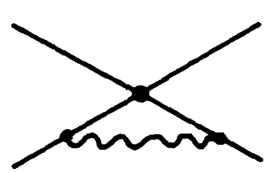

FIG. 5. Gauge-dependent contribution to $V(\hat{\phi})$.

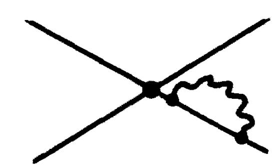

FIG. 6. External wave-function renormalization graph which removes gauge dependence of Fig. 5.

$$V_{\text{eff}}(\phi_c) = -\sum_{n=0}^{\infty} \frac{1}{n!} \phi_c^n \Gamma^{(n)}(p_i = 0)$$

Not one-particle-irreducible

SM gauge dependent EP (1)

- Classical SM Lagrangian

$$\mathcal{L}_{\rm C} = \mathcal{L}_{\rm YM} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F}$$

$$\mathcal{L}_{\rm YM} = -\frac{1}{4}\left(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \ldots\right)^2 - \frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)^2$$

$$\mathcal{L}_{\rm H} = \left(D_\mu H\right)^\dagger \left(D^\mu H\right) - V(H) \qquad V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$\mathcal{L}_{\rm F} = \ldots$$

- Gauge fixing: e.g. Fermi gauge (for unbroken phase problems)

$$\mathcal{L}^{\rm Fermi}_{\rm g.f.} = -\frac{1}{2\xi_W}\left(\partial^\mu W^a_\mu\right)^2 - \frac{1}{2\xi_B}\left(\partial^\mu B_\mu\right)^2$$

- Shift the Higgs doublet in a specific SU(2)xU(1) direction

$$H(x) \to \frac{1}{\sqrt{2}}\begin{pmatrix} \chi^1(x) + i\chi^2(x) \\ \phi + h(x) + i\chi^3(x) \end{pmatrix}$$

SM gauge dependent EP (2)

- Work out the quadratic part of the SM Lagrangian after the shift (see BFM)

$$\mathcal{L}^{\rm quad}_{\rm YM} = \tfrac{1}{2} W^a_\mu \left(\Box\, g^{\mu\nu} - \partial^\mu \partial^\nu \right) \delta^{ab} W^b_\nu + \tfrac{1}{2} B_\mu \left(\Box\, g^{\mu\nu} - \partial^\mu \partial^\nu \right) B_\nu$$

$$\mathcal{L}^{\rm quad}_{\rm H} = \tfrac{1}{2} h \left(-\Box - \bar{m}_h^2 \right) h + \tfrac{1}{2} \chi^a \left(-\Box - \bar{m}_\chi^2 \right) \delta^{ab} \chi^b + \tfrac{1}{2} \bar{m}_W^2 W^a_\mu W^{a\mu} + \tfrac{1}{2} \bar{m}_B^2 B_\mu B^\mu + \bar{m}_W \bar{m}_B W^3_\mu B^\mu$$
$$- \bar{m}_W \partial_\mu \chi^1 W^{2\mu} - \bar{m}_W \partial_\mu \chi^2 W^{1\mu} + \bar{m}_W \partial_\mu \chi^3 W^{3\mu} + \bar{m}_B \partial_\mu \chi^3 B^\mu$$

$$\mathcal{L}^{\rm quad}_{\rm F} = \bar{t} \left(i \partial\!\!\!/ - \bar{m}_t \right) t + \ldots$$

$$\mathcal{L}^{\rm Fermi}_{\rm g.f.} = -\frac{1}{2\xi_W} \left(\partial^\mu W^a_\mu \right)^2 - \frac{1}{2\xi_B} \left(\partial^\mu B_\mu \right)^2$$

- Field-dependent masses

$$\bar{m}_h^2 = -m^2 + 3\lambda\phi^2$$
$$\bar{m}_\chi^2 = -m^2 + \lambda\phi^2$$
$$\bar{m}_W = \tfrac{1}{2} g \phi$$
$$\bar{m}_B = \tfrac{1}{2} g' \phi$$
$$\bar{m}_t = \frac{y_t}{\sqrt{2}} \phi$$

