Traces of BSM physics in the SM effective potential

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with M. Lewicki and P. Olszewski
arXiv:1402.3826, JHEP

Outline:

- SM effective potential
- Tunneling and Lifetime
- SM phase diagram
- magnitude of suppression scale
- Derivative interactions
- Nearly-scale invariant SM
- Summary

SM Effective potential

Standard Model Effective potential

$$V_{SM}(\mu) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum_i \frac{n_i}{64\pi^2} M_i^4 \left[\ln\left(\frac{M_i^2}{\mu^2}\right) - C_i \right]$$

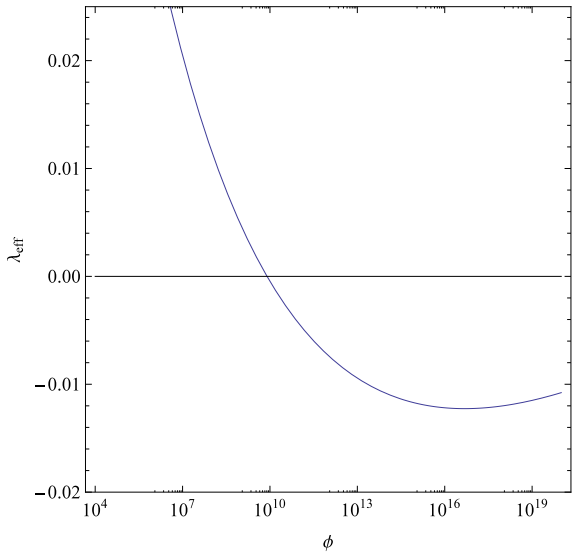
For large field values $m^2 << \phi^2$ and $\mu = \phi$ the potential is very well approximated by

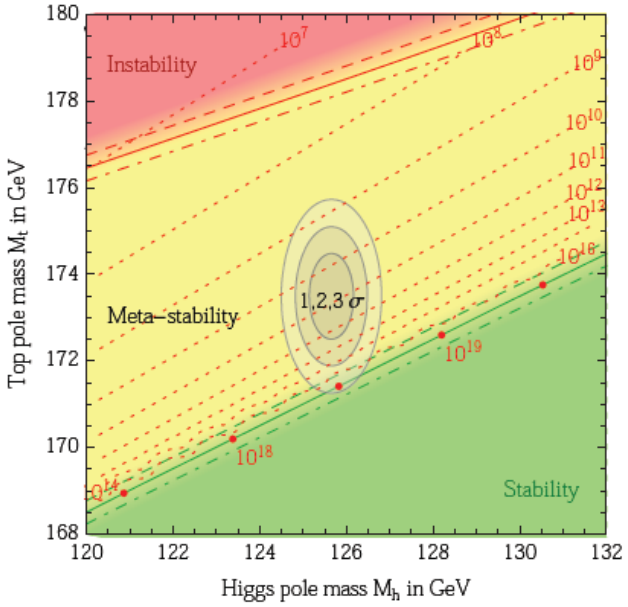
$$V_{SM}(\phi) \approx \phi^4 \left\{ \frac{\lambda}{4} + \frac{1}{64\pi^2} \left[6\left(\frac{g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_2^2}{4}\right) - \frac{5}{6}\right) + 3\left(\frac{g_1^2+g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_1^2+g_2^2}{4}\right) - \frac{6}{5}\right) \right.\right.$$
$$\left.\left. -12\left(\frac{y_t^2}{2}\right)^2 \left(\ln\left(\frac{y_t^2}{2}\right) - \frac{3}{2}\right) + \left(\frac{3\lambda}{2}\right)^2 \left(\ln\left(\frac{3\lambda}{2}\right) - \frac{3}{2}\right) + 3\left(\frac{\lambda}{2}\right)^2 \left(\ln\left(\frac{\lambda}{2}\right) - \frac{3}{2}\right) \right] \right\}$$

$$V_{SM}(\phi) \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4$$

SM Metastability

$$\lambda_{eff} < 0 \Longrightarrow \text{Metastability}$$





D. Buttazzo, et al. [arXiv:1307.3536].
G. Degrassi, et al. [arXiv:1205.6497].

Tunneling

Standard semiclassical formalism

S. R. Coleman, Phys. Rev. D **15** (1977) 2929.
C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D **16** (1977) 1762.

$O(4)$ symmetric solution to euclidean equation of motion

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$
$$s = \sqrt{\vec{x}^2 + x_4^2}.$$

with

- $\dot{\phi}(s = 0) = 0$ at the true vacuum
- $\phi(s = \infty) = \phi_{min}$ at the false vacuum

Tunneling

Action of the bounce solution

$$\begin{aligned} S_E &= \int d^4x \left\{ \frac{1}{2} \sum_{\alpha=1}^{4} \left(\frac{\partial \phi(\mathbf{x})}{\partial x^\alpha} \right)^2 + V(\phi(\mathbf{x})) \right\} \\ &= 2\pi^2 \int ds s^3 \left(\frac{1}{2} \dot{\phi}^2(s) + V(\phi(s)) \right), \end{aligned}$$

allows us to calculate decay probability dp of a volume d^3x

$$dp = dt d^3x \frac{S_E^2}{4\pi^2} \left| \frac{det'[-\partial^2 + V''(\phi)]}{det[-\partial^2 + V''(\phi_0)]} \right|^{-1/2} e^{-S_E}.$$

Simplifying

- normalisation factor replaced with width of the barrier $\propto \phi_0$
- size of the universe is $T_U = 10^{10}$yr

we can calculate the lifetime of the false vacuum ($p(\tau) = 1$)

$$\frac{\tau}{T_U} = \frac{1}{\phi_0^4 T_U^4} e^{S_E}.$$

Analytical solution

Analytical solutions for simple potentials

K. M. Lee and E. J. Weinberg, Nucl. Phys. B **267** (1986) 181.

Quartic potential:

$$V(\phi) = \tfrac{\lambda}{4}\phi^4 \qquad \Longrightarrow \qquad S_E = \tfrac{8\pi^2}{3|\lambda|}$$

for $\lambda < 0$.

Quartic and linear potential :

$$V_\eta(\phi) = \begin{cases} \frac{\lambda}{4}\phi^4 \,, & \phi \leqslant \eta \\ \frac{\lambda}{4}\eta^4 - K\,(\phi - \eta)\,, & \phi \ > \eta \end{cases} , \qquad \Longrightarrow \qquad \begin{array}{c} S_E = \frac{8\pi^2}{3|\lambda|}(1-(\gamma+1)^4) \\ \gamma = \frac{|\lambda|\eta^3}{K} \end{array}$$

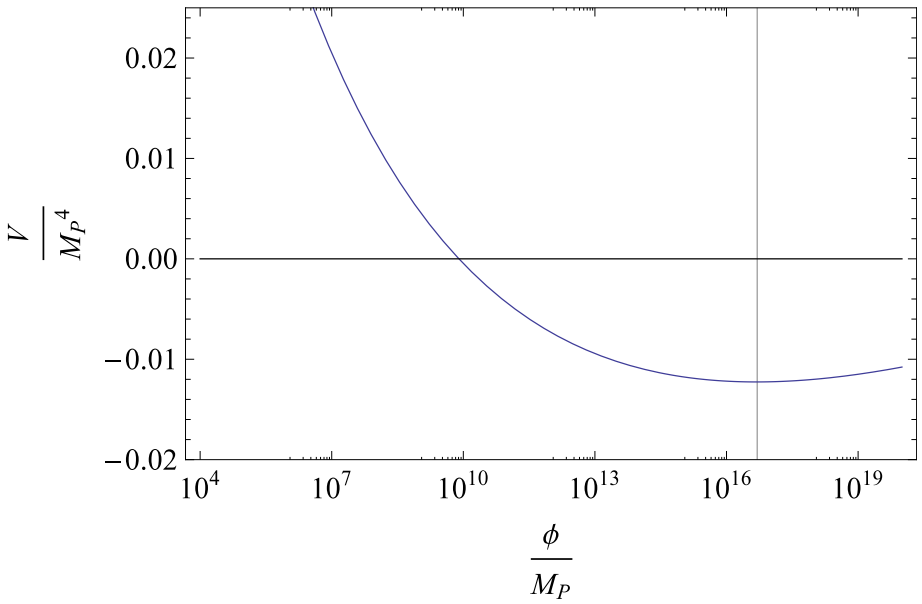
for $\lambda < 0$ and $-1 < \gamma < 0$

Standard Model

Approximating by a quartic potential:

$$\frac{\tau}{T_U} = \frac{1}{\phi^4(\lambda_{min})\, T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}} \approx 10^{540}.$$

lifetime is minimal for ϕ that minimizes $\lambda_{eff}(\phi)$.



Effective potential with nonrenormalisable interactions

We add new nonrenormalisable couplings
(similar to V. Branchina and E. Messina, [arXiv:1307.5193].)

$$V \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4 + \frac{\lambda_6}{6!}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8!}\frac{\phi^8}{M_p^4}.$$

That modify the potential around the Planck scale:

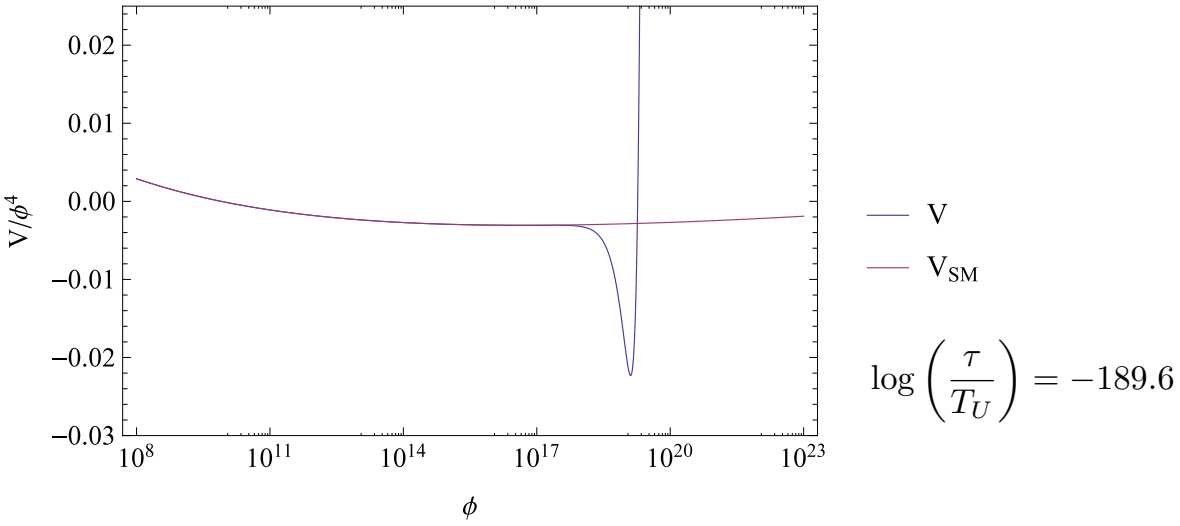


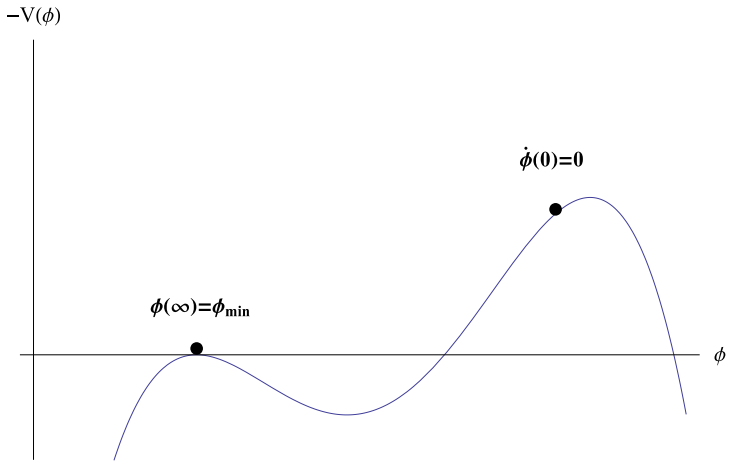
Figure: effective potential with $\lambda_6 = -1$ and $\lambda_8 = 1$.