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- Work out the quadratic part of the SM Lagrangian after the shift (see BFM)

$$
\begin{aligned}
\mathcal{L}_{\rm YM}^{\rm quad} &= \tfrac{1}{2} W^a_\mu \left(\Box\, g^{\mu\nu} - \partial^\mu \partial^\nu\right) \delta^{ab} W^b_\nu + \tfrac{1}{2} B_\mu \left(\Box\, g^{\mu\nu} - \partial^\mu \partial^\nu\right) B_\nu \\
\mathcal{L}_{\rm H}^{\rm quad} &= \tfrac{1}{2} h \left(-\Box - \bar{m}_h^2\right) h + \tfrac{1}{2} \chi^a \left(-\Box - \bar{m}_\chi^2\right) \delta^{ab} \chi^b + \tfrac{1}{2} \bar{m}_W^2 W^a_\mu W^{a\mu} + \tfrac{1}{2} \bar{m}_B^2 B_\mu B^\mu + \bar{m}_W \bar{m}_B W^3_\mu B^\mu \\
&\quad - \bar{m}_W \partial_\mu \chi^1 W^{2\mu} - \bar{m}_W \partial_\mu \chi^2 W^{1\mu} + \bar{m}_W \partial_\mu \chi^3 W^{3\mu} + \bar{m}_B \partial_\mu \chi^3 B^\mu \\
\mathcal{L}_{\rm F}^{\rm quad} &= \bar{t} \left(i \not{\partial} - \bar{m}_t\right) t + \ldots \\
\mathcal{L}_{\rm g.f.}^{\rm Fermi} &= -\frac{1}{2\xi_W} \left(\partial^\mu W^a_\mu\right)^2 - \frac{1}{2\xi_B} \left(\partial^\mu B_\mu\right)^2
\end{aligned}
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- Field-dependent masses
- Ghosts decouple at one loop

SM gauge dependent EP (2)

- Work out the quadratic part of the SM Lagrangian after the shift (see BFM)

$$\mathcal{L}^{\text{quad}}_{\text{YM}} = \tfrac{1}{2} W^a_\mu \left(\Box\, g^{\mu\nu} - \partial^\mu \partial^\nu\right) \delta^{ab} W^b_\nu + \tfrac{1}{2} B_\mu \left(\Box\, g^{\mu\nu} - \partial^\mu \partial^\nu\right) B_\nu$$

$$\mathcal{L}^{\text{quad}}_{\text{H}} = \tfrac{1}{2} h \left(-\Box - \bar{m}^2_h\right) h + \tfrac{1}{2} \chi^a \left(-\Box - \bar{m}^2_\chi\right) \delta^{ab} \chi^b + \tfrac{1}{2} \bar{m}^2_W W^a_\mu W^{a\mu} + \tfrac{1}{2} \bar{m}^2_B B_\mu B^\mu + \bar{m}_W \bar{m}_B W^3_\mu B^\mu$$

$$\boxed{- \bar{m}_W \partial_\mu \chi^1 W^{2\mu} - \bar{m}_W \partial_\mu \chi^2 W^{1\mu} + \bar{m}_W \partial_\mu \chi^3 W^{3\mu} + \bar{m}_B \partial_\mu \chi^3 B^\mu}$$

$$\mathcal{L}^{\text{quad}}_{\text{F}} = \bar{t} \left(i \not{\partial} - \bar{m}_t\right) t + \ldots$$

$$\mathcal{L}^{\text{Fermi}}_{\text{g.f.}} = -\frac{1}{2\xi_W} \left(\partial^\mu W^a_\mu\right)^2 - \frac{1}{2\xi_B} \left(\partial^\mu B_\mu\right)^2$$

- Field-dependent masses
- Ghosts decouple at one loop
- Goldstone-Gauge boson mixing is retained

$$\bar{m}^2_h = -m^2 + 3\lambda\phi^2$$
$$\bar{m}^2_\chi = -m^2 + \lambda\phi^2$$
$$\bar{m}_W = \tfrac{1}{2} g\phi$$
$$\bar{m}_B = \tfrac{1}{2} g'\phi$$
$$\bar{m}_t = \frac{y_t}{\sqrt{2}}\phi$$

$$X^T = \left(W^1_\mu, W^2_\mu, W^3_\mu, B_\mu, \chi^1, \chi^2, \chi^3\right) \longrightarrow \frac{1}{2} X^T \left(i\mathcal{D}^{-1}_X\right) X$$