Numerical calculations

Equation we need to solve

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$

is an equation of motion of a particle in potential $-V(\phi)$ with a "time" dependent friction $\frac{3}{s}\dot{\phi}$.



We will use a simple Overshot Undershot algorithm

Numerical vs Analytical

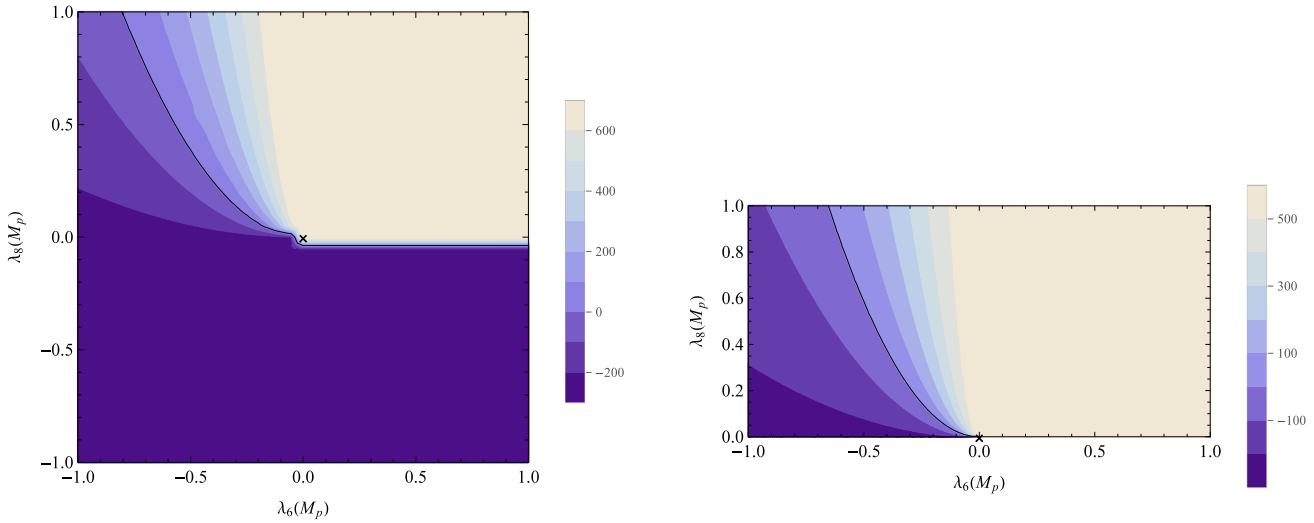


Figure: Decimal logatihm of lifetime of the universe in units of T_U as a function of the nonrenormalisable λ_6 and λ_8 couplings, calculated numerically (left panel) and analytically (right panel).

RG improvement

The correction to the running of the quatric Higgs coupling is of the form

$$\Delta\beta_\lambda = \frac{\lambda_6}{16\pi^2}\frac{m^2}{M_p^2}.$$

One-loop beta functions of new couplings take the form

$$\begin{aligned} 16\pi^2\beta_{\lambda_6} &= \lambda_8\frac{m^2}{M_p^2} + 15\lambda_6 6\lambda - 6\lambda_6\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2\right), \\ 16\pi^2\beta_{\lambda_8} &= 35\lambda_6^2 + 28\lambda_8 6\lambda - 8\lambda_8\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2\right). \end{aligned}$$

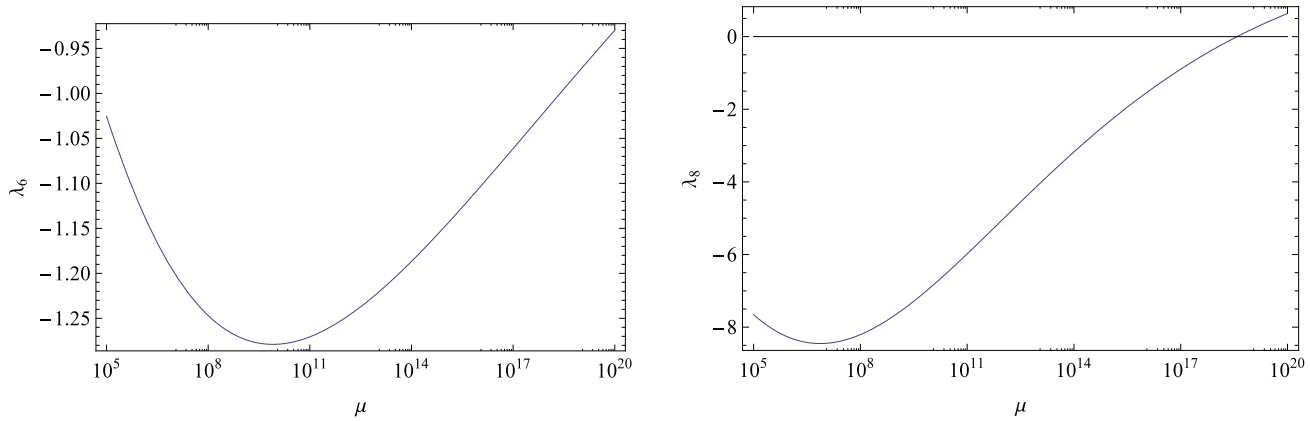


Figure: Example solution with $\lambda_6(M_p) = -1$ and $\lambda_8(M_p) = -0.1$.