SM gauge dependent EP (3)

- After some standard manipulations … [DL, Mihaila (2014)]

$$V_{\text{eff}}^{(1)} = \frac{1}{4(4\pi)^2}\Bigg[-12\bar{m}_t^4\left(\log\frac{\bar{m}_t^2}{\mu^2}-\frac{3}{2}\right)+6\bar{m}_W^4\left(\log\frac{\bar{m}_W^2}{\mu^2}-\frac{5}{6}\right)+3\bar{m}_Z^4\left(\log\frac{\bar{m}_Z^2}{\mu^2}-\frac{5}{6}\right)+\bar{m}_h^4\left(\log\frac{\bar{m}_h^2}{\mu^2}-\frac{3}{2}\right)$$
$$+2\bar{m}_{A+}^4\left(\log\frac{\bar{m}_{A+}^2}{\mu^2}-\frac{3}{2}\right)+2\bar{m}_{A-}^4\left(\log\frac{\bar{m}_{A-}^2}{\mu^2}-\frac{3}{2}\right)+\bar{m}_{B+}^4\left(\log\frac{\bar{m}_{B+}^2}{\mu^2}-\frac{3}{2}\right)+\bar{m}_{B-}^4\left(\log\frac{\bar{m}_{B-}^2}{\mu^2}-\frac{3}{2}\right)\Bigg]$$

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$$\left.+2\bar{m}_{A+}^4\left(\log\frac{\bar{m}_{A+}^2}{\mu^2}-\frac{3}{2}\right)+2\bar{m}_{A-}^4\left(\log\frac{\bar{m}_{A-}^2}{\mu^2}-\frac{3}{2}\right)+\bar{m}_{B+}^4\left(\log\frac{\bar{m}_{B+}^2}{\mu^2}-\frac{3}{2}\right)+\bar{m}_{B-}^4\left(\log\frac{\bar{m}_{B-}^2}{\mu^2}-\frac{3}{2}\right)\right]$$

$$\bar{m}_{A\pm}^2 = \frac{1}{2}\bar{m}_\chi\left(\bar{m}_\chi \pm \sqrt{\bar{m}_\chi^2 - 4\xi_W\bar{m}_W^2}\right)$$
$$\bar{m}_{B\pm}^2 = \frac{1}{2}\bar{m}_\chi\left(\bar{m}_\chi \pm \sqrt{\bar{m}_\chi^2 - 4(\xi_W\bar{m}_W^2+\xi_B\bar{m}_B^2)}\right)$$