Numerical vs Analytical again

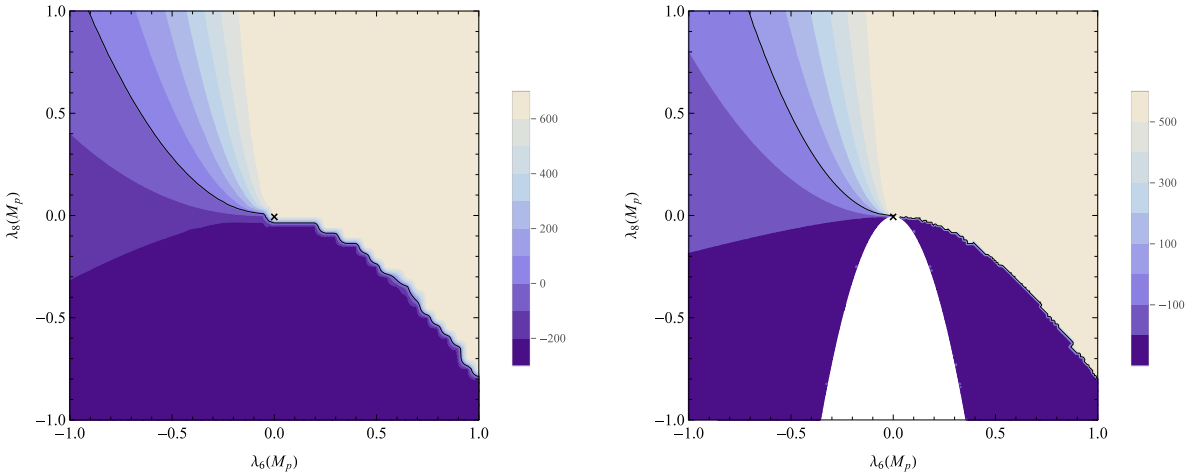
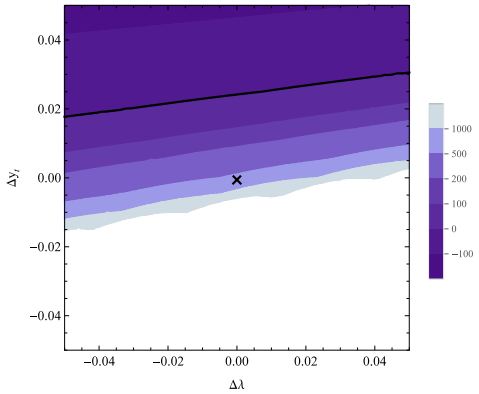


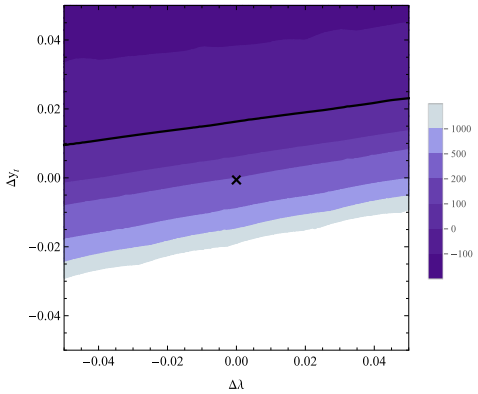
Figure: Decimal logatihm of lifetime of the universe in units of T_U as a function of the nonrenormalisable $\lambda_6(M_p)$ and $\lambda_8(M_p)$ couplings, calculated numerically (left panel) and analytically (right panel).

SM phase diagram

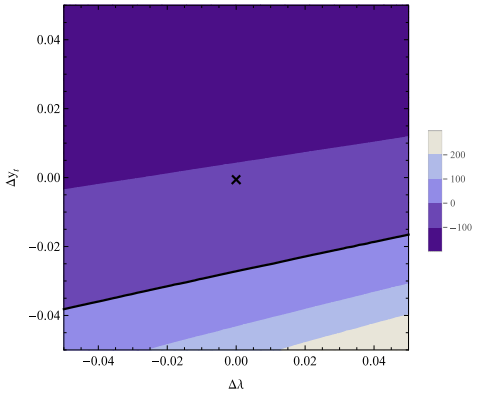
$\lambda_6(M_p) = 0,\ \lambda_8(M_p) = 0$



$\lambda_6(M_p) = -1/2,\ \lambda_8(M_p) = 1$



$\lambda_6(M_p) = -1,\ \lambda_8(M_p) = 1/2$



Magnitude of the suppression scale

Approximate lifetime:

$$\frac{\tau}{T_U} = \frac{1}{\mu^4(\lambda_{min})\, T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}}.$$

Positive λ_6 and $\lambda_8 \rightarrow$ stabilizing the potential

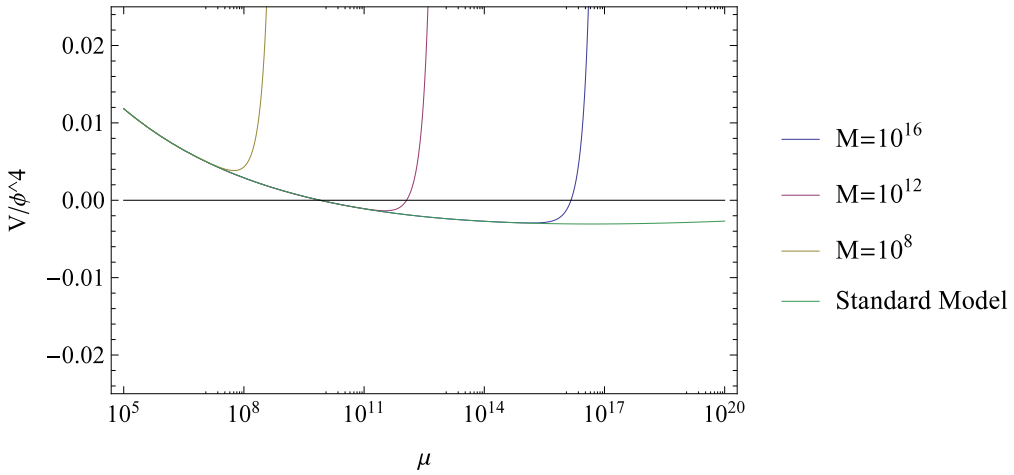


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = \lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$ are, respectively, $\log_{10}(\frac{\tau}{T_U}) = \infty, 1302, 581$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Magnitude of the suppression scale

Positive λ_8 and negative $\lambda_6 \rightarrow$ New Minimum

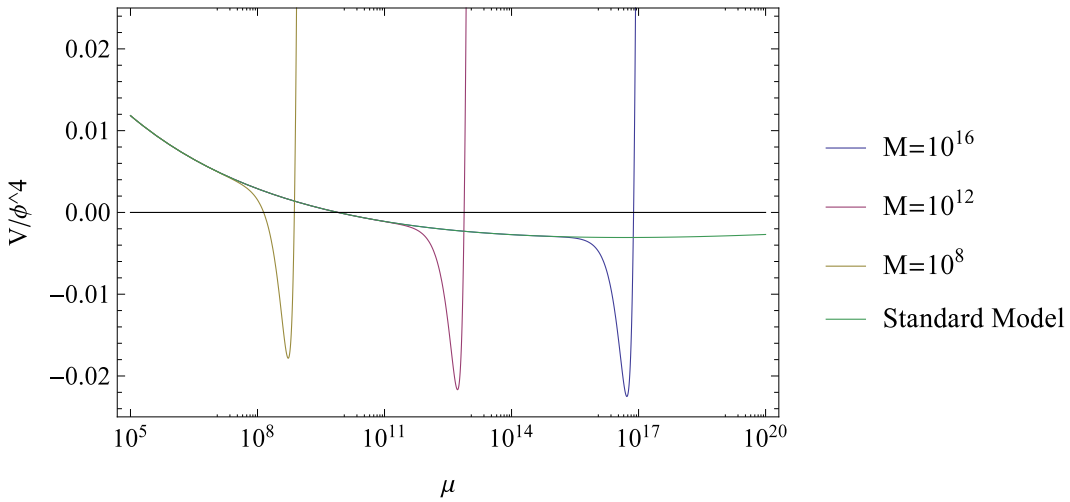


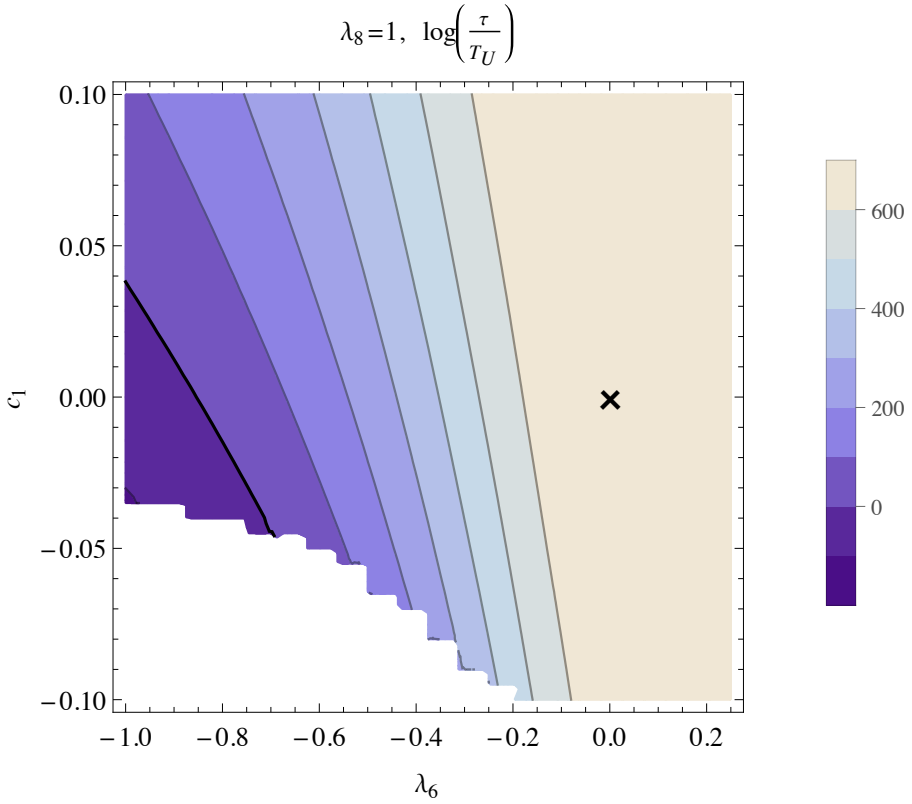
Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = -1$ and $\lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$, are, respectively, $\log_{10}(\frac{\tau}{T_U}) = -45, -90, -110$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Summary I

- Analytical approximation of vacuum lifetime is fairly accurate
- RG improvement stabilizes significant parts of the parameter space
- SM vacuum can be stabilized by nonremormalisable interactions only if they appear at low enough energy scale
- SM vacuum lifetime can be dramatically shortened by nonrenormalisable interactions for any suppression scale