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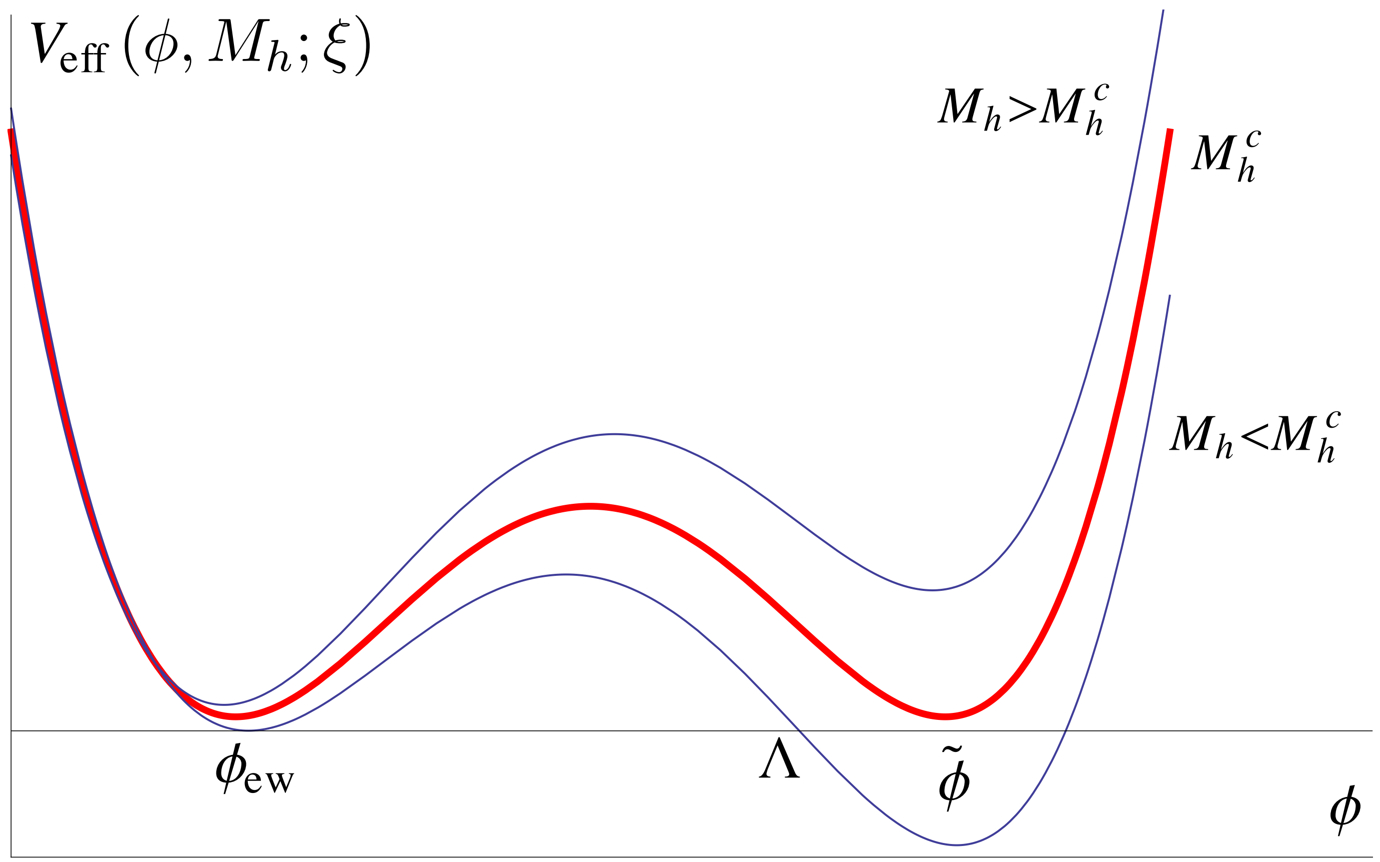
- Landau gauge: $\xi_W = \xi_B = 0$ → $\bar{m}_{A+} = \bar{m}_{B+} = \bar{m}_\chi$, $\bar{m}_{A-} = \bar{m}_{B-} = 0$

- Tree-level minimum: $\bar{m}_\chi = 0$ → $\bar{m}_{A\pm} = \bar{m}_{B\pm} = 0$

Vacuum stability bound

- Take all the parameters of the SM fixed but the Higgs mass:



→ Critical Higgs mass

$$V_{\rm eff}(\phi_{\rm ew}, M_h^c; \xi) - V_{\rm eff}(\tilde{\phi}, M_h^c; \xi) = 0$$

$$\left.\frac{\partial V_{\rm eff}}{\partial \phi}\right|_{\phi_{\rm ew}, M_h^c} = \left.\frac{\partial V_{\rm eff}}{\partial \phi}\right|_{\tilde{\phi}, M_h^c} = 0$$

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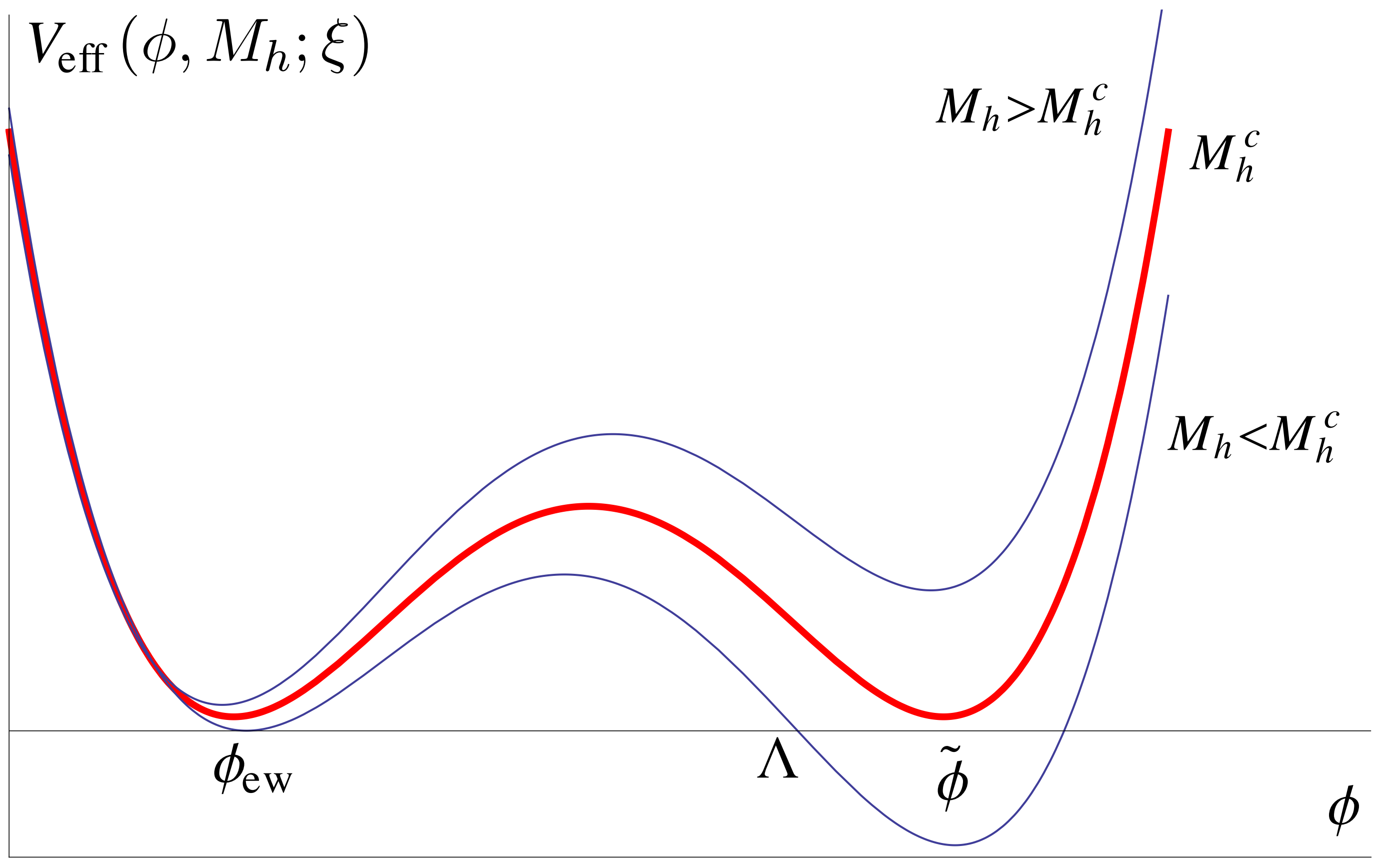

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- Which are the physical observables ?

Nielsen Identity

- Nielsen Identity (NI) [Nielsen (1975), Aitchison, Fraser (1984), Johnston (1985), Metaxas, Weinberg (1996), …]

$$\delta_{\mathrm{BRST}} \left\langle \overline{\eta}\, \Delta F \right\rangle = 0 \quad \longrightarrow \quad \boxed{\left(\frac{\partial}{\partial \xi} + C(\phi, \xi) \frac{\partial}{\partial \phi} \right) V_{\mathrm{eff}}(\phi, \xi) = 0}$$

- Interpretation:

  - the value of the EP at the extrema is gauge independent

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  - the value of the EP at the extrema is gauge independent

- By using NI one can formally prove: [DL, Mihaila (2014), Patel, Ramsey-Musolf (2011)]

1. $\frac{\partial M_h^c}{\partial \xi} = 0$ (gauge indep. of the critical Higgs mass)
2. $\frac{\partial \tilde{\phi}}{\partial \xi} = C(\tilde{\phi}, \xi)$ (gauge dep. of the extrema of the EP)
3. $\frac{\partial \Lambda}{\partial \xi} = C(\Lambda, \xi)$ (gauge dep. of the instability scale)

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3. $\boxed{\dfrac{\partial \Lambda}{\partial \xi} = C(\Lambda, \xi)}$ (gauge dep. of the instability scale)