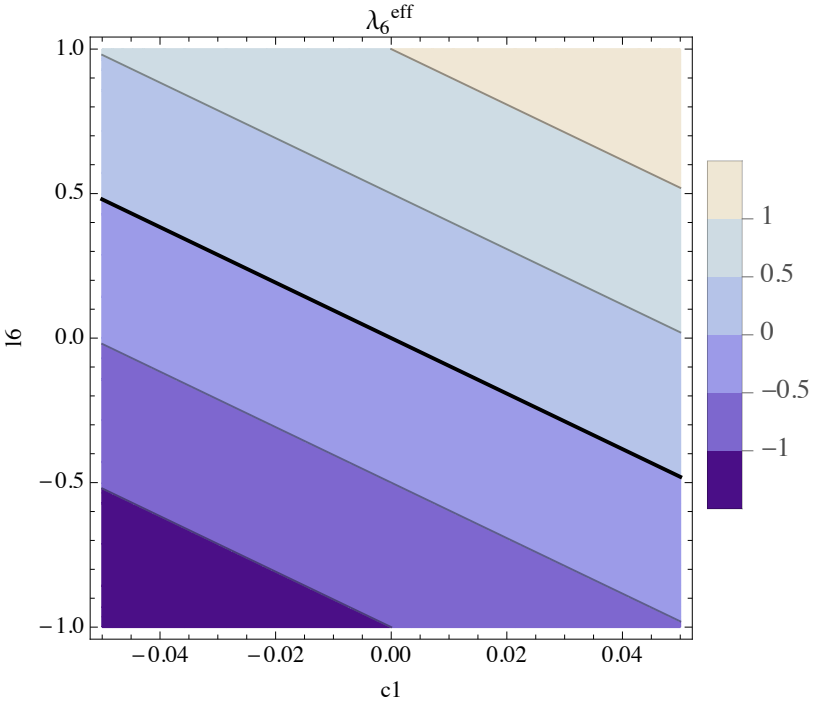
Derivative terms

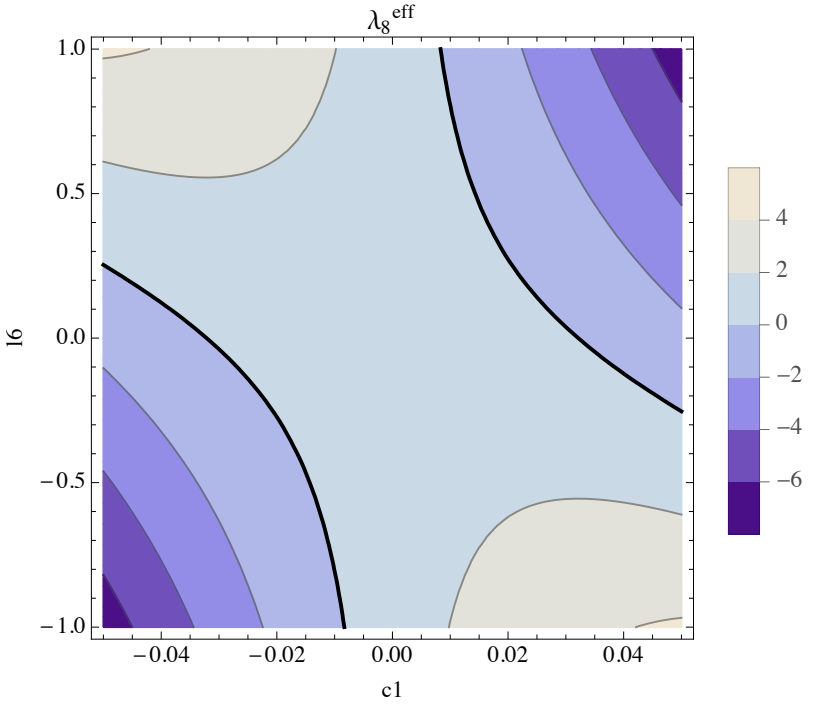
$$\partial_\mu \phi \partial^\mu \phi \left(c_1 \frac{\phi^2}{M^2} + c_2 \frac{\phi^4}{M^4} \right)$$

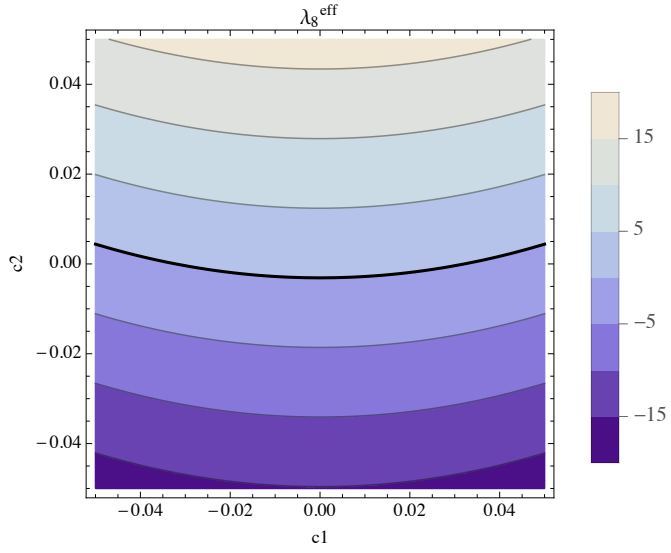


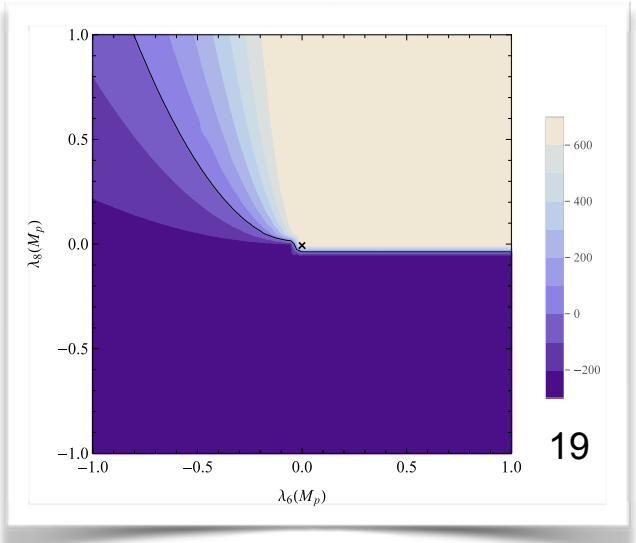
$$\partial_s^2 \phi + \frac{3}{s} \partial_s \phi = \frac{\partial V}{\partial \phi} \rightarrow \partial_s^2 (1 + c_1 \phi^2 / M^2) \left(\phi + \frac{3}{s} \partial_s \phi \right) = \frac{\partial V}{\partial \phi}$$

$$\phi \longrightarrow \tilde{\phi}\left(1 + a\frac{\tilde{\phi}^2}{M^2} + b\frac{\tilde{\phi}^4}{M^4}\right) \longrightarrow \text{shifted couplings } \lambda_6 \to \tilde{\lambda}_6 \text{ and } \lambda_8 \to \tilde{\lambda}_8$$









Nearly scale invariant SM

W.Buchmueller and C.Busch,
W.Buchmueller and N.Dragon

$$V = a + \mu\Phi^\dagger\Phi + \lambda/2(\Phi^\dagger\Phi)^2 + \frac{1}{(8\pi)^2}\left(3/2(\mu+\lambda\Phi^\dagger\Phi)^2(\log\frac{(\mu+\lambda\Phi^\dagger\Phi)^2}{M^4} - 1)\right.$$
$$+1/2(\mu+3\lambda\Phi^\dagger\Phi)^2(\log\frac{(\mu+3\lambda\Phi^\dagger\Phi)^2}{M^4} - 1)$$
$$\left. + (3/2g^4 + 3/4(g^2+g'^2)^2 - 12h_t^4)(\Phi^\dagger\Phi)^2(\log\frac{\Phi^\dagger\Phi}{M^2} - 1/2)\right),$$

scale invariance broken by the dilaton kinetic term

$$a \to ae^{4\sigma/f},\ \mu \to \mu e^{2\sigma/f}$$

But:

$$m_\sigma^2 = -\frac{4}{f^2}M\frac{\partial V}{\partial M} = \frac{1}{8\pi^2 f^2}\,Str\,\mathcal{M}^4$$

In SM

$$Str\,\mathcal{M}^4 < 0$$

Improvement

additional breaking of scale invariance

$$ae^{4\sigma/f} \to ae^{4\sigma/f} - be^{p\sigma/f}$$

this results in an acceptable solution

$$m^2_{\sigma\, tr} = \frac{4(4-p)}{f^2} ae^{4\sigma_0/f},$$

$$\frac{\sigma_0}{f} = \frac{1}{4-p} \log(\frac{bp}{4a}).$$

tuning

$$V(s) = \frac{1}{2s}\left(Ae^{-\frac{s}{N_1}}(N_1+2s) - Be^{-\frac{s}{N_2}}(N2+2s) + w\right)^2 - \frac{3}{2s}\left(AN_1e^{-\frac{s}{N_1}} - BN_2e^{-\frac{s}{N_2}} + w\right)^2$$

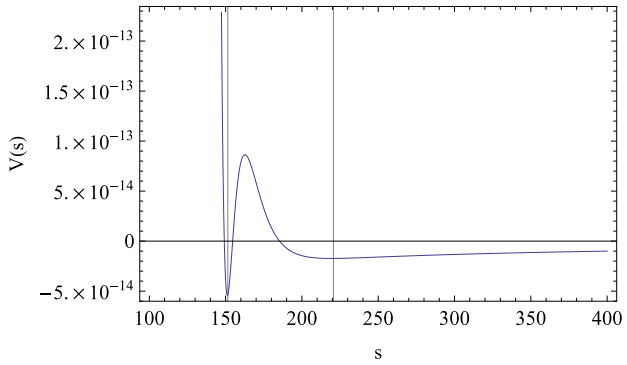


Figure 1: Example of potential (1.1) with $A = 1.5$, $B = 8.2$, $N1 = 9$, $N2 = 10$ and $w = -2\times10^{-6}$, with black lines showing positions of its minima.

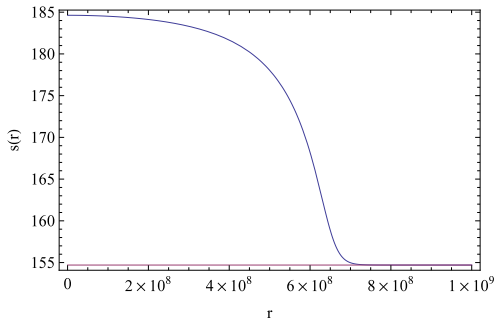


Figure 2: Instanton connecting the two vacua shown in Figure 1.

The potential has two minima as shown in Figure 1. We have numerically calculated the action of the bounce solution corresponding to decay of the false vacuum (shallow minimum) to the true vacuum (global minimum). The instanton connecting the two vacua has been shown in Figure 2. The action connected with this bounce is $S \approx 10^{23}$ and since the expected lifetime of the false vacuum depends on it exponentially ($\tau \propto e^S$), the false vacuum is very stable. This happens mostly due to a very small difference between positions of the two minima.

RGE improved scalar potential

By dimensional analysis and using $\frac{dV_{SM}}{dM} = 0$ one finds

$$V(\Phi, \sigma) = e^{4\sigma/f} V_{SM}(\xi^2(\bar{M})\tilde{\Phi}^\dagger\tilde{\Phi};\, \mu(\bar{M}), \lambda(\bar{M});\, M)$$

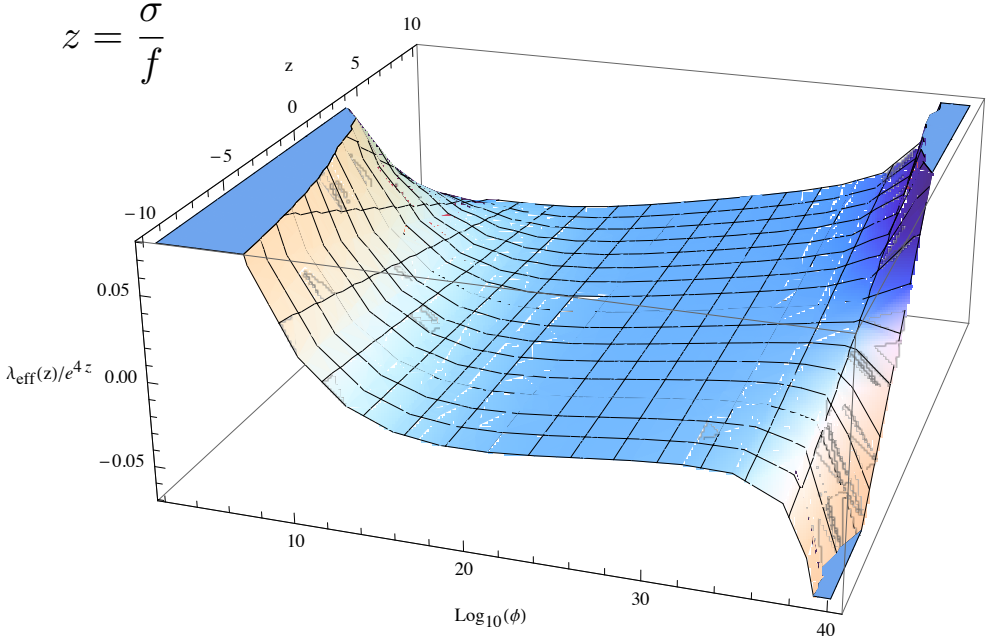
where $\tilde{\Phi} = e^{-\sigma/f}\Phi$ and $\bar{M} = e^{\sigma/f} M$

and $\xi(M) = e^{-\int^M \frac{dM'}{M'}\gamma(M')}$

This makes it possible to study the RGE improved effective potential in the (nearly) scale invariant case