RGE improvement

- Resum the large logs by means of RGEs

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\lambda_i} - \gamma\phi\frac{\partial}{\partial\phi}\right)V_{\text{eff}} = 0 \qquad \beta_i = \mu\frac{d\lambda_i}{d\mu} \qquad \gamma = -\frac{\mu}{\phi}\frac{d\phi}{d\mu}$$

- Formal solution: $V_{\text{eff}}(\mu, \lambda_i, \phi) = V_{\text{eff}}(\mu(t), \lambda_i(t), \phi(t))$

$$\mu(t) = \mu e^t \qquad \phi(t) = e^{\Gamma(t)}\phi \qquad \Gamma(t) = -\int_0^t \gamma(\lambda(t'))\,dt' \qquad \frac{d\lambda_i(t)}{dt} = \beta_i(\lambda_i(t)) \qquad \lambda_i(0) = \lambda_i$$

RGE improvement

- Resume the large logs by means of RGEs

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\lambda_i} - \gamma\phi\frac{\partial}{\partial\phi}\right)V_{\text{eff}} = 0 \qquad \beta_i = \mu\frac{d\lambda_i}{d\mu} \qquad \gamma = -\frac{\mu}{\phi}\frac{d\phi}{d\mu}$$

- Formal solution: $V_{\text{eff}}(\mu, \lambda_i, \phi) = V_{\text{eff}}(\mu(t), \lambda_i(t), \phi(t))$

$$\mu(t) = \mu e^t \qquad \phi(t) = e^{\Gamma(t)}\phi \qquad \Gamma(t) = -\int_0^t \gamma(\lambda(t'))\, dt' \qquad \frac{d\lambda_i(t)}{dt} = \beta_i(\lambda_i(t)) \qquad \lambda_i(0) = \lambda_i$$

- $\phi \gg m$ limit → $V_{\text{eff}}(\phi, t) \approx \frac{\lambda_{\text{eff}}(\phi, t)}{4}\phi^4$

$$\lambda_{\text{eff}}(\phi, t) \approx e^{4\Gamma(t)}\left[\lambda(t) + \frac{1}{(4\pi)^2}\sum_p N_p\kappa_p^2(t)\left(\log\frac{\kappa_p(t)e^{2\Gamma(t)}\phi^2}{\mu(t)^2} - C_p\right)\right]$$

- Choose t so that the convergence of perturbation theory is improved: e.g. $\mu(t) = \phi$

RGE improvement

- e.g. for the Fermi gauge

p	t	W	Z	h	$A^{\pm}$	$B^{\pm}$
N_p	-12	6	3	1	2	1
C_p	$\frac{3}{2}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
κ_p	$\frac{y_t^2}{2}$	$\frac{g^2}{4}$	$\frac{g^2+g'^2}{4}$	3λ	$\frac{1}{2}\left(\lambda \pm \sqrt{\lambda^2 - \lambda\xi_W g^2}\right)$	$\frac{1}{2}\left(\lambda \pm \sqrt{\lambda^2 - \lambda(\xi_W g^2 + \xi_B g'^2)}\right)$

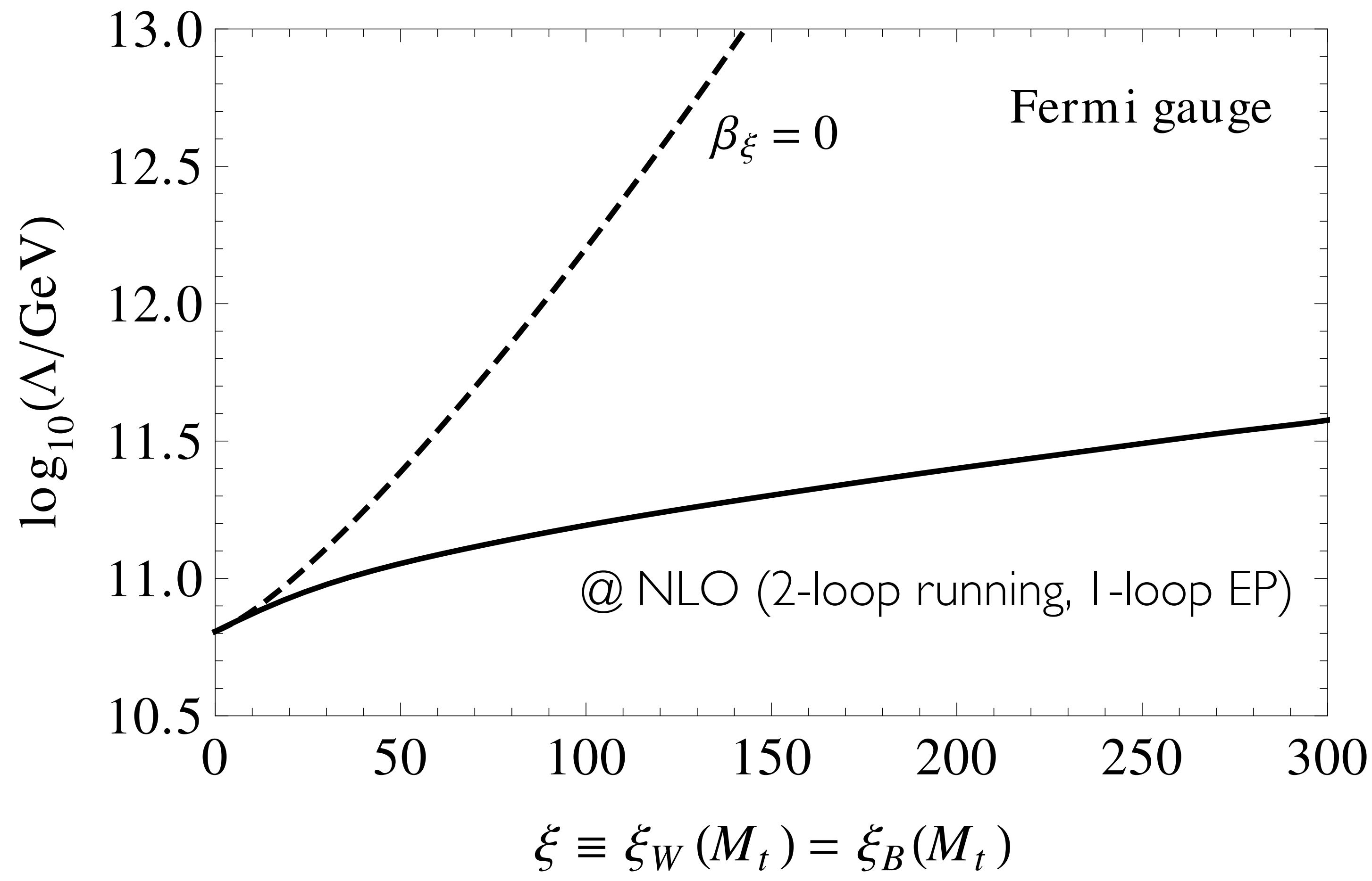
- Gauge dependence is twofold: κ_p for $p \in A^{\pm}, B^{\pm}$ and $\Gamma(t) = -\int_0^t \gamma(\lambda(t'))\, dt'$

$$\gamma = -\frac{9}{80}\frac{\alpha_1}{\pi} - \frac{9}{16}\frac{\alpha_2}{\pi} + \frac{3}{4}\frac{\alpha_t}{\pi} + \frac{3}{80}\frac{\xi_B\alpha_1}{\pi} + \frac{3}{16}\frac{\xi_W\alpha_2}{\pi}$$

$$\lambda_{\text{eff}}(\phi,t) \approx e^{4\Gamma(t)}\left[\lambda(t) + \frac{1}{(4\pi)^2}\sum_p N_p\kappa_p^2(t)\left(\log\frac{\kappa_p(t)e^{2\Gamma(t)}\phi^2}{\mu(t)^2} - C_p\right)\right]$$

SM instability scale in the Fermi gauge

- Instability scale operatively defined as $\lambda_{\text{eff}}(\Lambda) = 0$



$$|\xi_W(M_t)| < \frac{4\pi}{\alpha_2(M_t)} \approx 376 \qquad \text{(perturbativity)}$$

Conclusions

- The fate of the EW vacuum is a physical statement
  - Critical Higgs mass (analogously for Top mass ...) is gauge independent
  - Tunnelling probability of the EW vacuum is gauge indep. as well [Einhorn, Sato (1981), Metaxas, Weinberg (1996), Isidori, Ridolfi, Strumia (2001)]

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- $\Lambda \neq \Lambda_{\rm SM}$ - physical identification of Λ should be addressed with care