$$V(\Phi,\sigma) = e^{4\sigma/f} V_{SM}(\xi^2(\bar{M})\tilde{\Phi}^\dagger\tilde{\Phi};\ \mu(\bar{M}),\lambda(\bar{M});\ M)$$

$$z = \frac{\sigma}{f}$$



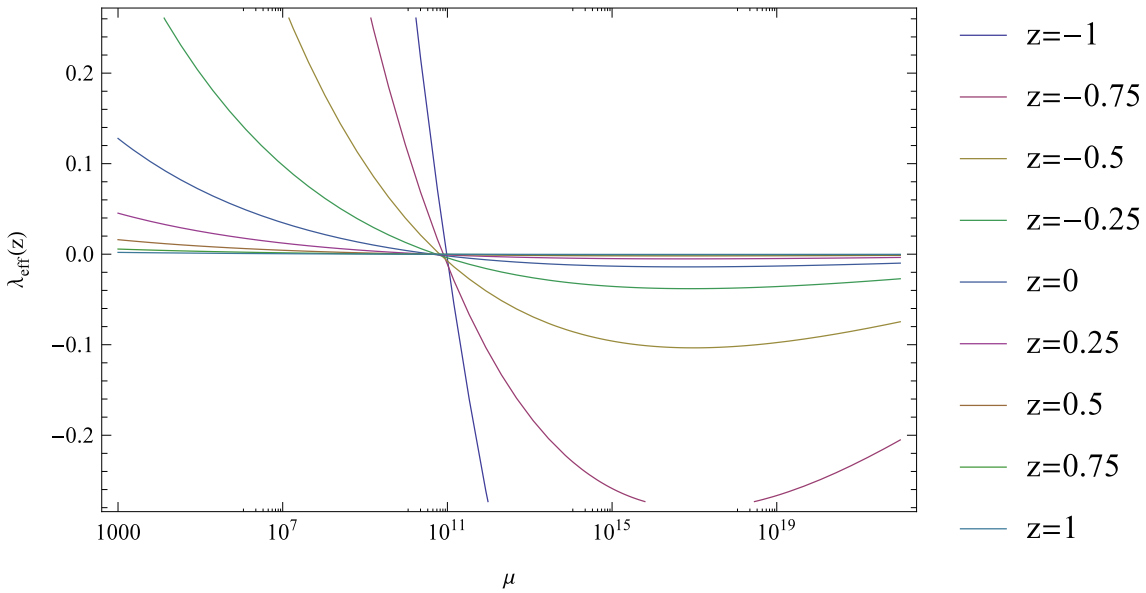


Figure 3: quartic coupling in the effective potential $\lambda_{eff}(z)$.

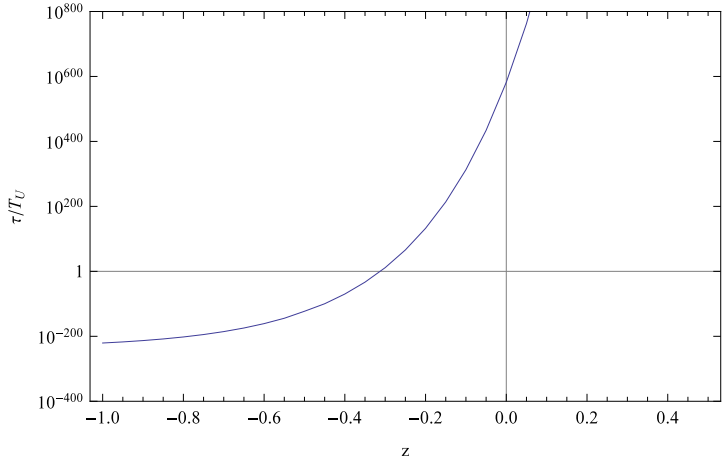
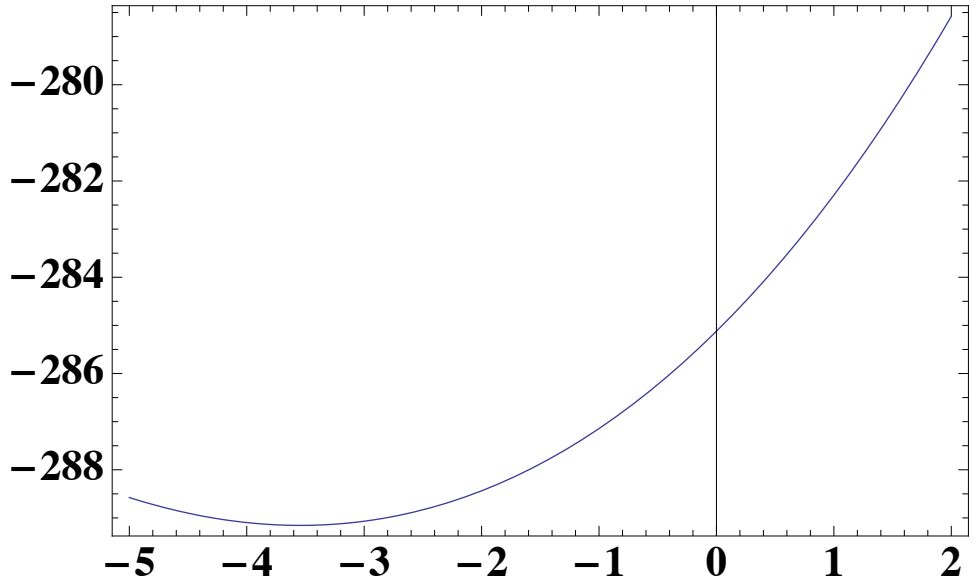


Figure 4: Lifetime of the electroweak vacuum as a function of z.

$$\mathcal{P}(z=\sigma/f) = (\tau_U Q_{min}(z))^4 e^{-\frac{8\pi^2}{3|\lambda_{eff}(Q_{min}(z))|}}$$

$\log_{10}(\mathcal{P})$



SM

Summary

- SM vacuum can be stabilized by nonrenormalisable interactions if they appear at suffciently low energy scale
- SM vacuum lifetime can be dramatically shortened by nonrenormalisable interactions for any suppression scale
- RG improvement stabilizes significant parts of the parameter space
- Higher-order terms with derivatives destabilize metastable vacua